

Identification and Estimation in a Class of Potential Outcomes Models

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Class of Models

Observables

- A discrete treatment $T \in \mathbf{T} \equiv \{t_1, \dots, t_d\}$.
- A scalar outcome $Y \in \mathbf{Y}$.
- A set of instruments $Z \in \mathbf{Z}$.
- A set of covariates $X \in \mathbf{X}$.

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- A vector of potential outcomes $Y^* \equiv (Y^*(t_1), \dots, Y^*(t_d))$ satisfying

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- A vector of potential outcomes $Y^* \equiv (Y^*(t_1), \dots, Y^*(t_d))$ satisfying

$$Y = Y^*(T)$$

- Type $T^* : \mathbf{Z} \rightarrow \mathbf{T}$ describing how Z affects treatment choice satisfying

$$T = T^*(Z)$$

The Assumptions

Assumption B

- $(Y^*, T^*, Z, X) \sim Q_0$ with $Y^* \in \mathbf{Y}^*$, $Z \in \mathbf{Z}$, $X \in \mathbf{X}$, and $T^* \in \mathbf{T}^*$.
- We observe $T = T^*(Z)$, $Y = Y^*(T)$, Z , and X with $(Y, T, Z, X) \sim P$.

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- (Exogeneity) $(Y^*, T^*) \perp\!\!\!\perp Z|X$ under Q_0 .

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- (Exogeneity) $(Y^*, T^*) \perp\!\!\!\perp Z|X$ under Q_0 .
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 - **Example:** No Defiers in Imbens and Angrist (1994).

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- (Support) $Q_0 \ll \mu$ for some probability measure μ .
 - **Example:** No Defiers in Imbens and Angrist (1994).
- (Convex) $dQ_0/d\mu \in \mathcal{Q} \subseteq L^1(\mu)$ for $\mathcal{Q}$ convex, closed in $(\mathbf{Q}, \|\cdot\|_{\mathbf{Q}})$.
 - **Example:** Sequential Ignorability in Mediation.

Results apply to any model that have this structure.

Example

Imbens and Angrist (1994)

- Outcome $Y \in \mathbb{R}$, treatment $T \in \{0, 1\}$, instrument $Z \in \{0, 1\}$.
- Unobservables: $Y^* \equiv (Y^*(0), Y^*(1))$ and type $T^* \equiv (T^*(0), T^*(1))$.
- Monotonicity requirement: $T^*(1) \geq T^*(0)$.

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- Monotonicity requirement: $T^*(1) \geq T^*(0)$.

Map To Our Model

- Impose monotonicity by setting the dominating measure μ to satisfy

$$\mu(T^*(1) \geq T^*(0)) = 1$$

- Impose Y^* has finite first moment through assumption $dQ_0/d\mu \in \mathcal{Q}$.

Example

Kline and Walters (2016)

- Instrument $Z \in \{0, 1\}$ indicates offer to attend Head Start school.
- Possible Treatments: Head Start (h), Other School (c), Home Care (n).
- Key Assumption: Offer Z does not dissuade students from Head Start.

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Map To Our Model

- A type T^* maps $\mathbf{Z} \equiv \{0, 1\}$ to $\mathbf{T} \equiv \{h, c, n\}$.
- Write $T^* \equiv (T^*(0), T^*(1))$ and set μ to satisfy $\mu(T^* \in \mathbf{R}^*) = 1$ where

$$\mathbf{R}^* \equiv \left\{ \begin{pmatrix} n \\ h \end{pmatrix}, \begin{pmatrix} c \\ h \end{pmatrix}, \begin{pmatrix} n \\ n \end{pmatrix}, \begin{pmatrix} c \\ c \end{pmatrix}, \begin{pmatrix} h \\ h \end{pmatrix} \right\}$$

- Use $\mathcal{Q}$ to impose regularity conditions such as finite moments for Y^* .

Other Examples

Monotonicity: Angrist & Imbens (1995), Rose & Shem-Tov (2021).

Revealed Preferences: Heckman & Pinto (2018), Pinto (2021).

