Bottom-up Markup Fluctuations*

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Abstract

We study markup cyclicality in a granular macroeconomic model with oligopolistic competition. We characterize the comovement of firm, sectoral, and economy-wide markups with sectoral and aggregate output following firm-level shocks. We then quantify the model's ability to reproduce salient features of the cyclical properties of markups in French administrative firm-level data, from the bottom (firm) level to the aggregate level. Our model helps rationalize various, seemingly conflicting, measures of markup cyclicality in the French data.

Keywords: Markup Cyclicality; Oligopolistic Competition; Firm Dynamics; Granularity; Aggregate Fluctuations

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Introduction

A long tradition in the business-cycle literature evaluates models on their ability to account for salient moments of business-cycle data, such as the relative volatility and correlation with GDP of key macroeconomic aggregates. Although there exists a broad consensus on moments concerning, for example, the behavior of consumption, investment, or unemployment over the business cycle, disagreement lingers, both in terms of theory and measurement, over the cyclical behaviour of markups (see, e.g., Bils et al., 2018 and Nekarda and Ramey, Forthcoming).[1]

In this paper, we re-examine this question, studying the cyclical properties of markups at the firm, sector, and aggregate level, both theoretically and empirically, based on French administrative data. We consider a model of oligopolistic competition in which granular firm-level shocks result not only in sector and economy-wide output changes, as in Gabaix (2011), but also in markup dynamics. We characterize the model's implications for comovement between output and markups, that is "markup cylicality", at various levels of disaggregation from the bottom (firm) level up to the sector and aggregate levels, and show how this comovement is mediated by market structure within and across sectors. We then assess the quantitative ability of our granular oligopolistic setting to reproduce salient measures of the cyclical properties of markups in the French data at the firm, sector, and aggregate levels.

To model in a tractable way the determination and aggregation of markups in an economy featuring a large but finite number of sectors with a discrete number of firms, we use a nested CES demand structure studied in Atkeson and Burstein (2008).[2] Firms compete under flexible prices, setting markups that are increasing in within-sector sales shares.[3] Firm-level shocks follow a random growth process that generates empirically plausible firm dynamics, firm-size distributions, and granular sectoral and aggregate fluctuations (Carvalho and Grassi 2019). The model yields predictions for the joint behavior of within-sector market shares, markups, and output following exogenous changes in firm-level shifters. Furthermore, the model's convenient equilibrium aggregation yields simple sectoral and aggregate counterparts to many of these firm-level objects.

[1] Other studies contributing to the active debate on the sign and magnitude of markup cyclicality include Bils (1987), Hall (1988), Anderson et al. (2018), and Stroebel and Vavra (2019). Additionally, a growing literature provides measures of lower-frequency trends in markups, such as De Loecker et al. (2020) and De Loecker and Eeckhout (2018).

[2] A similar framework has been used in a number of macro applications to quantify the welfare costs of market power (Edmond et al., 2018 and Berger et al., 2019), trends in market power (De Loecker et al., 2018), pro-competitive gains from trade (Edmond et al., 2015), exchange-rate pass-through (Amiti et al., 2019), and granularity in trade (Gaubert and Itskhoki, 2018). Other prominent work featuring fluctuations in market power in macroeconomic models include Gali (1994), Kimball (1995), Jaimovich and Floetotto (2008), and Bilbiie et al. (2012).

[3] Much of the literature on markup cyclicality is motivated by the implications of models with nominal rigidities (e.g., Rotemberg and Woodford (1999) for a comprehensive early survey), which depend on the nature of nominal rigidities (prices vs. wages) and on the source of aggregate shocks (e.g., monetary vs. productivity). By contrast, we assume prices are flexible, and focus on the implications for markup cyclicality of granular, firm-level shocks. See Mongey (2017) and Wang and Werning (2020) for recent analyses of money non-neutrality in an oligopolistic model like ours with price rigidities.

Our first theoretical contribution is to provide simple analytic expressions showing how the sign of markup cyclicality depends on the level of aggregation, the market structure within and across sectors, and the set of shocked firms.[4] We show sectoral output and markups comove positively in response to shocks to large firms in the sector, whereas they comove negatively in response to shocks to small firms. In turn, the effect of such shocks on the aggregate markup depends on the distribution of sector-level markups and sectoral expenditure shares. Under the additional assumption that shocks are uncorrelated across firms (such that large firms drive the cycle in each sector), we provide sufficient conditions for a positive asymptotic correlation between markups and output at the sectoral and aggregate levels.

Second, we compare, theoretically, the implications of our model to an alternative specification in which firm-level markups are heterogeneous but constant in response to shocks (i.e., complete pass-through) so that sectoral and aggregate markups only change due to between-firm reallocation and not within-firm markup changes. We show that, although within-firm markup changes account for half of sectoral markup fluctuations in the variable markup model, changes in sectoral and aggregate markups can be larger or smaller than in the constant markup model, depending on parameter values, because the extent of between-firm reallocation falls with incomplete pass-through. Additionally, we provide analytical formulas for sectoral and aggregate output volatility, which generalize those in Gabaix (2011) to an oligopolistic setting with variable markups, and show how the introduction of variable markups dampens granular aggregate volatility, by acting in a similar way to a decline in the Herfindahl index. Intuitively, when pass-through rates are lower for larger firms, the weight of large firm shocks in the price index is effectively reduced relative to constant markup models.

Our quantitative analysis is based on French administrative firm-level data over a 23-year period (1994-2016) covering approximately 500,000 firms per year. We use this firm census data to compute empirical distributions of firm market shares, sectoral output and concentration, and aggregate output over our annual sample. Further, we use balance-sheet information to estimate firm-level markups – following the methodologies introduced in De Loecker and Warzynski (2012) and De Loecker et al. (2016) – which we again also aggregate at the sectoral and aggregate levels.[5] We employ a rich set of empirical moments at the firm, sector, and aggregate levels both to calibrate our model and to assess its quantitative implications for (non-targeted) markup cyclicality.

We start by analyzing a basic mechanism in our oligopolistic setting: within a narrowly de-

[4] Grassi (2018) studies the role of input-output linkages and endogenous markups in shaping comovement of sector-level variables, providing an analytical characterization of the impact of microeconomic shocks on aggregate output using an approximation with respect to the deep parameters of the model. Our analytic results make use of a different approximation with respect to firm-level idiosyncratic shocks, similar to the one used in, for example, Gopinath et al. (2010), Burstein and Gopinath (2014), and Amiti et al. (2019) in the context of exchange rate shocks.

[5] Given the level of aggregation in our baseline dataset, we do not measure markups at the level of geographic regions, as in Anderson et al. (2018), or products, as in De Loecker et al. (2016). For consideration of pricing with multi-product firms in a similar framework to ours, see Hottman et al. (2016).

fined sector, a firms' market power is increasing in its market share. This relationship implies a positive correlation between firm-level markup and firm-level market share, both in the cross section and over time. Moreover, aggregating firm-level outcomes implies the same correlation between sectoral markups and sectoral concentration. We find support for these predicted correlations in the French census data, both at the firm and at the sector levels, in the cross section, and over time.[6]

Second, we show firm-level changes in markups play a sizable role in accounting for changes in sectoral markups. Namely, as discussed above, our model of oligopolistic competition with endogenous markups predicts that within-firm markup dynamics account for half of the changes in sectoral markups, and equilibrium reallocation accounts for the remaining half. Implementing a simple within-between decomposition in the French census data, we find that, for the median sector, the within-firm term accounts for 65% of the changes in sector markups in France. Furthermore, for half of the sectors, the contribution of the within term lies between 42% and 82%.[7]

Third, we examine in the model and data three measures of markup cyclicality that the literature has considered. We first consider a notion of firm-level markup cyclicality proposed by Hong (2017). In particular, we ask whether firm markups covary systematically with respect to sector-level output. We find this reduced-form relation is "counter-cyclical" for the average firm in the French data, but switches sign for large firms. Relatedly, large firms are "pro-cyclical" in that their market share increases during sectoral expansions, which is a key implication of our granular model. We then proceed to evaluate notions of sector-level markup cyclicality. Following Nekarda and Ramey (2013) (the working paper version of Nekarda and Ramey, Forthcoming), we ask whether sector markups comove with sector output over the business cycle. Like Nekarda and Ramey (2013) for the US, we find evidence for a positive systematic comovement between the two measures, or "pro-cyclicality", in the French data. Finally, we follow the recent work of Bils et al. (2018), who investigate yet another notion of cyclicality: the extent to which sector level markups comove with aggregate output. According to this measure, we find weak (statistically insignificant) pro-cyclical point estimates. Bils et al. (2018) document a negative correlation for the US. Note we obtain seemingly conflicting measures of markup cyclicality across different layers of aggregation despite using a single dataset and measure of firm-level markups. This suggests that by exploiting different reduced-form measures of markup cyclicality, two researchers may arrive at opposing conclusions even within a single dataset. Our model can reproduce qualitatively and, with a few exceptions, quantitatively different measures of markup cyclicality.

[6]Relatedly, Brooks et al. (2016) and Kikkawa et al. (2019) provide evidence of a positive relation between market shares and markups in the time series using firm-level data from China and Belgium, respectively.

[7]Relatedly, Figure 3 in Baqaee and Farhi (2019) shows within-firm changes in markups in the US are quantitatively important in accounting for cyclical (e.g., high frequency) movements in aggregate markup.

Finally, we examine the model's implications for aggregate markup and output fluctuations. In our baseline calibration, we abstract from aggregate shocks that leave the firm-size distribution unchanged because, in our model, they do not affect markups. Our model with granular firm-level shocks generates roughly 40% (on average, across 23-year simulated samples) of the volatility of aggregate output in the French data. The ratio of markup volatility relative to output volatility is roughly 0.5 in the data and 0.3 in the model. Note that much of the work on the granular origin of business cycles abstracts from these movements in desired markups that can partly offset the impact of own firm-level shocks or magnify the impact from shocks to competing firms.[8,9] Turning to aggregate markup cyclicality, our model implies a counterfactually large and positive point estimate for the correlation between aggregate output and markups relative to the data. However, there is large variation in point estimates across small 23-year simulated samples. This is because, as our analytic expressions show, the sign and magnitude of markup cyclicality depends on the set of shocked firms, which can vary substantially across small samples. Moreover, superimposing aggregate productivity shocks to account for the overall aggregate volatility reduces this correlation significantly. Finally, we show the magnitude and cyclicality of aggregate markups in our model is not too different when we counterfactually fix markups at their initial, heterogeneous equilibrium level. Of course, rather than exogenously fixing markups, our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes.

The paper is organized as follows. In section 1, we present our granular oligopolistic setup and describe the equilibrium from the bottom (firm) level to the aggregate level. In section 2, we characterize analytically various measures of markup cyclicality at various aggregation levels. In section 3, we discuss our French administrative firm data, the markup-estimation strategy and model calibration. In section 4, we compare a host of markup-related moments in the data and in model-generated data. Section 5 concludes. In the Appendix, we more fully discuss markup estimation, provide additional results and proofs, and present robustness checks.

[8] Gaubert and Itskhoki (2018) study the granular origins of a country's comparative advantage in an oligopolistic framework that is similar to ours. For work on the granular origins of business-cycle fluctuations – but featuring either perfect competition or constant markups – see Carvalho and Gabaix (2013) on the evolution of business-cycle volatility over time and across countries and di Giovanni and Levchenko (2012) or di Giovanni et al. (2018) for granular settings linking trade, aggregate volatility, and cross-country comovement. di Giovanni et al. (2014) provide an empirical benchmark for the role of granularity in aggregate fluctuations. Our emphasis on the micro origins of aggregate fluctuations is also related to the literature on production networks. See Acemoglu et al. (2012) for an initial benchmark, and Grassi (2018) for an analysis of how market power distorts the propagation of shocks along input linkages. Finally, Pasten et al. (2020) examine aggregate granular fluctuations in a multi-sector model allowing for changes in markups due to nominal price rigidities.

[9] Baqaee and Farhi (2019) provide a very general characterization of the impact of microeconomic shocks on aggregate productivity and output in a large class of models in which productivities and wedges (e.g., markups) are exogenous primitives. Baqaee and Farhi (2020) study the role of variable markups in shaping the aggregate implications of changes in market size.

1 Model

In this section, we first describe our assumptions on preferences, technologies, and market structure. We then characterize the equilibrium, first within a sector and then at the aggregate level.

1.1 Preferences and technologies

Households have preferences at time t over consumption of a final composite good, Y_t, and labor, L_t, given by

$$U(Y_t, L_t) = \frac{1}{1-\eta} Y_t^{1-\eta} - \frac{f_0}{1+f^{-1}} L_t^{1+f^{-1}},$$

where $\eta \leq 1$ and $f \geq 0$ are, respectively, the constant relative risk aversion and the Frisch elasticity of labor supply. The final good aggregates output of N sectors according to a constant-elasticity-of-substitution (CES) aggregator:

$$Y_t = \left[\sum_{k=1}^{N} A_k^{\frac{1}{\sigma}} Y_{kt}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where Y_{kt} denotes sector k output, A_k is a demand shifter for sector k (which we assume is constant over time, and we set it in our calibration to target the average size of each sector), and $\sigma \geq 1$ is the elasticity of substitution across sectors.

As in Atkeson and Burstein (2008), each sector k is itself a CES aggregator of the output of N_k individual firms given by

$$Y_{kt} = \left[\sum_{i=1}^{N_k} A_{kit}^{\frac{1}{\varepsilon}} Y_{kit}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Y_{kit} denotes the output of firm i in sector k, A_{kit} is a firm-demand shock (independently drawn across firms), and ε is the elasticity of substitution between the output of firms in sector k. We assume $\sigma \leq \varepsilon$, so that goods are more substitutable within sectors than across sectors. With a finite number of sectors and a discrete number of firms per sector, firm-level shocks can generate aggregate fluctuations as in Gabaix (2011). By contrast, with a continuum of sectors, as in Atkeson and Burstein (2008), firm-level shocks would not generate aggregate fluctuations.

Firm i in sector k produces output according to the constant-returns-to-scale technology:

$$Y_{kit} = Z_{kit} L_{kit}, \tag{1}$$

where Z_{kit} denotes the productivity of firm i in sector k and L_{kit} is a variable input, employment, that is perfectly mobile across firms.[10] Labor market clearing requires that the sum of

[10] In appendix A.8, we provide analytic results allowing for decreasing returns to scale at the firm level.

employment across all firms equals aggregate labor, L_t.

Our analytic results in section 2 on firm-level and sectoral-level outcomes are unchanged if the variable input, L_{kit}, is a composite of multiple inputs (e.g., labor, intermediate goods, and capital) that is common across firms in the sector. The specific assumptions on the composition of this variable input matter only for the aggregate response of the economy to given firm-level shocks. When estimating markups in section 3, we assume the input L_{kit} is a translog combination of labor, capital, materials, and services inputs with parameters that vary by sector. We then compare measures of cyclicality of estimated markups in the data with measures of cyclicality implied by our model.

Note we have not yet made assumptions about the stochastic process of firm-level shocks A_{kit} and Z_{kit}. We introduce these assumptions in the analytic section (for our asymptotic results) and quantitative section (in describing our model calibration).

1.2 Market structure and sector equilibrium

We now describe the equilibrium in a sector. Given the nested CES structure, firm i in sector k faces demand $Y_{kit} = A_k A_{kit} \left(P_{kit}\right)^{-\varepsilon} \left(P_{kt}\right)^{\varepsilon-\sigma} P_t^{\sigma} Y_t$, where the sector k price is

$$P_{kt} = \left[\sum_{i=1}^{N_k} A_{kit} P_{kit}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

and the aggregate price is

$$P_t = \left[\sum_{k=1}^{N} A_k P_{kt}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

The markup for firm i in sector k, which we characterize below, is defined as the ratio of price to marginal cost,

$$\mu_{kit} \equiv \frac{Z_{kit} P_{kit}}{W_t}, \tag{3}$$

where W_t is the price of the variable input (i.e., the wage). This markup determines how the firm's revenues are split into labor payments and profits, such that

$$L_{kit} W_t = \mu_{kit}^{-1} P_{kit} Y_{kit}, \qquad \text{and} \qquad \Pi_{kit} = \left(1 - \mu_{kit}^{-1}\right) P_{kit} Y_{kit}.$$

The market share of firm i in sector k, $s_{kit} \equiv \frac{P_{kit} Y_{kit}}{P_{kt} Y_{kt}}$, can be expressed in terms of markups and *firm shifters*, which are defined as a composite of demand and productivity shifters, $V_{kit} \equiv A_{kit} Z_{kit}^{\varepsilon-1}$. Specifically,

$$s_{kit} = \frac{V_{kit} \mu_{kit}^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't} \mu_{ki't}^{1-\varepsilon}}. \tag{4}$$

One can consider two alternative market structures. Firms maximize profits by choosing price, taking other firms' prices as given (Bertrand competition), or by choosing quantity, taking other firms' quantities as given (Cournot competition). In both cases, firms take into account that they are non-atomistic in their sector, and hence their choices affect sectoral output and prices. We assume, however, that individual firms behave as if the sector they produce in is atomistic in the aggregate economy (as in the case of a continuum of sectors).

Under these assumptions, equilibrium markups and market shares in each sector k solve the non-linear system of equations given by (4) and

$$\mu_{kit} = \begin{cases} \frac{\varepsilon}{\varepsilon-1}\left[1-\left(\frac{\varepsilon/\sigma-1}{\varepsilon-1}\right)s_{kit}\right]^{-1} & \text{under Cournot,} \\ \frac{\varepsilon}{\varepsilon-1}\left[\frac{1-\left(\frac{\varepsilon-\sigma}{\varepsilon}\right)s_{kit}}{1-\left(\frac{\varepsilon-\sigma}{\varepsilon-1}\right)s_{kit}}\right] & \text{under Bertrand.} \end{cases} \tag{5}$$

Under both formulations, if $\varepsilon > \sigma$, markups are increasing in market shares,[11] with $\lim_{s_{kit}\to 0}\mu_{kit} = \dfrac{\varepsilon}{\varepsilon-1}$ and $\lim_{s_{kit}\to 1}\mu_{kit} = \dfrac{\sigma}{\sigma-1}$. If $\varepsilon = \sigma$, markups are common across firms and constant over time as in the standard monopolistically competitive model. In our analytic and quantitative results, we focus on the case of Cournot competition because it generates more markup variation than Bertrand and is thus better able to match the estimates of incomplete pass-through and the markup-size relationship. In appendix A, we provide analytic results under Bertrand.

Two remarks are in order about firm shifters. First, firm-level market shares and markups in sector k depend only on relative firm shifters across firms within this sector. This result implies market shares and markups in sector k do not vary in response to proportional changes in shifters to all firms in sector k (including sectoral demand shifters A_k), shocks in other sectors, or changes in the aggregate wage. It follows that aggregate shocks to firms in all sectors generate fluctuations in aggregate output but not in aggregate markups. For this reason, in our baseline quantitative analysis, we abstract from standard aggregate productivity shocks.

Second, the split of firm shifters V_{kit} into demand and productivity does not matter for the solution of any model outcome (at the firm, sector, or aggregate level) except for firm-level quantities and prices. In mapping the model to data, however, one needs to take into account that some discrepancies may exist in how certain variables are constructed in the model and data. For example, price deflators that are used to construct sectoral output typically do not take into account high-frequency changes in demand or quality shifters. Therefore, in our quantitative analysis, we only consider firm-level productivity shocks and abstract from shocks to demand shifters.

