Using Instrumental Variables for Inference about Policy Relevant Treatment Parameters

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Basic Question

IV and Heterogeneity

- IV estimand interpretable as LATE (Imbens & Angrist 1994).
- Sometimes LATE has clear policy relevance.
- Other times, different parameters are of interest (external validity).
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- Other times, different parameters are of interest (external validity).

Our Paper

- Framework for extrapolation in IV model.
- Use insight of marginal treatment effect (Heckman & Vytlacil 2005).
- Allow flexible specifications and computational tractability.

**Goal:** Allow for different choices of parameters and assumptions.
The Model

Outcome
- Treatment $D \in \{0, 1\}$, potential outcomes $(Y_0, Y_1)$, and actual outcome

$$Y = DY_1 + (1 - D)Y_0.$$
The Model

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• Treatment $D \in \{0, 1\}$, potential outcomes $(Y_0, Y_1)$, and actual outcome

\[ Y = DY_1 + (1 - D)Y_0. \]

Selection
• For $U \sim U[0, 1]$ and $(Y_0, Y_1, U)$ independent of observable instrument $Z$

\[ D = 1\{U \leq p(Z)\} \]
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$$D = 1\{U \leq p(Z)\}$$

Comments
- $U \sim U[0, 1]$ normalization $\Rightarrow p(Z) = P(D = 1|Z)$ (propensity score).
- Instrument monotonicity equivalent to separability (Vytlacil 2002).
- Covariates omitted for simplicity but easily incorporated.
The Model

\[ D = 1\{U \leq p(Z)\} \]

⇒ Individuals with smaller unobservable \( U \) more likely to receive treatment.

Example: Suppose \( Z \in \{0, 1\} \) is binary and that \( p(1) > p(0) \), then we have

- \( U \in [0, p(0)] \): always-takers
- \( U \in (p(0), p(1)] \): compliers
- \( U \in (p(1), 1] \): never-takers

Comments

- Unobservable \( U \) determines likelihood of receiving treatment.
- Key Concern: \( U \) may not be independent of \( (Y_0, Y_1) \) (selection).
The Model


\[ \text{MTE}(u) \equiv E[Y_1 - Y_0 | U = u] \]

⇒ summary of unobserved heterogeneity in average treatment effects.
The Model


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⇒ summary of unobserved heterogeneity in average treatment effects.

**Key:** HV show that many parameters are weighted averages of the MTE

$$\beta^* = E[\int_0^1 \text{MTE}(u) \times \omega^*(u, Z) \, du]$$

parameter identified weights
The Model


\[ \text{MTE}(u) \equiv E[Y_1 - Y_0 | U = u] \]

⇒ summary of unobserved heterogeneity in average treatment effects.

**Key:** HV show that many parameters are weighted averages of the MTE

\[ \hat{\beta}^* = E \left[ \int_0^1 \text{MTE}(u) \times \omega^*(u, Z) \, du \right] \]

parameter identified weights

**Comments**

- \( \hat{\beta}^* \) may (or may not) be identified.
- Recall \( U \sim U[0, 1] \) so that \( \hat{\beta}^* \) is a weighted average.
Example: LATE

\[ \beta^* = E\left[ \int_0^1 \text{MTE}(u) \times \omega^*(u, Z) \, du \right] \]
Example: LATE

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Example: Suppose \( Z \in \{0, 1\} \) is binary and that \( p(1) > p(0) \), then we have

\[
E[Y_1 - Y_0 | p(0) < U \leq p(1)] = \int_0^1 E[Y_1 - Y_0 | U = u] \frac{1\{p(0) < u \leq p(1)\}}{p(1) - p(0)} du
\]

Comments

- LATE identified as IV estimand (i.e. identification of MTE not needed).
- Hence, LATE imposes restrictions on possible values of MTE.
Example: ATE

\[ \beta^* = E\left[ \int_0^1 MTE(u) \times \omega^*(u, Z) du \right] \]
Example: ATE

\[ \beta^* = E[\int_0^1 \text{MTE}(u) \times \omega^*(u, Z) du] \]

Example: Suppose \( Z \in \{0, 1\} \) is binary and that \( p(1) > p(0) \), then we have

\[ E[Y_1 - Y_0] = \int_0^1 E[Y_1 - Y_0|U = u] (\times 1) du \]

\( \beta^* \) (ATE) \hspace{1cm} \text{MTE} \hspace{1cm} \omega^*(u)