Structural Equations: Heckman & Vytlacil (1999), Lee & Salanie (2018).

Multiple IVs: Mogstad, Torgovitsky, & Walters (2021), Mountjoy (2022).

Mediation: Imai, Keele, & Yamamoto (2010), Frolich & Huber (2017).

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New Model: Mental health as mediator in Moving to Opportunity.

What Do We Study?

The distribution Q_0 is typically not identified, but instead restricted to the set

$$\Theta_0 \equiv \{Q : Q \text{ obs. equiv. } Q_0, (Y^*, T^*) \perp\!\!\!\perp Z|X \text{ under } Q, Q \ll \mu, \frac{dQ}{d\mu} \in \mathcal{Q}\}$$

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Parameter of Interest

- Expectations of some function ℓ of (Y^*, T^*, X) , which we denote by

$$\lambda_{Q_0} \equiv E_{Q_0}[\ell(Y^*, T^*, X)].$$

- **Examples:** Joint or marginal distribution of (Y^*, T^*) .
- Results also apply to local average treatment effects.
- Paper examines more general functionals defined by sequence $\{\ell_j\}$
 $\Rightarrow$ Includes marginal treatment effects, densities of Y^* given X .

Contributions

The Model

- Framework unifies and extends multiple existing identifying strategies.

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- Characterization yields a generalization of Abadie's kappa.

Estimation

- Need distribution of Z given $X \Rightarrow$ Obtain double robust estimators.
- Obtain efficiency bound and characterize when it is finite.

Today: Will (mostly) cover identification results only.

- 1 Identification: Sufficient Condition
- 2 Identification: Necessary Condition
- 3 Identification: Implications for Estimation
- 4 Moving to Opportunity

Sufficient Condition

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Note: $\Upsilon(f)$ depends on $P_{Z|X}$, which is identified.

Sufficient Condition

Define a map Υ from functions of (Y, T, Z, X) to functions of (Y^*, T^*, X) by

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Lemma K: If Assumptions B and I hold, and ℓ satisfies $\Upsilon(\kappa) = \ell$ for some function κ of (Y, T, Z, X) , then it follows that for all $Q \in \Theta_0$ we have

$$E_Q[\ell(Y^*, T^*, X)] = E_P[\kappa(Y, T, Z, X)]$$

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Comments

- Result shows $\Upsilon(\kappa) = \ell$ is sufficient condition for identification.
- Suggests estimation strategy: Estimate κ , then use sample moments.

- 1 Identification: Sufficient Condition
- 2 Identification: Necessary Condition**
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Informal Preview

What we have so far ...

- A **sufficient condition** for identification: If $\Upsilon(\kappa) = \ell$ then we have that

$$E_P[\kappa(Y, T, Z, X)] = E_{Q_0}[\ell(Y^*, T^*, X)]$$

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Question

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Question

- Is this simple estimation approach without loss of generality? I.e. are identification conditions in fact necessary and sufficient? **Almost!**

Key: The estimation strategy applies whenever parameter is identified.

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$$\lambda_{Q_0} \equiv E_{Q_0}[\ell(Y^*, T^*, X)]$$

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What closure?

- In terms of a topology τ defined in the paper.
- Topology reflects the strength of the assumptions imposed.

Last Assumption

$$\lambda_{Q_0} \equiv E_{Q_0}[\ell(Y^*, T^*, X)]$$

Assumption F

- (i) ℓ is integrable with respect to any $Q \in \Theta_0$.
- (ii) $dQ/d\mu$ belongs to the interior of $\mathcal{Q}$ for some $Q \in \Theta_0$.

Comments

- Assumptions F(i) can be verified through choice of $\mathcal{Q}$.
- Assumption F(ii) ensures $\mathcal{Q}$ does not yield identification.
- Assumption F(ii) can be dropped at cost of only partial converse.

Identification Result

Theorem If Assumptions B, I, and F hold, then the parameter λ_{Q_0} is identified if and only if ℓ belongs to the τ -closure of $\mathcal{R}$.

