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Inflation and output dynamics with state-dependent pricing decisions

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Abstract

This paper studies a state-dependent pricing model in which firms face a fixed cost of changing their pricing plans. A pricing plan specifies an entire sequence of time-varying future prices. Allowing firms to choose a pricing plan rather than a single price generates inflation inertia in the response of the economy to small changes in the growth rate of money. Allowing firms to choose when to change their pricing plan generates a non-linear response of inflation and output to small and large changes in the money growth rate. The non-linear solution method also reveals that the model generates an asymmetric response of output and inflation to monetary expansions and contractions.

Keywords: Inflation, Nominal Rigidities, State-Dependent Pricing, Inflation Inertia.

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1 Introduction

There is a large literature that studies the effects of monetary policy on output and inflation in models with sticky prices. The models in the literature can be classified into two broad classes. In the first one, commonly referred as *time-dependent* pricing models, the number of firms changing prices is fixed exogenously. Firms only control the degree to which they change their price once they have the opportunity to do so.[1] However, time-dependent models are often viewed as an approximation of more complicated firm behavior. As an alternative, a second class of models, commonly referred as *state-dependent* pricing models, endogenizes the number of firms changing prices. Typically, this extensive margin is modeled by assuming that firms face a fixed cost of changing their nominal price. Dotsey, King, and Wolman (1999) develop a tractable way to incorporate state-dependent pricing models into a quantitative general equilibrium framework.[2]

In this paper, we study a variation of a standard state-dependent pricing model, in which firms choose dynamic pricing plans. Once a firm pays a fixed cost, it can choose not only its current price, but also a plan specifying an entire sequence of future prices. Nominal rigidities arise because changing the plan is costly, and prices in the plan can be made contingent on the current information set but cannot be made contingent upon future aggregate variables. This pricing behavior is consistent with the fact that the costs of changing prices are broader than menu costs that have prevailed in the literature. There are other costs associated to implementing a new pricing plan such as communication and negotiation costs, as documented by Zbaracki, Ritson, Levy, Dutta, and Bergen (2004).[3] This pricing assumption resembles Fischer's (1977) contracting model with pre-determined prices. Mankiw and Reis (2002), Calvo, Celasun, and Kumhof (2001),

[1] See Taylor (1999) for a comprehensive literature review on time-dependent sticky price models.

[2] Other papers that study state-dependent pricing models include Caballero and Engel (1993), Caplin and Leahy (1991), Caplin and Spulber (1987), and Ireland (1997). These papers make simplifying assumptions to gain analytical tractability.

[3] In their case studies, Zbaracki et al. (2004) find that only 4% of the costs associated with changing prices are related to physical menu costs. Seasonalities and varying week/weekend prices in restaurants are illustrations of the assumed pricing behavior.

and Devereux and Yetman (2003) are recent papers that study related time-dependent pricing models.[4] In order to isolate the implications of our pricing assumptions, we abstract from other costs of changing prices such as information gathering costs, which are implicit in the analysis of Mankiw and Reis (2002) and Woodford (2001a).

Compared to the previous sticky-price literature, our model has two desirable properties. First, it generates inflation inertia in the response of the economy to small changes in the growth rate of money. Conventional time-dependent and state-dependent pricing models in which firms choose a single price do not.[5] Second, the model is consistent with the view that large, persistent changes in the growth rate of money, have relatively small effects on output. In contrast, standard time-dependent models in which the number of firms changing prices is constant, are not. Since we do not rely on linear approximations, in contrast to much of the literature, we can study the differences in the response of the model to changes in the growth rate of money of different magnitudes.

In order to understand the aggregate implications of our model, we focus on two monetary experiments. According to conventional wisdom:

1. Small temporary increases in the growth rate of money lead to a hump-shaped response of output and inflation. Large changes in the growth rate of money have relatively smaller effects on output.[6]

[4] In the model studied by Mankiw and Reis (2002), firms have flexible prices but only a fraction of firms update their information set every period. In the model studied by Calvo, Celasun, and Kumhof (2001), pricing plans are constrained to consist of an initial price level and a constant growth rate of the price over time. Devereux and Yetman (2003) study a time-dependent model with predetermined prices and focus on persistence associated with small changes in monetary policy. Our model extends these papers by combining the ability to set a price plan, versus a single price, and the ability to choose when to change the plan itself. For small changes in the growth rate of money, we obtain similar results to these papers.

[5] See Chari, Kehoe, and McGrattan (2000) and Mankiw and Reis (2002) for a criticism of conventional sticky price models. See also Christiano, Eichenbaum, and Evans (2005) and Dotsey and King (2001) for richer sticky price models that can account for the response of the U.S. economy to small monetary shocks.

[6] This conventional wisdom is not free of controversies, but roughly speaking it has been widely accepted. See for example Friedman (1968). For more recent studies, see Christiano et al. (2005), Fuhrer and Moore (1995), and references therein for a discussion of small shocks to the money growth rate in postwar U.S. data. See Sargent (1982) for a discussion of the relation of money and prices during hyperinflations. See also Ravn and Sola (1996), Weise (1999), and Fischer, Sahay, and Vegh (2002) for other empirical evidence supporting a differential response to small and large monetary shocks.

2. Permanent credible disinflations in low inflation environments generate a temporary contraction in output. Credible disinflations where the initial level of inflation is very high cause a relatively smaller decline in output.[7]

Our model is consistent with (1) and (2). The central features underlying its predictions are as follows. First, firms face a fixed cost of changing their pricing plan rather than their current price level. This feature is key to understanding the response of inflation and output to small changes in the growth rate of money. Suppose that firms expect nominal marginal costs will increase over time after an unforeseen monetary expansion. If a firm could only choose a single, non-time-varying price, it would choose a price that is higher than the current marginal cost. This is because, depending on the future costs of changing prices, it might not raise its price in the near future when higher marginal costs materialize. We refer to this type of forward looking behavior as 'front-loading'. Other models in the literature, such as Nelson (1998) and Woodford (2001b), reduce front-loading incentives by imposing frictions on the firm's problem so that the rate of change of its price is small. In contrast, firms in our model have smaller front-loading incentives because they have the freedom to choose a sequence of prices. Then firms can plan future price increases in advance without actually having to implement them today. The fact that firms have small incentives to front-load their price in anticipation of increases in marginal cost is the key feature that allows our model to generate an inertial response of inflation to small changes in the growth rate of money.

Second, the fraction of firms changing their pricing decisions every period is endogenous. This drives the differential response of output and inflation to small and large changes in the growth rate of money. In response to a small change, only a small number of firms decide to adjust their pricing decisions. But, after a large change, many firms find it optimal to adjust their pricing decisions. In contrast, the response of output and inflation in time-dependent models is

[7] See Gordon (1982), Ball (1994), Dornbusch and Fischer (1993), and references therein for a discussion of credible disinflations where initial inflation rates are relatively moderate. See Sargent (1982) for a discussion of credible disinflations where the initial level of inflation is very high.

roughly proportional to the size of the change in the money growth rate. Hence, for large enough changes in the growth rate of money (a change of roughly five percentage points in the quarterly growth rate under our parametrization), the ability of a time-dependent model to approximate state-dependent models deteriorates.

Our model economy also responds asymmetrically to monetary expansions and contractions. In particular, the rise in output after a monetary expansion is smaller than the fall in output that occurs after a monetary contraction. This is because in our model, firms are more averse to having a relative price that is too low than too high. In the extreme, if the price is low enough, a firm can have negative profits. On the contrary, if the price is too high, profits will be low but never negative. As a result, monetary expansions are more likely to induce price adjustment than monetary contractions. This generates an asymmetric response in output to monetary expansions and contractions.[8]

The remainder of the paper is organized as follows. Section 2 lays out the model and two different pricing assumptions: sticky prices and sticky plans. Section 3 discusses the chosen parameter values, and describes some properties of the steady state. Section 4 presents the sticky price and sticky plan models' aggregate implications for temporary and permanent changes in the growth rate of money. Section 5 concludes.