Comments

- ATE is not necessarily identified with binary instrument \( Z \).
- But! LATE still provides information on MTE and hence on ATE.
Basic Idea

\[ \beta^* = E[\int_0^1 \text{MTE}(u) \times \omega^*(u, Z) du] \] (\*)
Basic Idea

\[ \beta^* = E \left[ \int_0^1 \text{MTE}(u) \times \omega^*(u, Z) \, du \right] \]  

(*)

Step 1
- Find parameters \( \beta^* \) that are separately identified (e.g. LATE).
- Employ relationship (\( \star \)) to restrict possible values of MTE function.
Basic Idea

\[ \beta^* = E[\int_0^1 \text{MTE}(u) \times \omega^*(u, Z) du] \]  

Step 1

- Find parameters \( \beta^* \) that are separately identified (e.g. LATE).
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Step 2

- Define the parameter of interest (e.g. ATE).
- Impose desired restriction on MTE (optional).
Basic Idea

\[ \beta^* = E\left[ \int_0^1 \text{MTE}(u) \times \omega^*(u, Z) du \right] \]  

\((\star)\)

Step 1

• Find parameters \(\beta^*\) that are separately identified (e.g. LATE).
• Employ relationship \((\star)\) to restrict possible values of MTE function.

Step 2

• Define the parameter of interest (e.g. ATE).
• Impose desired restriction on MTE (optional).

Step 3

• Conduct inference on possible values of parameter of interest.
• Values must be consistent with MTE restrictions from Steps 1 and 2.
Related Literature

Extrapolation


⇒ We avoid: parametric assumptions, support conditions.
⇒ We allow for: treatment heterogeneity, no closed form solutions.
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Extrapolation

⇒ We avoid: parametric assumptions, support conditions.
⇒ We allow for: treatment heterogeneity, no closed form solutions.

Inference

⇒ Our problem: specific convex programming problem.
⇒ Allows for: uniformly valid inference under weak assumptions.
1. General Framework

2. Numerical Illustration

3. Inference: Basic Outline

4. Efficacy of Price Subsidies for Bed Nets
Notation

**Assumption M:** $(Y, D, Z)$ are generated according to the model

\[ Y = DY_1 + (1 - D)Y_0 \]
\[ D = 1\{U \leq p(Z)\} \]
\[ (Y_0, Y_1, U) \perp Z \]
\[ U \sim U[0, 1] \]

**Note:** Covariates ignored for notational simplicity but simple to add.
Notation

**Assumption M:** \((Y, D, Z)\) are generated according to the model

\[
Y = DY_1 + (1 - D)Y_0 \\
D = 1\{U \leq p(Z)\}
\]

\((Y_0, Y_1, U) \perp Z\)

\(U \sim U[0, 1]\)

**Note:** Covariates ignored for notational simplicity but simple to add.

**Marginal Treatment Response (MTR)**

\[
m_d(u) \equiv E[Y_d|U = u] \text{ for } d \in \{0, 1\}
\]

where \(m = (m_0, m_1) \in \mathcal{M}\) for some known set \(\mathcal{M}\) (prior assumptions)

**Note:** By definition, \(\text{MTE}(u) = m_1(u) - m_0(u)\) (but + flexibility with \(m_d\)).
Target Parameter

\[ \beta^* = E[ \int_0^1 m_0(u)\omega_0^*(u, Z)du ] + E[ \int_0^1 m_1(u)\omega_1^*(u, Z)du ] \]

where \( \omega_d^* \) are known or identified weighting functions for \( d \in \{0, 1\} \).
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where \( \omega_d^* \) are known or identified weighting functions for \( d \in \{0, 1\} \).

Example 1: Average Treatment Effect (ATE)

\[ E[Y_1 - Y_0] = \int_0^1 E[Y_0|U = u] \times (-1) \, du + \int_0^1 E[Y_1|U = u] \times (+1) \, du \]

\[ m_0(u) \quad \omega_0^*(u, Z) \quad m_1(u) \quad \omega_1^*(u, Z) \]
Target Parameter

\[ \beta^* = E\left[ \int_0^1 m_0(u)\omega^*_0(u, Z)du \right] + E\left[ \int_0^1 m_1(u)\omega^*_1(u, Z)du \right] \]

where \( \omega^*_d \) are known or identified weighting functions for \( d \in \{0, 1\} \).

Example 2: Average Treatment on Treated (ATT)

\[ E[Y_1 - Y_0 | D = 1] \]
\[ = E\left[ \int_0^1 m_0(u)\left(-\frac{1\{u \leq p(Z)\}}{P(D = 1)}\right)du \right] + E\left[ \int_0^1 m_1(u)\left(\frac{1\{u \leq p(Z)\}}{P(D = 1)}\right)du \right] \]

\[ \omega^*_0(u, Z) \quad \omega^*_1(u, Z) \]
What We Know

Problem: Target parameter identified when MTR $m = (m_0, m_1)$ identified ...  
... but the MTR functions $m = (m_0, m_1)$ may not be identified ...