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Stronger Assumptions $\Rightarrow$ Weaker Topology

$\Rightarrow$ Easier to Satisfy $\Upsilon(\kappa_j) \approx \ell \Rightarrow$ More Parameters Identified

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Key Implication: Estimation based on $\{\kappa_j\}$ applies to all identified λ_{Q_0}

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$$\lambda_{Q_0} = E_P[\kappa(T, Z, X)]$$

Comments

- Function κ now only depends on (T, Z, X) only.
- When Z is discrete, solving κ is linear algebra problem.

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Corollary: Suppose Z is discrete, Assumptions B and I hold with $Q = L^\infty(\mu)$, and that regularity conditions hold. Then these are equivalent:

- (i) $E_{Q_0}[\rho(Y^*(t), X)\ell(T^*, X)]$ is identified.
- (ii) $E_{Q_0}[f(Y^*(t), X)\ell(T^*, X)]$ is identified for all bounded f .
- (iii) There is a $\kappa(T, Z, X) = 1\{T = t\}g(Z, X)$ such that $\Upsilon(\kappa) = \ell$.

Key: We can only now use observations with $T = t$ to identify λ_{Q_0} .

Example

Kline and Walters (2016)

- Instrument: $Z \in \{0, 1\}$ indicates offer to attend Head Start school.
- Possible Treatments: Head Start (**h**), Other School (**c**), Home Care (**n**).
- Write $T^* \equiv (T^*(0), T^*(1))$ and restrict possible values of T^* to the set

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Goal: Identify and estimate outcome distribution for different types $t^* \in \mathbf{T}^*$

$$E_{Q_0} \left[\underbrace{f(Y^*(t)) 1\{T^* = t^*\}}_{\ell(Y^*, T^*, X)} \right]$$

Examples: Probability of t^* , expected outcome conditional on type t^* , ...

Example

$$E_{Q_0}[f(Y^*(t))1\{T^* = t^*\}]$$

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Our Result: Says that to identify the parameter we need κ satisfying

$$\underbrace{E_{Q_0}[\kappa(Y, T, Z, X)|Y^*, T^*, X]}_{\Upsilon(\kappa)} = \underbrace{f(Y^*(t))1\{T^* = t^*\}}_{\ell}$$

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Idea: **Step 1.** Solve case $Z \perp\!\!\!\perp X$ and uniformly distributed on $\{z_1, \dots, z_q\}$
Step 2. Obtain true solution by adjusting by propensity score.

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Step 2. For $(s_1^*, \dots, s_q^*)$ the coefficients from the regression in Step 1 set

$$g(z_j, X) = \frac{s_j^*}{P(Z = z_j | X)} \times \frac{1}{q}$$

Note: Estimating g only requires estimating the distribution of Z given X .

- 1 Identification: Sufficient Condition
- 2 Identification: Necessary Condition
- 3 Identification: Implications for Estimation**
- 4 Moving to Opportunity

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Focus On: Applications in which μ , Q , and ℓ do not depend on P .

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Setup

Moving to Opportunity (MTO)

- Experiment offered voucher to relocate to low poverty neighborhoods.
- Significant impacts on mental health. Nuanced on economic impact.

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Mediation Model

- $Z \in \{0, 1\}$ indicates voucher offer.
- $D \in \{0, 1\}$ indicates relocation and $M \in \{0, 1\}$ good mental health.
- $T = (D, M)$ and potential outcomes indexed by $Y^*(t)$ with $t = (d, m)$.
- X is set of baseline covariates.

MTO: Types

Type Restrictions

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	NN	NA	CN	CC	CA	AN	AA
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Our Results

- Characterizes the identification of distribution of types.
- Gives us estimator for joint distribution of types and covariates.