2 The model

The model consists of a representative household, a government that follows an exogenous policy, a competitive sector producing the final consumption good, and a continuum of monopolistic producers of intermediate goods.

[8] See Maklem, Paquet, and Phaneuf (1996) and references therein for evidence of this asymmetry in the U.S. economy.

Representative Household

Lifetime utility is defined over expected sequences of consumption and hours worked:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right). \tag{1}$$

The function $u(.,.)$ is decreasing in the total number of hours worked (N_t), and increasing in consumption (C_t). The parameter β is the discount factor, with $0 < \beta < 1$.

The agent faces the following budget constraint in period t:

$$P_t^C(1 - \tau_t^C)C_t + B_t + M_t - M_{t-1} = W_t N_t + (1 + i_t)B_{t-1} + R_t K + \Pi_t + T_t \tag{2}$$

Here M_{t-1} and B_{t-1} denote the household's beginning of period t holdings of cash and nominal bonds. The latter pay a nominal interest rate equal to i_t. P_t^C represents the price of the aggregate consumption good, T_t denotes nominal lump sum transfers from the government, and τ_t^C denotes an ad-valorem subsidy to consumption purchases. We assume that households hold a constant amount of capital, K, which earns nominal rent R_t for its services in period t. Nominal profits are denoted by Π_t.

Finally, we assume that agents face the following cash in advance constraint on consumption purchases:

$$P_t^C C_t \leq M_t\ . \tag{3}$$

The consumer maximizes (1) subject to (2) and (3). Under our parametrization, the nominal interest rate is positive, so (3) holds with equality.

Government

The exogenous growth rate of money in period t is μ_t, and is equal to M_t/M_{t-1}. In order to isolate the effects of monetary policy that operate through sticky prices, we abstract from the costs of inflation that would arise in a flexible price version of this economy. Specifically, we assume the government chooses τ_t^C so as to imply:

$$\tau_t^C = \frac{i_{t+1}}{1 + i_{t+1}}.$$

By subsidizing consumption, the government eliminates the distortions present in the choice of consumption versus leisure, and in the valuation of future versus present nominal profits. In addition, we assume the government chooses lump sum transfers, T_t, to satisfy the following period by period balanced budget constraint:

$$M_t - M_{t-1} = T_t + \tau_t^C P_t^C C_t \ . \tag{4}$$

Final Good Producing Firms

This sector is composed of perfectly competitive firms. The final good, Y_t, is produced with a constant returns to scale CES production function in intermediate inputs, Y_{it}:

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \ . \tag{5}$$

Final goods producers take the schedule of intermediate goods' prices P_{it} as given and minimize the unit cost of producing a unit of Y_t. Individual demands for intermediate goods are given by:

$$Y_{it} = \left(\frac{P_{it}}{P_t^C} \right)^{-\varepsilon} Y_t \ . \tag{6}$$

Under perfect competition the price of the final good firms is equal to:

$$P_t^C = \left[\int_0^1 {P_{it}}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} . \tag{7}$$

Intermediate Goods Producers

The intermediate goods sector is composed of a continuum of monopolistic producers indexed by $i \in [0, 1]$. These goods are produced with capital (K_{it}) and labor (N_{it}) using a constant returns to scale Cobb-Douglas production function:

$$Y_{it} = A \left(N_{it} \right)^{\alpha} \left(K_{it} \right)^{1-\alpha} . \tag{8}$$

Profits at time t for firm i are:

$$\Pi_{it} = P_{it} Y_{it} - W_t N_{it} - R_t K_{it} - \text{cost of changing price scheme.} \tag{9}$$

Our assumption that the production function is homogenous of degree one implies that every monopolist faces the same marginal cost of production, equal to $\kappa W_t^{\alpha} R_t^{1-\alpha}$, where $\kappa = A^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. Intermediate producers set prices in the way described below, and meet demand at these prices. The assumption of constant elasticity of substitution in demand implies that, in a flexible price world, each monopolist would set its price equal to a fixed markup over the marginal cost.

Adjusting the price scheme is costly. As in Dotsey et al. (1999), we assume that each period a firm draws a random labor cost ξ. Costs of changing the price scheme are either ξW_t in case the firm decides to change the price scheme, and zero otherwise. The cost ξ is i.i.d. across time and firms, with c.d.f. $G(\xi)$ that has support $[0, \bar{\xi}]$. Two particular functional forms of $G(.)$ that have received considerable interest in the literature are:

$$\text{Time-dependent (Calvo) Pricing: } \xi = \begin{cases} 0 \text{ with probability } q \\ \infty \text{ with probability } 1-q \end{cases}$$

$$\text{(S,s) Pricing: } \xi = \bar{\xi} \text{ with probability } 1 \,.$$

We study two particular environments regarding the nature of these adjustment costs. In the first one, denoted as *sticky prices,* the period's fixed cost is incurred in order to change the current price. This is the menu cost interpretation that has prevailed in the literature. In the second environment, which we denote as *sticky plans,* the period's fixed cost is incurred in order to choose a time-varying path of predetermined prices rather than a single fixed price. The decision to change the plan is state contingent. Prices in the plan can be made conditional on the current information set, but cannot be made contingent upon future aggregate variables. This pricing behavior is consistent with the fact that costs of changing prices are broader than the menu costs that have prevailed in the literature. They also include other costs associated with implementing a new pricing plan such as communication and negotiation costs, as documented by Zbaracki et al. (2004). While in reality there is a mixture of firms that choose fixed price plans and time-varying price plans, in this paper we study the aggregate implications of two extreme settings where all firms choose a fixed price plan or all firms choose a time-varying plan.

We make the nominal variables stationary by normalizing prices with respect to the current money supply. We denote by lower case letters those nominal variables that have been rescaled by the current money supply. For example, $p_t^C = P_t^C / M_t$.

Recursive formulation of a firm's decision problem

For tractability and computational purposes we present the recursive formulation of a firm's decision problem. The aggregate state of the economy is $s = (\theta, \mu)$. μ denotes the growth rate of money, evolving according to the Markovian transition function $P(\mu, \mu')$. θ denotes the cumulative distribution of firms over the rescaled price p.[9] Let S be the state space that contains all possible realizations of s. The law of motion of s is given by the transition function Q^A. We first describe the firm's problem in the sticky price environment. We then discuss the changes to this problem in the sticky plans environment.

Firm's decision problem with sticky prices

Let $v(p, \xi; s)$ be the real value of a firm with rescaled price p that has drawn a cost of changing price ξ, when the aggregate state is s. Let $v^0(s)$ denote the real value of a firm that changes its price when the aggregate state is s, excluding costs of price adjustment. Then $v(p, \xi; s)$ and $v^0(s)$ must satisfy the following Bellman equations:

$$v(p, \xi; s) = \max \left\{ \pi(p; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') \, v(p/\mu', \xi'; s') \, G(d\xi') \, Q^A(s, ds'), \; v^0(s) - \xi w(s) \right\} \tag{10}$$

and

$$v^0(s) = \max_{p^*} \left[\pi(p^*; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') \, v\left(\frac{p^*}{\mu'}, \xi'; s'\right) G(d\xi') \, Q^A(s, ds') \right],$$

[9] Note that Dotsey et al. (1999) define the distribution of firms θ over the number of periods since the last price adjustment, rather than on the rescaled price p. Their approach requires a lower number of possible states for a firm (i.e: the maximum number of periods before adjustment occurs with probability one), and this simplifies their log-linearization procedure. Our approach, besides providing hazard functions with which the model's dynamics can be easily analyzed, also allows us to solve for the case with very low steady state inflation.

where

$$\begin{aligned} \pi(p;s) &= \max_{n,k}\ py - w(s)n - r(s)k \text{ , subject to} \\ An^{\alpha}k^{1-\alpha} &= \left(\frac{p}{p^{C}(s)}\right)^{-\varepsilon} Y(s) \end{aligned}$$

Functions $w(s)$ and $r(s)$ denote the rescaled factor prices, $p^{C}(s)$ denotes the rescaled aggregate price of consumption, and $d(s,s')$ denotes the discount factor with respect to future state s', all as a function of the current aggregate state. Firms take these functions as given. The choice of n and k are functions of the rescaled price and the aggregate state s, but not of the ξ realization. Therefore, we can define the functions $n(p;s)$ and $k(p;s)$ that solve the static maximization in (10). In addition, the optimal price is independent of the initial p and the ξ realization, so we can define the function $p^{*}(s)$. This price is a fixed markup over expected real marginal costs, weighted by the current and future levels of demand and the probability of price adjustment in future periods.

