On the Testability of Identification in Some Nonparametric Models with Endogeneity

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Three Nonparametric Models

Conditional Mean IV: Let $(Y, X, Z) \in \mathbf{R} \times \mathbf{R}^{d_x} \times \mathbf{R}^{d_z}$ have distribution P:

$$Y = \theta_0(X) + \epsilon \qquad\qquad E_P[\epsilon|Z] = 0$$

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Non-separable IV: Let $(Y, X, Z) \in \mathbf{R} \times \mathbf{R}^{d_x} \times \mathbf{R}^{d_z}$ have distribution P:

$$Y = \theta_0(X, \epsilon) \qquad P(\theta_0(X, \epsilon) \leq \theta_0(X, \tau)|Z) = \tau$$

where in addition $\tau \mapsto \theta_0(X, \tau)$ is assumed strictly monotonic almost surely.

Identification

In conditional mean IV, identification requires a unique solution (in θ) to:

$$E_P[Y|Z] = E_P[\theta(X)|Z]$$

Since Newey & Powell (2003), identification through completeness condition

$$E_P[\theta(X)|Z] = 0 \quad P-a.s. \quad \Rightarrow \quad \theta(X) = 0 \quad P-a.s.$$

Comments

- More general: bounded completeness or $L^q(P)$ completeness.
- Sometimes referred to as nonparametric rank condition.
- Also used in identification of quantile and nonseparable models.

Testability

Problems

- Completeness conditions are difficult to interpret.
- Hard to motivate from economic theory.

Questions

- Are completeness assumptions testable under reasonable conditions?
- More generally: is point identification testable in these three models?

Answers

- We show no nontrivial tests for completeness exist.
- We show no nontrivial tests for identification exist in these three models.

Linear Model Intuition

Linear IV: Suppose $(Y, X, Z) \in \mathbf{R}^3$ with distribution $P \in \mathbf{P}$, and satisfy:

$$Y = X\theta_0 + \epsilon \qquad\qquad E_P[Z\epsilon] = 0$$

$\Rightarrow \theta_0$ is identified if and only if $E_P[XZ] \neq 0$ – i.e. $\theta_0 = E_P[XY]/E_P[XZ]$.

Testing Rank Condition

$$H_0 : E_P[XZ] = 0 \qquad\qquad H_1 : E_P[XZ] \neq 0$$

Bahadur and Savage (1956)

- Negative: If $\mathbf{P}$ is rich enough, only test is the trivial test.
- Positive: Learn how to restrict $\mathbf{P}$ for tests to exist (example bounded).

General Setup

$$H_0 : P \in \mathbf{P}_0 \qquad\qquad H_1 : P \in \mathbf{P}_1$$

where $\mathbf{P}_1 \equiv \mathbf{P} \setminus \mathbf{P}_0 = \{$distributions that are complete (or model identified)$\}$.

Main Result

Any test ϕ_n that controls asymptotic size at level $\alpha \in (0, 1)$, in the sense:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha \ ,$$

(for $P^n \equiv \bigotimes_{i=1}^n P$) will have no power against any alternative, in the sense:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \alpha \ .$$

Conclusion holds for all three models, under common assumptions on $\mathbf{P}$.

Literature Review

Nonparametric IV

Newey & Powell (2003), Hall and Horowitz (2005), Blundell et al. (2007), Darolles et al. (2011), Hu and Schennach (2008), Berry & Haile (2010), d'Haultfoeuille (2011), Andrews (2011), Hoderlein et al. (2012).

Quantile/Nonseparable IV

Chernozhukov & Hansen (2005), Horowitz & Lee (2007), Chen & Pouzo (2008), Chernozhukov et al. (2010), Imbens & Newey (2009), Berry & Haile (2009, 2010), Torgovitzky (2011), d'Haultfoeuielle & Fevrier (2011).

Uniformly Valid Inference

Bahadur & Savage (1956), Romano (2004), and many others ...

General Outline

Setup

- Notation and Assumptions.
- Useful Lemma.

Testing Completeness

- The null and alternative hypothesis.
- Main result and proof strategy.