[11] The property is satisfied in a variety of models with variable elasticity of demand (see, e.g., the reviews in Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017))

1.3 Sectoral outcomes

We now describe how the model aggregates outcomes from the firm level to the sector level. We define sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt} Y_{kt}}{W_t L_{kt}}, \tag{6}$$

where sectoral employment is $L_{kt} = \sum_{i=1}^{N_k} L_{kit}$. Sectoral markups can be expressed as an harmonic mean (weighted by market shares) of firm-level markups,

$$\mu_{kt} = \left[\sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit} \right]^{-1}. \tag{7}$$

Substituting the markup-market-share relationship (equation 5) under Cournot competition, we can express the sectoral markup, μ_{kt}, as a simple function of the sector's Herfindahl-Hirschman index, $HHI_{kt} = \sum_{i=1}^{N_k} s_{kit}^2$:[12]

$$\mu_{kt} = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1} \right) HHI_{kt} \right]^{-1}. \tag{8}$$

Note the positive relationship between sectoral markup and HHI takes the same form as the firm-level relationship between markup and market share in equation (5). In the same way that a firm with a large market share charges a higher markup, a sector with a large average market share, that is, a high HHI, has a high sectoral markup as long as $\varepsilon > \sigma$.[13]

Sectoral markups can be expressed as the standard ratio between sectoral price and marginal cost (i.e., the ratio of wage to sectoral productivity), $\mu_{kt} = P_{kt} Z_{kt} / W_t$. Sectoral productivity, $Z_{kt} \equiv Y_{kt} / L_{kt}$, can be expressed in terms of firm-level markups and firm shifters as

$$Z_{kt} = \frac{\left(\sum_{i=1}^{N_k} V_{kit} \mu_{kit}^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon - 1}}}{\sum_{i=1}^{N_k} V_{kit} \mu_{kit}^{-\varepsilon}}. \tag{9}$$

[12]The HHI is an average of market shares, weighted by market shares themselves, and hence ranges between 0 and 1.

[13]A similar mapping between sectoral markups and concentration indices can be obtained under Bertrand competition (see Grassi, 2018).

1.4 Aggregate outcomes

We now describe how the model aggregates outcomes from the sector level to the aggregate level. We define aggregate markup as the ratio of aggregate revenues and labor payments,

$$\mu_t \equiv \frac{P_t Y_t}{W_t L_t} = \left[\sum_{k=1}^{N} s_{kt} \mu_{kt}^{-1} \right]^{-1}. \tag{10}$$

As indicated by the second equality, aggregate markups can be expressed as a harmonic weighted average of sectoral markups, where sectoral expenditure shares are determined by sectoral markups and sectoral shifters $V_{kt} \equiv A_k Z_{kt}^{\sigma-1}$,

$$s_{kt} \equiv \frac{P_{kt} Y_{kt}}{P_t Y_t} = \frac{V_{kt} \left(\mu_{kt}\right)^{1-\sigma}}{\sum_{k'} V_{kt'} \left(\mu_{kt'}\right)^{1-\sigma}}. \tag{11}$$

Alternatively, under Cournot, we can express the aggregate markup as a simple function of average sectoral HHI (weighted by sectoral expenditure shares) that mirrors the expressions for firm-level and sector-level markups in equations (5) and (8), respectively:

$$\mu_t = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1} \right) \sum_{k=1}^{N} s_{kt} HHI_{kt} \right]^{-1}.$$

The weighted average of sectoral HHIs is equal to the average market share across firms weighted by firms' expenditure share in the whole economy. When this weighted-average market share in the economy is high, the aggregate markup is high.

Aggregate markups can also be expressed as the standard ratio between aggregate price and aggregate marginal cost, $\mu_t = P_t Z_t / W_t$, where aggregate productivity, $Z_t \equiv Y_t / L_t$, can be expressed in terms of sectoral markups and sectoral shifters as

$$Z_t = \frac{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{-\sigma} \right)}. \tag{12}$$

Finally, aggregate output and labor are given by

$$Y_t^{\eta + \frac{1}{f}} = \frac{Z_t^{1+\frac{1}{f}}}{f_0 \mu_t} \quad \text{and} \quad L_t = \frac{Y_t}{Z_t}, \tag{13}$$

where the aggregate markup, μ_t, distorts the leisure/consumption choice relative to the optimal allocation.

In sum, our model aggregates outcomes in a very parsimonious manner from the firm level to the sector level, and from the sector level to the aggregate level. In appendix A, we summarize how to solve for prices and quantities at every time period as a function of the realized shocks to firm shifters.

In the following section, we use a first-order approximation to characterize the equilibrium response to firm-level shocks at the firm, sectoral, and aggregate levels.

2 Analytic results

In this section, we characterize, up to a first-order approximation, the equilibrium response of markups, prices, and output to firm-level shocks at the firm, sectoral, and aggregate levels.[14]

We first introduce a first-order approximation to solve for changes in firm-level markups and market shares in a sector. We then develop expressions for changes in prices, markups, and output in response to firm-level shocks, first at the sector level and then at the aggregate level. We provide expressions for asymptotic covariances between markup and output changes at different aggregation levels under the additional assumption that firm-level shocks are i.i.d. and equally distributed across firms with variance $\sigma_v^2 \equiv \mathbb{V}ar\left[\widehat{V}_{kit}\right]$. We focus on the case of Cournot competition, and present results under Bertrand in the appendix. We highlight the role of variable markups versus constant markups in shaping markup cyclicality, as well as the impact of variable markups on aggregate output volatility. We return to these formulas in our quantitative analysis in section 4.4.

2.1 Firm-level outcomes

Consider an initial equilibrium in sector k with market shares $\{s_{ki}\}$ and markups $\{\mu_{ki}\}$ where, for simplicity, we omit time subscripts in the initial equilibrium. Taking a first-order approximation of equations (4) and (5), changes in equilibrium market shares and markups are the solution to

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\widehat{\mu}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Gamma_{ki'}\widehat{s}_{ki't}\right), \tag{14}$$

$$\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}. \tag{15}$$

Variables with hats denote log differences at time t relative to the initial equilibrium, that is, $\widehat{V}_{kit} \equiv \log V_{kit} - \log V_{ki}$, and Γ_{ki} denotes the *markup elasticity* with respect to market share for firm i in sector k, evaluated at the initial equilibrium.

[14]We thank Dmitry Mukhin for his valuable input in deriving these analytic results.

Markup elasticities under Cournot are, by equation (5),

$$\Gamma_{ki} \equiv \frac{\partial \ln \mu_{ki}}{\partial \ln s_{ki}} = \frac{\left(\frac{\varepsilon}{\sigma} - 1\right) s_{ki}}{\varepsilon - 1 - \left(\frac{\varepsilon}{\sigma} - 1\right) s_{ki}}.$$

As discussed above, if $\varepsilon > \sigma$, markups are increasing in market shares (i.e., $\Gamma_{ki} \geq 0$, with strict inequality if $s_{ki} > 0$). Moreover, markup elasticities are also increasing in market shares. This property whereby markup elasticities are increasing in market shares is satisfied by a variety of demand models with variable elasticity, as discussed in, for example, Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017).

We now introduce pass-through elasticities, which are not required to solve for sectoral market shares and markups but, nevertheless, we use in our analytic results that follow. Changes in firm-level prices at time t are given by $\widehat{P}_{kit} = -\widehat{Z}_{kit} + \widehat{\mu}_{kit}$ where, without loss of generality, we choose the wage as the numeraire. Combined with equations (15) and $\widehat{s}_{kit} = \widehat{A}_{kit} + (1-\varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right)$, we obtain

$$\widehat{P}_{kit} = \alpha_{ki}\left(-\widehat{Z}_{kit} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1-\alpha_{ki})\,\widehat{P}_{kt}, \tag{16}$$

where α_{ki} is the *pass-through rate* governing how firm-level prices respond to idiosyncratic shocks (for given changes in sectoral prices, $\widehat{P}_{kt}$),

$$\alpha_{ki} = \frac{1}{1 + (\varepsilon - 1)\,\Gamma_{ki}}. \tag{17}$$

Conversely, $1 - \alpha_{ki}$ governs how prices respond to changes in sectoral price (due to variable markups).[15] Because markup elasticities are increasing in market shares (if $\varepsilon > \sigma$), pass-through rates are decreasing in market shares.[16] To isolate the role of changes in markups in response to shocks, we consider the case in which markups are fixed at the initial equilibrium levels, imposing $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$.

2.2 Sectoral outcomes

In this subsection, we characterize how sectoral prices, markups, and output respond to firm-level shocks, and provide expressions for variances and covariances of markup and output changes over long realizations of shocks.

[15] In the appendix, we provide expressions for the elasticity of market shares with respect to firm-level shifters and for the variance of market shares.

[16] We can further solve for $\widehat{P}_{kit}$ using $\widehat{P}_{kt} = s_{ki}\widehat{P}_{kit} + (1-s_{ki})\widehat{P}_{k-it}$, where $\widehat{P}_{k-it}$ is the competitors' price index defined in Amiti et al. (2019). We can rewrite (16) as $\widehat{P}_{kit} = \tilde{\alpha}_{ki}\left(-\widehat{Z}_{ki} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1-\tilde{\alpha}_{ki})\,\widehat{P}_{k-it}$, where $\tilde{\alpha}_{ki} = \frac{\alpha_{ki}}{1-(1-\alpha_{ki})s_{ki}}$, which is a U-shaped function of market shares s_{ki}.

Sectoral prices As a first step in understanding changes in sectoral output, we characterize changes in sectoral prices (relative to the the numeraire, i.e., wage), which are related to sectoral output by CES demand, $Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t$.

Taking a first-order approximation of the sectoral price definition (2) and using firm-level price changes (16), log changes in sectoral prices can be expressed as a weighted average of firm shifters,

$$\widehat{P}_{kt} = -\frac{1}{\varepsilon - 1} \frac{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki}}, \tag{18}$$

where the weights are given by the product of market shares, s_{ki}, and pass-through rates, α_{ki}. Because $\varepsilon \geq 1$, sectoral prices fall in response to an increase in firm shifter.

To understand how sectoral price changes are shaped by pass-through rates, note that if $\alpha_{ki} = \alpha_k$, $\widehat{P}_{kt}$ is independent of α_k for given market shares s_{ki} in the initial equilibrium. That is, the response in sectoral price is identical to that if markups were fixed at their initial level ($\alpha_{ki} = 1$). Intuitively, as pass-through α_k falls, the larger markup change by a firm to an own shock is exactly offset by a larger change in markup, in the opposite direction, of its competitors.

With heterogeneity in pass-through rates, because α_{ki} is decreasing in s_{ki}, a single value $\bar{s}_k^p$ exists such that a positive shock to firm i with $s_{ki} > \bar{s}_k^p$ results in a smaller reduction in sectoral prices than if markups were fixed at their initial level. Intuitively, firm i's increase in markup more than offsets the markup decrease of its competitors. Conversely, a positive shock to firm i with $s_{ki} < \bar{s}_k^p$ results in a larger reduction in sectoral prices than if markups were fixed at their initial level.[17]

From equation (18), the asymptotic variance of price changes in sector k assuming firm-level shifters are i.i.d. with common variance σ_v^2 is

$$\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon - 1}\right)^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2. \tag{19}$$

If markups are fixed at their initial level (or, more generally, if $\alpha_{ki} = \alpha_k$), this variance is proportional to the sectoral HHI, as in Gabaix (2011): $\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon-1}\right)^2 \sum_{i=1}^{N_k} s_{ki}^2$. Comparing this expression with (19), we note $\mathbb{V}ar\left[\widehat{P}_{kt}\right]$ is lower under variable markups than under constant markups if and only if the variance of $\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}$ is lower than the variance of s_{ki}. Because α_{ki} is decreasing in s_{ki}, this condition is satisfied if $s_{ki}\alpha_{ki}$ is increasing in s_{ki} (see condition 53 below).

Intuitively, under the condition presented above, the variance of sectoral prices is lower under variable markups, because pass-through rates are lower for larger firms, effectively reducing the weight of large firm shocks in the price index (with similar effects on volatility as a decline in the HHI).

[17] The threshold $\bar{s}_k^p$ is defined implicitly by $\alpha_k(\bar{s}_k^p) = \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}$.

Sectoral markups Changes in sectoral markups, defined in equation (7), can be decomposed into changes in markups *within* firms and the reallocation of expenditures *between* firms with heterogeneous markups:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \frac{\mu_k}{\mu_{ki}} \left(\widehat{\mu}_{kit} - \widehat{s}_{kit} \right). \tag{20}$$

In appendix A, we derive the following expression for changes in sectoral markups:[18]

$$\widehat{\mu}_{kt} = 2 \left(\frac{1}{\sigma} - \frac{1}{\varepsilon} \right) \mu_k \sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \left[s_{ki} - \frac{\sum_{i'} s_{ki'}^2 \alpha_{ki'}}{\sum_{i'} s_{ki'} \alpha_{ki'}} \right] \widehat{V}_{kit}. \tag{21}$$

The following proposition states that a positive shock to firm i results in an increase in the sectoral markup if and only if firm i is sufficiently large in its sector.

Proposition 1 *Consider a positive shock to firm i in sector k, $\widehat{V}_{kit} > 0$. Then, under Cournot competition, sector k markup increases, $\widehat{\mu}_{kt} > 0$, if and only if $s_{ki} > \sum_{i'} s_{ki'}^2 \alpha_{ki'} / \sum_{i'} s_{ki'} \alpha_{ki'}$.*

Intuitively, recall from equation (20) that changes in sectoral markups reflect changes in firm-level markups (within term) and between-firm reallocation (between term). Consider first the within term. A positive shock to firm i raises firm i's markup and reduces it for competing firms. The former dominates if firm i is large, whereas the latter dominates if firm i is small. Consider now the between term. A positive shock to firm i reallocates market shares towards firm i, increasing the sectoral markup if firm i's markup is sufficiently high (or, equivalently, if its market share is sufficiently large). Therefore, the within and between terms push the sectoral markup in the same direction.

The "2" in front of (21) reflects the fact that the magnitude of the within term is equal to the magnitude of the between term (and hence each accounts for 50% of changes in sectoral markups). A change in parameters (e.g., an increase in $\varepsilon - \sigma$) that increases the sensitivity of markups to firm-level shocks (increasing the within term) also increases the dispersion of markups across firms (increasing the between term). In appendix A, we show this 50-50 within/between decomposition of changes in sectoral markups under Cournot competition holds globally (not only up to a first order).

How do changes in sectoral markups compare in the specification with variable markups versus the specification with constant markups (in which sectoral markups change only due to between-firm reallocation)? If firm-level markups are fixed at their initial level (setting $\Gamma_{ki} = 0$

[18] Ex-ante firm heterogeneity is a necessary condition for sectoral markups to change in response to firm-level shocks. To see this, if s_{ki} and μ_{ki} are equal across all firms in sector k, equations (15), (20), and $\sum_{i=1}^{N_k} s_{ki} \widehat{s}_{ki} = 0$ imply that $\widehat{\mu}_{kt} = 0$.

and $\alpha_{ki} = 1$), changes in sectoral markups in equation 49) are:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \left(1 - \frac{\mu_k}{\mu_{ki}}\right) \widehat{V}_{kit}. \tag{22}$$

In response to a positive shock to firm i, sectoral markups increase if and only if $\mu_{ki} > \mu_k$.

In general, we do not obtain a simple characterization comparing (22) with (21). To make analytic progress, in appendix A, we restrict the extent of ex-ante firm heterogeneity to two types. We provide a simple sufficient condition for sectoral markups to change by more (and display a higher variance) under variable markups than under constant markups. Intuitively, changes in sectoral markups can be smaller under variable markups than under constant markups because the larger response of sectoral markups due to changes in firm-level markups is more than offset by a smaller extent of between-firm reallocation due to incomplete pass-through.

To summarize, even though changes in sectoral markups under variable markups are twice as large as the between-firm reallocation term for any firm-level shocks, variable markups do not necessarily magnify changes in sectoral markups relative to the model specification with constant markups, because incomplete pass-through mutes the extent of between-firm reallocation

Covariance between sectoral prices and sectoral markups Recall from previous results that in response to a positive shock to firm i in sector k, the sectoral price falls, whereas sectoral markup can increase or decrease depending on the firm's initial markup. We now calculate the asymptotic covariance between sectoral price and markup changes, which shapes the covariance between sectoral output and markup that we examine below.

First, to build intuition, in the case of constant markups,

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\frac{1}{\varepsilon - 1} \sum_{i=1}^{N_k} s_{ki}^2 \left[1 - \frac{\mu_k}{\mu_{ki}}\right] \times \sigma_v^2. \tag{23}$$

Thus, sectoral markups and prices are negatively correlated as long as large firms within sector charge higher markups. Intuitively, shocks to small firms induce a positive comovement, whereas shocks to large firms induce a negative comovement. Overall, comovement is negative because shocks to large firms induce larger changes in sectoral price than shocks to small firms.

With variable markups, using the expressions for the change in sectoral price (18) and markup (21),

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\left(\frac{2\mu_k}{\varepsilon - 1}\right)\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right) \sum_{i'=1}^{N_k} s_{ki'}^2 \alpha_{ki'} \sum_{i=1}^{N_k} \left[\frac{s_{ki}^2 \alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2 \alpha_{ki'}} - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right] \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \times \sigma_v^2. \tag{24}$$

Therefore, when $\varepsilon > \sigma$, sectoral prices and markups comove negatively in long samples if and only if

$$\sum_{i=1}^{N_k} \left[\frac{s_{ki}^2 \alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2 \alpha_{ki'}} - \frac{s_{ki} \alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right] \frac{s_{ki} \alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} > 0. \tag{25}$$

If firms are ex-ante homogeneous, equation (25) holds with equality and sectoral markups are constant over time. If firms are heterogeneous in the initial equilibrium, inequality (25) may or may not hold. The following proposition, proven in the appendix, states that if pass-through rates do not fall too strongly with market shares, inequality (25) holds, so sectoral prices and markups comove negatively.

Proposition 2 *Under Cournot competition, if firms are ex-ante heterogeneous and $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, sectoral markup and price comove negatively, $\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] < 0$.*

In appendix A.4 we show that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} provided that market shares are not too large. Intuitively, the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} implies, by equation (18), that sectoral prices are more responsive to large firm shocks than to small firm shocks (i.e., the lower pass-through rate by large firms does not fully offset their higher weight in the price index). The fact that sectoral markups increase in response to large firm shocks and decrease in response to small firm shocks implies a negative covariance between sectoral price and markup, as in the case of constant markups.

Covariance between sectoral output and markups In appendix A, we derive the following expression for changes in sector k output in response to sector k shocks:

$$\widehat{Y}_{kt} = -\left[\sigma\left(1 - s_k\right) + \left(\frac{f+1}{f\eta+1} + \left(\frac{\sigma-1}{f\eta+1}\right)\left(1 - \frac{\mu}{\mu_k}\right)\right) s_k\right] \widehat{P}_{kt} + \frac{s_k \mu}{\mu_k} \frac{\widehat{\mu}_{kt}}{f\eta+1}. \tag{26}$$

A sufficient condition for sectoral output and price to move in opposite directions is that sector k is small in the aggregate ($s_k \to 0$) or that disutility of labor is linear ($f \to \infty$). In this case, the previous results on sectoral price apply immediately to sectoral output (with the opposite sign).[19] Specifically, in response to a positive shock to firm i in sector k, sectoral output increases whereas sectoral markup can increase or decrease depending on the firm's initial markup.

Taking into account a long sequence of firm shocks in sector k, the covariance between changes

[19] If f finite and sector k is sufficiently large in the aggregate, it is *possible* that sectoral output and price both fall in response to positive sector k firm level shocks if sectoral markup μ_k is very low relative to the aggregate markup and/or if sector k markup falls substantially when the sectoral price falls.

in sectoral output and sectoral markup is:

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]=-\left[\sigma\left(1-s_k\right)+\frac{f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)}{f\eta+1}s_k\right]\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]+\frac{s_k\mu}{\mu_k}\frac{1}{f\eta+1}\mathbb{V}ar\left[\widehat{\mu}_{kt}\right],$$

where $\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]$ is defined above.

The following proposition provides sufficient conditions for pro-cyclical sectoral markups with respect to sectoral output.