However: We do have some information about $m = (m_0, m_1)$ through LATE

Example: Suppose $Z \in \{0, 1\}$ is binary and that $p(1) > p(0)$, then we have

$$\text{Cov}(Y, Z) = \text{Cov}(D, Z) = E\left[ Y_1 - Y_0 | p(0) \leq U \leq p(1) \right] = \int_0^1 \left\{ m_1(u) - m_0(u) \right\} 1 \{ p(0) \leq u \leq p(1) \} p(1) - p(0) du$$

Key: LATE imposes a linear restriction on MTR ... are there others?

Mogstad, Santos, and Torgovitsky. November 10, 2017. UCLA
What We Know

**Problem:** Target parameter identified when MTR \( m = (m_0, m_1) \) identified ... ... but the MTR functions \( m = (m_0, m_1) \) may not be identified ...

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**Example:** Suppose \( Z \in \{0, 1\} \) is binary and that \( p(1) > p(0) \), then we have

\[
\frac{\text{Cov}(Y, Z)}{\text{Cov}(D, Z)} = E[Y_1 - Y_0 | p(0) < U \leq p(1)]
\]

\[
= \int_0^1 \{m_1(u) - m_0(u)\} \frac{1\{p(0) < u \leq p(1)\}}{p(1) - p(0)} du
\]

**Key:** LATE imposes a linear restriction on MTR ... are there others?

Mogstad, Santos, and Torgovitsky. November 10, 2017. UCLA
Proposition: If Assumption M holds and $E[s^2(D, Z)] < \infty$, then it follows

$$E[Ys(D, Z)] = E[\int_0^1 m_0(u)\omega_{0s}(u, Z)du] + E[\int_0^1 m_1(u)\omega_{1s}(u, Z)du]$$

where $\omega_{0s}(u, Z) \equiv s(0, Z)1\{u > p(Z)\}$
and $\omega_{1s}(u, Z) \equiv s(1, Z)1\{u \leq p(Z)\}$.

Comments
- LATE corresponds to $s(D, Z) = (Z - E[Z]) / \text{Cov}(D, Z)$.
- Additional choices of $s(D, Z) \Rightarrow$ more restrictions on MTR.
- Sufficiently many $s(D, Z) \Rightarrow$ reflect all information in conditional means.
Using MTR Restrictions

What We Want

\[ \beta^* = E \left[ \int_0^1 m_0(u)\omega^*_0(u, Z)du \right] + E \left[ \int_0^1 m_1(u)\omega^*_1(u, Z)du \right] \]

Note: \( \beta^* \) is linear in \( m \) – i.e. \( \beta^* = \Gamma^*(m) \) where \( \Gamma^* : \mathcal{M} \rightarrow \mathbb{R} \) equals

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Using MTR Restrictions

What We Want

\[ \beta^* = E\left[ \int_0^1 m_0(u)\omega_0^*(u, Z)du \right] + E\left[ \int_0^1 m_1(u)\omega_1^*(u, Z)du \right] \]

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What We Know

\[ E[Y_s(D, Z)] = E\left[ \int_0^1 m_0(u)\omega_{0s}(u, Z)du \right] + E\left[ \int_0^1 m_1(u)\omega_{1s}(u, Z)du \right] \]

Note: Linear restrictions – i.e. \( E[Y_s(D, Y)] = \Gamma_s(m) \) where \( \Gamma_s(m) \) equals

\[ \Gamma_s(m) = E\left[ \int_0^1 m_0(u)\omega_{0s}(u, Z)du \right] + E\left[ \int_0^1 m_1(u)\omega_{1s}(u, Z)du \right] \]
Proposition: Let $\mathcal{M}$ be convex, $S$ a set of functions of $(D, Z)$, and define

$$
\underline{\beta^*} \equiv \inf_{m \in \mathcal{M}} \Gamma^*(m) \text{ s.t. } E[Y s(D, Z)] = \Gamma_s(m) \text{ for all } s \in S
$$

$$
\bar{\beta}^* \equiv \sup_{m \in \mathcal{M}} \Gamma^*(m) \text{ s.t. } E[Y s(D, Z)] = \Gamma_s(m) \text{ for all } s \in S
$$

Then closure of feasible (s.t. linear constraints) values of $\beta^*$ equals $[\underline{\beta^*}, \bar{\beta}^*]$. 