Quantile/Nonseparable IV

- Quantile IV: Main result and proof strategy.
- Nonseparable IV: Main result.

Notation

Let $\mathbf{M}$ be the set of all probability measures on $\mathbf{R} \times \mathbf{R}^{d_x} \times \mathbf{R}^{d_z}$, and define:

$$\mathbf{M}(\nu) \equiv \{P \in \mathbf{M} : P \ll \nu\}$$

We will require $\mathbf{P} \subseteq \mathbf{M}(\nu)$ for some measure ν satisfying the following:

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We will require $\mathbf{P} \subseteq \mathbf{M}(\nu)$ for some measure ν satisfying the following:

Main Assumption (A)

- ν is a σ-finite Borel measure on $\mathbf{R} \times \mathbf{R}^{d_x} \times \mathbf{R}^{d_z}$.
- $\nu = \nu_y \times \nu_x \times \nu_z$ for ν_y, ν_x and ν_z Borel measures on $\mathbf{R}$, $\mathbf{R}^{d_x}$ and $\mathbf{R}^{d_z}$.
- The measure ν_x is atomless on $\mathbf{R}^{d_x}$.

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- The measure ν_x is atomless on $\mathbf{R}^{d_x}$.

Comments

- Support restrictions imposed through ν (example $(X, Z) \in [0, 1]^{d_x + d_z}$).
- ν product measure does not require $P \in \mathbf{P}$ to be product measure.

Discussion

ν_x atomless

- May be relaxed, but ν_x cannot be purely discrete.
- If $d_x > 1$, then sufficient for one coordinate to be atomless.

Example

- Suppose ν_x and ν_z have discrete support $\{x_1, \ldots, x_s\}$ and $\{z_1, \ldots, z_t\}$.

$$\Pi(P) \equiv \{s \times t \text{ matrix with } \Pi(P)_{j,k} = P(X = x_j | Z = z_k)\}$$

- Newey & Powell (2003) showed P is complete iff $\text{rank}(\Pi(P)) = s$.
- Test can be constructed through uniform confidence region for $\Pi(P)$.

Useful Lemma

$$\|P_1 - P_2\|_{TV} \equiv \sup_{g:|g|\leq 1} \frac{1}{2}\Big|\int g dP_1 - \int g dP_2\Big|$$

Lemma If for all $P \in \mathbf{P}_1$, there is $\{P_k\}$ with $P_k \in \mathbf{P}_0$ and $\|P - P_k\|_{TV} = o(1)$

$$\sup_{P \in \mathbf{P}_1} E_{P^n}[\phi_n] \leq \sup_{P \in \mathbf{P}_0} E_{P^n}[\phi_n]$$

for every sequence of test functions $\{\phi_n\}$ and for every n.

Comments

- Small modification of Theorem 1 in Romano (2004).
- Result implies its asymptotic analogue.
- Intuition: If every $P \in \mathbf{P}_1$ is in the boundary of $\mathbf{P}_0$, then we conclude

Size Control $\Rightarrow$ No Power

Useful Lemma

Key Idea: Since $|\phi_n| \leq 1$ for any test function, $\|P - P_k\|_{TV} = o(1)$ implies:

$$\Big|\int \phi_n dP_k^n - \int \phi_n dP^n\Big| \leq \sup_{g:|g|\leq 1} \frac{1}{2}\Big|\int g dP_k^n - \int g dP^n\Big| = o(1)$$

Comments

- Total Variation distance plays no role in the definition of $\mathbf{P}_0$ and $\mathbf{P}_1$.
- Metrics compatible with weak topology may be too weak for result.
- Stronger metric, implies harder to show $P_k \to P$.

Goal: Show in problems we study, lack of identification $(\mathbf{P}_0)$ is "dense".