Proposition 3 *Under the conditions of Proposition 2, sectoral markup and sectoral output comove positively,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$*, if at least one of these three conditions holds: (i)* $s_k\to 0$*, (ii)* $f\to\infty$*, (iii)* $\sigma\to 1$*. If all three conditions (i)-(iii) are violated,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$ *as long as sectoral markup* μ_k *is not too low relative to aggregate markup.*

In our empirical analysis, we also consider the cyclicality between sector output and firm-level markups. In appendix A, we show that, for the case of $f\to\infty$, the covariance between changes in firm i markup and sector k output is

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]=\left(\sigma\left(1-s_k\right)+\eta^{-1}s_k\right)\frac{\alpha_{ki}\Gamma_{ki}}{(\epsilon-1)\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\left[s_{ki}\alpha_{ki}-\frac{\sum_{i'=1}^{N_k}\left(s_{ki'}\alpha_{ki'}\right)^2}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\times\sigma_v^2. \tag{27}$$

The following proposition states that firm-level markups are procyclical for large firms and counter-cyclical for small firms:

Proposition 4 *If* $s_{ki}\alpha_{ki}$ *is increasing in* s_{ki} *and* $f\to\infty$*, firm-level markups and sectoral output comove positively,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]>0$*, if and only if* $s_{ki}>\bar{s}_k^{\mu}$*, and comove negatively if and only if* $s_{ki}<\bar{s}_k^{\mu}$*, where* $\bar{s}_k^{\mu}$ *is defined by the condition that the square bracket in (27) is equal to 0.*

Intuitively, firm-level markups are positively correlated with sectoral output in response to own-shocks and negatively correlated in response to competitors' shocks. Because large firms have a disproportionate impact on sectoral price and output (if $s_{ki}\alpha_{ki}$ is increasing in s_{ki}), firm-level markups are pro-cyclical for large firms and counter-cyclical for small firms. The cutoff $\bar{s}_k^{\mu}$ differs from the cutoff defined in Proposition 1, because the condition in Proposition 1 is based on a shock to one firm only, whereas the asymptotic covariance in Proposition 4 takes into account shocks to all firms in the sector.

2.3 Aggregate outcomes

We now describe how changes in sectoral markup, price, and output that we characterized above shape changes in aggregate price (i.e., the negative of the real wage given the choice – without loss of generality – of the wage as a numeraire), markup, productivity, and output.

We provide expressions for sectoral and aggregate markup cyclicality with respect to aggregate output, which we consider in our empirical analysis. We also examine how the variance of aggregate output compares under variable markups and constant markups.

Up to a first order, changes in the aggregate price are $\widehat{P}_t = \sum_k s_k \widehat{P}_{kt}$. Based on our results above, any positive firm-level shock in sector k reduces the corresponding sectoral price and therefore reduces the aggregate price (or increases the real wage) proportionately to the share in expenditures of sector k. Whether the real wage increases more or less under variable markups relative to constant markups depends, as discussed above, on the shocked firm's relative size in its sector.

Changes in aggregate markup can be decomposed into a within-sector markup term and a reallocation term, analogous to the decomposition of sectoral markups in equation (20):

$$\widehat{\mu}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} + (1-\sigma) \sum_k s_k \left(1 - \frac{\mu}{\mu_k}\right) \widehat{P}_{kt}. \tag{28}$$

In response to a positive shock to a firm in sector k, aggregate markup can increase or decrease. The first (within) term in (28) is positive if the shocked firm is relatively large (and sets a higher markup) in sector k. The second (between) term in (28) is positive, when $\sigma > 1$, if sector k has a relatively high markup relative to the aggregate markup.

Changes in aggregate productivity, using $\widehat{Z}_t = \widehat{\mu}_t - \widehat{P}_t$, can be expressed in terms of changes in sectoral markups and prices as

$$\widehat{Z}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} - \sum_k s_k \left[1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)\right] \widehat{P}_{kt}. \tag{29}$$

Recall that in response to positive firm-level shocks, the sectoral price decreases ($\widehat{P}_{kt} < 0$). Aggregate productivity typically increases, but can decrease if shocked firms are relatively small in their sector (such that the sectoral markup falls) or belong to low-markup sectors and $\sigma > 1$.

Finally, by equation (13), changes in aggregate output are

$$\widehat{Y}_t = (f^{-1} + \eta)^{-1} \left[f^{-1} \widehat{Z}_t - \widehat{P}_t\right]. \tag{30}$$

With inelastic labor supply ($f \to 0$), $\widehat{Y}_t = \widehat{Z}_t$. With linear disutility of labor ($f \to \infty$), the aggregate productivity term drops, so $\widehat{Y}_t = -\eta^{-1} \sum_k s_k \widehat{P}_{kt}$. A positive firm-level shock in sector k reduces the sectoral price and increases aggregate output. Based on the discussion above on the role of variable markups for the response of sectoral prices, the increase in aggregate output is smaller under variable markups compared to constant markups if and only if the shocked firm has a high market share.

Variance of aggregate output The variance of aggregate output (when $f \to \infty$) is

$$\mathbb{V}ar\left[\widehat{Y}_t\right] = \eta^{-2} \sum_k s_k^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right] = \frac{\sigma_v^2}{\eta^2 \left(\varepsilon - 1\right)^2} \sum_k s_k^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right)^2, \tag{31}$$

where the second equality used equation (19). Based on the discussion after equation (19) above, aggregate output is less volatile under variable markups than under constant markups when pass-through rates are decreasing in size, effectively reducing the weight of large firms in the price index (with similar effects on volatility as a reduction in market-share concentration).

In appendix A, we provide an expression for the variance of aggregate output without imposing $f \to \infty$, as well as for the variance of aggregate markups.

Covariance between aggregate output and markups We first calculate the covariance between aggregate output and sector k markup, which is one of the measures of cyclicality in our empirical analysis. When calculating this covariance, we use the fact that sector k markups are affected only by shocks to sector k firms and not by shocks to firms in other sectors. We can thus express this covariance as

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] = \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right] + \sigma \left(1 - s_k\right) \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right]. \tag{32}$$

The following proposition states that the covariance between aggregate output and sector k markups is positive and lower than the covariance between sector k output and sector k markup:[20]

Proposition 5 *Under the conditions of Proposition 3,*

$$0 < \mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] \leq \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right], \tag{33}$$

where the second inequality holds strictly if the economy has more than one sector (i.e. $s_k < 1$).

Next, we calculate the covariance between aggregate output and aggregate markups (when $f \to \infty$):

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = -\frac{\mu}{\eta} \sum_k \frac{s_k^2}{\mu_k} \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{\sigma - 1}{\eta} \sum_k s_k^2 \left(1 - \frac{\mu}{\mu_k}\right) \mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{34}$$

[20] To prove the first inequality in Proposition 5, we write equation (32) as

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] = -s_k \left[\frac{f + 1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)}{f\eta + 1}\right] \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{s_k \mu}{\mu_k} \frac{1}{f\eta + 1} \mathbb{V}ar\left[\widehat{\mu}_{kt}\right].$$

To prove the second inequality, we note that under the conditions of Proposition 3, $\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] \leq 0$. Note the second inequality does not extend immediately from covariances to correlations, because for some sectors, the variance of aggregate output is smaller than the variance of sectoral output.

The first term in (34) is positive if sectoral markups and sectoral prices comove negatively, which we discussed above. The second term in (34) is positive unless larger sectors have relatively lower markups.

So far, we have calculated measures of markup cyclicality considering only i.i.d firm-level shocks. In our quantitative analysis, we also allow for aggregate productivity shocks to firms in all sectors. In our model, in which firm-level markups are functions of market shares, markups do not respond to aggregate shocks. Therefore, incorporating aggregate shocks leaves the covariance of aggregate markups and output unchanged but decreases the correlation, because the volatility of aggregate output increases with these shocks.

From these theoretical results, we see the sign of markup cyclicality depends on the level of aggregation, market structure within and across all industries, and the set of shocked firms. Moreover, the sign and magnitude of covariances in finite samples may differ from those of the asymptotic covariances we derived.

In what follows, we calibrate the model to match salient features of the French firm-level data. We evaluate quantitatively its implications for the cyclicality of markups, as well as its ability to generate aggregate fluctuations in output and markups in response to idiosyncratic firm-level shocks.

3 Data, Estimation, and Calibration

For the remainder of this paper, we use the model above as a data-generating process – from which we simulate firm, sector, and aggregate time series – and then proceed to compare the resulting model-implied moments to their empirical counterparts. In this section, we describe how we use French administrative firm-level data to parametrize our model and provide the empirical moments of interest. We start by describing the data and how we estimate markups. We then describe how we parametrize the firm shock process and how we calibrate the model. Appendix B provides additional details.

3.1 Data

Our empirical analysis deploys French firm-level data between 1994 and 2016. We use two main datasets: the FICUS data covering the period 1994-2007 and the FARE dataset covering the period 2008-2016. The datasets cover the universe of French firms and originate from the French tax administration (DGFiP) that collects yearly tax statements for each firm, including income statements, balance-sheet, and demographic information. The Institut National de la Statistique et des Etudes Economiques (INSEE) uses these statements to construct the FICUS-FARE datasets.

We assign firms to sectors according to the Nomenclature d'Activités Française (NAF2008) five-digit classification, which is a French industry classification similar to the NACE Rev. 2 industry classification at four-digit.[21]

We use a subset of the variables available in the FICUS-FARE dataset: total firm revenues, wage bill (sum of wages and social security payments), capital (measured by fixed assets), and expenditures on inputs. Our baseline measure of materials (which is our choice of variable input in the estimation of markups) is the sum of expenditures and stock variation of materials and merchandises (ACHAMPR and ACHAMAR, respectively). The variable ACHAMPR is defined as "everything that the firm purchases in order to be transformed," and ACHAMAR is defined as "everything that the firm purchases to be sold as is." We consider as a separate input expenditures on service inputs (AUTACH), which includes research expenditures, outsourcing costs, and external personnel cost (including temporary workers). We also use GDP deflators and two-digit sector-price indices provided by EU-KLEMS.

We drop firms with non-positive revenue, materials, services expenditure, wage bill, or capital.[22] We also drop firms in Finance and Insurance industries and, to comply with confidentiality rules, drop sectors with fewer than 12 firms in a given year. Finally, we keep firms that were government owned early in our sample because most of them switched to private ownership during the period we consider.[23] After these treatments, we end up with $10,928,469$ firm-year observations across 23 years and 504 sectors.

For a subsample of manufacturing firms – and only from 2009 onwards – we can additionally construct measures of firm-level prices based on the Enquête Annuelle d'Entreprise (EAE) survey. Although much smaller in coverage, this information is nevertheless useful below as a robustness check to our baseline markup-estimation strategy.

Although this dataset is extremely rich, it misses some important information that limits the extent of our analysis. First, we do not use information on imports and exports in the corresponding sector. Specifically, when we compute market share as the ratio of a firm's revenue relative to the sum of all French firms' revenue in this sector, we do not take into account the sales of foreign firms in this market. Moreover, when we estimate markups, we do not exclude sales to foreign countries, because we do not know the share of inputs expenditure accounted for by exports.

[21] In 2008, the NACE and NAF industry classification changed. To construct a panel of firms between 1994 and 2016 with a consistent industry classification over time, we proceed as follows. For firms with both the old and the new industry codes are observed, we apply the new code to all years. For firms for which only one industry code on either side of 2008, we assign the code that is most frequently associated with the observed industry code (using the sample of firms where we do observe the two sectoral codes). We thank Isabelle Mejean for sharing the code to help merge the FICUS and FARE datasets.

[22] Due to a regulation change, the number of firms with fewer than two employees increases markedly over time in our sample. We drop these firms when we estimate production-function parameters.

[23] Information about government-owned firms can be found in the variable APPGR of FICUS-FARE, which is available only before 2009. Government-owned firms represented 0.12% of the total number of firms in 1994 and 0.05% in 2008. Over the same period, their share of revenue went from 7.2% to 4.2%.

Second, because firm-level revenues in our dataset are reported at the national level, we do not have information on revenues at the local level. This limitation is important for non-tradeable goods, whose markets are most likely local.[24] Because our definition of a market is at the national level, for non-tradable goods, we likely underestimate the concentration in the local market relevant for the firm.

Table 1 presents some descriptive statistics of our data at the firm, sector, and aggregate levels. We take 2014 as a representative year for the cross section of firms and sectors that we use for our model's calibration. As Table 1 shows, the 2014 cross-section is qualitatively similar to other years in our sample. Note, first, that the average market share across all firms and years, defined as the revenue of a firm divided by the total revenue of firms in the same sector, is very low at about 0.1%. However, the distribution of market shares is highly skewed, with the top 0.01% having a market share of about 61%. Second, among the 504 sectors in 2014, the median number of firms is 324, while the average is 1,291. This finding reflects the large heterogeneity across sectors: In 2014, 25% of sectors have less than 91 firms, while 25% of sectors have more than 1,037 firms. Third, in 2014, the median HHI across sectors is 4.8%, but 1% of sectors have a HHI higher than 71.8%. The skewness of the HHI distribution across sectors reflects the skewness of firm-level market shares, because recall that the HHI is a sales-weighted average of market shares. Finally, in 2014, the total revenue of manufacturing sectors (two-digit NACE codes ranging from 10 to 35) represents 29% of the total revenue in the economy.

3.2 Markup Estimation

We now describe how we estimate firm-level markups using the FICUS-FARE data described above. We use our markup estimates for two purposes. First, when we calibrate the model, we target the relationship between sectoral markups and HHI (in appendix C.1, we also discuss an alternative strategy that does not use markup estimates). Second, we calculate measures of markup growth in the data, which we compare with markup dynamics implied by our model.

Our empirical framework to estimate markups in the data is more general than our theoretical framework described above, where labor was the only factor of production. Specifically, we introduce intermediate inputs and other factors of production, some of which may be subject to adjustment frictions.[25] Following Hall (1988) and De Loecker and Warzynski (2012), the first-order condition in the cost-minimization problem by firm i in sector k for variable input M that is not subject to adjustment costs implies

$$\mu_{kit} = \theta^M_{kit} \frac{P_{kit} Y_{kit}}{P^M_{kit} M_{kit}}, \tag{35}$$

[24]See Rossi-Hansberg et al. (2020) for a study of diverging local and national market-concentration trends.

[25]We maintain the assumption that firms are price takers in the input market. Morlacco (2019) relaxes this assumption to estimate markdowns on inputs.

Table 1: Descriptive Statistics

Description	Value		Description	Value	
Firm-Level					
Market share (pp)			St. dev. of market share growth rate		
Mean	0.1		Mean	0.28	
top 1%	1.28		25th percentile	0.13	
top 0.5%	2.59		Median	0.23	
top 0.1%	12.62		75th percentile	0.38	
top 0.01%	61.01		95th percentile	0.85	
Sector-Level					
# firms in a sector in 2014			# firms in a sector in 1994-2016		
Mean	1,291		Mean	1,192	
25th percentile	91		25th percentile	98	
Median	324		Median	328	
75th percentile	1,037		75th percentile	987	
95th percentile	5,485		95th percentile	4,969	
HHI in 2014 (pp)			HHI in 1994-2016 (pp)		
Mean	10.59		Mean	9.86	
25th percentile	1.60		25th percentile	1.47	
Median	4.84		Median	4.48	
75th percentile	12.90		75th percentile	12.38	
95th percentile	41.25		95th percentile	39.58	
top 1%	71.75		top 1%	70.16	
Median effective number of firms in 2014 ($\equiv HHI_k^{-1}$)	20.67		Median effective number of firms in 1994-2016	22.34	
Top 5 largest sectors in 2014 (Revenue share and HHI in pp)			Top 5 concentrated sectors in 2014 (Revenue share and HHI in pp)		
"Sale of cars and light motor vehicles"	3.60	1.09	"Production of electricity"	1.71	92.68
"Hypermarket (larger supermarket)"	2.10	2.40	"Wholesale of tobacco products "	0.57	90.20
"Supermarkets"	2.07	3.50	"Gambling and betting activities "	0.56	83.57
"Activities of head offices"	1.81	15.08	"Manufacture of steam generators"	0.11	81.75
"Production of electricity"	1.71	92.68	"Shopping mall"	0.18	73.66
Aggregate-Level					
# of firms	10,928,469		# of sectors with >11 firms	504	
# of firms in 2014	650,423		Revenue Share of Manufacturing in 2014 (pp)	29.02	

where $P^M_{kit}M_{kit}$ denotes expenditure on input M by firm i in sector k, $P_{kit}Y_{kit}$ is the revenue of this firm, and θ^M_{kit} is the output elasticity with respect to input M. We use the above equation to recover markups using data on firm-level revenues, expenditures on a variable input, and estimates of the output elasticity with respect to this variable input.

We assume firms combine four inputs (labor, capital, materials, and services) according to a flexible translog production function, and that expenditure in material inputs is the variable input available to a firm. Further, all firms in the same two-digit sector share the same production-function parameters, and inputs are homogeneous across firms within sector. This choice is led by the availability of two-digit-level price indices for output and intermediate inputs for our main baseline sample. Note that with a translog specification, the output elasticity θ^M_{kit} depends not only on parameter values but also on firm-level input use, and is therefore heterogeneous across firms.

We estimate output elasticities θ^M_{kit} using an iterative generalized method of moments (GMM) in which unobserved productivity is controlled for using conditional demand of material input, following the literature (see e.g. Ackerberg et al., 2007, 2015; De Loecker and Warzynski, 2012; De Loecker et al., 2016, 2020).

There are at least two concerns with this methodology (for a more detailed discussion, see, e.g., Bond et al., 2020). The first concern is due to the unavailability of firm-level quantity data when estimating output elasticities. To address this concern, we construct firm-level price proxies for the restricted EAE manufacturing firm sample.[26] Because these prices are only a proxy for quality-adjusted firm-level prices, we use them as a control in our production-function estimation, as in De Loecker et al. (2016).

The second concern relates to the identification of the production function under imperfect competition. To address this concern, we assume an oligopolistic competition structure in which firm-level markups are a function of market share, as in our theoretical model. As described in the appendix, we obtain estimates of output elasticities under imperfect competition by using firm-level price proxies and the relationship between markups and market shares to control for marginal cost.

Given these concerns, we implement the estimation of θ^M_{kit} in four different ways. First, in our baseline case, we ignore the lack of price data for a large fraction of firms in our sample and treat deflated revenue (using two-digit price indices) as quantity. Second, for a subset of manufacturing firms during the subperiod 2009-2016, we are able to use as a control the proxy for firm-level prices described above. Third, we use the market share of a firm as a control for the

[26] To construct firm-level prices for this subsample, we first measure product prices at the firm level as the ratio of firm revenues and quantity produced by a product-firm pair in a given year. To address eventual quality differences across firms producing the same product, we then calculate standardized prices as the ratio of the product-firm price described above to the quantity-weighted average price across all firms producing the same product in the same year. Finally, we calculate firm-level prices as value-weighted averages of standardized prices across all products produced by a firm in a given year.

full sample. Fourth, we use as controls both market share and the proxy for firm-level prices on the restricted sample in which the latter information is available.

Across these four measures of θ^M_{kit}, we find the pairwise correlation of the implied firm-level markups is very high, as reported in appendix B.1. In particular, we find the pairwise correlations for firm markups in levels range between 0.8 and 0.9. Importantly, these correlations are even higher when we consider the first differences of firm-level markups, ranging between 0.91 and 0.99 across specification pairs. Guided by the similarity of estimated markups under the various procedures, we choose the first specification as a baseline in order to maximize our sample size and obtain full coverage of the French economy.

Alternatively, as described in appendix C.1, by combining cost minimization and the market structure assumed in our model, we can construct a structural relationship between market share and variable input expenditure share, conditional on input usage, that does not require estimates of markups or output elasticities. We use this relationship as an untargeted moment to evaluate our calibrated model.