It is simple to see that $v(p,\xi;s)$ is decreasing in ξ. For firms with rescaled price p, there is a threshold function $\hat{\xi}(p;s)$ such that only those that draw a cost, ξ, lower than $\hat{\xi}(p;s)$ change their price. This threshold function is defined as follows:

$$v^{0}(s) - \hat{\xi}(p;s)\, w(s) = v\left(p, \hat{\xi}(p;s); s\right).$$

It is also useful to define the hazard function for price changes. $h(p;s)$ is the fraction of firms with a rescaled price p that adjust their price when the aggregate state is s. The hazard function is defined as:[10]

$$h(p;s) = G\left(\hat{\xi}(p;s)\right)$$

Finally, we define the function $n_M(p;s)$ to denote labor used in changing prices by p firms when the aggregate state is s:

$$n_M(p;s) = \int_0^{\hat{\xi}(p;s)} \xi dG(\xi).$$

[10] Two standard hazard functions in the literature are: time-dependent (Calvo) pricing, $h(p;s) = \bar{h}$, and (S,s) pricing, $h(p;s) = 0$ if $p \in (\bar{s}, \bar{S})$, and $= 1$ otherwise.

In equilibrium, the transition function Q^A is defined by the law of motion for μ from the transition function $P(\mu, \mu')$, and by $\theta'(p) = \omega(p\mu')$, where:

$$\omega(p) = \begin{cases} [1 - h(p; s)]\,\theta(p) \text{ , for all } p \neq p^*(s) \\ \\ \theta(p^*) + \int_0^\infty h(z; s)\,\theta(dz) \text{ , for } p = p^*(s) \end{cases} .$$

Firm's problem with sticky plans

The state of a firm at the beginning of period t is x, where $x = \{x_j\}_{j=0}^{\infty}$ is a plan of nominal prices for each future period j normalized by the current money supply. We assume x_j is not state contingent, so it cannot be indexed to aggregate variables such as the money supply. We can think of sticky prices as a specific case in which prices in the plan are restricted to be constant. We define x' as the continuation of plan x from next period onwards. x_0 is the first element of x. The aggregate state of the economy is $s = \left(\tilde{\theta}, \mu\right)$, where $\tilde{\theta}$ denotes the cumulative distribution of firms over plan x. The firm's problem is:

$$\begin{aligned} v(x, \xi; s) &= \max \left\{ \pi(x_0; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds')\, v\left(x'/\mu', \xi'; s'\right) G\left(d\xi'\right) Q^A(s, ds'), \right. \qquad (11) \\ &\qquad \left. v^0(s) - \xi w(s) \right\} \end{aligned}$$

$$v^0(s) = \max_{x^*} \left[\pi(x_0^*; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds')\, v(x'^*/\mu', \xi'; s') G\left(d\xi'\right) Q^A(s, ds') \right]$$

and

$$\begin{aligned} \pi(x_0; s) &= \max_{n,k}\ x_0 y - w(s)\, n - r(s)\, k \text{ , subject to} \\ y &= A n^{\alpha} k^{1-\alpha} \\ y &= \left(\frac{x_0}{p^C(s)} \right)^{-\varepsilon} Y(s). \end{aligned}$$

The aggregate price level is defined as:

$$p^C(s) = \left[\int (x_0)^{1-\varepsilon}\, \omega(dx) \right]^{\frac{1}{1-\varepsilon}} .$$

The transition function Q^A is defined by the law of motion for μ, and by $\tilde{\theta}'(x') = \omega\,(x\mu')\ \forall\ x_0$, where $\omega(x)$ is the analogous of $\omega(p)$ in the sticky prices environment.

Equilibrium and monetary experiments

A full description of the recursive competitive equilibrium is in an appendix available upon request. In equilibrium, labor demand (employment from production and costs of changing prices) is equal to the representative household's labor supply, and the household's consumption is equal to final good production. In addition, the law of motion of the aggregate state has to be consistent with firms' individual decisions.

We assume that the law of motion for μ_t is:

$$\mu_t = (1-\rho)\,\mu_S + \rho\mu_{t-1} + \nu_t.$$

For computational reasons, we consider one-time, unanticipated, changes in ν_t. The economy is initially in a non-stochastic steady state with money growing at the constant rate μ_S. At $t = 0$, there is an unanticipated change in the growth rate of money (i.e: $\nu_0 \neq 0$). After this shock, agents know with perfect foresight the realizations of the future money path ($\nu_t = 0$ for $t > 0$). We study temporary ($\rho < 1$), and permanent ($\rho = 1$) changes in the growth rate of money.

3 Parameter values and steady state

We now discuss how values were assigned to the model's parameters. The length of the period is a quarter. The discount factor is set equal to 0.984, as Dotsey et al. (1999). The function $u(C, N)$ takes the form proposed in Greenwood, Hercowitz, and Huffman (1988):

$$u(C, N) = \log\left(C - \frac{\phi_0}{1+\phi} N^{1+\phi}\right).$$

In order for the sticky price model to have the best chance of generating an inertial response of inflation to a money growth shock, real marginal costs should be fairly insensitive to fluctuations in output. In our framework, this can be achieved by choosing a high labor supply elasticity and a relatively low share of capital in production. Concretely, ϕ is chosen so as to imply an infinite Frisch labor supply elasticity as in Hansen (1985), and α is set to 0.8. Even with our extreme specification, the sticky price model is unable to generate a hump-shaped response of inflation. The simple specification is a first step in the direction of a richer model that includes additional elements such as investment, variable capital utilization, and sticky wages, which can dampen movements in real marginal costs.

The parameter ϕ_0 is calibrated so that, conditional on the assigned values for the other parameters, agents work 25% of their time endowment. The capital stock is normalized to unity. The elasticity of substitution between intermediate goods is chosen so that the implied flexible price markup is 20%, as suggested by Hornstein (1993). In the benchmark model, the steady state quarterly growth rate of money is set equal to 1.5%. This is consistent with the average growth rate of money in the post-war U.S. economy. We also consider the case where the steady state money growth rate is equal to zero. In our baseline model, we set $\rho = 0$. We also consider cases where $\rho = 0.5$, and $\rho = 1$.

Our experiments suggest that the dynamics of the response of output and inflation to monetary shocks are sensitive to the shape of $G(.)$. In this paper we use the beta distribution, which is very flexible as a function of only two parameters.[11] In order to make our analysis comparable to Dotsey et al. (1999), our benchmark model uses $\gamma_1 = 0.3$ and $\gamma_2 = 0.1$, so that the implied fixed-costs distribution is similar to the assumed in that paper.[12] This specification is consistent with microeconomic data on price adjustment that suggests that hazard rates are an increasing function of $|p - p^*|$, as estimated by Caballero and Engel (1993) and Willis (2000). The parameter $\bar{\xi}$ is chosen so that in the steady state of the sticky price economy, prices are on average 3 quarters old (the average hazard rate is 0.23). This implies that the expected labor costs of price adjustment for an individual firm are 4.5% of its average steady state employment level. Weighted by the

[11] I want to thank Alexander Wolman for suggesting this family of cost distributions.