Completeness

$$H_0 : P \in \mathbf{P}_0 \qquad\qquad H_1 : P \in \mathbf{P}_1$$

where $\mathbf{P} = \mathbf{M}(\nu)$ for some $\nu \in \mathbf{M}$, $\mathbf{P}_0 \equiv \mathbf{P} \setminus \mathbf{P}_1$ and we additionally define:

$$\mathbf{P}_1 \equiv \{P \in \mathbf{P} : E_P[\theta(X)|Z] = 0 \text{ for } \theta \in L^\infty(P) \Rightarrow \theta(X) = 0 \ P - a.s.\}$$

Comments

- Using $L^\infty(P) \Rightarrow$ test for bounded completeness.
- Replacing $L^\infty(P)$ with $L^q(P)$ for $1 \leq q < \infty$ just enlarges $\mathbf{P}_0$.
- No power in this setting $\Rightarrow$ no power in test of $L^q(P)$ completeness.

Completeness

Theorem Let $\mathbf{P} = \mathbf{M}(\nu)$ and Assumption (A) hold. Then if $\{\phi_n\}$ satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha \ ,$$

for $P^n \equiv \bigotimes_{i=1}^n P$ and level $\alpha \in (0,1)$, then it follows that it also satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \alpha \ .$$

Comments

- If ν has compact support, then support of $P \in \mathbf{P}$ uniformly bounded.
- In contrast, ν with compact support suffices in linear IV model.

Proof Outline

Step 1 Fix $P \in \mathbf{P}_1$, let $f \equiv dP/d\nu$, show $\sup_{g:|g|\leq 1} |\int g(f_k - f)d\nu| = o(1)$:

$$f_k(x,z) \equiv \sum_{i=1}^{K_k} \pi_{ik} 1\{(x,z) \in S_{ik}\} \qquad f_k \geq 0 \qquad \int f_k d\nu = 1$$

Step 2 $\{S_{ik}\}_{i=1}^{K_k}$ can be chosen to be the product of two collections of sets:

- $\{U_{ik}\}$ a partition of the set $[-M_k, M_k]^{d_x}$ some $M_k \in (0,\infty)$.
- $\{V_{ik}\}$ a partition of the set $[-M_k, M_k]^{d_z}$ same $M_k \in (0,\infty)$.

Step 3 Since ν_x is atomless, we can partition each U_{ik} into $(U_{ik}^{(1)}, U_{ik}^{(2)})$:

$$\nu_x(U_{ik}^{(1)}) = \nu_x(U_{ik}^{(2)}) = \frac{1}{2}\nu_x(U_{ik})$$

Proof Outline

Step 4 Let P_k be measure with $dP_k/d\nu = f_k$, and define the function:

$$\theta_k(x) \equiv \sum_{i=1}^{D_k}(1\{x \in U_{ik}^{(1)}\} - 1\{x \in U_{ik}^{(2)}\})$$

Step 5 Then: (i) θ_k is bounded, (ii) $\theta_k(X) \neq 0 \; P_k - a.s.$, and (iii):

$$\begin{aligned}\int_{V_{nk}}\int_{U_{tk}} &\psi(z)\theta_k(x)\nu_x(dx)\nu_z(dz) \\ &= \int_{V_{nk}}\psi(z)\int_{U_{tk}}(1\{x \in U_{tk}^{(1)}\} - 1\{x \in U_{tk}^{(2)}\})\nu_x(dx)\nu_z(dz) \\ &= 0\end{aligned}$$

However, recall $dP_k/d\nu = \sum_{i=1}^{K_k}\pi_{ik}1\{(x,z) \in S_{ik}\}$ with $S_{ik} = V_{nk} \times U_{tk}$ ...

Proof Outline

Step 6 Therefore, $E_{P_k}[\psi(Z)\theta_k(X)] = 0$ for all P_k-integrable ψ, and hence:

$$E_{P_k}[\theta_k(X)|Z] = 0 \quad P_k - a.s.$$

Step 7 Therefore, $P_k \in \mathbf{P}_0$ for all k, and $\|P_k - P\|_{TV} = o(1)$. By Lemma,

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha$$

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Comments

- The sequence $\{\theta_k\}$ developed in the proof is not differentiable.
- Proof may be modified so $\{\theta_k\}$ is infinitely differentiable.
- $\Rightarrow L^\infty(P)$ may be replaced by Sobolev space or Ball.
- Similarly, we may also impose smoothness restrictions on $dP/d\nu$.