3.3 Calibration

In this section, we describe how we parametrize the model to match salient features of the French data in 2014, which we take as a representative cross-section.

We first introduce the firm-level productivity stochastic process, which follows a discrete Markov chain giving rise to random productivity growth as in the literature on firm dynamics (see, e.g., Luttmer, 2010). We then describe how we target the size and concentration of each of the 504 sectors, the estimated markup-market-share relationship in our data, and pass-through estimates in the literature. Finally, we discuss the model's implications for other empirical moments that are not based on our estimates of markups.

Firm-Level Productivity Process

We assume firm-level demand shocks, A_{kit}, are fixed over time. It follows that the composite V_{kit} is driven only by productivity shocks.[27] Following Carvalho and Grassi (2019), we assume firm-level productivity, Z_{ikt}, follows the discretized random growth process introduced by Córdoba (2008). Specifically, firm productivity in sector k evolves on an evenly spaced log grid, $\Phi_k = \{1, \varphi_k, \varphi_k^2, \dots, \varphi_k^S\}$, where φ_k is greater than 1 and where S is an integer. Note $\varphi_k^n = \varphi_k \varphi_k^{n-1}$. A firm's productivity follows a Markov chain on this grid where the associate matrix of transition

[27] Although our analytic results do not take a stand on the importance of productivity versus demand firm-level shocks, in the data, we construct sectoral output by deflating nominal value-added by industry price indices. The latter typically do not take into account high-frequency changes in demand or quality shifters. Therefore, for consistency, we abstract from shocks to demand shifters.

probabilities is equal to:

$$P = \begin{pmatrix} a_k + b_k & c_k & 0 & \cdots & \cdots & 0 & 0 \\ a_k & b_k & c_k & \cdots & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_k & b_k & c_k \\ 0 & 0 & 0 & \cdots & 0 & a_k & b_k + c_k \end{pmatrix}$$

with $a_k < 1, b_k, c_k > 0$ and $a_k + b_k + c_k = 1$. The stationary distribution of this process is Pareto with a tail index equal to $\delta_k = \ln(\frac{a_k}{c_k})/\ln(\varphi_k)$. Given these restrictions, this productivity process yields three parameters (φ_k, δ_k, and c_k) to calibrate.

As shown in Córdoba (2008) and Carvalho and Grassi (2019), this process satisfies Gibrat's law for productivity such that, away from the boundaries, firms' productivity growth is independent of its current level. Additionally, this law generates a stationary Pareto distribution of productivity (Gabaix 1999). Note, however, that in our environment, this does not immediately imply firm size satisfies these properties, due to the finite number of firms within sectors and endogenous markups.

In section 4.4, we discuss an exercise where we add aggregate TFP shocks. We leave the discussion on the calibration of these shocks to this section.

Calibration Strategy

We now describe how we assign values to the model's parameters: the two demand elasticities ε and σ, the two macro parameters relative risk aversion η and the Frisch labor-supply elasticity f, the number of firms N_k, the demand shifter A_k, and the productivity parameters φ_k, δ_k, and c_k for each of our 504 sectors. Table 2 summarizes parameter values and targets.

We assume that in all sectors, firms compete à la Cournot.[28] We set the two demand elasticities $\sigma = 1.704$ and $\varepsilon = 7$ to target the two following moments. First, according to equation (8), the slope between the inverse of a sector's markup and its HHI is equal to $-\left(\frac{\varepsilon}{\sigma} - 1\right)/\varepsilon$. In section 4.1, we present results for different specifications of this relationship in our data. We target a coefficient of -0.44, reported in column (4) of Table 4.[29] Second, own-cost pass-through rates $\tilde{\alpha}_{ki}$, defined in footnote 16, are shaped by our two demand elasticities. Our choice of σ and ε implies pass-through is roughly 0.5 for large firms (those with a market share of 40%), consistent with estimates in Amiti et al. (2019).[30]

[28]Under Bertrand competition, no elasticity values are able to match our two calibration targets, given a flatter relationship between firm size and markups (both in levels and over time).

[29]In appendix C.2, we report results for the alternative calibration $\sigma = 2.0298$ and $\varepsilon = 7$. For this alternative calibration, we target the slope between the inverse of a sector's markup and its HHI in first differences (rather than in level as in our baseline). This coefficient equals -0.35 as reported in Table 13.

[30]About 2200 firm-year observations have a market share above 40%, which represents the top 0.02% of the market-share distribution.

Table 2: Baseline Calibration

Parameters	Value	Description	Target/Source
ε	7	substitution across firms	pass-through rate for large firms
σ	1.7040	substitution across sectors	slope of μ_{kt}^{-1} on HHI_{kt}
η	1	relative risk aversion	log utility
f	1	Frisch elasticity of labor supply	
f_0	1	labor disutility parameter	does not affect results
S	70	number of productivity bins	see main text
φ_k	1.1201	median firm-level pdty process	vol. 25% and HHI_k
a_k, c_k	0.09,0.05	median firm-level pdty process	vol. 25% and HHI_k
A_k	0.17	median HH preference shifts (pp)	revenue share

In terms of macro parameters, we set the relative risk aversion to 1 (log utility) and the Frisch labor-supply elasticity to 1, both of which are standard values in the business-cycle literature.

We now discuss how we assign the parameter values that vary across sectors. We set the number of firms per sector, N_k, to that observed in the data in 2014. We calibrate the constant sector-level demand shifter, A_k, to target the revenue share of each sector in the data in 2014. We choose the tail parameter of the stationary distribution, δ_k, to match the HHI in the data in 2014.[31] The grid parameter φ_k determines the range of values that the HHI can take as we vary δ_k. We choose the lowest φ_k such that this range of values contains the value of the HHI in the data for this sector.[32] Finally, we set the remaining parameter of the productivity process, c_k, such that in each sector, the volatility of productivity growth is 4.3%. The resulting market-share volatility is roughly 25% for small firms, which roughly matches that in the data (see Panel B of Figure 1). Our calibrated model underestimates the volatility of large firms in the data, as we discuss below.

Panel A of Figure 1 reports HHI in the data against the model counterpart computed in the calibration procedure. Each dot represents a sector whose size is proportional to this sector's revenue in 2014. All the dots are on the 45-degree line represented by the black dashed line. The calibration procedure is successful at reproducing the HHI in each of the 504 sectors.

Panel B of Figure 1 shows the relationship between firm volatility and size. Using both data and simulated data from the model, we compute firm growth rates in each period. We then compute the standard deviation of this growth rate across firms in a market-share bin.[33] In

[31] Given a guess of δ_k, we draw 1,000 samples of N_k firm-level productivities from the Pareto distribution characterized by δ_k. For each of these samples, we solve for firm-level market shares and compute the implied HHI. We then calculate the median HHI across the 1,000 samples, and iterate over δ_k until we match the HHI for a given sector in the data. We repeat this procedure for each of the 504 sectors.

[32] We choose the number of productivity bins $S = 70$. Higher values of S do not make a significant difference for our results.

[33] Computing for each firm, over time, the standard deviation of its market share and then binning gives very

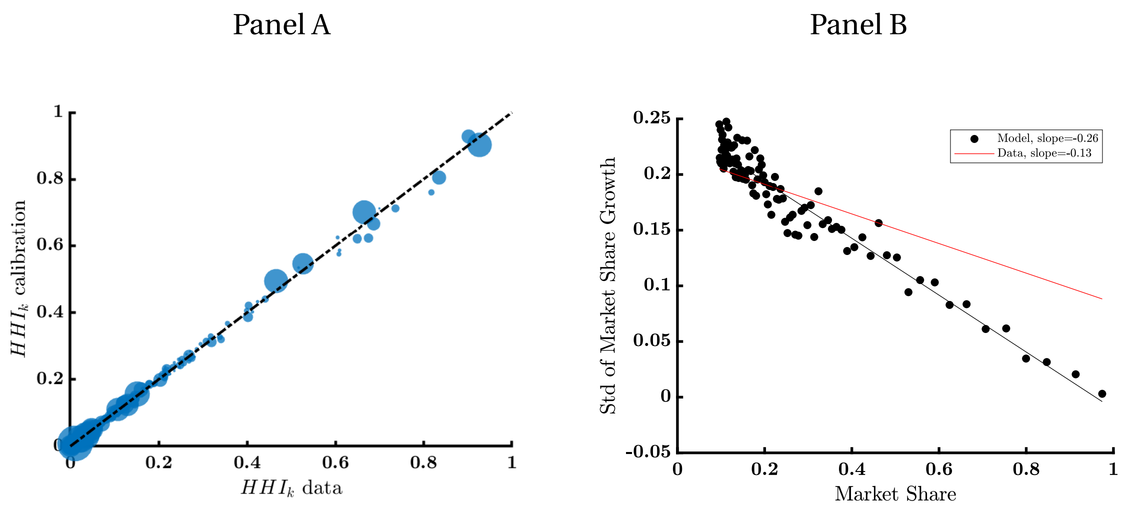

Figure 1: Model Fit

NOTE: Panel A shows, for each sector, the HHI in the data (x-axis) against the median HHI computed over 1,000 samples drawn from the baseline calibration (y-axis). The size of each dot is proportional to the sector's revenue share in 2014. Panel B reports the standard deviation of market-share growth for each market-share bin in the cross-section. The black dots and the black line are computed on simulated data from the model. The red line is computed using the FICUS-FARE data.

Panel B, black dots are the simulated data, the black line is a linear fit on simulated data, and the red line is a linear fit on the data (we are unable to report individual data points due to confidentiality rules). Although the intercept of the black line is a target of our calibration, the slope is not. Note our calibrated model underestimates the volatility of large firms compared to the data, and hence the exercises below likely give a lower bound on the aggregate volatility arising from large firm shocks.

Note our calibration strategy relies on estimates of markups only through the coefficient of the HHI in the sector-level inverse-markup regression. Alternatively, we can consider a moment of the data that does not require estimates of markups but imposes the market structure in our model. As described in appendix C.1, under our baseline calibration, the model roughly matches this untargeted moment.

In what follows, we use the calibrated model as a data-generating process to simulate firm-level, sector-level, and aggregate-level time series. We use the simulated sector-level and firm-level panels to run the corresponding regressions that we run on actual data. We also compute aggregate business-cycle statistics using the simulated aggregate time-series, which we then compare with counterparts in the data.

similar results.

4 Model meets Data

In this section, we interpret firm, sector, and aggregate markup dynamics through the lenses of our framework. We start by providing empirical evidence on the behavior of markups as a function of concentration measures, both within and across sectors. We additionally document that within-firm markup dynamics account for a non-negligible fraction of sector-level and aggregate markup dynamics. We then study some reduced-form notions of markup cyclicality, showing our model can rationalize the sign and magnitude of alternative, reduced-form moments in data. We conclude by quantifying the magnitude of aggregate markup movements in our granular setup and discussing the importance of endogenous markup dynamics in this context.

4.1 Inspecting the Mechanism: Firm and Sector-level Evidence

Hardwired into our model are two key relationships between markups and concentration. At the firm-level, and following the discussion in section 1.2, markups increase with a firm's market share. In turn, this immediately gives rise to a notion of markup pro-cyclicality at the micro-level: A firm's markup increases whenever its market share increases. At the sector-level, as discussed in section 1.3, equilibrium aggregation of firm-level outcomes yields that sectoral markups increase in the HHI of that sector. By the same token, this yields a simple notion of markup cyclicality at the sector level: A sector's markup increases whenever its level of concentration increases. We now evaluate whether these two relations, at the firm and sector levels, hold empirically in the data.

Starting at the micro-level, note that taking the inverse of equation (5) for Cournot yields a simple relation between the firm's market share and its markup,

$$\mu_{kit}^{-1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} s_{kit}, \tag{36}$$

where μ_{kit}^{-1} is the inverse of the (gross) markup of firm i in sector k at time t and s_{kit} is its market share. In turn, this suggests the following simple empirical specification,

$$\mu_{kit}^{-1} = \gamma_i + \alpha_t + \beta s_{kit} + \epsilon_{kit}, \tag{37}$$

where β is the coefficient of interest, which the model predicts to be negative. We include firm fixed-effects γ_i to control for unobserved heterogeneity, and year fixed-effects α_t to control for unobserved markup shifters that are common across all firms. Note these fixed effects are not present in our model. As a robustness check, in appendix B.2, we report estimates based on a first-difference specification with no firm fixed effects.

We start by inspecting these firm-level relations in the French data. Recall from our discussion

Table 3: Firm Inverse Markup and Market Share: Level

	Dependent Variable: μ_{kit}^{-1}			
Coefficient	(1)	(2)	(3)	(4)
s_{kit}	-4.853	-4.852	-.663	-.589
	(.273)	(.273)	(.103)	(.101)
Year FE	N	Y	N	Y
Firm FE	N	N	Y	Y
Number of Firms	1 284 905	1 284 905	1 154 181	1 154 181
Observations	10 928 469	10 928 469	10 727 745	10 727 745

NOTE: μ_{kit}^{-1} is the inverse of firm i sector k gross markup in year t, and s_{kit} gives the market share of firm i in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parentheses) are clustered at the firm level. Markups are winsorized at the 2% level.

in the previous section that we have estimated firm-level markups for the population of French firms over the period 1994-2016. Firm-level market shares are immediate to calculate in data by dividing firm-level revenue by the corresponding five-digit NAF sector revenue (abstracting, recall, from sales by foreign firms). This yields time series for μ_{kit} and s_{kit} for each firm in data.

Table 3 summarizes the results. Column (1) displays the firm-level relation between inverse markup and market share in the data without fixed effects. Pooling all firm-level data (across sectors and years) for a total of over 10 million observations of markups and market shares gives a negative and statistically significant coefficient. Further, including year fixed effects (column 2) does not alter this relation.

In our baseline specification, when we additionally include firm-fixed effects (columns 3 and 4), we again find negative and significant estimates. Point estimates are now smaller, indicating the importance of controlling for unobserved heterogeneity in the data. Results for the specification in first-differences, displayed in Table 12 in the appendix, are very similar to those in levels (with fixed effects) in columns (3) and (4) of Table 3.

Turning to our sector-level predictions, by the same logic as above, note that taking the inverse of equation (8) yields the following relation between inverse sectoral markup and the sector's HHI:

$$\mu_{kt}^{-1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} HHI_{kt}. \tag{38}$$

To assess this relationship in the data, we consider the following empirical specification:

$$\mu_{kt}^{-1} = \gamma_k + \alpha_t + \beta HHI_{kt} + \epsilon_{kt}, \tag{39}$$

where γ_k is a sector fixed-effect and α_t is a time fixed-effect (which are present in our model

Table 4: Sector Inverse Markup and Sector HHI: Level

	Dependent Variable: μ_{kt}^{-1}			
Coefficient	(1)	(2)	(3)	(4)
HHI_{kt}	-.7267	-.7281	-.4335	-.4436
	(.2310)	(.2315)	(.1102)	(.1094)
Year FE	N	Y	N	Y
Sector FE	N	N	Y	Y
Number of Sectors	504	504	504	504
Observations	11 592	11 592	11 592	11 592

NOTE: μ_{kt}^{-1} is sector k (inverse) markup in year t, HHI_{kt} is the HHI in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1994-2016) data, aggregated to the sector level. Standard errors (in parentheses) are clustered at the sector level.

only if ε varies across sectors). In appendix B.2, we report estimates based on a first-difference specification. To construct sectoral markups, we aggregate firm-level markups to their sector-level counterparts by taking an harmonic weighted average of firm-level markups - as indicated by the model equation (7) - within 504 narrowly defined five-digit NAF sectors.

Table 4 displays estimates of a pooled regression across all sectors and years, for different fixed-effect combinations. Our estimates indicate a negative and significant relation between the level of concentration and the inverse of sector markups and are stable across fixed-effect configurations. We use the specification with time and sector fixed effects (column 4) as a target in the calibration of our model. Finally, note that estimates are very similar for the alternative specification in first-differences, as shown in Table 13 in the appendix. Taken together, these firm- and sector-level estimates confirm a basic qualitative prediction of our model in the French data.

Our model additionally imposes cross-equation restrictions. Comparing equations (37) and (38), note the slope coefficients of these two relations - that is, the slope of the inverse of firm markup on market share and the slope of the inverse sector markup on HHI - should coincide. We can formally test for the equality of the two slope coefficients in the data by forming a simple Z-score. We focus on the specifications controlling for sector and time fixed effects. Despite differences in the point estimates, we cannot reject the null that the two coefficients are the same (Z-score of 1.20)

4.2 Markup Decomposition over Time

We now consider a within-between decomposition of sectoral and aggregate markups in the data and in the model. Specifically, we report how much of year-on-year changes in sector- and aggregate-level markups is due to changes in firm-level markups and how much is due to reallocation across firms. We first consider changes in markups at the sector level and then at the aggregate level.

Sector-Level Decomposition By equation (7), the change in sectoral markups between two time periods is

$$\Delta\mu_{kt}^{-1} = \sum_{i=1}^{N_k} \Delta\mu_{kit}^{-1}\, \bar{s}_{kit} + \sum_{i=1}^{N_k} \Delta s_{kit}\, \overline{\mu}_{kit}^{-1},$$

where Δ denotes the year-on-year difference and bars denotes averages over two consecutive years. The first term on the right-hand-side is the within term: It measures change in the (inverse) sectoral markup due to changes in firm-level markups, evaluated at the average market share. The second term on the right-hand-side is the between or reallocation term: It measures the change in the (inverse) sectoral markup due to the changes in firm market share, evaluated at the average (inverse) markup. As discussed in section 2.2 and shown in appendix A.2, in the model under Cournot competition, the within and between terms are equal to each other in every sector. From this result, it follows that the contribution of the within and between/reallocation terms are both equal to 50%.

Given time series of firm-level markups and market shares in the data and model, we calculate sectoral markups as well as the within and between terms. For each sector, we regress the within term over time on changes in sector-level markup. The coefficient of this regression is the contribution of the within term to the evolution of sector-level markups. For the median sector, the within term accounts for 65% of the changes in sector markups. For half of the sectors, the contribution of the within term lies between 42% and 82%.[34]

Aggregate-Level Decomposition By equation equation (10), the change in the aggregate markup between two time periods is

$$\mu_t^{-1} = \sum_{k=1}^{N} s_{kt}\mu_{kt}^{-1} = \sum_{k=1}^{N}\sum_{i=1}^{N_k} (s_{kt} \times s_{kit})\, \mu_{kit}^{-1},$$

where s_{kt} is the revenue share of sector k and s_{kit} is the market share of firm i in sector k. Note that $s_{kt} \times s_{kit} = (P_{kit}Y_{kit})/(P_tY_t)$ is equal to revenues of firm i as a share of aggregate revenues.

[34] In the data, the reallocation term is not only due to changes in market share across firms, but also due to churning as some firms enter and exit the market each year. We define the reallocation term as the residual obtained from the difference between the change in (inverse) sectoral markup and the within term.

Analogously to the sectoral-markup decomposition, changes in the (inverse) aggregate markup can again be decomposed into a within and a reallocation term. The within term is defined as the change in the (inverse) aggregate markup due to changes in firm-level markups, evaluated at the average firms' aggregate revenue share. The between term is the change in the (inverse) aggregate markup due to changes in firms' aggregate revenue share, evaluated at the average (inverse) markup.

When computing the contribution of the within and reallocation term in the data, we find that 67.1% of year-on-year changes in the (inverse) aggregate markup is due to the within term, and the remaining 32.9% is due to the reallocation term. In the model, and under our baseline calibration, we find that 79.7% of year-on-year changes in the (inverse) aggregate markup is due to the within term, and 20.3% is due to the reallocation term. That is, both in the data and the model, more than two-thirds of the year-on-year movement in the aggregate markup is due to within firms' change in markups.

We conclude that both in the model and in our data, changes in firm-level markups account for a sizeable fraction of year-on-year changes in sectoral and aggregate markups.