[12] As explained in Dotsey et. al. (1999), a uniform distribution (i.e: $\gamma_1 = \gamma_2 = 1$) would generate more variation in the average hazard in response to small changes in the growth rate of money.

fraction of firms changing their price, total labor used in changing pricing schemes is only 0.35% of total employment. This corresponds to 0.25% of average revenues. This is smaller than Levy, Bergen, Dutta, and Venable (1997) and Dutta, Bergen, Levy, and Venable's (1999) estimates of the physical costs of price adjustment, which average 0.70% of store revenues in their supermarket and drugstore samples. In the steady state of the sticky plans model, the fraction of firms changing their pricing scheme is zero. Our parametrization of $G(.)$ in this environment is such that, under zero steady state money growth, the response of the average hazard rate to a 1% temporary increase in the growth rate of money is the same as in the sticky price environment.

The baseline parameter values are summarized in Table 1. In order to solve the model, we use an iterative non-linear method described in the appendix.

Steady state

Figure 1, Panel A, displays the steady state hazard function $h\left(p;s^{SS}\right)$ and price distribution $\theta\left(p\right)$ of the sticky price economy, in our benchmark calibration with positive money growth rate. The hazard function is V shaped around the target price. This is due to the fact that firms dislike real prices that are too low or too high relative to the target price, and are thus more willing to pay higher costs when the gap between the actual and target price is higher. Very low or very high prices are associated with hazard rates equal to 1.

In our benchmark calibration, there are 7 vintages of firms in the steady state: every firm changes its price no more than 7 periods after the previous price change. In the figure, firms that have changed their price 7 periods ago are in the section of the hazard function where the probability of adjustment is equal to 1. The resulting average hazard rate is 0.23, and the average price is 3 quarters old. Note that $\theta\left(p\right)$ is decreasing for $p \leq p^{*}$ because the hazard function is positive. If there was steady state deflation, then $\theta(p)$ would lie to the right of p^{*}. The effects of an increase in the money supply are a leftward shift in $\theta\left(p\right)$ because $p = P/M$ falls for any given P, and a change in p^{*}.

Panel B displays the steady state hazard function and price distribution when steady state money growth rate is zero. In this case, all firms have price equal to p^* (i.e: $\theta(p^*) = 1$). Moreover, the hazard function becomes steeper. Under higher inflation, firms are more willing to let their real price erode before re-adjusting the price.

The hazard function is asymmetric around the target price. This can be more clearly seen in Panel C, which assumes a higher elasticity of substitution across intermediate goods such that the flexible price markup is 3%. This asymmetry arises from the global asymmetry of the profit function. To understand this, suppose an extreme case in which the elasticity of substitution between intermediate goods is high enough so that the average markup is close to zero and $p^* \simeq mc$. Given the demand function arising from the CES aggregator, a firm with $p < p^*$ would have profits tending to $-\infty$, so it would change its price with probability one. Now take a firm with $p > p^*$. Its profits equal zero if it doesn't change the price. Given that profits at $p = p^*$ are also zero, it will not change its price unless $\xi = 0$.

Finally, Panel D illustrates that the shape of the hazard function depends on the underlying fixed cost distribution $G(.)$. This will affect the response of output and inflation to aggregate shocks.

4 Changes in the money growth rate

In this section we study the response of the model to temporary and permanent changes in the growth rate of money. For each experiment, we discuss the differences between the sticky price and the sticky plan models, and we assess their potential to account qualitatively for the conventional view about the response of actual economies to these monetary experiments.

Experiment 1: Temporary increase in the money growth rate

Sticky prices

Figure 2 displays the response of inflation, output, and the average hazard rate in the sticky price model to a temporary 1% increase in the money growth rate (i.e: $\rho = 0$ and $\nu_0 = 0.01$). The solid lines refer to the model where the timing of pricing decisions is endogenous (the state-dependent pricing model). The broken lines refer to the time-dependent (Calvo) pricing model, where the hazard rate is exogenous and equal to the steady state average hazard rate (i.e. $h(p) = 0.23\ \forall p$). Under state-dependent pricing, the fraction of firms adjusting their price increases at $t = 0$ from 23% to roughly 25%. This implies that, relative to time-dependent pricing, the state-dependent pricing model generates a larger contemporaneous increase in inflation (0.2% versus 0.1%) and a smaller response in output (0.8% versus 0.9). Moreover, in the state-dependent pricing model the bunching of price changes at $t = 0$ generates cycles in output and inflation that vanish as the economy converges back to the steady state. We also observe that the sticky price model does not produce a long-lived hump-shape in the response of inflation.

Figure 3 displays the differential effects at $t = 0$ on inflation, output, and the average hazard rate (y axis) of changes in the growth rate of money of different sizes (x axis). Panels A, B, and C correspond to the sticky price environment. Solid and dashed lines trace the behavior of these variables under the state and time-dependent pricing models, respectively. For small changes in the growth rate of money, the response of output is similar across both models. This is not the case for large changes in the growth rate of money. In the time-dependent pricing model, the response of output is roughly proportional to the size of the change in the growth rate of money. In the state-dependent pricing model, as the size of the change in the growth rate of money gets larger, the fraction of firms adjusting prices increases and so does the implied inflation rate. Consequently, the increase in output becomes smaller relative to the size of the change in the money growth rate. The maximum increase in output corresponds to a 3% increase in the growth rate of money. Furthermore, the response of output is negative for changes larger than 5%. Adjusting firms start with low prices, so inflation can be larger than the increase in the money stock. Real money balances fall, and so does output.[13] This illustrates one of the non-linearities of this model: as the size of the change in the growth rate of money increases, the relative response is higher for inflation and lower for output.[14]

Another source of non-linearities is the asymmetric response of the economy to monetary expansions and contractions. The rise in output after a monetary expansion is smaller than the fall in output after a monetary contraction. This can be observed in figure 3. For example, a decline of 3% in the growth rate of money reduces output by 2.2% . An increase of 3% in the growth rate of money increases output by only 1.17%. There are two sources of asymmetries. First, the hazard function is asymmetric around p^*. This is because, as discussed above, the profit function of a firm is asymmetric. Second, under positive steady state inflation, firms' prices are lower than or equal to p^*. The distribution $\theta(p)$ is in the section of the hazard function that is decreasing in p. A monetary expansion implies a leftward shift in $\theta(p)$, so the average hazard rate increases. A monetary contraction implies a rightward shift in $\theta(p)$, so the average hazard rate declines if the money contraction is small. Many firms do not pay the fixed cost to reduce their price because the positive inflation rate erodes it costlessly.

Sticky plans

Figure 4 compares the state-dependent sticky price (solid lines) and sticky plan (broken lines) models under zero steady state inflation. In this case, steady state allocations are identical in both models. The hump in inflation is more pronounced under sticky plans, with inflation now reaching a peak 12 periods after the shock.

[13] This effect does not take place when the steady state inflation rate is 0%, where large changes in the growth rate of money have a minimal effect on output.

[14] Larger shocks also reduce the persistence of output to the shock. This is consistent with Kiley (2000) and Fischer, Sahay, and Vegh (2002), who present evidence that the autocorrelation of output is lower in high inflation countries. In order to make this link in our model, note that the mean and standard deviation of inflation are highly correlated in the data.

The key difference is that under sticky plans, incentives to front-load prices are reduced relative to the sticky price model. Firms expect the marginal cost to increase over time, and they can plan future price increases in advance without actually having to set today these higher prices. The initial increase in nominal marginal cost is roughly equivalent in both economies (Panel E). In the sticky plan case, firms paying the fixed cost change their current price one to one with changes in the nominal marginal cost. Conversely, the initial increase in P^* under sticky prices is larger than the increase in nominal marginal cost (Panel F). The hump-shaped response of inflation in the sticky plans model is driven by the fact that firms changing their plan in later periods increase their price by the accumulated increase in the marginal cost, which is larger than the initial rise in the marginal cost. This effect is partly offset by the fact that fewer firms adjust their plan in later periods. The two key conditions that produce the hump shape in inflation are that the initial increase in the nominal marginal cost is not too large (this requires that real marginal costs are fairly insensitive to output), and that the fraction of firms changing their plans in early periods is not too large.