Comments

Two Important Features

- Completeness may be testable under alternative specifications of $\mathbf{P}$. However, standard "nonparametric" approaches do not seem to apply.
- Assumptions routinely employed that are non testable but "reasonable".

Genericity Arguments

- Alternative justification in favor of completeness assumptions.
- Andrews (2011) shows set of distributions for which it fails is "shy".
- Chen et al. (2012) show certain measures (over conditional expectation operators) assign zero probability to completeness failure.

Quantile IV

$$H_0 : P \in \mathbf{P}_0 \qquad\qquad H_1 : P \in \mathbf{P}_1$$

for $\mathbf{P}$ the subset of $\mathbf{M}(\nu)$ consisting of $P \in \mathbf{M}(\nu)$ such that $\exists\, \theta_0 \in L^\infty(P)$:

$$Y = \theta_0(X) + \epsilon \qquad\qquad P(\epsilon \leq 0|Z) = \tau \; P - a.s.$$

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As before, $\mathbf{P}_0 \equiv \mathbf{P} \setminus \mathbf{P}_1$, where now $\mathbf{P}_1 \subset \mathbf{P}$ is given by the set of measures:

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Comments

- Uniqueness of $\theta \in L^\infty(P)$ understood up to sets of P-measure zero.
- No easy necessary conditions for identification from completeness:

$\Rightarrow$ We test for identification directly

Quantile IV

Theorem Let $\mathbf{P}$ be as defined, and Assumption (A) hold. If $\{\phi_n\}$ satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha \ ,$$

for $P^n \equiv \bigotimes_{i=1}^n P$ and level $\alpha \in (0,1)$, then it follows that it also satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \alpha \ .$$

Comments

- We show $\mathbf{P}_0$ is dense in $\mathbf{M}(\nu)$ (not just $\mathbf{P}_1$) w.r.t Total Variation.
- Theorem holds for $L^q(P)$ in place of $L^\infty(P)$ as well.

Proof Outline

Step 1 Fix $P \in \mathbf{P}_1$, let $f \equiv dP/d\nu$, show $\sup_{g:|g|\leq 1} |\int g(f_k - f)d\nu| = o(1)$:

$$f_k(y,x,z) \equiv \sum_{i=1}^{K_k} \pi_{ik} 1\{(y,x,z) \in S_{ik}\} \qquad f_k \geq 0 \qquad \int f_k d\nu = 1$$

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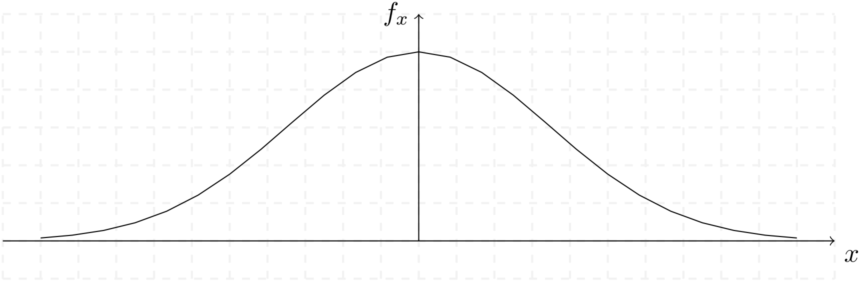
- $\{V_{ik}\}$ a partition of the set $[-M_k, M_k]^{d_z}$ same $M_k \in (0,\infty)$.
- $\{L_{ik}\}$ a partition of the set $[-M_k, M_k]$ same $M_k \in (0,\infty)$.