4.3 Reduced-Form Varieties of Markup Cyclicality

Our theoretical framework yields a simple relation between markups and size: The level of a firm's markup is determined by its market share within a sector. Further, aggregation of this relation yields a relation between a sector's markup and its level of concentration, both across sectors and dynamically over time. As we have seen, the data broadly supports these relations.

By contrast, a large applied literature investigates different definitions of "markup cyclicality." This literature yields a variety of results, with some contributions arguing for pro-cyclicality and others concluding in favor of counter-cyclicality.

In this section, we argue these conflicting empirical results can be largely ascribed to the alternative reduced-form exercises pursued and, in particular, to the reduced-form definitions of markup cyclicality being deployed in the literature. Importantly, as we show, our model with firm-level shocks only can go a long away in accounting for these seemingly conflicting reduced-form relations in the data.

4.3.1 Firm-level Evidence

We start by analyzing a firm-level notion of reduced-form markup cyclicality and ask how do firm markups covary with the respective sector-level output.

Before going to the data, recall that our setting is a granular one in which extensive within-sector heterogeneity in the firm-size distribution enables large firm dynamics to lead the sector business cycle. In particular, in our setting with idiosyncratic firm-level shocks – and if pass-through rates do not fall too strongly with market shares – sector-output fluctuations are necessarily led by shocks to very large firms. To make matters concrete, consider a positive idiosyncratic (demand or technology) shock hitting a large market-share firm. Given the granular nature of the economy, the corresponding sector output will typically increase (see equation 26). In addition, the large shocked firm will increase its market share and markup. This implies, as indicated in Proposition 4, that markups of large firms should comove positively with sector output.

By the same token, the average (small) firm in a given sector loses market share to the very largest firms: If sector-output expansions are led by large firms, the latter will increase their market share whereas the average firm loses competitiveness - as evaluated by its market share within the sector. Again, due to the markup-market-share relation in our setting, this implies the average firm-markup is expected to comove negatively with sector output, as summarized by Proposition 4.

To evaluate this implication of the model, we implement the following reduced-form regression, both in the data and in our model-simulated data:

$$\ln(\mu_{kit}) = \alpha_i + \gamma_t + \beta_1 \widehat{Y}_{k,t} + \beta_2 \widehat{Y}_{k,t} \times s_{kit} + X_{kit} + \epsilon_{it}, \tag{40}$$

where μ_{kit} is firm i sector k gross markup in year t, $\widehat{Y}_{kt}$ is the deviation of sector k (log) value-added in year t from its HP trend, and X_{kit} is a set of firm-level controls for firm i in sector k at year t, which includes market share.[35] Finally, α_i is a firm fixed effect, which controls for time-invariant firm-level unobservables determining the average level of a firm's markup, while γ_t is a year fixed effect. In this specification, β_1 captures the average correlation between firm markups and their respective sector output, and coefficient β_2, in the interaction term, captures heterogeneity in this relation as a function of a firm's market share.[36]

Before proceeding, note Hong (2017) runs a version of this regression, where Y is aggregate (rather than sector) value-added, using data for four large European countries. For these data, Hong (2017) estimates a negative β_1 and a positive β_2 estimate, concluding that (i) in the data "markups are countercyclical" and (ii) that there is "substantial heterogeneity in markup cyclicality across firms, with small firms having significantly more countercyclical markups than large firms."

Columns (1) and (2) of Table 5 summarize the estimates obtained when implementing the above reduced-form regression on our French firm-level data with and without firm-level controls. We see that, for the average firm, markups are weakly "countercyclical" with respect to

[35] To obtain sector value-added, we sum firm-level nominal value-added to the NAF five-digit level and deflate using EUKLEMS sectoral price deflators.

[36] Note that according to our model, given the parameters ε and σ, market share determines markup (equation 5). For this reason, we also estimate equation (40) without firm-level controls.

Table 5: Firm Markup and Sector Output

Dependent variable: $\ln(\mu_{kit})$				
Coefficient	(1) Data	(2) Data	(3) Model	(4) Model
$\widehat{Y}_{kt}$	-0.093	-0.068	-0.093	-0.014
	(0.051)	(0.042)		
$\widehat{Y}_{kt} * s_{kit}$	0.209	0.192	0.598	0.089
	(0.092)	(0.082)		
Firm Controls	N	Y	N	Y
Firm FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Firms	1 154 181	1 154 181	650 423	650 423

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\widehat{Y}_{kt}$ is the deviation of sector k (log) value-added in year t from its HP trend. Columns (1) and (2) report empirical estimates for the FICUS-FARE (1994-2016) data. Columns (3) and (4) report estimates based on model-simulated data. This regression is weighted by average revenue. Standard errors in the data are clustered at the sector$\times$year level. Calculating standard errors across various samples of model-generated data is computationally (memory) intensive. We will report them in the next draft.

own-sector output. Further, we additionally confirm that there is substantial heterogeneity in this relation. In particular, the estimates on the interaction term imply large firms, roughly with market shares above 0.35, are procyclical with respect to the dynamics of sectoral output.[37]

Columns (3) and (4) of Table 5 implement the same reduced-form regressions on model-simulated data for 650,423 firms distributed across 504 sectors. The model is able to reproduce the patterns observed in the data. Consistent with Proposition 4, markups for the average firm are weakly countercyclical with respect to own-sector output, whereas large firms' markups are procyclical. Furthermore, point estimates in the model-simulated and French data are of the same order of magnitude.

Underlying this prediction of the model for the heterogeneous cyclicality of markups with sectoral output is that with firm-level shocks, large firms' market shares are correlated positively with sector output whereas small firms' market shares are countercyclical. To assess this mechanism, we estimate the following regression:

$$s_{kit} = \alpha_i + \gamma_t + \beta \widehat{Y}_{kt} + \epsilon_{kit},$$

where s_{kit} is the market share of firm i in sector k, $\widehat{Y}_{kt}$ is as above the sector k value-added, α_i is a firm-level fixed effect, and γ_t is a year fixed effect. In this specification, β measures the relation

[37] Hong (2017) also reports another version of this regression where aggregate value-added is interacted with an indicator for large firm. When we estimate a version of this reduced-form specification where sector value-added is interacted with an indicator for market-shares over 30%, we find a coefficient of 0.168 (0.07) on the interaction term.

Table 6: Firm Market Share and Sector Output

Dependent variable: s_{kit}						
Coefficient	(1) Data (all data)	(2) Data ($s_{kit} < 0.30$)	(3) Data ($s_{kit} > 0.30$)	(4) Model (all data)	(5) Model ($s_{kit} < 0.30$)	(6) Model ($s_{kit} > 0.30$)
$\widehat{Y}_{kt}$	-.002	-.002	.042	-.000	-.001	.482
	(.0002)	(.0002)	(.027)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Firms	1 154 181	1 154 057	124	650 423	650 335	88

NOTE: s_{kit} gives the market share of firm i in sector k, year t, and $\widehat{Y}_{kt}$ is the deviation of sector k (log) value-added in year t from its HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) reports estimates based on model-simulated data. Standard errors in the data are clustered at the sector×year level. Calculating standard errors across various samples of model-generated data is computationally (memory) intensive. We will report them in the next draft.

between market share and sector value-added. We implement this regression (i) on the whole sample of firms, (ii) on the subsample of firms whose average market share is lower than 30%, and (iii) on the subsample of firms whose average market share is above 30%.

Columns (1) and (4) in Table 6 report estimates of β on the full sample of the data and on the model-simulated data, respectively. Both in the data and in the model, the average firm's market share is counter-cyclical. Columns (2) and (5) report estimates for the subsample of firms whose market share is lower than 30%. Estimates of β are negative both in the data and in the model. Finally, columns (3) and (6) report estimates for the subsample of firms whose market share is above 30%. Estimates of β are positive both in the data and in the model, although the magnitude is smaller in the data (significant at the 11.8% level).

Taken together, the results in Tables 5 and 6 provide support for a key implication of our granular model with firm-level shocks. The average firm's market share and markup are countercyclical with respect to its own sector value-added, whereas large firms' market share and markups are procyclical.

4.3.2 Sector-Level Evidence

We now explore sector-level notions of markup cyclicality. We first ask how sector markups covary with own-sector output.

Recall that in our granular setting sectoral business cycles are driven by large firm dynamics, and that shocks to large firms induce a positive relationship between changes in sector-level output and markups. As encoded in Proposition 3, we should expect a positive correlation between sector markup and sector output.

Table 7: Sector Markup and Sector Output

Dependent variable: $\widehat{\mu}_{kt}$		
Coefficient	(1) Data	(2) Model
$\widehat{Y}_{kt}$	0.102	0.132
	(0.028)	[0.033; 0.195]
Sector FE	Y	Y
Year FE	Y	Y
Number of Sectors	504	504

NOTE: Regression of sector-level (log) change markup ($\widehat{\mu}_{kt}$) on sector value-added (log) change ($\widehat{Y}_{kt}$). Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data, and standard errors (in parentheses) are clustered at the sector level. Columns (2) reports estimates based on model-simulated data. The point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 independent simulated samples, each of the same length (23 years) as the French data. Terms in square brackets give, respectively, the 5 and 95 quantiles of coefficient estimates across simulated samples.

To assess this relationship in the model and data, we consider the following sector panel regression:

$$\widehat{\mu}_{kt} = \alpha_k + \gamma_t + \beta \widehat{Y}_{kt} + \epsilon_{kt}, \tag{41}$$

where $\widehat{\mu}_{kt}$ denotes sector k's log change in markup (output) between $t-1$ and t, and $\widehat{Y}_{kt}$ denotes the log change in sector k value-added. Sector-level markups are aggregated from firm-level estimates according to a harmonic weighted average, as indicated by the model equation (7). We measure sector value-added in the data as in the previous section (see footnote 35). Finally, we include sector and year fixed effects to control for unobservables.

Nekarda and Ramey (2013) consider a similar version of this regression based on US sectoral data. They find β is robustly positive and significant and therefore that "markups are generally procyclical (...) hitting troughs during recessions and reaching peaks in the middle of expansions." Column (1) in Table 7 reports estimates of β in our data. Sector markups comove positively and significantly with sector output, which is consistent with the findings in Nekarda and Ramey (2013) despite differences regarding the country of analysis, sample period, and the methods deployed to estimate markups.

Column (2) in Table 7 reports estimates of β in model-simulated data. In particular, we report the median (as well as 5th and 95th percentiles) estimate of β over 5,000 independent samples of 23 years each. Figure 2 displays the full histogram of estimated coefficients over the 5,000 simulated samples. As in the data, the model implies a positive correlation between sector markup and sector output.

The more recent work by Bils et al. (2018) explores yet another reduced-form notion of markup

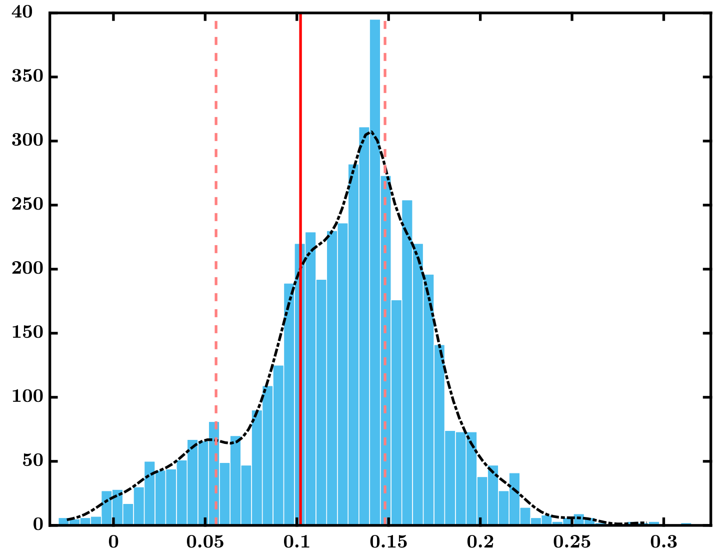

Figure 2: Histogram of Sector Markup on Sector Output Slopes in Model Simulations

NOTE: Kernel density of estimated regression coefficient on model-simulated data from equation (41) based on 5,000 repetitions of independent samples. The solid red lines is the estimate in the data while the dashed red lines reports the lower and upper bound of the 10% confidence interval.

cyclicality: Do sector-level markups comove systematically with aggregate GDP fluctuations? That is, unlike the previous specification, markup cyclicality is evaluated with respect to its comovement with aggregate GDP rather than sector-level output.

To understand this form of comovement in the context of our model, note that sector markups only react to within-sector firm shocks. Over long samples, as summarized in Proposition 5, the model implies positive comovement of a sector's markup with aggregate GDP. Over a given cyclical episode, a positive comovement is expected if the fluctuation in aggregate economic activity is due to large firm dynamics in the same sector. More generally, aggregate output movements reflect shocks hitting other sectors in the economy. If a sector comoves negatively with aggregate output, a negative correlation of that sector's markup with aggregate output will obtain. Overall, we should expect a weak correlation between the average sector's markup and aggregate GDP fluctuations.

To explore this logic, we implement the following regression:

$$\widehat{\mu}_{kt} = \alpha_k + \beta \widehat{Y}_t + \epsilon_{kt}, \tag{42}$$

where $\widehat{\mu}_{kt}$ is the deviation of sector k's markup in year t from its HP trend, $\widehat{Y}_t$ gives the HP-trend deviation of (log) aggregate value-added in year t, and α_k is a sector fixed effect. Sector-level markups are computed as above by taking a weighted harmonic mean of firm-level markups,

Table 8: Sector Markup and Aggregate Output

Dependent variable: $\widehat{\mu}_{kt}$		
Coefficient	(1) Data	(2) Model
$\widehat{Y}_t$	0.191	0.009
	(0.161)	[-0.041, 0.217]
Sector FE	Y	Y
Number of Sectors	504	504

NOTE: Regression of sector k's markup in year t in deviation from its HP trend $\widehat{\mu}_{kt}$ on $\widehat{Y}_t$ the HP-trend deviation of (log) aggregate real value-added in year t. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parentheses) are clustered at the sector level. Column (2) reports estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 independent simulated samples, each of the same length (23 years) as the French data. Terms in square brackets give, respectively, the 0.05 and 0.95 quantiles of coefficient estimates from the simulated data.

and aggregate value-added is computed by summing firm-level value-added deflated by the respective EU-KLEMS sector-level deflator.

Bils et al. (2018) consider a version of this regression based on US data. They conclude that "the price markup is estimated to be highly countercyclical" with the possible exception of service industries, for which they find evidence favoring procyclicality.

Column (1) of Table 8 summarizes our estimates based on French data. We do not find a statistically significant relation between sectoral markups and aggregate GDP. For the average French sector, the data suggests that this relation is acyclical.

We now consider the relationship between sectoral markups and aggregate output implied by our model. Column (2) of Table 8 reports the median (as well as 5th and 95th percentiles) estimate of β over 5,000 independent samples of 23 years each. Figure 3 displays the full histogram of estimated coefficients over the 5,000 simulated samples. Our model implies a positive median slope between the average sector markup and aggregate GDP. However, as in the data, the histogram points to considerable uncertainty over the magnitude and sign of this slope. This finding is consistent with the intuition above: Over small samples, we may expect either a positive or negative correlation to arise, depending on whether the sectors driving within-sample aggregate dynamics have, respectively, higher or lower levels of sectoral markups with respect to the aggregate. Thus, both the model and the data imply markups are acyclical (with large dispersion across samples) when evaluated through this particular reduced-form statistic.

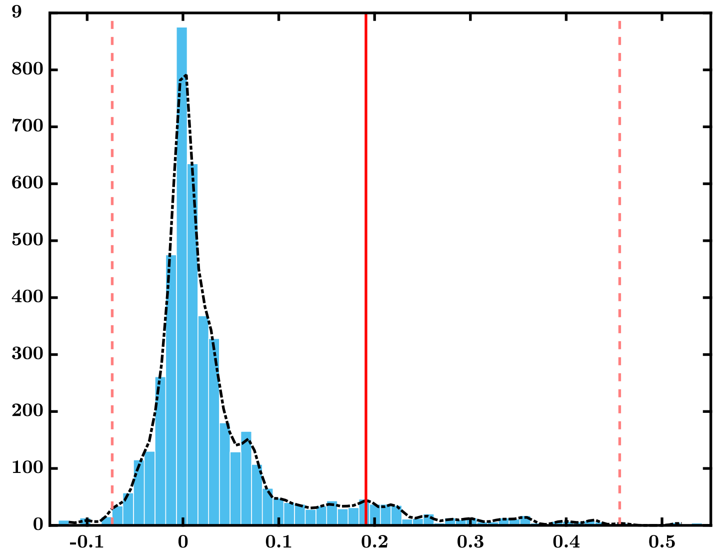

Figure 3: Histogram of Sector Markup on Aggregate Output Slopes in Model Simulations

NOTE: Kernel density of estimated regression coefficient on model-simulated data from equation (42). 5,000 repetitions of independent samples. The solid red lines are the estimate in the data, and the dashed red lines report the lower and upper bound of the 10% confidence interval.

4.4 Aggregate Markup Cyclicality and Output Fluctuations

In this final section, we turn our attention to fluctuations in aggregate markups and output. We first consider only idiosyncratic firm-level shocks according to the Markov process introduced above. Recall that in our environment, these shocks constitute the only source of markup and output fluctuations at the firm, sector, and aggregate levels. We then introduce aggregate productivity shocks to fully account for aggregate output volatility in our data.

Using our FICUS-FARE data, we construct aggregate markups, μ_t, as a weighted harmonic mean of firm-level markups, and aggregate GDP, Y_t, as the sum of firm-level value-added. We detrend these variables using an HP-filter. Using our calibrated model, we simulate 5,000 samples of 23-year firm-level histories. We implement the same procedure to construct detrended time-series of simulated aggregate output and markup. Table 9 presents data- and model-based estimates of the correlation and standard deviation of aggregate output and markups.

We first consider aggregate markup cyclicality. Recall from expression (34) that our model implies a positive comovement between aggregate output and aggregate markups, unless larger sectors have lower markups or, for finite samples, if a particular expansionary episode is driven by a sector with a sufficiently low markup, in which case negative comovement may obtain. That is, whereas we should observe positive comovement over sufficiently long samples, in any given short sample, comovement may be absent or negative depending on sectors driving the aggregate dynamics.

In Table 9, we can see that both in the data and in the model, aggregate markup is procyclical with respect to aggregate output. Our model predicts, however, much higher aggregate procyclicality than that observed in the data: The correlation between the aggregate markup and aggregate output is 0.13 in the data and 0.87 (on average, across shock realizations) in the model.

Our model predicts large variation in this correlation coefficient across small samples, depending on which sectors are driving aggregate dynamics and their relative levels of sectoral markups. To see this variation at play, Panel A of Figure 4 plots the histogram of correlation coefficients $\rho(\mu_t, Y_t)$ across our 5,000, 23-year samples. A non-negligible number of samples display only weak procyclicality (as in the data) because, over small samples, aggregate output dynamics may be driven by positive shocks to large firms in large sectors that nevertheless have relatively lower markups, inducing a negative correlation.

Next, we examine the magnitude of aggregate, granular fluctuations in output and markups implied by our model. Recall from our analytic results that incomplete pass-through weakens (relative to the specification of the model with heterogeneous but constant markups) the response of aggregate output to firm-level shocks, as implied by equation (31) — derived under parameter restrictions that we relax in our quantitative analysis.

Table 9 shows the standard deviation of aggregate output is 1.71% in our French data and 0.71% in our model (average across samples). That is, the volatility of aggregate output in our model is 42% of the volatility in the data.

How large are granular movements in aggregate markups? The ratio of the standard deviation of aggregate markup to that of aggregate output is 57% in the data and 33% in our calibrated model (average across samples). However, across samples, we see large variation in this ratio, with many instances where this ratio exceeds that in the data, as can be seen in Panel B of Figure 4.