Figure 5 displays the response of the sticky price and sticky plan models under the assumption that the growth rate of money follows an AR1 process, where $\rho = 0.5$ (as in Christiano et al. 2005). Under this assumption, the sticky plans model generates a hump-shaped response in both inflation and output.

In the sticky plans economy, the initial response of output and inflation is non-linear with respect to changes in the money supply. This can be seen in Panels D, E, and F in Figure 3. For expansions in the money growth rate smaller than 6.4%, the initial response of output and inflation in the state-dependent model is very well approximated by a time-dependent model. Once the increase in the growth rate of money is larger than 6.4%, every firm adjusts its plan, prices increase by the magnitude of the monetary expansion, and so real allocations are almost unaffected (there is a small increase in labor used to change price plans). The maximum increase in output corresponds to a 6% increase in the money growth rate.

Moreover, as in the sticky price model, monetary expansions and contractions have asymmetric effects on output and inflation. For example, the decline in the growth rate of money has to be larger than 7.5% (versus 6.4% for money expansions) in order for all firms to adjust their plans.

Finally, the effects of a change in the growth rate of money of a given size will differ according to the history of previous monetary shocks. For example, suppose that after a 1% unanticipated monetary expansion at $t = 0$, we conduct another 1% unexpected increase in the growth rate of money at $t = 1$. In the time-dependent pricing model, the change in output and inflation is roughly equal at $t = 0$ and $t = 1$. This is because the fraction of firms adjusting their plan remains constant. In the state-dependent model the changes in inflation and output are 0.023% and 0.98% at $t = 0$, and 0.092% and 0.91% at $t = 1$, respectively. The response of output is smaller after a 1% increase in the money growth rate that follows a previous change in the money growth rate of the same size. This is due to the positively sloped hazard functions, which induce an increase in the mass of adjusting firms from 8.3% at $t = 0$ to 10.6% at $t = 1$. This illustrates another source of non-linearities in the model.[15]

Experiment 2: Permanent credible disinflation

Sticky prices

The solid lines in figure 6 display the response of various variables to a permanent unexpected decline in the money growth rate from 1.5% to 0% ($\rho = 1$ and $\nu_0 = -0.015$). The rate of inflation falls abruptly at $t = 0$, and output increases permanently by 0.7%. This result is at odds with the conventional wisdom that small disinflations are contractionary. This result is not surprising because the state-dependent sticky price model behaves similarly to the time-dependent pricing model for small changes in the growth rate of money. The inability of the latter models to generate inflation inertia has been extensively discussed in previous work (see for example Ball 1994). It is explained by the fact that although the price level is sticky, the rate of inflation is not. Firms

[15] This non-linearity has also been emphasized by Caplin and Leahy (1991), and Caballero and Engel (1999) in the context of investment dynamics.

changing their price increase it by a lower rate relative to the initial steady state, because they anticipate that their price could be too high in the future if they draw high costs of adjustment. In fact, under our parametrization P^* falls by 4%, and this generates a reduction of 1.5% in the overall inflation rate. The increase in output across the two steady states is the result of a decrease in the average markup, a reduction in the dispersion of prices, and a reduction in the labor used in price adjustment as a fraction of total employment. The model displays a similar pattern for large disinflations (i.e.: $v_0 = -0.1$), as can be seen in Figure 7.

Sticky plans

Figure 6 show that the sticky plans model (broken lines) is capable of generating a contraction in output after a permanent reduction in quarterly money growth from 1.5% to 0%. This is because only 17% of the firms pay the fixed cost of plan adjustment to reduce the growth rate of their individual prices. The remaining mass of firms that do not change their plan continue increasing their prices at the old steady state's high inflation rate. In addition, front-loading incentives are reduced. This is because firms that do adjust can plan on gradually reducing the growth rate of future prices rather than immediately reducing the growth rate of their price.[16] The reduction of front loading incentives implies that inflation is above zero for the first 6 quarters after the policy implementation. The reduction in real money balances entails output losses that last 7 quarters.[17]

Things change when the reduction in money growth is larger. The broken lines in figure 7 show the response of the sticky plans economy when the quarterly money growth rate is permanently reduced from 10% to 0%. In this case, at $t = 0$, every firm pays the fixed cost and changes its pricing plan to the lower rate of money growth. Therefore, output is almost unaffected by this

[16] We can neutralize the first effect (i.e: firms that do not change their plan increase their individual prices by the pre-disinflation rate of inflation) by starting in a 0% inflation steady state. In this case, it is still true that the sticky plans model generates inflation inertia, and the sticky price model does not.

[17] In addition, contrary to the implication of the sticky price model, pre-announced credible disinflations do not generate short term booms in output in the sticky plan model.

policy change (there is a small increase in labor used in changing price plans). This is consistent with the conventional wisdom that suggests that large inflations can be stopped at low output costs.

5 Concluding remarks

This paper studies a dynamic general equilibrium model in which firms face a fixed cost of changing their pricing plan. We argue that the model's predictions are consistent with the conventional wisdom about temporary and permanent changes in monetary policy that have been widely studied in the literature, as well as the notion that changes in the growth rate of money that are large enough have relatively smaller effects on output.

In the context of our monetary experiments, we see that time-dependent models are a good approximation of state-dependent models in environments where the size of the change in the growth rate of money is not very large. While changes in the money growth rate have been small in the U.S. in the post-war period, monetary shocks tend to be larger in the context of developing countries. For example, in developing countries nominal exchange rates are very volatile. Time-dependent sticky price models, often used in international economics, appear ill suited to study environments subject to large exchange rate fluctuations. An open economy version of the model studied in this paper has the potential to explain the response of various prices to both small and large exchange rate fluctuations.

It would also be interesting to use non-linear solution methods to study the stochastic equilibrium for this economy. As we saw above, the effects of a monetary shock of a given size will differ according to the history of previous shocks. This mechanism might reinforce the role that non-linearities have in explaining US inflation and output dynamics.

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Appendix: Numerical Algorithm[18]

Given the nature of our policy experiments, it seems natural to think of the numerical solution method in terms of sequences of objects rather than time independent functions. We now outline the algorithm we use to solve the sticky price model. We use a very similar algorithm to solve the sticky plans model.[19]

1. Solve for the initial steady state and the new steady state of the economy after the shock (which can be different from the initial one in the case of permanent shocks). This involves solving a system of non-linear equations.

2. Compute $v\left(p, \xi; s^{SS}\right)$ and $h\left(p, s^{SS}\right)$, where s^{SS} denotes the aggregate state in the steady state.

3. Guess that the economy is in the new steady state at time T.

4. Guess an initial sequence for w_t, mc_t and Y_t:

$$\{w_t, mc_t, Y_t\}_{t=0}^{T}$$

   The initial steady state is usually a good initial guess.

5. Using the aggregate variables guessed in (4), compute optimal price rules, hazard functions and factor demand functions between $t = 1$ and $t = T - 1$. This requires solving for value functions in problem (10), which can be well approximated using splines. Given that we know the value functions at $t = T$, we start from $t = T - 1$ and we move backwards up to $t = 1$.

6. Using these optimal firm responses, compute aggregate quantities and equilibrium prices from the consumer's maximization problem and market clearing conditions.

[18] This method was developed jointly with Ivan Werning.

[19] In order to check the solution method, we also solve the model using a linearization algorithm similar to the one used by Dotsey et. al. (1999). For small shocks, the outcome of both techniques are almost equivalent. However, once the size of the shock increases, the linearization algorithm runs into the problem of occasionally binding constraints. This is when firms from older vintages want to change their price with probability one.

7. If the old and new aggregates are not the same, we re-start (4) with updated aggregate quantities and prices which are computed as a function of the old and new ones (usually with some weighting scheme).

8. Check that the economy has converged to the new steady state before period T. If not, choose a new T and re-start from (2).

