Welfare with product entry-exit and taste shocks

David Baqaee Ariel Burstein

This short note shows how results in Baqaee & Burstein (2021) can be applied when there are changes in the set of consumed goods (extensive margin) between t_0 and t_1 (time or space). Consumers can start or stop consuming a good either because of changes in the good's availability or due to changes in tastes. We show how to calculate welfare changes in the presence of taste shocks allowing for both sources of changes in the extensive margin. In particular, we provide assumptions under which the new goods adjustment in Feenstra (1994) can be used even though there are taste shocks.

Consider a consumer whose preferences are homothetic CES with taste shifters. The expenditure function is

$$e(p,u;x) = \left(\sum_i x_i p_i^{1-\theta_0}\right)^{\frac{1}{1-\theta_0}} u, \tag{1}$$

where $\theta_0 > 1$ is the elasticity of substitution. Good i is *available* to the consumer if its price is finite, $p_i < \infty$. Given prices p and tastes x, the budget share of good i is

$$b_i(p;x) = \frac{x_i p_i^{1-\theta_0}}{\sum_j x_j p_j^{1-\theta_0}}. \tag{2}$$

Note that the budget share is zero if either the good is unavailable ($p_i = \infty$) or if the consumer does not value the good ($x_i = 0$).

Consider a change in prices from p_{t_0} to p_{t_1}, income from I_{t_0} to I_{t_1}, and tastes from x_{t_0} to x_{t_1}. Split the set of goods that consumers value at t_1 and are available in either t_0 or t_1 into three sets:

- $\mathcal{C}$: continuing goods consumed in both periods, $b_{it_0} > 0$ and $b_{it_1} > 0$;
- $\mathcal{N}$: newly consumed goods that either were unavailable at t_1 ($p_{it_0} = \infty$ and $p_{it_1} < \infty$) or that were available ($p_{it_0} < \infty$) but are valued at t_1 and not at t_0 ($x_{it_0} = 0$ and $x_{it_1} > 0$);

- $\mathcal{X}$: exiting goods that become unavailable at t_1, $p_{it_0} < \infty$ and $p_{it_1} = \infty$.[1]

Proposition 1. *The change in equivalent variation at t_1 tastes is given by*

$$EV^m = \log \frac{I_{t_1}}{I_{t_0}} - \log \frac{P_{t_1}}{P_{t_0}}, \tag{3}$$

where

$$\frac{P_{t_1}}{P_{t_0}} = \left(\frac{b^c_{t_1} \left(\frac{P^c_{t_1}}{P^c_{t_0}} \right)^{\theta_0 - 1} + \left(1 - b^c_{t_1}\right) \left(\frac{P^n_{t_1}}{P^n_{t_0}} \right)^{\theta_0 - 1}}{1 - b^{x_{t_1}}_{t_0}} \right)^{\frac{1}{\theta_0 - 1}}. \tag{4}$$

In this expression $b^c_{t_1}$ is the expenditure share on continuing goods in t_1, and $P^c_{t_1}/P^c_{t_0}$ and $P^n_{t_1}/P^n_{t_0}$ is the change in the CES price index for continuing and new goods under t_1 tastes. Finally, $b^{x_{t_1}}_{t_0}$ is the expenditure share on exiting goods under t_1 tastes and p_{t_0} prices.

In order to apply (4) we require three pieces of information:

- The t_1 share of continuing goods, $b^c_{t_1}$, and changes in the price index for continuing goods $P^c_{t_1}/P^c_{t_0}$. Constructing the price index for continuing goods is straightforward given expenditures at t_1 and changes in prices between t_0 and t_1 for continuing goods.

- The price index for newly consumed goods $P^n_{t_1}/P^n_{t_0}$, which combines newly available goods, for which $p_{i_{t_0}}/p_{i_{t_1}} = 0$, and goods that were available but consumers did not have tastes for at t_0, for which $p_{i_{t_0}}/p_{i_{t_1}}$ is finite. It many applications, it is reasonable to think that $P^n_{t_1}/P^n_{t_0} \in [0, P^c_{t_1}/P^c_{t_0}]$. The lowerbound assumes that newly consumed goods were priced at infinity in t_0. The upperbound assumes that changes in prices of newly consumed goods are not higher than changes in prices of continuing goods (in practice, if one has access to changes in the prices of newly consumed goods, then those can be used directly).