Step 3 Since ν_x is atomless, we can pick $U_{ik}^{(1)}(\tau) \subset U_{ik}$, and $U_{ik}^{(2)}(\tau) \subset U_{ik}$:

$$\nu_x(U_{ik}^{(1)}(\tau)) = \nu_x(U_{ik}^{(2)}(\tau)) = \tau\nu_x(U_{ik}) \qquad \nu_x(U_{ik}^{(1)}(\tau) \triangle U_{ik}^{(2)}(\tau)) > 0$$

Proof Outline



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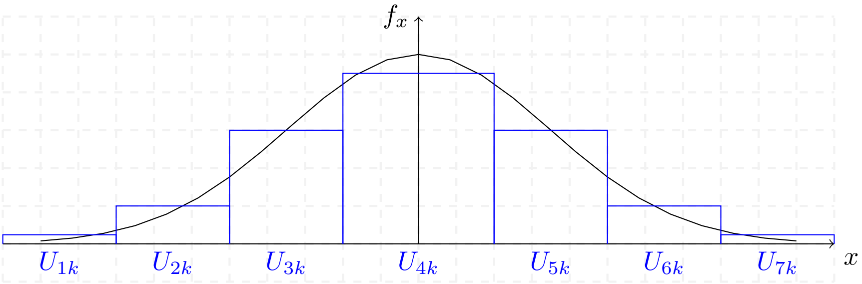

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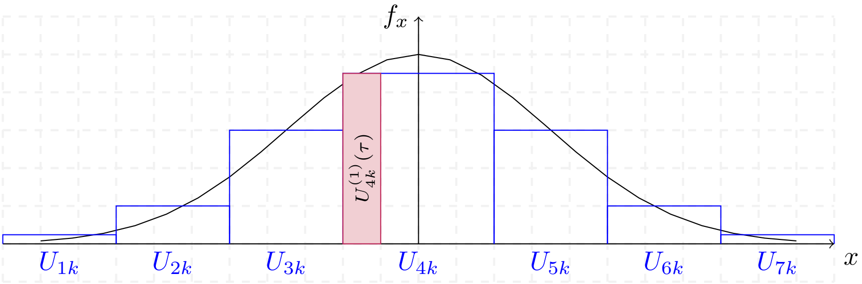

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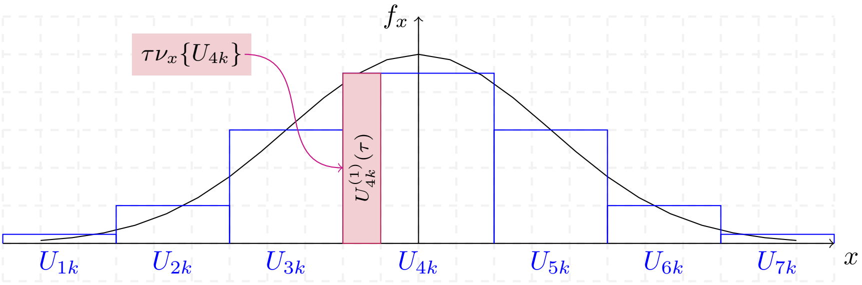

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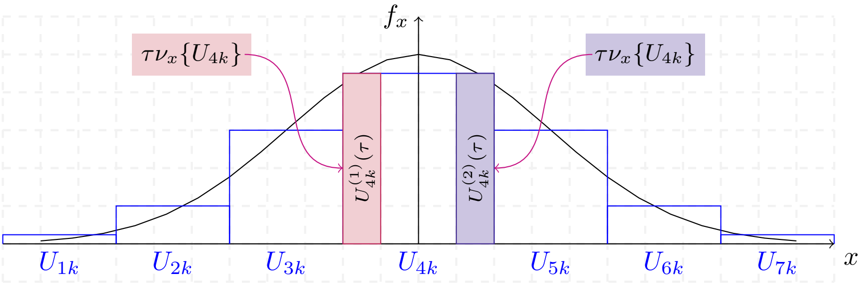

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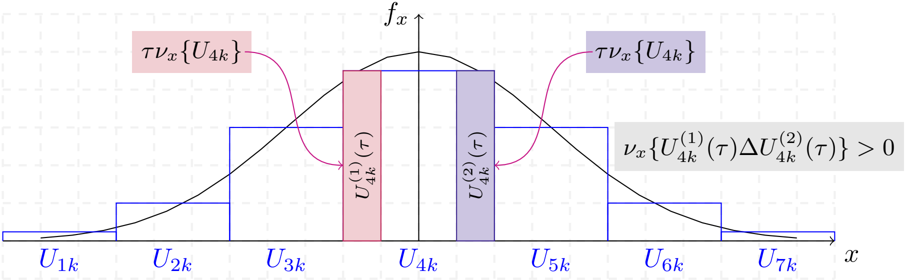