Table 1

Parameter Values in

Baseline Calibration

β	0.984
ϕ	0.01
ϕ_0	0.89
ε	6
α	0.8
μ_S	0.015 and 0
ρ	0
$\bar{\xi}$	0.015
γ_1	0.3
γ_2	0.1

Figure 1: Hazard Function and Price Distribution, Sticky Prices

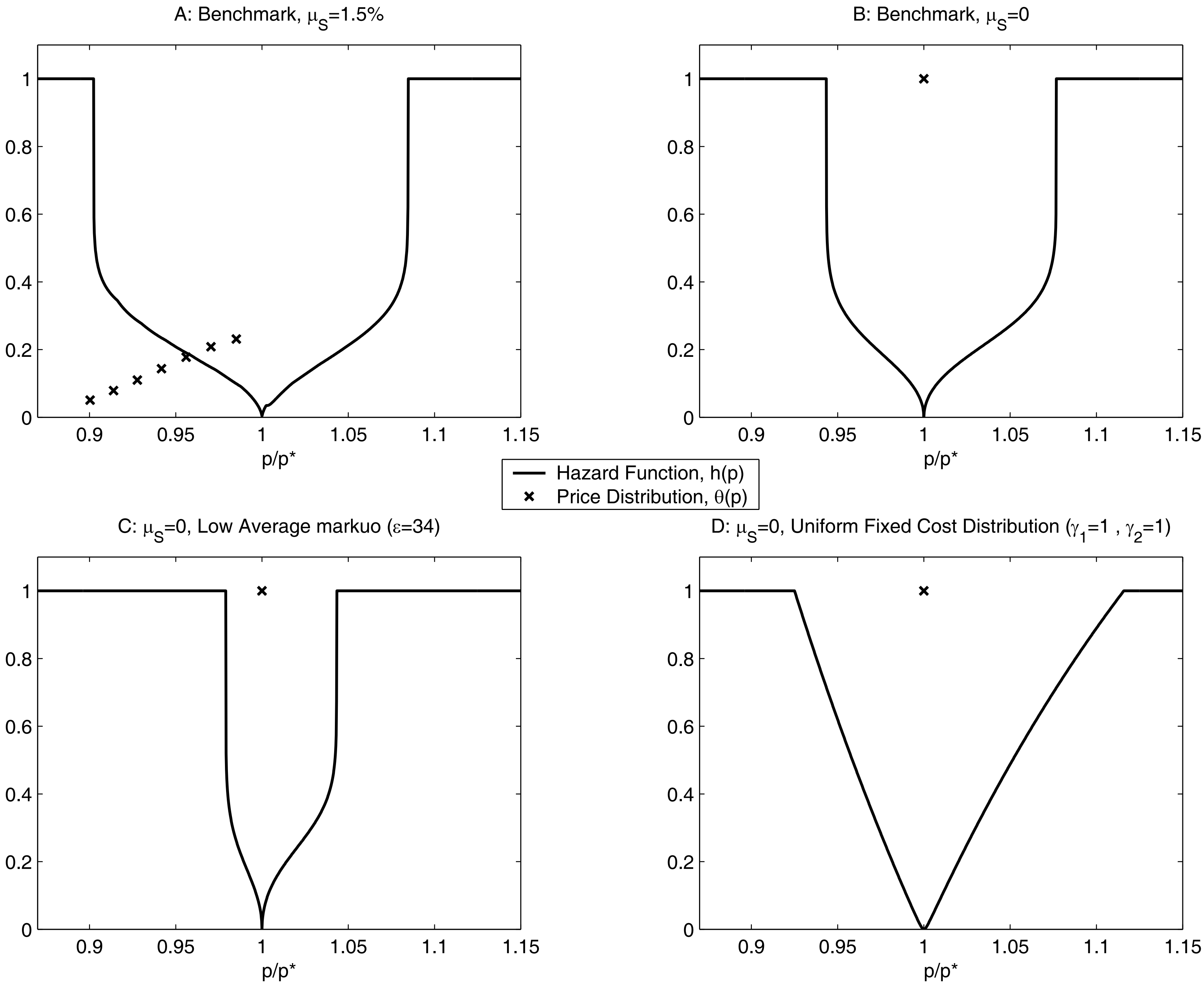

Figure 2: Temporary 1% Increase in Money Growth Rate, Sticky Prices, μ_S=1.5%
State-Dependent (-) and Time-Dependent (--)

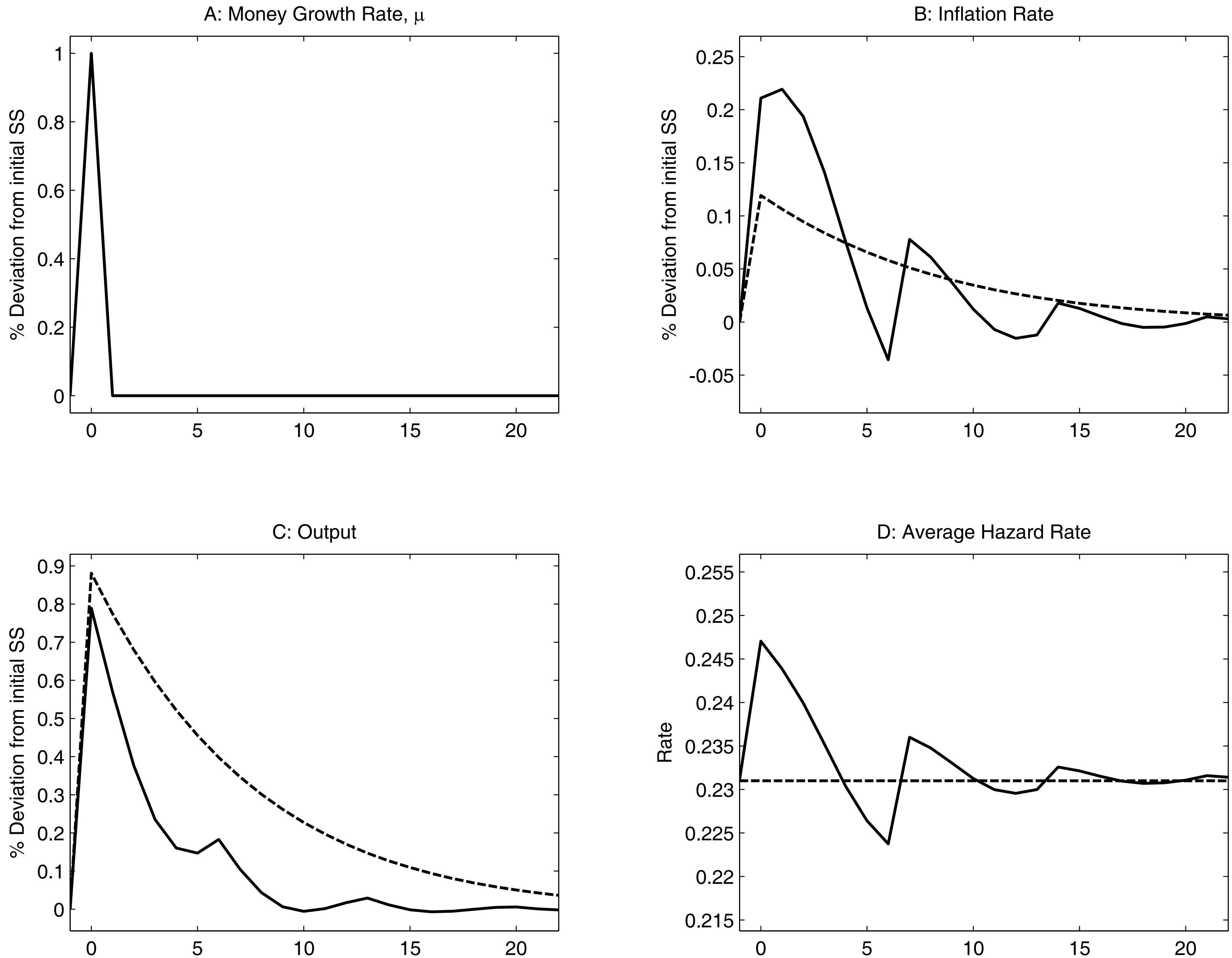

Figure 3: Contemporaneous Impact for Different Size of Changes in Money Growth Rate