Comments

- Convex optimization problem $\Rightarrow$ simple estimators (when feasible).
- Constraints may be unfeasible $\Rightarrow$ model is misspecified.
- For appropriate $S$ can exhaust information in conditional means.  
  ... but still may not correspond to identified set (unless $Y$ binary).
1 General Framework

2 Numerical Illustration

3 Inference: Basic Outline

4 Efficacy of Price Subsidies for Bed Nets
Basic Design

Data Generating Process

- Instrument takes three values $Z \in \{0, 1, 2\}$.
- Outcome $Y$ is binary $Y \in \{0, 1\}$.
- Propensity score $p(0) = 0.35$, $p(1) = 0.6$, and $p(2) = 0.7$.

Parameter of Interest

$$\text{LATE}(0.35, 0.9) \equiv E[Y_1 - Y_0 | U \in (0.35, 0.9)] = 0.046$$

Comments

- Three LATEs nonparametrically identified.
- Parameter of interest measures sensitivity to expanding complier group.
- MTR functions not identified (unless $M$ is restricted).
Basic Information

Weights (where $\neq 0$)

$d = 0$

$d = 1$

Figure: MTRs Used in the Data Generating Process (DGP)
Nonparametric bounds: [-0.421, 0.500]

Weights (where $\neq 0$)

$d = 0$

$d = 1$

MTR

Figure: Maximizing MTRs When Using Only the IV Slope Coefficient
Adding Information

Nonparametric bounds: [-0.411, 0.500]

Weights
(where \( \neq 0 \))

\( d = 0 \)

\( d = 1 \)

MTR

Maximizing MTRs

LATE(0.35, 0.90)

IV slope

OLS slope

Figure: Maximizing MTRs When Using Both the IV and OLS Slope Coefficients
Adding All Information

Nonparametric bounds: [-0.138, 0.407]

Figure: Maximizing MTRs When Using All IV–like Estimands (Sharp Bounds)
Nonparametric bounds, MTRs decreasing: [-0.095, 0.077]

Weights (where $\neq 0$)

$u = 0$ $0.2$ $0.4$ $0.6$ $0.8$ $1$

$-2$ $0$ $2$

$\text{MTR}$

Maximizing MTRs

$LATE(0.35, 0.90)$

$(1 - D)1[Z = 1]$ $(1 - D)1[Z = 2]$ $(1 - D)1[Z = 3]$


Figure: Maximizing MTRs When Restricted to be Decreasing
Adding Smoothness

Order 9 polynomial bounds, MTRs decreasing: \([0.000, 0.067]\)

Figure: Maximizing MTRs When Further Restricted to be a 10th Order Polynomial
Available Information

- Valuable information in the data beyond identified LATEs.
- Parameters can be informative without being interesting (e.g. OLS).

Shape Restrictions

- When credible, they can substantially improve bounds.
- Value in parametric and nonparametric restrictions.

Additional Comments

- Computational approach allows flexibility without analytical solution.
- Different information for different parameters (e.g. LATE(0.35, \( \bar{u} \))).
General Framework

Numerical Illustration

Inference: Basic Outline

Efficacy of Price Subsidies for Bed Nets
Setup

**Goal:** Build confidence regions and/or conduct specification tests.
Setup

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Formally: Given i.i.d. sample \( \{Y_i, Z_i, D_i\}_{i=1}^n \) with \((Y, Z, D) \sim P \in \mathcal{P}\), we test

\[
H_0 : P \in \mathcal{P}_0 \quad \quad H_1 : P \in \mathcal{P} \setminus \mathcal{P}_0
\]

where for some linear map \( \Gamma_P \) and element \( \beta_P \) we define \( \mathcal{P}_0 \) to equal

\[
\mathcal{P}_0 \equiv \{ P \in \mathcal{P} : \Gamma_P(m) = \beta_P \text{ for some } m \in \mathcal{M} \}
\]

Allows For

- Confidence regions for target parameter.
- Specification tests for different maintained assumptions.
Test Statistic

\[ P_0 \equiv \{ P \in \mathbf{P} : \Gamma_P(m) = \beta_P \text{ for some } m \in \mathcal{M} \} \]
$P_0 \equiv \{ P \in P : \Gamma_P(m) = \beta_P \text{ for some } m \in M \}$

Test Statistic

$T_n \equiv \inf_{m \in M} \sqrt{n} \| \hat{\beta} - \hat{\Gamma}(m) \|$

Comments

- Theory allows for use of sieve $\mathcal{M}_n$ (if needed for computation).
- Linear minimum distance problem, complications arise from $\mathcal{M}$.
- “Irregular” behavior $\Rightarrow$ Bootstrap failure (Fang and Santos (2016)).
Inference Results

Test Statistic

\[ T_n \equiv \inf_{m \in M} \sqrt{n} \| \hat{\beta} - \hat{\Gamma}(m) \| \]

Main Result Propose Bootstrap critical values and establish size control.