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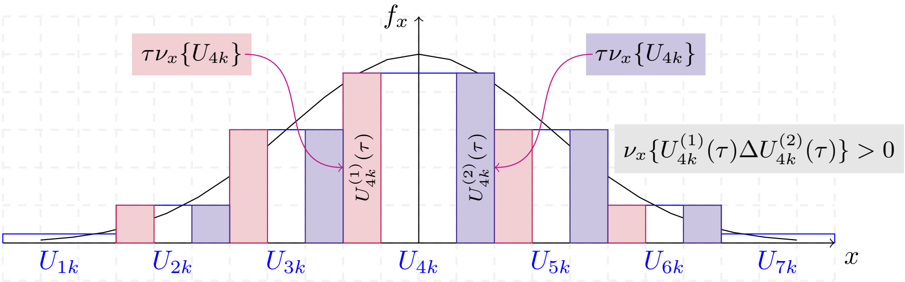

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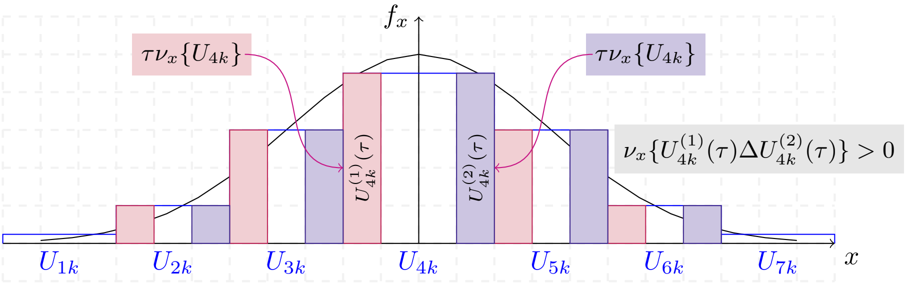

Step 4 Under P_k, Y has support contained in $[-M_k, M_k]$. Hence, letting:

$$\theta_k^{(l)}(x,\tau) = \sum_{i=1}^{D_k}\{2M_k 1\{x \in U_{ik}^{(l)}(\tau)\} - 2M_k 1\{x \in U_{ik} \setminus U_{ik}^{(l)}(\tau)\}\}$$

we get that $1\{Y \leq \theta_k^{(l)}(X,\tau)\} = \sum_i 1\{X \in U_{ik}^{(l)}\}$, almost surely under P_k.

Proof Outline

Step 5 Then: (i) $\theta_k^{(1)}$ and $\theta_k^{(2)}$ are bounded, (ii) for any $L_{jk} \times V_{nk} \times U_{tk}$:

$$\begin{aligned}
&\int_{L_{jk}} \int_{V_{nk}} \int_{U_{tk}} \psi(z)(1\{y \leq \theta_k^{(l)}(x,\tau)\} - \tau)\nu_x(dx)\nu_z(dz)\nu_y(dy) \\
&\qquad = \int_{L_{jk}} \int_{V_{nk}} \psi(z) \int_{U_{tk}} (1\{x \in U_{tk}^{(l)}(\tau)\} - \tau)\nu_x(dx)\nu_z(dz)\nu_y(dy) \\
&\qquad = 0
\end{aligned}$$

However, $dP_k/d\nu = \sum_{i=1}^{K_k} \pi_{ik} 1\{(x,z) \in S_{ik}\}$ with $S_{ik} = L_{jk} \times V_{nk} \times U_{tk}$ ...