- We need the (counterfactual) share of exiting goods at t_0 prices but t_1 preferences $b^{x_{t_1}}_{t_0}$. It is reasonable to think that $b^{x_{t_1}}_{t_0} \in [0, b^{x_{t_0}}_{t_0}]$. The lowerbound takes the position that goods that exited are no longer valuable to the consumer with t_1 tastes, and the

[1] We can ignore goods that the consumer does not value at t_1 ($x_{it_1} = 0$) because those goods do not matter for welfare at t_1 preferences.

upperbound takes the position that goods that exited are not relatively more valuable to the consumer with t_1 tastes than the consumer with t_0 tastes (i.e. demand curves for those goods that exited did not shift out).

We discuss three noteworthy special cases of expression (4):

i. **No extensive margin**: no new goods ($b^c_{t_1} = 1$) and no exiting goods ($b^{x_{t_1}}_{t_0} = 0$), so

$$\frac{P_{t_1}}{P_{t_0}} = \frac{P^c_{t_1}}{P^c_{t_0}} = \left(\sum_{i \in \mathcal{C}} b_{it_1} \left(\frac{p_{it_0}}{p_{it_1}} \right)^{1-\theta_0} \right)^{\frac{1}{\theta_0 - 1}} \tag{5}$$

as in the Nielsen application in Baqaee & Burstein (2021) in which we compute the price index only for continuing goods.

ii. **Feenstra (1994) with taste shocks**: Suppose that newly consumed goods at t_1 were unavailable at t_0 ($P^n_{t_0} = \infty$). Furthermore, suppose that, under t_1 tastes but t_0 prices, the share of expenditures on exiting goods equals the observed share of exiting goods at t_0: ($b^{x_{t_1}}_{t_0} = b^{x_{t_0}}_{t_0} = 1 - b^c_{t_0}$). In this case, (4) reduces to

$$\frac{P_{t_1}}{P_{t_0}} = \left(\frac{b^c_{t_1}}{b^c_{t_0}} \right)^{\frac{1}{\theta_0 - 1}} \frac{P^c_{t_1}}{P^c_{t_0}}.$$

The term in front of the continuing price index coincides with the well-known new-goods adjustment with CES preferences due to Feenstra (1994). Note that this is still not the same as Feenstra (1994) because $P^c_{t_1} / P^c_{t_0}$ is calculated for fixed x_{t_1} tastes and hence is not the Sato-Vartia price index. In other words, $P^c_{t_1} / P^c_{t_0}$ is the price index in (5).

iii. **Entry and exit only due to taste shocks**: Suppose all goods were available in t_0 and t_1, but some goods are consumed in t_1 and not t_0 because of changes in tastes. In this case, $b^{x_{t_1}}_{t_0} = 0$, so

$$\frac{P_{t_1}}{P_{t_0}} = \left(b^c_{t_1} \left(\frac{P^c_{t_1}}{P^c_{t_0}} \right)^{\theta_0 - 1} + (1 - b^c_{t_1}) \left(\frac{P^n_{t_1}}{P^n_{t_0}} \right)^{\theta_0 - 1} \right)^{\frac{1}{\theta_0 - 1}}. \tag{6}$$

In this case, the price index is a weighted CES aggregator of the price index for continuing goods and the price index for newly consumed goods. The price index increases less than the price index for continuing goods if inflation for newly consumed goods is less than inflation for continuing goods.[2]

Proposition 1 also applies in the case of non-homothetic CES with taste shocks. As we emphasize in Baqaee & Burstein (2021), the price index for continuing goods must be computed using budget shares (as a function of prices) evaluated at the t_1 indifference curve ($t_1$the tastes and t_1 utility). Similarly, $b_{t_0}^{x_{t_1}}$ is the share of exiting goods at t_0 prices under t_1 tastes and t_1 utility.