Contributions

- Size control is uniform in large class of distributions \( P \).
- Allow shape restrictions, and parametric/nonparametric specifications.
- Employ with finite or infinite number of moment restrictions.
- More generally applicable to linear programming problems.
- Bootstrap statistic obtained as a bilinear optimization problem.
General Framework

Numerical Illustration

Inference: Basic Outline

Efficacy of Price Subsidies for Bed Nets
Background

The Data

- Randomized control experiment in Kenya by Dupas (2014).
- Households randomly assigned a price (out of 17) for antimalaria net.
- Total of 1200 households in six villages.
- Follow up check to see if malaria net was in use.

Policy Concerns

- Subsidising inframarginal consumers that would have purchased.
- Unwilling to purchase may be unwilling to use (nonmonetary cost).
- Higher price may exclude poor or credit constrained individuals.
Background

The Setup

- $Y$ indicator for whether net is in use.
- $Z$ randomly assigned price.
- $D$ indicator for whether net was purchased.

Figure: Impact of Price on the Household’s Purchase of Bed Net (logit regression)
Policy Examination

First Target Parameter

- Obtain the average treatment effect.
- Interpretable as comparing no availability of net with free nets.

Second Target Parameter

- Obtain LATE from no net to propensity score at Ksh 150 (avg price).
- Interpretable as the effect of introducing the net into the market.
- Propensity score at 150 estimated via logit prediction.
### Information Specification

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### Panel A. Population Average Treatment Effect

**Bounds**

| K (polynomial order) | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP |
|----------------------|---|---|----|----|----|---|---|----|----|----|---|---|----|----|----|----|
| Lower                | .6521 | .4646 | .3857 | .3275 | .2533 | .6521 | .4956 | .4700 | .4537 | .3954 | .6365 | .5602 | .5269 | .4487 |
| Upper                | .6772 | .7269 | .7362 | .7445 | .7515 | .6521 | .7269 | .7362 | .7445 | .7515 | .7104 | .7178 | .7229 | .7253 |

**90% Confidence Interval**

| K (polynomial order) | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP |
|----------------------|---|---|----|----|----|---|---|----|----|----|---|---|----|----|----|----|
| Lower                | .5486 | .3761 | .2995 | .2421 | .4282 | .4032 | .3511 | .3204 | .5206 | .4130 | .3652 | .3260 |
| Upper                | .7462 | .8019 | .8102 | .8139 | .7516 | .8093 | .8179 | .8209 | .7491 | .7910 | .7941 | .7978 |

### Panel B. PRTE at Free Provision versus a Price of 150 Ksh

**Bounds**

| K (polynomial order) | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP |
|----------------------|---|---|----|----|----|---|---|----|----|----|---|---|----|----|----|----|
| Lower                | .6600 | .5881 | .5626 | .5444 | .4817 | .6600 | .5881 | .5626 | .5444 | .4856 | .6758 | .6506 | .6214 | .5573 |
| Upper                | .7049 | .8140 | .8469 | .8817 | .9732 | .6600 | .7085 | .7172 | .7275 | .7941 | .6895 | .6988 | .7140 | .7492 |

**90% Confidence Interval**

| K (polynomial order) | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP | 2 | 6 | 10 | 20 | NP |
|----------------------|---|---|----|----|----|---|---|----|----|----|---|---|----|----|----|----|
| Lower                | .5417 | .5005 | .4695 | .4479 | .3890 | .3472 | .3414 | .3320 | .5079 | .4755 | .4584 | .4281 |
| Upper                | .7686 | .9161 | .9519 | .9746 | .7732 | .9263 | .9616 | .9838 | .7713 | .9093 | .9291 | .9511 |

### Specifications of the IV-like Estimands

<table>
<thead>
<tr>
<th>Intercept</th>
<th>(s(d, z) = 1)</th>
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<tbody>
<tr>
<td>Linear in (p(Z))</td>
<td>(s(d, z) = p(z))</td>
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Conclusion

Summary

• General method for inference on a specified target parameter.
• Does not require continuous/large support instruments (though help).
• Allows for specification testing.
• Computation is fast and reliable – linear/bilinear programming.
• Uniformly valid inference in linear programs.

In Progress

• Extensive Monte Carlo experiments.
• Developing R package (with Bradley Setzler).
• Additional guidance on bandwidth selection.