Although our model with firm-level shocks generates non-negligible fluctuations in aggregate output and markups, it only accounts for a fraction of aggregate fluctuations in the data. Moreover, as we discussed above, the correlation of markups and output is much higher than that in the data. In what follows, we show that if we superimpose on our calibrated model aggregate productivity shocks, in order to match aggregate output volatility in the data, the markup pro-cyclicality of markups is much lower and closer to the data.

Specifically, we assume firm-level productivity is given by $\widetilde{Z}_t \times Z_{ikt}$, where $\widetilde{Z}_t$ is normally distributed and independent across periods. We choose the standard deviation of $\widetilde{Z}_t$ to exactly match the volatility of aggregate output in the data. Panel (2) of Table 10 shows the implied business-cycle moments for the median 23-year sample. As discussed in section 1, aggregate

Table 9: Aggregate Markup and Aggregate Output

	(1) Data			(2) Model		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
Y_t	1.71	1	1	0.71	1	1
μ_t	0.96	0.57	0.13	0.21	0.33	0.87

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for aggregate output Y_t and aggregate markup μ_t, both in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) reports the average over 5,000 independent simulated samples, each of 23 years.

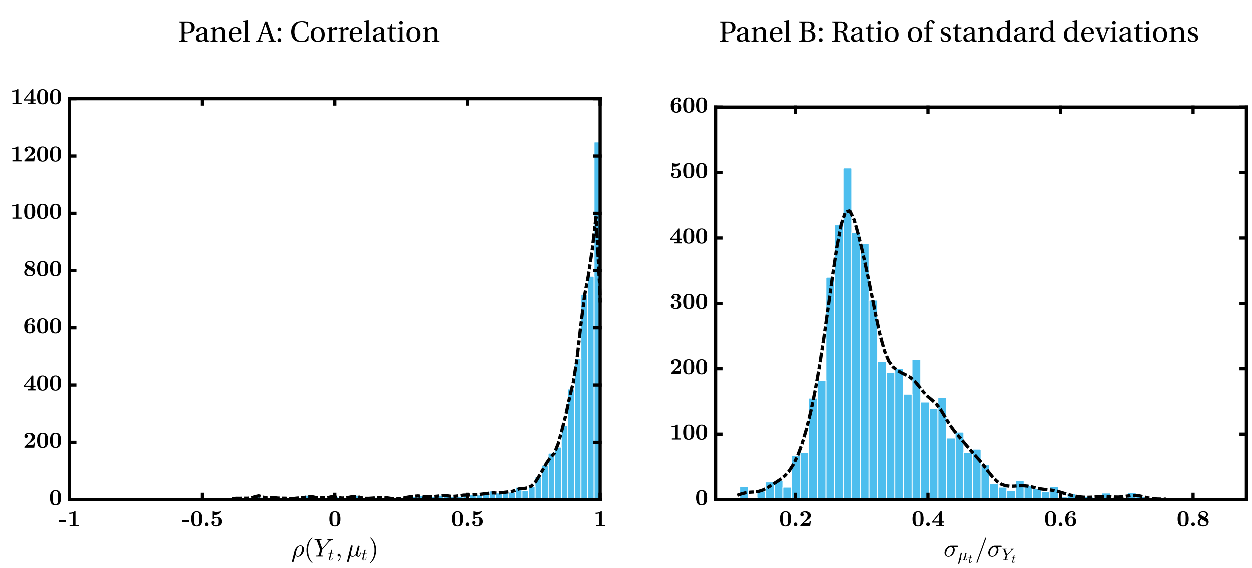

Figure 4: Histogram of Correlation and Relative Standard Deviations of Aggregate Markups and Output in Model-Simulated Data

NOTE: Kernel density of $\rho(\mu_t, Y_t)$, the correlation coefficient between aggregate markups and aggregate output, and $\sigma(\mu_t)/\sigma(Y_t)$, the ratio of standard deviation of aggregate markups and aggregate output, on model-simulated data based on 5,000 repetitions of 23 period samples.

Table 10: Aggregate Markup and Aggregate Output with Aggregate Productivity Shocks

	(1) Data			(2) Model (Productivity shocks)		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
Y_t	1.71	1	1	1.71	1	1
μ_t	0.96	0.57	0.13	0.21	0.12	0.33

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for aggregate output Y_t and aggregate markup μ_t, both in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) reports the average over 5,000 independent simulated samples from a model with TFP, each of 23 years. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output in France.

shocks do not affect firm market shares and markups, and hence the volatility of aggregate markups is unchanged relative to the model with only firm-level shocks. Because output-movements-driven aggregate productivity shocks are uncorrelated with markups, the correlation between aggregate markup and output falls to 0.33. Accordingly, the histogram of this correlation across 23-year samples (Figure 6 in the appendix D) is now shifted to the left. In roughly 15% of our 23-year samples, aggregate markups are countercyclical.[38]

Role of changes in firm-level markups for aggregate results Whereas the within/between decomposition in section 4.2 demonstrates the importance of changes in firm-level markups to account for changes in aggregate markups in the model and data, as we discussed in section 2, it is not necessarily informative of how different movements in aggregate output and markups would be if firm-level markups were heterogeneous but fixed over time.

To answer this question, we apply quantitatively the first-order-approximation analytic expressions in section 2. In Appendix A.7, we provide expressions for correlations and volatilities under variable markups versus constant markups, given market shares and markups in the initial equilibrium. Because in our model, the distribution of firms by sector changes every period, we consider 1,000 independent samples drawn from the equilibrium in our quantitative model.[39]

Consider first movements in aggregate output. We compare the standard deviation of aggregate

[38] By the same token, one could introduce additional forces or shocks to exactly match aggregate markup volatility. As a crude first step to explore the implications of additional markup shocks on markup cyclicality, we consider an extension in which aggregate markup is $\mu_t = \widetilde{\mu}_t \left(\sum_{k=1}^{N} \mu_{kt}^{-1} s_{kt}\right)^{-1}$, where $\widetilde{\mu}_t$ is normally distributed and independent across periods. We calibrate the standard deviation of this shock to match the volatility of aggregate markups in the data. Aggregate markup shocks are contractionary and thus reduce the correlation between markup and output. The resulting correlation between aggregate markups and output combining both aggregate shocks is -0.17, with large variation across samples.

[39] The magnitudes of correlations and ratios of standard deviations based on the first-order approximations are remarkably close to those in our quantitative non-linear results. For the median sample, the standard deviation of aggregate markup relative to output is 0.386 (vs. 0.33 in our quantitative non-linear results) and the correlation between aggregate markup and output is 0.897 (vs. 0.87 in our quantitative non-linear results).

output under variable markups with that under heterogeneous but constant markups, given the same initial firm-level expenditure shares and markups and assuming the same volatility of firm-level shocks.[40] For the median sample, the standard deviation under variable markups is 0.92 times that under heterogeneous but constant markups (the 95% confidence interval is 0.80-1.00). As explained in section 2, incomplete pass-through (given pass-through rates that are decreasing in size) reduces aggregate output volatility as does a decline in firm concentration.[41,42]

Consider now movements in aggregate markups. For the median sample, the standard deviation of aggregate markups under variable markups is 0.96 times that under heterogeneous but constant markups (the 95% confidence interval is 0.84-1.00). As discussed in section 2, variable markups do not magnify changes in sectoral markups relative to the model specification with constant markups because incomplete pass-through mutes the extent of between-firm reallocation. The median volatility of aggregate markups relative to output is 0.386 under variable markups and 0.372 under heterogeneous but constant markups. The median correlation between markups and output is 0.897 under variable markups and 0.932 under constant markups (the 95% confidence interval for the difference in correlations is 0-0.09).

Overall, we see the magnitude and cyclicality of aggregate markups in our model is not too different when we counterfactually fix markups at their initial, heterogeneous level. Of course, rather than exogenously fixing markups, our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes. Further, as argued above, this theory is consistent with a number of observations about markup changes in the data, at the firm, sector, and aggregate levels.

[40] Market shares of large firms are less volatile under variable markups than under constant markups (see equation (47) in the appendix). One could compare aggregate volatilities under these two specifications after adjusting the size of firm-level shocks to keep the same average volatility of market-shares (which, recall, is a target in our baseline calibration). If we match an unweighted average of these market-share volatilities, our results remain roughly unchanged. If we target a weighted average of these market-share volatilities, aggregate volatility is slightly higher under variable markups. In all cases, variable markups have a limited impact on reducing aggregate output volatility.

[41] Whereas variable markups reduce the volatility of aggregate output, markup heterogeneity per se contributes to higher aggregate volatility. By equation (30), markup heterogeneity under constant markups does not affect output changes with linear disutility of labor ($f \to \infty$). However, with finite labor disutility, reallocation of economic activity across heterogeneous markup firms is an additional source of output fluctuations, as studied in detail in Baqaee and Farhi (2019). In our model, the median standard deviation under heterogeneous and constant markups is 1.2 times that under homogeneous and constant markups, for given initial expenditure shares (the 95% confidence interval is 1.18-1.26). Combining both results, the standard deviation under variable and heterogeneous markups is 1.1 times that under homogeneous and constant markups (the 95% confidence interval is 1-1.18). That is, variable markups result in higher output volatility than constant and homogeneous markups.

[42] We also consider an alternative initial equilibrium to evaluate our analytic expressions, using firm-level shares observed in the data in 2014 rather than using data from the calibrated model. The results are similar qualitatively but starker quantitatively. The standard deviation under variable markups is 0.79 times that under heterogeneous but constant markups, and 0.99 times that under homogenous markups. The correlation between aggregate markups and output is 0.04 lower under variable markups than under heterogeneous but constant markups.

5 Conclusion

In this paper, we examine markup cyclicality through the lens of a simple oligopolistic macroeconomic model with rich implications for the behavior of markups at the firm, sector, and aggregate levels. Working with administrative firm data for France, we show the model can reproduce qualitatively, and many times quantitatively, an array of markup-related empirical moments at various levels of disaggregation. Within our framework and measure of markups, we can reconcile seemingly conflicting variants of "markup-cyclicality" that have been considered in the literature. Finally, our granular oligopolistic setting produces non-negligible aggregate fluctuations, both in output and markups.

One obvious route for future work is to superimpose in our model alternative shocks and frictions. Along this line, prime candidates would be to consider price setting and customer-accumulation frictions (see, e.g., Gilchrist et al. 2017 and Afrouzi 2019), as well as aggregate monetary and financial shocks. Relatedly, we have focused on static, intra-temporal markup decisions in which movements in markups are the result of changes in the shape of the demand curve in response to firm-level shocks. These forces would remain relevant even if one were to extend the model to allow for richer inter-temporal dynamics that result in more complex dynamic markup strategies (see e.g., Rotemberg and Saloner 1986). Bringing the resulting firm, sector, and aggregate dynamics to data - and comparing them against the forces in our static benchmark - would then render possible an assessment of the empirical merits of this more general approach.

Finally, extensions to more realistic and richer product and market structures would allow us to more accurately map model objects to the increasingly detailed micro data available to researchers. Such extensions would likely include multi-product firms, interlinked through intermediate-inputs, with some firms competing only locally in spatially segmented (product and factor) markets and others doing so nationwide and/or internationally.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 09 2012, *80* (5), 1977–2016.

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer, "Identification Properties of Recent Production Function Estimators," *Econometrica*, 2015, *83* (6), 2411–2451.

Ackerberg, Daniel, C. Lanier Benkard, Steven Berry, and Ariel Pakes, "Econometric Tools for Analyzing Market Outcomes," in J.J. Heckman and E.E. Leamer, eds., *Handbook of Econometrics*, Vol. 6 of *Handbook of Econometrics*, Elsevier, January 2007, chapter 63.

Afrouzi, Hassan, "Firm Competition and the Cyclicality of Markups," 2019.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks, Variable Markups, and Domestic Prices," *The Review of Economic Studies*, 02 2019, *86* (6), 2356–2402.

Anderson, Eric, Sergio Rebelo, and Arlene Wong, "Markups Across Space and Time," Working Paper 24434, National Bureau of Economic Research March 2018.

Arkolakis, Costas and Monica Morlacco, "Variable Demand Elasticity, Markups, and Pass-Through," Technical Report, Yale University 2017.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, December 2008, *98* (5), 1998–2031.

Baqaee, David and Emmanuel Farhi, "The Darwinian Returns to Scale," Working Paper 27139, National Bureau of Economic Research May 2020.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 09 2019. qjz030.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey, "Labor Market Power," NBER Working Papers 25719, National Bureau of Economic Research, Inc March 2019.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, "Endogenous Entry, Product Variety, and Business Cycles," *Journal of Political Economy*, 2012, *120* (2), 304 – 345.

Bils, Mark, "The Cyclical Behavior of Marginal Cost and Price," *The American Economic Review*, 1987, *77* (5), 838–855.

_, Peter J. Klenow, and Benjamin A. Malin, "Resurrecting the Role of the Product Market Wedge in Recessions," *American Economic Review*, April 2018, *108* (4-5), 1118–46.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data," Working Paper 27002, National Bureau of Economic Research April 2020.

Brooks, Wyatt J, Joseph P Kaboski, and Yao Amber Li, "Growth Policy, Agglomeration, and (the Lack of) Competition," Working Paper 22947, National Bureau of Economic Research December 2016.

Burstein, Ariel and Gita Gopinath, *International Prices and Exchange Rates*, Vol. 4, Elsevier,

Carvalho, Vasco M. and Basile Grassi, "Large Firm Dynamics and the Business Cycle," *American Economic Review*, April 2019, *109* (4), 1375–1425.

_ and Xavier Gabaix, "The Great Diversification and Its Undoing," *American Economic Review*, August 2013, *103* (5), 1697–1727.

Córdoba, Juan Carlos, "A Generalized Gibrat's Law," *International Economic Review*, November 2008, *49* (4), 1463–1468.

De Loecker, Jan and Frederic Warzynski, "Markups and Firm-Level Export Status," *American Economic Review*, May 2012, *102* (6), 2437–71.

_ and Jan Eeckhout, "The Rise of Market Power and the Macroeconomic Implications," Working Paper 23687, National Bureau of Economic Research August 2017.

— **and** — , "Global Market Power," Technical Report 2018.

— **,** — **, and Simon Mongey**, "Quantifying the Effect of Market Power: An Oligopolistic Framework for Firm Dynamics," Technical Report 2018.

di Giovanni, Julian and Andrei A. Levchenko, "Country Size, International Trade, and Aggregate Fluctuations in Granular Economies," *Journal of Political Economy*, 2012, *120* (6), 1083 – 1132.

— **,** — **, and Isabelle Mejean**, "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 07 2014, *82* (4), 1303–1340.

— **,** — **, and** — , "The Micro Origins of International Business-Cycle Comovement," *American Economic Review*, January 2018, *108* (1), 82–108.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *American Economic Review*, October 2015, *105* (10), 3183–3221.

— **,** — **, and** — , "How Costly Are Markups?," Working Paper 24800, National Bureau of Economic Research July 2018.

Gabaix, Xavier, "Zipf'S Law For Cities: An Explanation," *The Quarterly Journal of Economics*, August 1999, *114* (3), 739–767.

— , "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 05 2011, *79* (3), 733–772.

Gali, Jordi, "Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand," *Journal of Economic Theory*, June 1994, *63* (1), 73–96.

Gaubert, Cecile and Oleg Itskhoki, "Granular Comparative Advantage," Working Paper 24807, National Bureau of Economic Research July 2018.

Gilchrist, Simon, Raphael Schoenle, Jae Sim, and Egon Zakrajšek, "Inflation Dynamics during the Financial Crisis," *American Economic Review*, March 2017, *107* (3), 785–823.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, March 2010, *100* (1), 304–36.

Grassi, Basile, "IO in I-O: Size, Industrial Organization, and the Input-Output NetworkMake a Firm Structurally Important," Working Papers 619, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University 2018.

Hall, Robert E., "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, 1988, *96* (5), 921–947.

Hong, Sungki, "Customer Capital, Markup Cyclicality, and Amplification," Working Papers 2017-33, Federal Reserve Bank of St. Louis April 2017.

Hottman, Colin J., Stephen J. Redding, and David E. Weinstein, "Quantifying the Sources of Firm Heterogeneity," *The Quarterly Journal of Economics*, 2016, *131* (3), 1291–1364.

Jaimovich, Nir and Max Floetotto, "Firm dynamics, markup variations, and the business cycle," *Journal of Monetary Economics*, 2008, *55* (7), 1238 – 1252.

Kikkawa, Ayumu Ken, Glenn Magerman, and Emmanuel Dhyne, "Imperfect Competition in Firm-to-Firm Trade," 2019.

Kimball, Miles S., "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 1995, *27* (4), 1241–1277.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 01 2020, *135* (2), 561–644.

_ , Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik, "Prices, Markups, and Trade Reform," *Econometrica*, 2016, *84* (2), 445–510.

Luttmer, Erzo G.J., "Models of Growth and Firm Heterogeneity," *Annual Review of Economics*, 09 2010, *2* (1), 547–576.

Mongey, Simon, "Market Structure and Monetary Non-neutrality," Staff Report 558, Federal Reserve Bank of Minneapolis October 2017.

Morlacco, Monica, "Market Power in Input Markets: Theory and Evidence from French Manufacturing," Technical Report 2019.

Nekarda, Christopher J. and Valerie A. Ramey, "The Cyclical Behavior of the Price-Cost Markup," NBER Working Papers 19099, National Bureau of Economic Research, Inc June 2013.

_ and _ , "The Cyclical Behavior of the Price-Cost Markup," *Journal of Money, Credit, and Banking: 50th Anniversary Issue*, Forthcoming.

Pasten, Ernesto, Raphael Schoenle, and Michael Weber, "Price Rigidity and the Granular Origins of Aggregate Fluctuations," Working Papers Central Bank of Chile 864, Central Bank of Chile January 2020.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Nicholas Trachter, "Diverging Trends in National and Local Concentration," in "NBER Macroeconomics Annual 2020, volume 35" NBER Chapters, National Bureau of Economic Research, Inc, 2020.

Rotemberg, Julio J and Garth Saloner, "A supergame-theoretic model of price wars during booms," *The American economic review*, 1986, *76* (3), 390–407.

Rotemberg, Julio J. and Michael Woodford, "The cyclical behavior of prices and costs," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, August 1999, chapter 16, pp. 1051–1135.

Stroebel, Johannes and Joseph Vavra, "House Prices, Local Demand, and Retail Prices," *Journal of Political Economy*, 2019, *127* (3), 1391–1436.

Wang, Olivier and Iván Werning, "Dynamic Oligopoly and Price Stickiness," Working Paper 27536, National Bureau of Economic Research July 2020.

Appendix to “Bottom-up Markup Fluctuations”

Ariel Burstein, Vasco M. Carvalho, and Basile Grassi

A Theory Appendix

A.1 Solving for an equilibrium

Consider a given realization at time t of firm shifters, $\{V_{kit}\}$, and sectoral demand shifters, $\{A_k\}$.

Equilibrium firm-level markups and market shares, μ_{kit} and s_{kit}, are the solution to equations (4) and (5). Sectoral markups and productivities, μ_{kt} and Z_{kt}, are solved for from equations (7) and (9), respectively, and sectoral expenditure shares, s_{kt}, from equation (11).

Aggregate markup, productivity, output, and employment, μ_t, Z_t, Y_t, and L_t, are solved for from equations (10), (12), and (13). Setting $W_t = \bar{W}$ as the numeraire, sectoral, and aggregate price levels, P_{kt} and P_t, are given by $P_{kt} = \mu_{kt} W_t / Z_{kt}$ and $P_t = \mu_{kt} W_t / Z_t$. Sectoral output is solved for from

$$Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t, \tag{43}$$

and sectoral employment using $L_{kt} = Y_{kt}/Z_{kt}$. Firm-level expenditures and employment, $P_{kit}Y_{kit}$ and L_{kit}, are solved from from $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$ and equation (6), respectively.