Step 6 Hence, $E_{P_k}[\psi(Z)(1\{Y \leq \theta_k^{(l)}(X,\tau)\} - \tau)] = 0$ for $\psi \in L^1(P_k)$, and:

$$E_{P_k}[1\{Y \leq \theta_k^{(l)}(X,\tau)\} - \tau | Z] = 0 \quad P_k - a.s.$$

Proof Outline

Step 7 Argue that $P_k(\theta_k^{(1)}(X,\tau) \neq \theta_k^{(2)}(X,\tau)) > 0$ for all k.

Step 8 Hence, $P_k \in \mathbf{P}_0$ for all k, and $\|P_k - P\|_{TV} = o(1)$. By Lemma,

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha$$

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Comments

- In the proof, we actually establish the stronger inequality:

$$E_{P_k}[(1\{Y \leq \theta_k^{(1)}(X,\tau)\} - 1\{Y \leq \theta_k^{(2)}(X,\tau)\})^2] > 0 \ .$$

- Results holds if identification is up to P equivalence of $1\{Y \leq \theta(X)\}$.

Nonseparable IV

$$H_0 : P \in \mathbf{P}_0 \qquad\qquad H_1 : P \in \mathbf{P}_1$$

for $\mathbf{P}$ the maximal subset of $\mathbf{M}(\nu)$ s.t. for each $P \in \mathbf{P}$, $\exists\, \theta_0 \in L^\infty(P)$, with:

$$Y = \theta_0(X, \epsilon) \qquad\qquad P(\theta_0(X,\epsilon) \leq \theta_0(X,\tau)|Z) = \tau \; P - a.s.$$

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As before, $\mathbf{P}_0 \equiv \mathbf{P} \setminus \mathbf{P}_1$, where now $\mathbf{P}_1 \subset \mathbf{P}$ is given by the set of measures:

$$\mathbf{P}_1 \equiv \{P \in \mathbf{P} : \exists! \theta \in L^{\infty}(P) \text{ s.t. } P(Y \leq \theta(X, \tau)|Z) = \tau \quad \forall \tau \ P - a.s.\}$$

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$$Y = \theta_0(X, \epsilon) \qquad\qquad P(\theta_0(X, \epsilon) \leq \theta_0(X, \tau)|Z) = \tau \ P - a.s.$$

As before, $\mathbf{P}_0 \equiv \mathbf{P} \setminus \mathbf{P}_1$, where now $\mathbf{P}_1 \subset \mathbf{P}$ is given by the set of measures:

$$\mathbf{P}_1 \equiv \{P \in \mathbf{P} : \exists! \theta \in L^\infty(P) \text{ s.t. } P(Y \leq \theta(X, \tau)|Z) = \tau \quad \forall \tau \ P - a.s.\}$$

Comments

- For each $\tau \in (0, 1)$ model is equivalent to previous one.
- $\tau \mapsto \theta(X, \tau)$ additionally strictly increasing $P - a.s.$

Nonseparable IV

Theorem For $\mathbf{P}$ be as defined, and under Assumption (A), if $\{\phi_n\}$ satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_0} E_{P^n}[\phi_n] \leq \alpha \ ,$$

for $P^n \equiv \bigotimes_{i=1}^n P$ and level $\alpha \in (0,1)$, then it follows that it also satisfies:

$$\limsup_{n\to\infty} \sup_{P\in\mathbf{P}_1} E_{P^n}[\phi_n] \leq \alpha \ .$$

Comments

- We show $\mathbf{P}_0$ is dense in $\mathbf{M}(\nu)$ not just $\mathbf{P}_1$ w.r.t. Total Variation.
- Theorem holds for $L^q(P)$ in place of $L^\infty(P)$ completeness as well.
- Essentially same steps, but add monotonicity in τ to construction.

Conclusion

Testability

- No nontrivial tests for identification exist in three IV models.
- $\mathbf{P}$ requirements are satisfied by usual assumptions in the literature.

However ...

- Valid tests may exist under more restrictive assumptions on $\mathbf{P}$.
- Valid tests may also exist under shape restrictions on θ_0.
- Results can aid develop nontrivial tests under additional requirements.

Two Constructive Points

- Highlight importance of alternative justifications – e.g. genericity.
- Emphasize value of procedures that are robust to partial identification.