State-Dependent (-) and Time-Dependent (--)

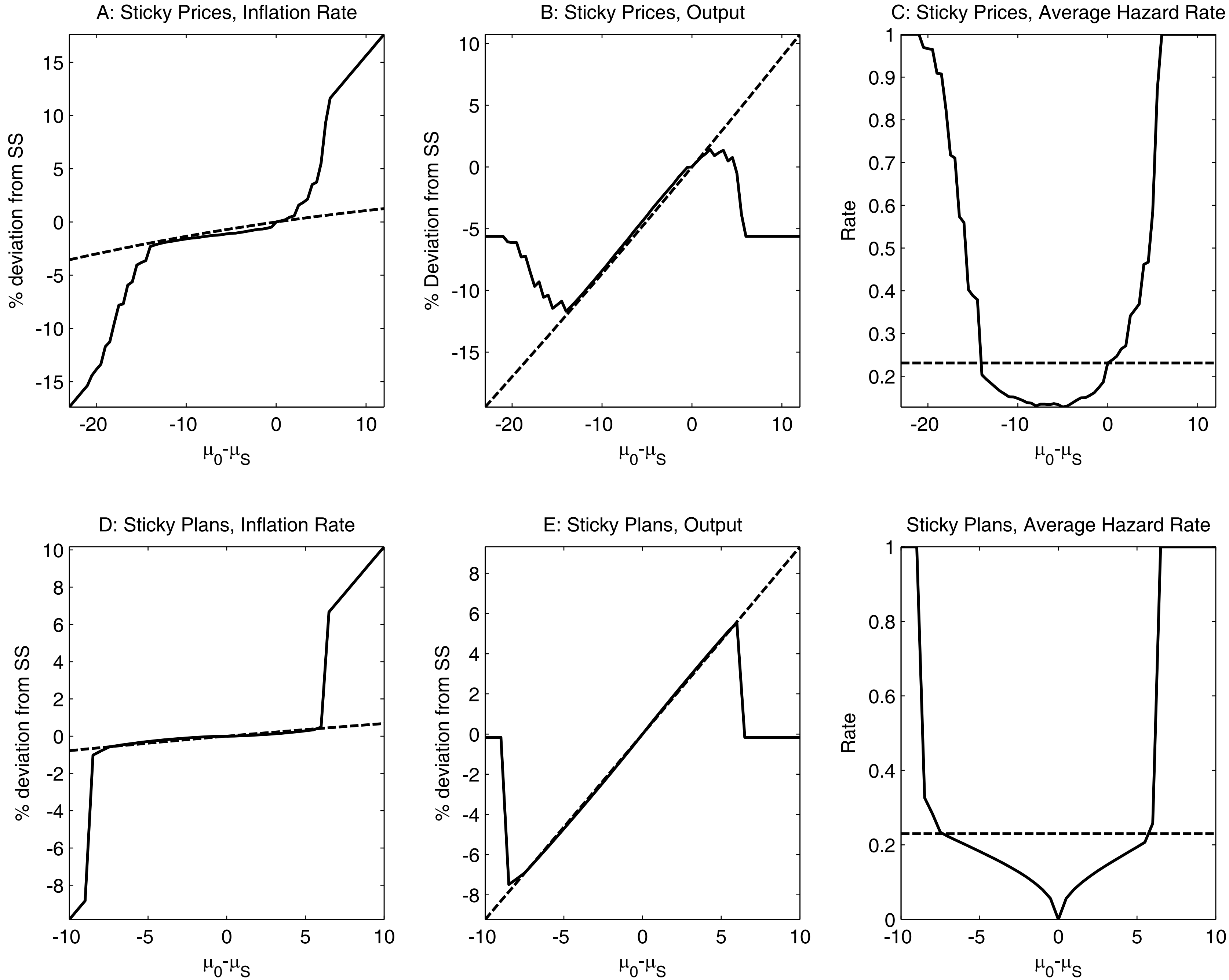

Figure 4: Temporary 1% Increase in Money Growth Rate, μ_S=0%

Sticky Prices (-) and Sticky Plans (--)

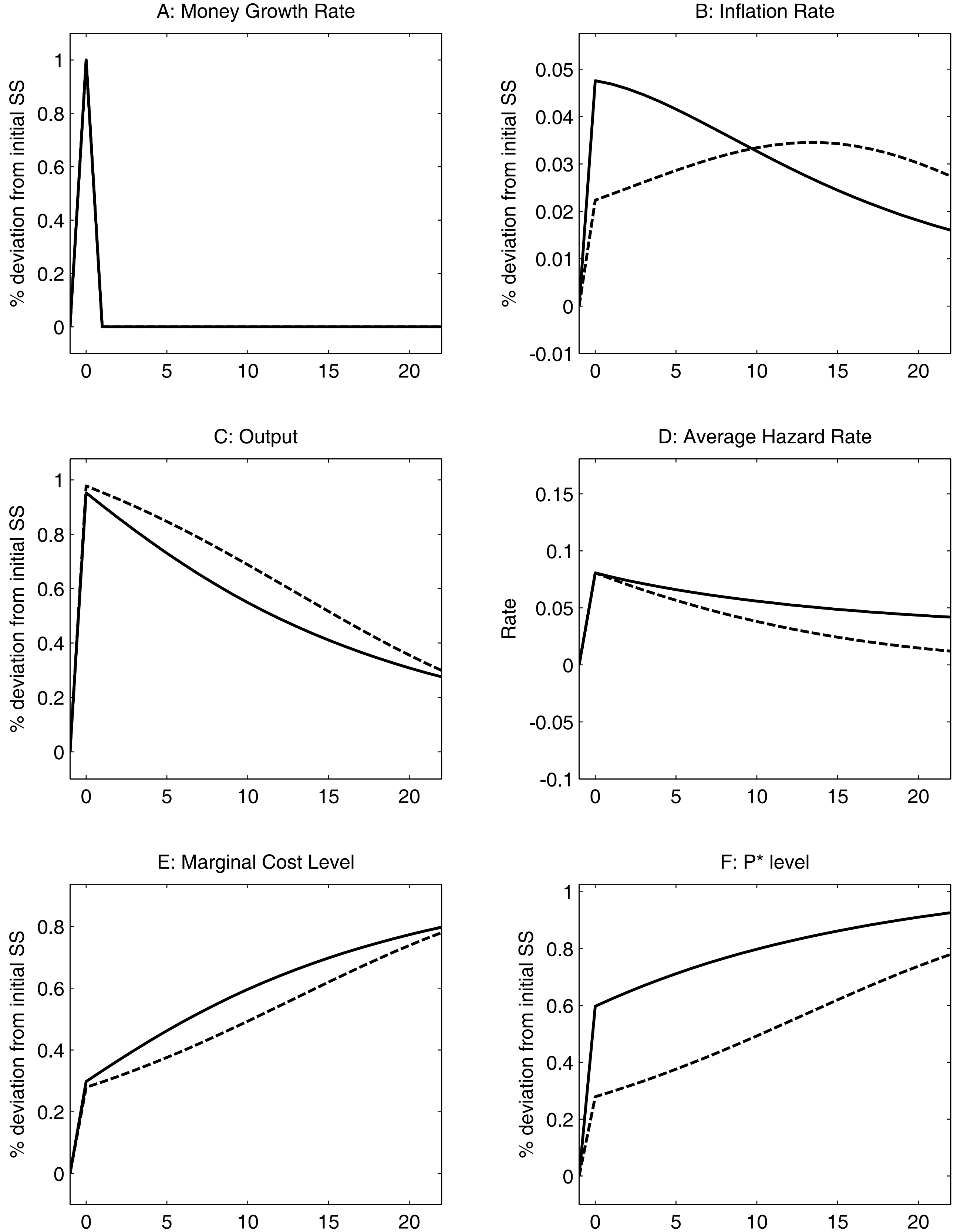

Figure 5: Temporary, AR1, 1% Increase in Money Growth Rate, μ_S=0%

Sticky Prices (-) and Sticky Plans (--)