Finally, given realization of firm-level productivities $\{Z_{kit}\}$, firm-level output and price are solved from equations (1) and (3), respectively. Therefore, the split of firm shifters V_{kit} into demand and productivity does not matter to solve for any model outcome (at the firm, sector, or aggregate level) except for firm-level quantity and prices.

A.2 Global between / within decomposition of changes in sectoral markups

The change in the inverse of the sectoral markup between two time periods is, by equation (7),

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \left(\frac{s_{kit'}}{\mu_{kit'}} - \frac{s_{kit}}{\mu_{kit}} \right)$$

This change in sectoral markups can be decomposed into a within term (i.e., changes in firm-level markups evaluated at firms' expenditure share averaged over both time periods) and a between term (i.e., changes in expenditure shares evaluated at firm-level markups averaged over both time periods) as follows:

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \frac{1}{2} \left[(s_{kit'} + s_{kit}) \left(\frac{1}{\mu_{kit'}} - \frac{1}{\mu_{kit}} \right) + \left(\frac{1}{\mu_{kit'}} + \frac{1}{\mu_{kit}} \right) (s_{kit'} - s_{kit}) \right] \tag{44}$$

Note that if markups are equal across firms (as is the case with $\sigma = \varepsilon$), then all terms in (44) are equal to zero.

It is straightforward to show that, by equation (5) under Cournot competition, the within and the between terms in (44) are equal to

$$\frac{1}{2}\sum_{i=1}^{N_k}\left(s_{kit'} - s_{kit}\right)\left(s_{kit'} + s_{kit}\right)\left(\frac{1}{\sigma} + \frac{1}{\varepsilon}\right).$$

Therefore, under Cournot competition, the contribution in changes in sectoral markups of the between and the within terms is 50% each, irrespective of the values of σ and ε (as long as $\sigma \neq \varepsilon$). If σ is close to ε, firm-level markups are less responsive to shocks (reducing the within term), but firm-level markups are also less heterogeneous across firms (reducing the between term).

With Bertrand competition, the within/between decomposition is not pinned down at 50-50.

A.3 Firm-level market shares

Combining $\widehat{s}_{kit} = \widehat{A}_{kit} + (1-\varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right)$, (16), and (18),

$$\widehat{s}_{kit} = \alpha_{ki}\left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}\widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]. \tag{45}$$

The response of firm i's expenditure share to a firm i shock is

$$\widehat{s}_{kit} = \alpha_{ki}\left[1 - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\widehat{V}_{kit}. \tag{46}$$

Finally, we can express the variance of expenditure shares as

$$\mathbb{V}ar\left[\widehat{s}_{kit}\right] = \left(\frac{\alpha_{ki}\sigma_v}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right)^2\left[\left(\sum_{i'\neq i}^{N_k} s_{ki'}\alpha_{ki'}\right)^2 + \sum_{i'\neq i}^{N_k}\left(s_{ki'}\alpha_{ki'}\right)^2\right]. \tag{47}$$

A.4 Changes in sectoral markups

Substituting equations (15), (16), (18), and

$$\widehat{s}_{kit} = \widehat{A}_{kit} + (1-\varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right), \tag{48}$$

into equation (20) yields

$$\widehat{\mu}_{kt} = \mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[\left(\frac{\Gamma_{ki} - 1}{\mu_{ki}} \right) - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'} \left(\frac{\Gamma_{ki'}-1}{\mu_{ki'}} \right)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}. \tag{49}$$

Setting $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$, we obtain the expression for changes in sectoral markups under constant markups, (22).

Under our assumptions on market structure,

$$\frac{\Gamma_{ki} - 1}{\mu_{ki}} = \frac{\varepsilon - 1}{\varepsilon} - \frac{2}{\mu_{ki}}. \tag{50}$$

$(\Gamma_{ki} - 1)/\mu_{ki}$ is increasing in markup μ_{ki} and hence also in market share s_{ki}. Substituting equation (50) into (49), we obtain expression (21).

Under Bertrand competition, markup elasticities Γ_{ki} are given by

$$\Gamma_{ki} \equiv \frac{\partial \ln \mu_{ki}}{\partial \ln s_{ki}} = \left[\varepsilon \left(\frac{\mu_{ki} - 1}{\mu_{ki}} \right) - 1 \right] (\mu_{ki} - 1),$$

and $(\Gamma_{ki} - 1)/\mu_{ki}$ by

$$\frac{\Gamma_{ki} - 1}{\mu_{ki}} = \varepsilon \left(\frac{\mu_{ki} - 1}{\mu_{ki}} \right)^2. \tag{51}$$

Both Γ_{ki} and $\frac{\Gamma_{ki}-1}{\mu_{ki}}$ are increasing in markups and market shares. Changes in sectoral markups under Bertrand competition are

$$\widehat{\mu}_{kt} = \mu_k \varepsilon \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[(\varepsilon - s_{ki}(\varepsilon - \sigma))^{-2} - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'} (\varepsilon - s_{ki'}(\varepsilon - \sigma))^{-2}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}.$$

As under Cournot, a positive shock to firm i results in an increase in sectoral markup if and only if firm i is sufficiently large in its sector.

In general, we do not obtain a simple characterization comparing changes in sectoral markups under constant markups (equation 15) and variable markups (equation 21). To make analytic progress, we restrict the extent of *ex-ante firm heterogeneity*. Specifically, we assume that sector k contains N_k^A type A firms and $N_k^B = N_k - N_k^A$ type B firms, and in the initial equilibrium, firms within each type have equal demand/productivity composite, V_{kit}. In the initial equilibrium, each firm of type $g = A, B$ has market share s_k^g, markup μ_k^g, and markup elasticity Γ_k^g. Firms of type A are indexed by $i = 1, ..., N_k^A$ and firms of type B are indexed by $N_k^A + 1, ..., N_k$. In this case, equation (49) under Cournot competition can be written as

$$\widehat{\mu}_{kt} = \frac{2}{1 + (\varepsilon - 1)\widetilde{\Gamma}_k} \left[s_k^A \left(1 - \frac{\mu_k}{\mu_k^A} \right) \sum_{i=1}^{N_k^A} \widehat{V}_{kit} + s_k^B \left(1 - \frac{\mu_k}{\mu_k^B} \right) \sum_{i=N_k^A+1}^{N_k} \widehat{V}_{kit} \right], \tag{52}$$

where

$$\widetilde{\Gamma}_k = N_k^B s_k^B \Gamma_k^A + N_k^A s_k^A \Gamma_k^B.$$

The term in square brackets in equation (52) corresponds to the change in the sectoral markup under fixed markups as expressed above. Therefore, given the same firm-level shocks, sectoral markups change by more (and the variance is higher) under variable markups than under constant markups if and only if the term in front of the square brackets in equation (52) is higher than 1, which is the case if $(\varepsilon-1)\widetilde{\Gamma}_k < 1$. This condition is violated if σ is sufficiently low and/or ε sufficiently high.

Proof of Proposition 2 Define $f(s)$ and $g(s)$ as probability density functions defined over market shares in sector k, $s = s_{k1}, ..., s_{kN_k}$, given by $f(s) = \frac{s\alpha(s)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}$ and $g(s) = sf(s)a$ with $a = \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} > 1$ and $\alpha(s)$ is defined in equation (17). Because the likelihood ratio $g(s)/f(s) = sa$ is increasing in s, $g(.)$ first-order stochastically dominates $f(.)$. If $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, $f(s)$ is increasing in s. It then follows that $\sum_{i=1}^{N_k} [g(s_{ki}) - f(s_{ki})] f(s_{ki}) > 0$, which corresponds to inequality (25). Note that if $s_{ki}\alpha_{ki}$ is decreasing in s_{ki}, inequality (25) is reversed.□

Under what conditions is $s_{ki}\alpha_{ki}$ increasing in market shares, as required by Proposition 2? Under Cournot competition,

$$s_{ki}\alpha_{ki} = \frac{\left(1-\frac{1}{\varepsilon}\right)s_{ki} - \left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)s_{ki}^2}{1-\frac{1}{\varepsilon}+(\varepsilon-2)\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)s_{ki}},$$

which is increasing in s_{ki} if and only if

$$2\left(\frac{\varepsilon-1}{\varepsilon}\right)s_{ki} + \left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)(\varepsilon-2)\,s_{ki}^2 < \frac{\sigma\,(\varepsilon-1)^2}{\varepsilon\,(\varepsilon-\sigma)}. \tag{53}$$

Because the left-hand side of this equation is increasing in s_{ik} (for $s_{ik} \le 1$), this inequality holds for $s_{ki} \le \tilde{s}_k$, where $\tilde{s}_k$ is a function of σ and ε. This implies inequality (25) is satisfied if all market shares in sector k are less than or equal to $s_{ki} \le \tilde{s}_k$. Note the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} is sufficient but not necessary for inequality (25) to hold. In particular, inequality (25) may hold (so that sectoral markups and prices comove negatively) even if $s_{ki}\alpha_{ki}$ is increasing in some range of the distribution of market shares in a sector but decreasing at the upper tail of the distribution.

A.5 Changes in sectoral productivity

By equations (9) and (15), changes in sectoral productivity are, up to a first order, given by

$$\widehat{Z}_{kt} = \sum_{i=1}^{N_k} s_{ki}\left[\left(\frac{\varepsilon}{\varepsilon-1}-\frac{\mu_k}{\mu_{ki}}\right)\widehat{V}_{kit}-\varepsilon\left(1-\frac{\mu_k}{\mu_{ki}}\right)\Gamma_{ki}\widehat{s}_{kit}\right],$$

where changes in market shares are given by (48). The term $s_{ki}\times\frac{\varepsilon}{\varepsilon-1}$ corresponds to the elasticity of sectoral productivity under monopolistic competition. The remaining terms reflect changes in efficiency due to reallocation across firms with heterogeneous markups, as discussed in detail in Baqaee and Farhi (2019).

A.6 Changes in sectoral and aggregate output

We now derive equation (30). We first calculate changes in aggregate output. Using equations (29) and (30), changes in aggregate output can be expressed in terms of changes in sectoral markup and price as

$$\widehat{Y}_t = (1+f\eta)^{-1}\sum_k s_k\left[-\left(f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)\right)\widehat{P}_{kt}+\frac{s_k\mu}{\mu_k}\widehat{\mu}_{kt}\right] \tag{54}$$

In response to sector k shocks only, changes in aggregate output are

$$\widehat{Y}_t = (1+f\eta)^{-1}s_k\left[-\left(f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)\right)\widehat{P}_{kt}+\frac{s_k\mu}{\mu_k}\widehat{\mu}_{kt}\right] \tag{55}$$

and change in aggregate price by $\widehat{P}_t = s_k\widehat{P}_{kt}$.

Changes in sectoral output are given by

$$\widehat{Y}_{kt} = -\sigma\widehat{P}_{kt}+\sigma\widehat{P}_t+\widehat{Y}_t. \tag{56}$$

In response to sector k shocks only, substituting changes in aggregate output and price using the expressions above, changes in sectoral output are given by equation (26).

Finally, expression (27) is obtained as follows. First, changes in firm-level markups are, combining equations (45) and (15),

$$\widehat{\mu}_{kit} = \Gamma_{ki}\alpha_{ki}\left[\widehat{V}_{kit}-\frac{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}\widehat{V}_{ki't}}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]. \tag{57}$$

Changes in sectoral output when $f \to \infty$ are, by equations (26) and (18),

$$\widehat{Y}_{kt} = -\left[\sigma\left(1 - s_k\right) + \eta^{-1} s_k\right] \widehat{P}_{kt} = \frac{\left[\sigma\left(1 - s_k\right) + \eta^{-1} s_k\right]}{\varepsilon - 1} \frac{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki}}. \tag{58}$$

Calculating $\mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kit}\right]$ in the presence of shocks to all firms (only those in sector k are relevant), we obtain expression (27).

A.7 Volatility and covariance of aggregate markups and output

In this section, we provide expressions for the variance of and covariance between aggregate markups and aggregate output. We do not impose $f \to \infty$, as we do in the main text. We use these expressions in section 4.4.

The covariance between sector prices and markups, $\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right]$, is given by (24) under variable markups and (23) under constant markups.

The variance of sectoral prices is

$$\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \frac{\sigma_v^2}{(\varepsilon - 1)^2} \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2. \tag{59}$$

Under constant markups, $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$. The variance of the aggregate price is

$$\mathbb{V}ar\left[\widehat{P}_t\right] = \sum_k s_k^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{60}$$

The variance of sectoral markups is

$$\mathbb{V}ar\left[\widehat{\mu}_{kt}\right] = \mu_k^2 \sum_{i=1}^{N_k} s_{ki}^2 \alpha_{ki}^2 \left[\left(\frac{\Gamma_{ki} - 1}{\mu_{ki}}\right) - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \left(\frac{\Gamma_{ki'} - 1}{\mu_{ki'}}\right)}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}}\right]^2 \sigma_v^2. \tag{61}$$

The variance of aggregate markups is

$$\mathbb{V}ar\left[\widehat{\mu}_t\right] = \sum_k s_k^2 \left[\left(\frac{\mu}{\mu_k}\right)^2 \mathbb{V}ar\left[\widehat{\mu}_{kt}\right] + (1 - \sigma)^2 \left(1 - \frac{\mu}{\mu_k}\right)^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right] - (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right) \mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right]\right] \tag{62}$$

The covariance between aggregate price and markup is

$$\mathbb{C}ov\left[\widehat{P}_t, \widehat{\mu}_t\right] = \mu \sum_k \frac{s_k^2}{\mu_k} \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] - (\sigma - 1) \sum_k s_k^2 \left(1 - \frac{\mu}{\mu_k}\right) \mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{63}$$

The variance of aggregate output is

$$\mathbb{V}ar\left[\widehat{Y}_t\right] = \left(\frac{1}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{\mu}_t\right] + \left(\frac{1+f}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{P}_t\right] - \frac{(1+f)}{(1+\eta f)^2}\mathbb{C}ov\left[\widehat{P}_t, \widehat{\mu}_t\right] \tag{64}$$

Finally, the covariance between aggregate output and markups is

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = \left(\frac{1}{1+\eta f}\right) \mathbb{V}ar\left[\widehat{\mu}_t\right] - \left(\frac{1+f}{1+\eta f}\right) \mathbb{C}ov\left[\widehat{P}_t, \widehat{\mu}_t\right]. \tag{65}$$

A.8 Decreasing returns to scale

The production function is now given by

$$Y_{kit} = Z_{kit} L_{kit}^{\beta}. \tag{66}$$

where $\beta \leq 1$. Marginal cost is

$$MC_{kit} = \beta^{-1} W_t \left(Y_{kit}\right)^{(1-\beta)/\beta} \left(Z_{kit}\right)^{-1/\beta}, \tag{67}$$

or, using $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$,

$$MC_{kit} = \beta^{-1} W_t \mu_{kit}^{\beta-1} \left(P_{kt}Y_{kt}s_{kit}\right)^{(1-\beta)} \left(Z_{kit}\right)^{-1}. \tag{68}$$

The firm-level markup, μ_{kit}, is defined as the ratio of price to marginal cost, and is related to expenditure shares by equation (5), which does not depend on β.

Labor payments of firm i in sector k are

$$L_{kit}W_t = \beta \mu_{kit}^{-1} P_{kit} Y_{kit},$$

and profits (revenues minus labor payments) are

$$\Pi_{kit} = \left(1 - \beta \mu_{kit}^{-1}\right) P_{kit} Y_{kit}.$$

We define the sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_t L_{kt}}, \tag{69}$$

which can be expressed as a function of firm-level markups and expenditure shares,

$$\mu_{kt}^{-1} = \beta \sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit}. \tag{70}$$

The 50-50 between/within decomposition of changes in sectoral markups under Cournot competition derived in the appendix holds irrespectively of the value of β.

The expenditure share of firm i in sector k, using $P_{kit} = \mu_{kit} MC_{kit}$, satisfies

$$s_{kit} = \frac{V_{kit}\left(\mu_{kit}^{\beta} s_{kit}^{1-\beta}\right)^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't}\left(\mu_{ki't}^{\beta} s_{ki't}^{1-\beta}\right)^{1-\varepsilon}}. \tag{71}$$

Equilibrium firm-level expenditure shares and markups are the solution to equations (5) and (71).

Log-linearizing this equation, and using $\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}$, we obtain the analog to equations (14) and (15):

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\Lambda_{ki}\widehat{s}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Lambda_{ki'}\widehat{s}_{ki't}\right), \tag{72}$$

where $\Lambda_{ki} = \beta\Gamma_{ki} + 1 - \beta$. Note $\Gamma_{ki} < \Lambda_{ki}$ if and only if $\Gamma_{ki} < 1$.

We can follow similar steps to obtain expressions for changes in sectoral markups and prices to firm-level shocks, as well as the implied variances and covariances.

B Empirical Appendix

B.1 Markup-Estimation Appendix

In this appendix, we describe the empirical framework we use to estimate production functions and firm-level markups. We also discuss its implementation in the FICUS-FARE French firm census data. This framework is based on the so-called production approach and builds on to the methodology of De Loecker and Warzynski (2012), De Loecker and Eeckhout (2017), and De Loecker et al. (2016, 2020).

We assume all firms within two digit sectors have a common production function, up to a firm-specific Hicks-neutral TFP. For simplicity, in what follows, we omit sector notation. The production function of firm i is

$$Y_{it} = Z_{it} F\left(L_{it}, K_{it}, M_{it}, X_{it}\right),$$

where Z_{it} denotes TFP, L_{it} denotes labor, K_{it} denotes capital, M_{it} denotes materials, and X_{it} denotes services. These inputs are homogenous across firms within sectors and traded in competitive markets. In our estimation of markups, we do not impose that F is constant returns to scale.

We now describe how we recover markups given production-function estimates, and then show how we estimate production functions.

B.1.1 Recovering Markups

When minimizing costs, we assume that materials is a variable input that is not subject to any adjustment cost or any intertemporal decision. Under these assumptions, the first-order-condition of the firms' cost-minimization problem for materials M_{it} can be rewritten as

$$P_t^M = \lambda_{it} Z_{it} \frac{\partial F}{\partial M} \quad \Leftrightarrow \quad \mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}},$$

where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$; that is output price is equal to markup over marginal cost. We denote by $\theta_{it}^M = \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}}$ the elasticity of the production function with respect to material input M_{it}. Markup is equal to the product of the ratio of sales to materials and the elasticity of the production function with respect to materials:

$$\mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \theta_{it}^M. \tag{73}$$

We calculate the ratio of sales to materials using the FICUS-FARE data on sales and input expenditures, and we estimate the elasticity as we describe below.

B.1.2 Production-function estimation

In this subsection, we describe the production-function estimation procedure. We implement a control-function approach, introduced by Ackerberg et al. (2007, 2015), in an oligopolistic competition environment.

We implement the estimation procedure described below at the two-digit sector level. Given our assumptions that inputs are homogeneous and that firms are price takers in the input markets, we deflate input expenditures by sector-level price indices to recover inputs' quantities.[43]

We write the revenue production function as

$$\log P_{it} Y_{it} = \log F\left(L_{it}, K_{it}, M_{it}, X_{it}\right) + \log Z_{it} + \log P_{it} + \epsilon_{it},$$

where ϵ_{it} denotes the (log) of measurement error. Hereafter, denote with small capital letters the logarithm of large capital letters: $z_{it} = \log Z_{it}$, $p_{it} = \log P_{it}$, $l_{it} = \log L_{it}$, $k_{it} = \log K_{it}$, $m_{it} = \log M_{it}$ and $x_{it} = \log X_{it}$, $py_{it} = \log P_{it} Y_{it}$, and $f(l_{it}, k_{it}, m_{it}, x_{it}) = \log F\left(L_{it}, K_{it}, M_{it}, X_{it}\right)$. With this notation, we write the revenue production function as

$$py_{it} = f\left(l_{it}, k_{it}, m_{it}, x_{it}\right) + z_{it} + p_{it} + \epsilon_{it}, \tag{74}$$

[43] Given our assumptions that firms produce differentiated goods and set heterogeneous markups (and prices), we cannot recover quantities by deflating firm-level revenue using sector-level output deflators.