Type Definitions and Probabilities							
	NN	NA	CN	CC	CA	AN	AA
$(D^*(0), M^*(0))$	(0,0)	(0,1)	(0,0)	(0,0)	(0,1)	(1,0)	(1,1)
$(D^*(1), M^*(1))$	(0,0)	(0,1)	(1,0)	(1,1)	(1,1)	(1,0)	(1,1)
Probability Point Estimate	0.258	0.253	0.194	0.065	0.203	0.014	0.013
s.e.	(0.014)	(0.014)	(0.013)	(0.023)	(0.022)	(0.004)	(0.003)
Expected Value of Covariate Conditional on Types							
	NN	NA	CN	CC	CA	AN	AA
Household member has a disability	0.183 (0.020)	0.145 (0.019)	0.156 (0.022)	0.388 (0.156)	0.076 (0.039)	0.091 (0.066)	0.077 (0.054)
No teens in the household	0.535 (0.028)	0.566 (0.029)	0.656 (0.030)	0.736 (0.176)	0.653 (0.050)	0.602 (0.121)	0.902 (0.071)
No friends in the neighborhood	0.355 (0.026)	0.400 (0.029)	0.410 (0.033)	0.699 (0.204)	0.360 (0.051)	0.341 (0.115)	0.659 (0.118)

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$\Rightarrow$ The causal effects LATE_N , LATE_A , and LATE_C are not identified

MTO: Outcomes

Solution: Impose “exogeneity of irrelevant mediator choices” requirement

$$Y^*(0,0) \perp M^*(1) | D^*(1) > D^*(0), M^*(0) = 0, X$$
$$Y^*(1,1) \perp M^*(0) | D^*(1) > D^*(0), M^*(1) = 1, X$$

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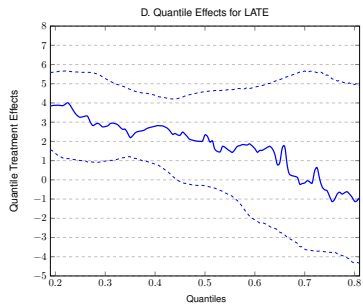
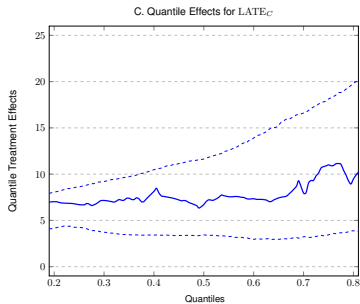
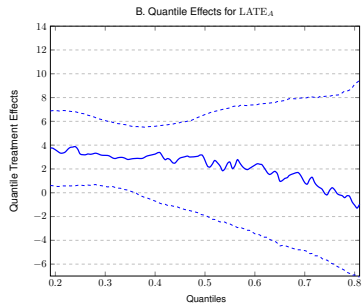
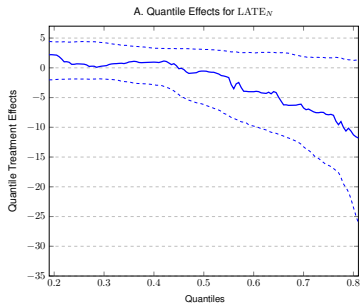
- If it is identified, then $E_{Q_0}[Y^*(0,0)1\{T^* = CN\}] = E_P[Y\kappa(T, Z, X)]$.

Outcome	$LATE_N$	$LATE_A$	$LATE_C$	LATE
Household is Economically Self-Sufficient	0.024 (0.050)	0.048 (0.063)	0.059 (0.078)	0.035 (0.039)
Sampled Adult not in Labor Force	-0.042 (0.071)	-0.074 (0.072)	-0.412** (0.144)	-0.113** (0.049)
Household Total Income	0.498 (2.262)	2.928 (2.191)	2.479 (3.014)	1.811 (1.523)

Comments

- Results support conjecture regarding mediating role of mental health.
- Not statistically significant effect on household total income ...

... but if we examine difference in quantiles of potential outcomes (QTE) ...



Takeaways

Identification

- Obtain necessary and sufficient conditions for identification.
(Intuitively) Identification through generalization of Abadie's kappa.
- Results give equation characterizing Abadie's kappa for the model.

Estimation

- Often requires estimating distribution of Z given X .
- Derive double robust moment conditions for parameter of interest.
- Obtain asymptotically normal estimators and show bootstrap validity.
- Characterize semi-parametric efficiency bound for parameters.