Proof. By Lemma 1 from Baqaee & Burstein (2021), welfare changes measured as the equivalent variation at t_1 preferences is given by[3]

$$EV^m = \log \frac{I_{t_1}}{I_{t_0}} - \log \frac{P_{t_1}}{P_{t_0}}, \tag{7}$$

where

$$\frac{P_{t_1}}{P_{t_0}} = \left(\frac{\sum_j x_{jt_1} p_{jt_1}^{1-\theta_0}}{\sum_i x_{it_1} p_{it_0}^{1-\theta_0}} \right)^{\frac{1}{1-\theta_0}} = \left(\frac{\sum_i x_{it_1} p_{it_1}^{1-\theta_0} \left(\frac{p_{it_0}}{p_{it_1}}\right)^{1-\theta_0}}{\sum_j x_{jt_1} p_{jt_1}^{1-\theta_0}} \right)^{\frac{1}{\theta_0-1}}. \tag{8}$$

We re-express (8) as

$$\begin{aligned} \left(\frac{P_{t_1}}{P_{t_0}}\right)^{\theta_0-1} &= \frac{\sum_{i\in\mathcal{C}} x_{it_1} p_{it_1}^{1-\theta_0} \left(\frac{p_{it_0}}{p_{it_1}}\right)^{1-\theta_0} + \sum_{i\in\mathcal{N}} x_{it_1} p_{it_1}^{1-\theta_0} \left(\frac{p_{it_0}}{p_{it_1}}\right)^{1-\theta_0} + \sum_{i\in\mathcal{X}} x_{it_1} p_{it_0}^{1-\theta_0}}{\sum_j x_{jt_1} p_{jt_1}^{1-\theta_0}} \\ &= b_{t_1}^c \sum_{i\in\mathcal{C}} b_{it_1}^c \left(\frac{p_{it_0}}{p_{it_1}}\right)^{1-\theta_0} + \left(1 - b_{t_1}^c\right) \sum_{i\in\mathcal{N}} b_{it_1}^n \left(\frac{p_{it_0}}{p_{it_1}}\right)^{1-\theta_0} + b_{t_0}^{x_{t_1}} \left(\frac{P_{t_1}}{P_{t_0}}\right)^{\theta_0-1} \end{aligned} \tag{9}$$

where $b_{t_1}^c \equiv \sum_{i\in\mathcal{C}} b_{it_1}$ denotes the expenditure share on continuing goods at t_1,

$$b_{it_1}^c = \frac{x_{it_1} p_{it_1}^{1-\theta_0}}{\sum_{i\in\mathcal{C}} x_{it_1} p_{it_1}^{1-\theta_0}}, \qquad b_{it_1}^n = \frac{x_{it_1} p_{it_1}^{1-\theta_0}}{\sum_{i\in\mathcal{N}} x_{it_1} p_{it_1}^{1-\theta_0}},$$

and

$$b_{t_0}^{x_{t_1}} \equiv \frac{\sum_{i\in\mathcal{X}} x_{it_1} p_{it_0}^{1-\theta_0}}{\sum_j x_{jt_1} p_{jt_0}^{1-\theta_0}}$$

[2]For example, the model in Arkolakis (2016) predicts that changes in tastes induced by advertising will be correlated with changes in physical productivity, whereby more productive firms will expend more resources on advertising. In this case price of newly consumed goods increase on average less than continuing goods.

[3]With homothetic preferences, equivalent and compensating variation are the same.

is the (unobserved) share of exiting goods at t_0 prices under t_1 preferences. Defining a price index for continuing goods under t_1 preferences,

$$\frac{P^c_{t_1}}{P^c_{t_0}} = \left(\sum_{i \in \mathcal{C}} b^c_{it_1} \left(\frac{p_{it_0}}{p_{it_1}} \right)^{1-\theta_0} \right)^{\frac{1}{\theta_0 - 1}}$$

and a price index for new goods under t_1 preferences,

$$\frac{P^n_{t_1}}{P^n_{t_0}} = \left(\sum_{i \in \mathcal{N}} b^n_{it_1} \left(\frac{p_{it_0}}{p_{it_1}} \right)^{1-\theta_0} \right)^{\frac{1}{\theta_0 - 1}},$$

we can express the change in the welfare-relevant price index as

$$\frac{P_{t_1}}{P_{t_0}} = \left(\frac{b^c_{t_1} \left(\frac{P^c_{t_1}}{P^c_{t_0}} \right)^{\theta_0 - 1} + \left(1 - b^c_{t_1}\right) \left(\frac{P^n_{t_1}}{P^n_{t_0}} \right)^{\theta_0 - 1}}{1 - b^{x_{t_1}}_{t_0}} \right)^{\frac{1}{\theta_0 - 1}} \tag{10}$$

□