This equation cannot be estimated by simple OLS, because of unobserved productivity that is likely correlated both with the left- and right-hand sides of the equation. To address this simultaneity problem, we need to build an instrument for productivity by introducing a conditional control function. To construct this control function, we use the first-order-condition with respect to the static input materials in the cost-minimization problem (as in equation 73)

$$P_t^M = \frac{P_{it}}{\mu_{it}} Z_{it} \frac{\partial F}{\partial M}. \tag{75}$$

Using the fact that $\frac{\partial F}{\partial M}$ is a function of the inputs' usage, $\frac{\partial F}{\partial M}(L_{it}, K_{it}, M_{it}, X_{it})$, equation (75) implicitly defines M_{it} as a function of productivity, Z_{it}, conditional on other inputs' usage L_{it}, K_{it}, X_{it}, material price, output price, and markup.[44] Inverting this relationship, we can write productivity as a function of time, input usage, output price, and markup.[45] Let us call this conditional control function h_t such that

$$z_{it} = h_t(m_{it}, l_{it}, k_{it}, x_{it}, \zeta_{it}) \quad \text{with} \quad \zeta_{it} = \{p_{it}, \mu_{it}\}. \tag{76}$$

The subscript t captures the dependence of the function h to input prices because these are the same for all firms at a given period. Assume for now that $\zeta_{it} = \{p_{it}, \mu_{it}\}$ is observed. Substituting equation (76) in the revenue production function (74) yields

$$py_{it} = f(l_{it}, k_{it}, m_{it}, x_{it}) + h_t(m_{it}, l_{it}, k_{it}, x_{it}, \zeta_{it}) + p_{it} + \epsilon_{it} \equiv \phi_t(l_{it}, k_{it}, m_{it}, x_{it}, \zeta_{it}) + \epsilon_{it}, \tag{77}$$

where ϕ_t is a function that we estimate non-parametrically.[46] The estimation of the function ϕ_t is the first-stage of our procedure.

In a second stage, we use a generalized method of moments (GMM) estimator. To construct the moment conditions, we define $err_{it} \equiv \phi_t(l_{it}, k_{it}, m_{it}, x_{it}, \zeta_{it}) - f(l_{it}, k_{it}, m_{it}, x_{it})$, which is equal to (log) revenue productivity, $z_{it} + p_{it}$.[47] We assume this measure of productivity is Markovian. Specifically,

$$err_{it} = g(err_{it-1}) + \xi_{it},$$

where g is an AR(1) function and ξ_{it} is the innovation.[48] By the Markovian assumption, ξ_{it} is orthogonal to the information set at time $t-1$. Formally, the moment condition is

$$\mathbb{E}\left[\xi_{it} | I_{it-1}\right] = 0, \tag{78}$$

where I_{it} is the information set at time t.

[44] For the Cobb-Douglas case in which $F = L_{it}^{\alpha_L} K_{it}^{\alpha_K} M_{it}^{\alpha_M} X_{it}^{\alpha_X}$, the implicit function between material as a function of productivity and input usage is given by $P_t^M = \frac{P_{it}}{\mu_{it}} Z_{it} \alpha_M L_{it}^{\alpha_L} K_{it}^{\alpha_K} M_{it}^{\alpha_M - 1} X_{it}^{\alpha_X}$.

[45] For the Cobb-Douglas case, this function is $Z_{it} = P_t^M \frac{\mu_{it}}{P_{it}} \frac{1}{\alpha_M} L_{it}^{-\alpha_L} K_{it}^{-\alpha_K} M_{it}^{1-\alpha_M} X_{it}^{-\alpha_X}$.

[46] For the Cobb-Douglas case, the function ϕ_{it} is equal to $p_t^M + \log \mu_{it} - \log \alpha_M + m_{it}$.

[47] For the Cobb-Douglas case, $err_{it} = p_t^M + \log \mu_{it} - \log \alpha_M + (1 - \alpha_M) m_{it} - \alpha_L l_{it} - \alpha_K k_{it} - \alpha_X x_{it}$.

[48] Our estimated elasticities are similar if we assume err_{it} follows the process $err_{it} = \rho_0 + \rho_1 err_{it-1} + \rho_2 err_{it-1}^2 + \xi_{it}$.

We use the first-stage and the moment condition to estimate the parameters of the production function as follows. First, we estimate non-parametrically ϕ_{it} using an OLS estimator where the function is approximated by a third-order polynomial. Second, for a given set of parameters characterizing the production function f and with a first-stage estimation of the ϕ_{it}, we compute the error term $err_{it} = \phi_{it} - f(l_{it}, k_{it}, m_{it}, x_{it})$. We then estimate the AR(1) Markovian process g and compute the innovation $\xi_{it} = err_{it} - g(err_{it-1})$. To do so, we need a set of instruments from the information set at time $t-1$. In our translog production-function specification, these instruments are the inputs $\{m_{it-1}, x_{it-1}, l_{it-1}\}$ and the predetermined variable $\{k_{it}\}$ together with their power (up to the third order) and cross-products. Finally, the parameters of the production function are chosen using a numerical solver to solve the moment condition (78).

Recall the procedure above assumed the variables $\zeta_{it} = \{p_{it}, \mu_{it}\}$ are observed. We discuss how we deal with each unobservable variable in turn.

To address the unobserved-markup problem, we assume the markup is a function of market share, $\mu_{it} = \mu_t(s_{it})$, as is the case under the nested CES demand system in our model under either Cournot or Bertrand competition. In the data, this market share is defined as the ratio between firm-level sales and the sum of the sales of all firms in the same NAF sector, where market shares are defined at the five-digit level of sectoral disaggregation. In practice, we substitute the variable μ_{it} by the market share s_{it} in the vector of controls ζ_{it}.

The second problem deals with unobserved firm-level prices. For our full dataset, as in many large firm censuses, firm-level prices are unobserved. However, for a subsample of firms in our data, we can construct a price proxy aggregating product-level prices, as described in section 3.1. We can thus include this proxy for firm-level price in the vector of controls z_{it}. Recall that this measure is only available for firms in the EAE sample, which is restricted to manufacturing and from 2009 onwards.

Using the methodology above, we then proceed to estimate firm-level markups using four different sets of control variables z_{it}: (i) firms' market share and price proxies; (ii) price proxies; (iii) market shares; and (iv) no controls. Note that for (i) and (ii), we have to restrict our estimation to the period 2009-2016 and manufacturing firms only, whereas for (iii) and (iv) we estimate on both the restricted and full sample.

As shown in Table 11, the pairwise correlations among these four different firm-level markup estimates are very high, both in levels and in annual changes, ranging between 0.80 and 0.99. With this in mind, we use the specification (iv) as a baseline choice in order to take advantage of the full sample, both cross-sectionally and over time. Using specification (iii), which we also can implement on the whole dataset, leaves results virtually unchanged, whereas specifications (i) and (ii) reduce considerably the scope of the analysis and size of the sample.

Table 11: Correlation between markup estimates from different specifications

	levels				first-diff (logs)			
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
(i) price proxy and market share	1				1			
(ii) price proxy	0.843	1			0.941	1		
(iii) market share	0.896	0.860	1		0.941	0.993	1	
(iv) no controls	0.830	0.801	0.890	1	0.909	0.944	0.940	1

NOTE: This table reports pairwise correlations, in levels and in log difference, across various specification of firm-level markup estimates. These estimates and correlations are computed on the restricted sample of firms (EAE) on the period 2009-2016. The correlation in levels (resp. first-difference) between firm-level markup for the specification (iii) and (iv) on the full sample is equal to 0.977 (resp. 0.977).

Table 12: Firm Inverse Markup and Market Share: First Difference

	Dependent Variable: $\Delta\mu_{kit}^{-1}$	
	(1)	(2)
Δs_{kit}	-.511	-.508
	(.156)	(.156)
Year FE	N	Y
Number of Firms	1 135 547	1 135 547
Observations	9 328 004	9 328 004

NOTE: $\Delta\mu_{ikt}^{-1}$ is the first difference of the inverse of firm i sector k gross markup in year t, Δs_{kit} gives the first difference of market share of firm i in sector k. Columns (1) and (2) report empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parentheses) are clustered at the firm level. Markups are winsorized at the 2% level.

B.2 Markups, Market Shares, and Concentration: Robustness

This appendix derives additional cross-sectional and within-firm predictions for the empirical relation between (a) firm markup and firm market share and (b) sector markup and sector concentration.

Proceeding in parallel to the results in the main text, taking the first difference of equation (36) yields

$$\Delta\mu_{kit}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon}\Delta s_{kit},$$

where $\Delta\mu_{kit}^{-1}$ and Δs_{kit} are the first difference across time of the inverse firm-level markup and market share respectively. Testing this relationship empirically yields the results in Table 12 that shows a negative and significant coefficient even after controlling for time effect.

Turning to our sector-level predictions, we follow the same methodology as above by taking the

Table 13: Sector Inverse Markup and Sectoral HHI: First Difference

	Dependent Variable: $\Delta\mu_{kt}^{-1}$	
	(1)	(2)
ΔHHI_{kt}	-.340	-.350
	(.147)	(.147)
Year FE	N	Y
Number of Sectors	504	504
Observations	11 088	11 088

NOTE: μ_{kt} is sector k gross markup in year t, HHI_{kt} gives the HHI of concentration k. Columns (1)-(2) report empirical estimates for the FICUS-FARE (1994-2016) data, aggregated to sector level. Standard errors (in parentheses) are clustered at sector level.

first difference of equation (39) which yields

$$\Delta\mu_{kt}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon}\Delta HHI_{kt}.$$

Table 13 reports the results with and without time fixed effects. This table shows, as predicted by the model, a negative and significant estimate.

C Simulation Appendix

C.1 Untargeted Moment Not based on Markup Estimation

When we calibrate the model, we also consider a specification of the markup equation (35) that does not require estimates of output elasticities but imposes the market structure assumed in our model. Substituting the relation between markups and market shares in our model (equation 5), we write equation (35) as

$$\log\frac{P_{kit}Y_{kit}}{P_{kit}^M M_{kit}} = \log\frac{\varepsilon}{\varepsilon - 1} - \log\left(1 - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon - 1}s_{kit}\right) - \log\theta_{kit}^M. \tag{79}$$

Recall that under a translog production function, output elasticities depend on sector level parameters and firm-level factor use. We thus consider the following empirical specification of (79):

$$\log\frac{P_{kit}Y_{kit}}{P_{kit}^M M_{kit}} = \gamma_k + \alpha_t + \beta s_{kit} + X_{kit} + \epsilon_{kit}, \tag{80}$$

where X_{kit} denotes a vector of firm-level factor use, γ_k is a sector-level fixed effect at the two digit level, and α_t is a year fixed effect. Approximating the log term in equation (79) yields $\beta = \frac{\frac{\varepsilon}{\sigma}-1}{\varepsilon-1}$ around $s_{kit} = 0$ and $\beta = \frac{\frac{\varepsilon}{\sigma}-1}{\varepsilon-\frac{\varepsilon}{\sigma}}$ around $s_{kit} = 1$. We estimate equation (80) weighting

Table 14: Firm Markup and Sector Output: Robustness

Dependent variable: $\ln(\mu_{kit})$				
Coefficient	(1) Data	(2) Data	(3) Model $\sigma = 2.0298$	(4) Model $\sigma = 2.0298$
$\widehat{Y}_{k,t}$	-0.093	-0.068	-0.135	-0.015
	(0.062)	(0.035)		
$\widehat{Y}_{k,t} * s_{kit}$	0.209	0.192	0.872	0.094
	(0.077)	(0.043)		
Firm Controls	N	Y	N	Y
Firm FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Firms	1 154 181	1 154 181	650 423	650 423

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\widehat{Y}_{k,t}$ is the deviation of sector k (log) value-added in year t from its HP trend. Columns (1) and (2) report empirical estimates for the FICUS-FARE (1994-2016) data. Columns (3) and (4) report estimates based on model-simulated data for an alternative calibration with $\varepsilon = 7$ and $\sigma = 2.0298$. This regression is weighted by average revenue. Standard errors clustered at the firm-level.

observations by average firm-level revenues. The OLS estimate of β is 0.593 (s.e 0.141).[49]

Our baseline calibration strategy relies on estimates of firm-level markups only through the coefficient of the HHI in the regression of sector-level inverse markup. Alternatively, equation (80) imposes the market structure in our model but does not require estimates of output elasticities and markups. When we run this regression on model-simulated data (where labor is the only factor of production), we obtain an estimate of β of 0.620, which is very close to 0.593 (s.e 0.141) in the data.

C.2 Alternative Calibration

As described in footnote 29 in section 3.3, we consider an alternative calibration in which we target a slope between sector inverse markup and HHI equal to -0.35 (as reported in colum 2 of Table 13), estimated in first difference (rather than -0.44 estimated in levels), setting $\varepsilon = 7$ and $\sigma = 2.0298$.

D Additional Figures Appendix

[49]The point estimate for the unweighted specification is 0.945 (s.e. 0.318).

Table 15: Sector Markup and Sector Output: Robustness

Dependent variable: $\widehat{\mu}_{kt}$			
Coefficient	(1) Data	(2) Model ($\sigma = 1.704$)	(3) Model ($\sigma = 2.0298$)
$\widehat{Y}_{kt}$	0.102	0.129	0.079
	(0.028)	[0.029; 0.198]	[0.012; 0.134]
Sector FE	Y	Y	Y
Year FE	Y	Y	Y
Number of Sectors	504	504	504

NOTE: Regression of sector-level (log) change markup ($\widehat{\mu}_{kt}$) on sector value-added (log) change ($\widehat{Y}_{kt}$). Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data, and standard errors (in parentheses) are clustered at the sector level. Columns (2) and (3) report estimates based on model-simulated data for our baseline and an alternative calibration, respectively. Point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 independent simulated samples, each of the same length (23 years) as the French data. Terms in square brackets give, respectively, the 0.05 and 0.95 quantiles of coefficient estimates across simulated samples.

Table 16: Sector Markup and Aggregate Output: Robustness

Dependent variable: $\widehat{\mu}_{kt}$			
Coefficient	(1) Data	(2) Model ($\sigma = 1.704$)	(3) Model ($\sigma = 2.0298$)
$\widehat{Y}_{t}$	0.191	0.014	0.009
	(0.161)	[-0.480, 0.553]	[-0.041, 0.217]
Sector FE	Y	Y	Y
Number of Sectors	504	504	504

NOTE: Regression of sector k's markup in year t in deviation from its HP trend $\widehat{\mu}_{kt}$ on $\widehat{Y}_t$ the HP-trend deviation of (log) aggregate real value-added in year t. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parentheses) are clustered at the sector level. Column (2) reports estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 independent simulated samples, each of the same length (23 years) as the French data. Terms in square brackets give, respectively, the 0.05 and 0.95 quantiles of coefficient estimates from simulated data.

Panel A Panel B

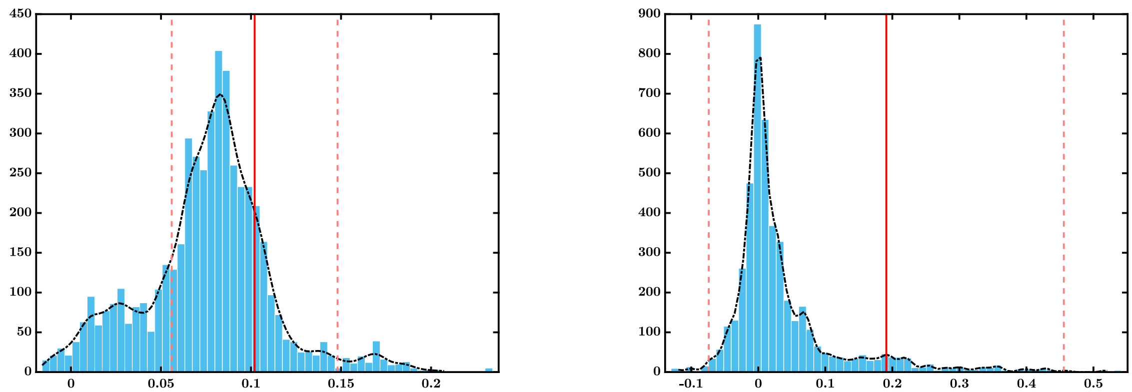

Figure 5: Histogram of Model-based Sector-level Regression: Robustness

NOTE: Panel A: Kernel density of estimated regression coefficient on model-simulated data from equation (41) based on 5,000 repetitions of independent samples for the alternative calibration with $\varepsilon = 7$ and $\sigma = 2.0298$. The solid red lines are the estimate in the data, and the dashed red lines reports the lower and upper bound of the 10% confidence interval. Panel B: Kernel density of estimated regression coefficient on model-simulated data from equation (42). 5,000 repetitions of independent samples for the alternative calibration with $\varepsilon = 7$ and $\sigma = 2.0298$. The solid red lines are the estimate in the data while the dashed red lines reports the lower and upper bound of the 10% confidence interval.

Table 17: Aggregate Markup and Aggregate Output: Robustness

	(1) Data			(2) Model $\sigma = 2.0298$		
	σ_x	σ_x/σ_Y	$\rho(x, Y)$	σ_x	σ_x/σ_Y	$\rho(x, Y)$
Y_t	1.71	1	1	0.99	1	1
μ_t	0.96	0.57	0.13	0.27	0.29	0.96

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x, Y)$, for aggregate output Y_t and aggregate markup μ_t, both in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) reports the average over 5,000 independent model-simulated samples each of the same length (23 years) for the alternative calibration with $\varepsilon = 7$ and $\sigma = 2.0298$.

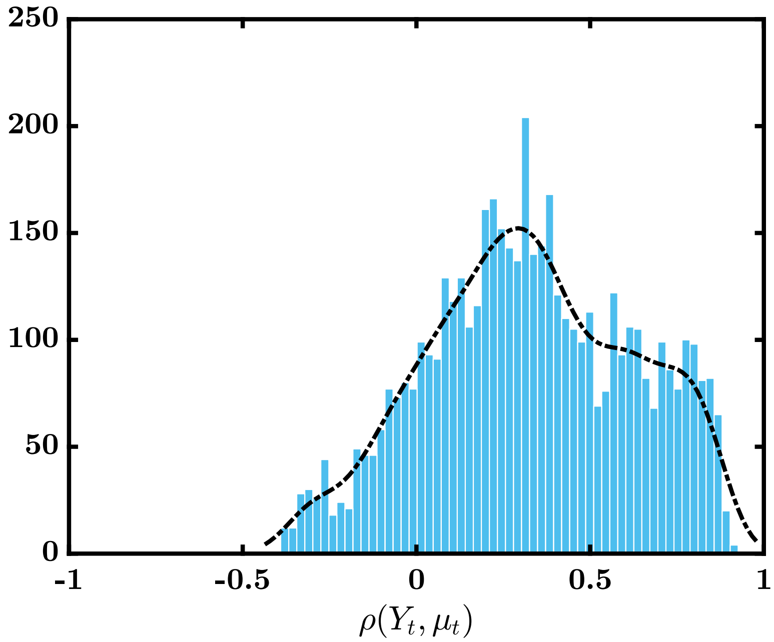

Figure 6: Histogram of $\rho(\mu_t, Y_t)$ in Model-simulated Data with Aggregate Shocks

NOTE: Kernel density of $\rho(\mu_t, Y_t)$, the correlation coefficient between aggregate markups and aggregate output on simulated data from the model with aggregate shocks based on 5,000 repetitions of independent 23 period samples. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of GDP in France.