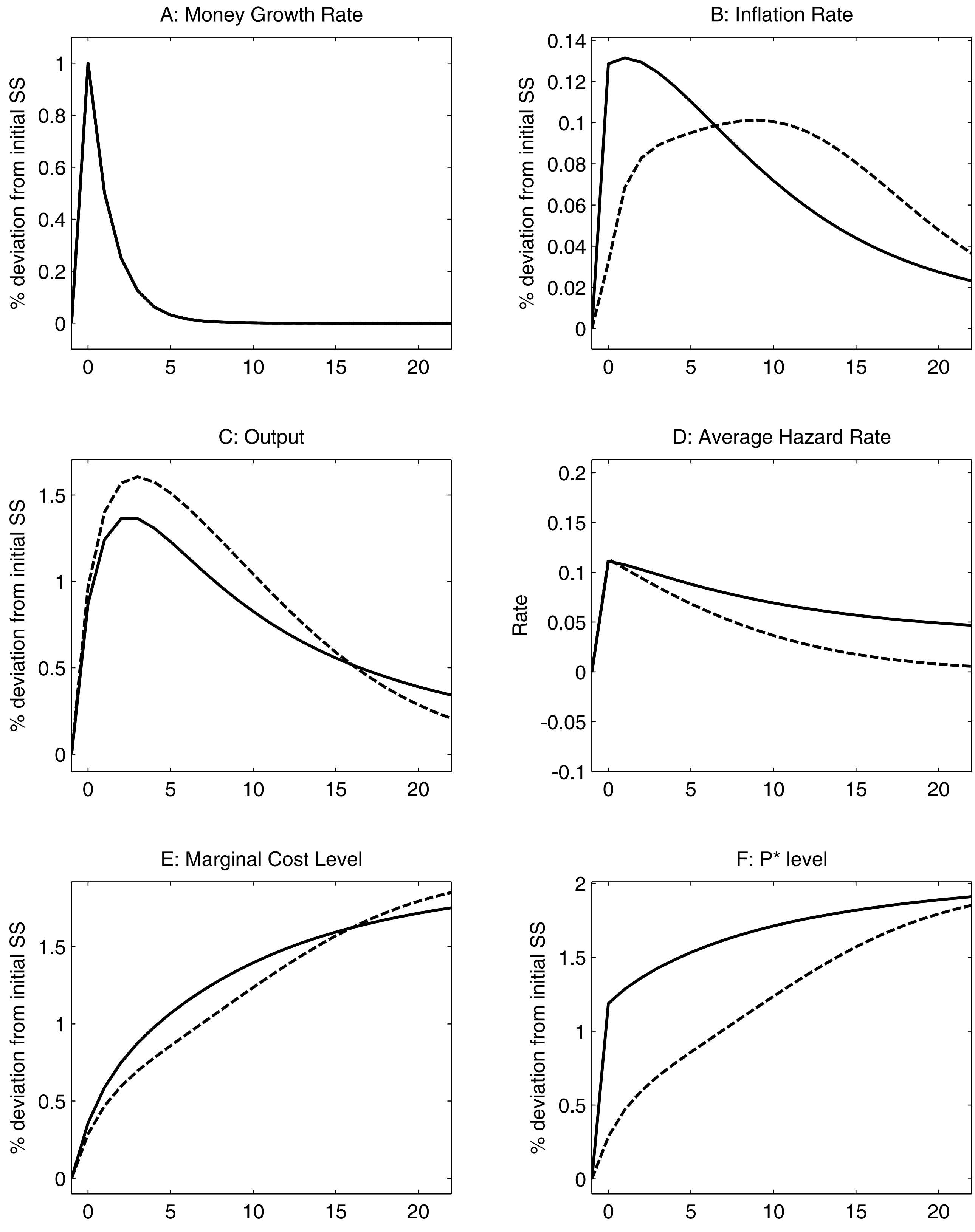

Figure 6: Permanent 1.5% Decline in Money Growth Rate, μ_S=1.5%

Sticky Prices (-) and Sticky Plans (--)

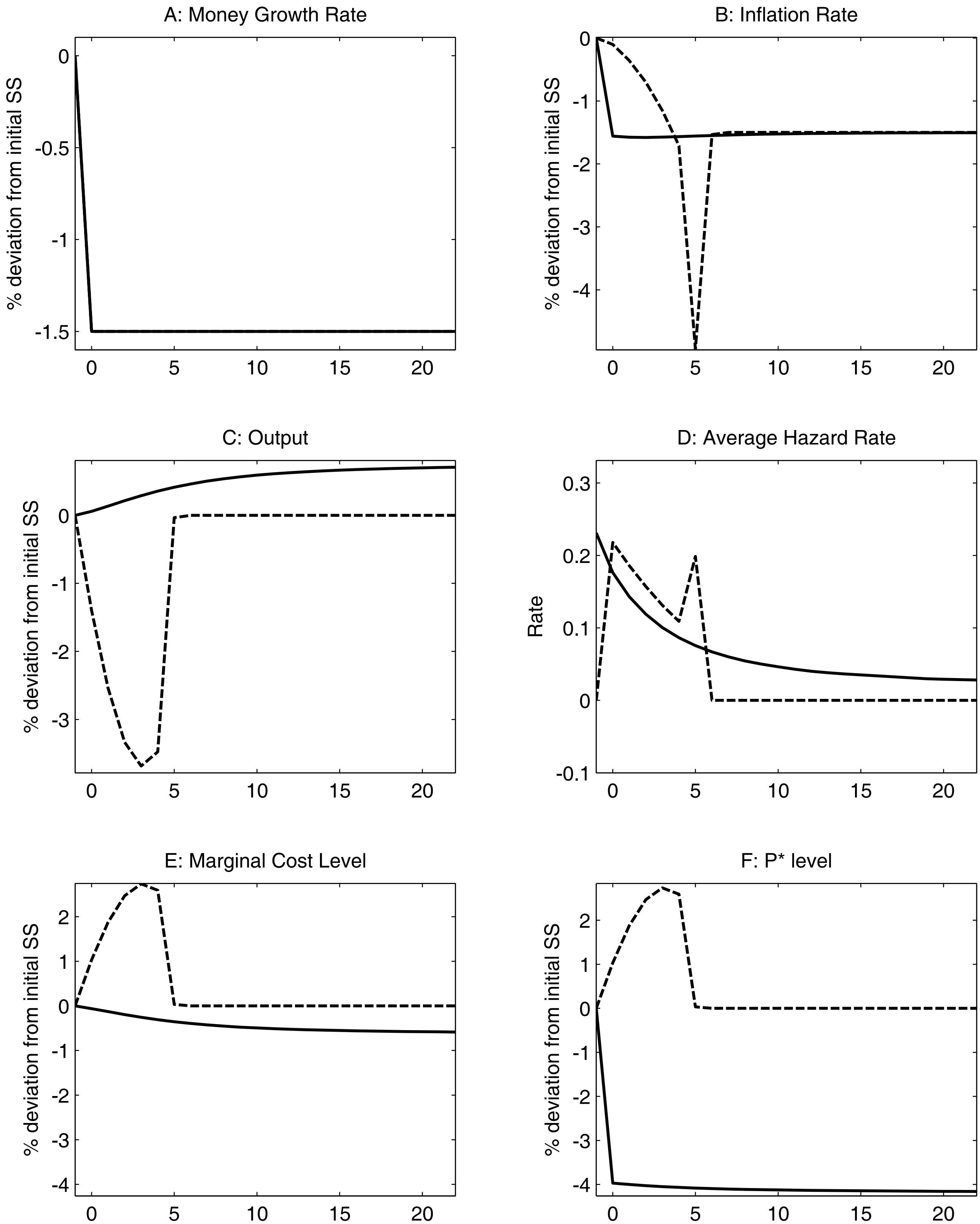

Figure 7: Permanent 10% Decline in Money Growth Rate, μ_S=10%

Sticky Prices (-) and Sticky Plans (--)