The Aggregate Implications of Innovative Investment in the Garcia-Macia, Hsieh, and Klenow Model

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Abstract

In this paper, we extend the model of firm dynamics of Garcia-Macia, Klenow, and Hsieh (2016) (GHK) to include a description of the costs of innovative investments as in the models of Klette and Kortum (2004), Luttmer (2007, 2011), and Atkeson and Burstein (2010). In this model, aggregate productivity (TFP) grows as a result of innovative investment by incumbent and entering firms in improving continuing products and acquiring new products to the firm. This model serves as a useful benchmark because it nests both Quality-Ladders based Neo-Schumpeterian models and Expanding Varieties models commonly used in the literature and, at the same time, it provides a rich model of firm dynamics as described in GHK. We show how data on firm dynamics and firm value can be used to infer the elasticities of aggregate productivity growth with respect to changes in incumbent firms' investments in improving their incumbent products, incumbent firms' investments in acquiring products new to the firm, and entering firms' investments in acquiring new products. As discussed in Atkeson and Burstein (2015), these elasticities are a crucial input in evaluating the extent to which it is possible to alter the medium term growth path of the macroeconomy through policies aimed at stimulating innovative investments by firms. Using these methods, we find elasticities that are moderately larger than those possible in Neo-Schumpeterian models, corresponding to modest rates of social depreciation of innovation expenditures. Our estimates are sensitive to the extent of business stealing, which is not well identified in our data.

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1 Introduction:

Garcia-Macia, Klenow, and Hsieh (2016) (henceforth GHK) present a tractable model that captures many features of the data on firm dynamics. This model allows for aggregate productivity growth to arise through innovation by incumbent firms to improve their own products, innovation by incumbent firms to obtain products new to the firm, and innovation by entering firms to obtain new products. Products that are new to a firm may be new to society or "stolen" from other firms. The goal of their paper is to use data on firm dynamics to estimate how much of the observed growth in aggregate productivity comes from these different types of innovation by firms.

In this paper, we extend the GHK model of firm dynamics to include a description of the costs of innovative investments that are left un-modelled in their paper. Our extended version of the GHK model then conveniently nests both the canonical Expanding Varieties models analyzed in Luttmer (2007), Luttmer (2011) and Atkeson and Burstein (2010) and the canonical Quality-Ladders based Neo-Shumpeterian models analyzed in Klette and Kortum (2004) and the many models based on that framework. As a result, it incorporates the increasing returns due to increased variety as well as the intertemporal knowledge spillovers from one firm's success in innovation to the social payoffs to another firm's innovative investment. We then use this extended version of the GHK model to consider the question of how an economist who has access to rich data on firm dynamics and firm value might identify the social returns to *increased* innovative investment by firms.

We measure the social returns to innovative investment by firms in terms of the increased growth of aggregate productivity (TFP) from one year to the next that would result from an increase in the level of real innovative investment undertaken by firms in the economy. We conduct our measurement using data on firm dynamics from the Business Dynamics Statistics database and the Integrated Macroeconomic Accounts for the U.S. non-financial corporate sector. Our measurement of firm value follows the work of Hall (2003), McGrattan and Prescott (2005, 2010, Forthcoming), and others.

When we use the baseline specification of our model to conduct the measurement, we find a moderate elasticity of aggregate TFP growth with respect to changes in aggregate innovative investment in the range of 0.027. This elasticity implies that a permanent 10% increase in the labor force devoted to innovative investment would produce a 27bp increase in the growth rate of aggregate TFP on impact and an increase in the level of aggregate TFP in 20 years relative to its baseline level of 2.9%.[1]

In our baseline specification of our model, we assume that there is no business stealing and, as a result, our estimates of the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment do not depend on how that additional innovative investment is allocated across different categories of investment. Moreover, there are no gains in productivity growth to be had by reallocating a given aggregate level of innovative investment across the three categories of investment. This finding is thus quite sensitive to our assumptions about the extent of business stealing. To illustrate this point, we consider alternative specifications of our model in which the maximum level of business stealing consistent with the firm dynamics data is assumed. Here we find that the elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment is unchanged relative to our baseline specification of the model if that change in innovative investment in concentrated exclusively on an increase investment by incumbent firms on improving their existing products. In contrast, the elasticity of aggregate TFP growth with respect to changes in aggregate innovative investment concentrated on investment by incumbent and entering firms on acquiring products new to these firms is much smaller that we found in our baseline. In addition, we find that there would be substantial gains in productivity growth to be had by reallocating a given aggregate level of innovative investment away from investment by entering firms and by incumbent firms in acquiring new products and towards investment by incumbent firms in improving existing products. We thus see our findings in the baseline specification of our model as a generous estimate of the productivity gains to be had from using general or untargeted policies to stimulate more innovative investment by firms. Further research would be needed to make firm conclusions about the possibilities for improving aggregate productivity through policies that reallocate a given level of innovative investment.

Atkeson and Burstein (2015) show that if one imposes the assumption that there is no social depreciation of innovative expenditures, as is done implicitly in Neo-Schumpeterian models based on the Quality Ladders framework such as Klette and Kortum (2004) and also in GHK, then the quantitative implications of the model for the elasticities of aggre-

[1]As discussed in Atkeson and Burstein (2015), to compute the dynamics of TFP growth beyond the first year, we must specify the extent of intertemporal knowledge spillovers. Here we use the estimate of these spillovers from Fernald and Jones (2014).

gate productivity growth with respect to changes in innovative investments in economies with low baseline levels of TFP growth are tightly restricted by that low baseline level of TFP growth (roughly 1.2% or less than half of what we find in the baseline specification of our model) regardless of the fit of the model to the data on firm dynamics and value. To conduct our measurement, we thus extend the GHK model to allow for two simple forms of social depreciation of innovative expenditures: we allow incumbent firms to lose products due to exogenous exit of products and we allow for the productivity with which incumbent firms can produce products to deteriorate over time in the absence of innovative investments by that firm. One implication of our measurement findings is that social depreciation of innovative expenditures must exist and be larger than roughly 50bp per year.

Now consider the logic of how we obtain these results. Our measurement of the model-implied elasticity of aggregate TFP growth with respect to a change in the level of innovative investment is based on two key implications of the model regarding the relationship, in equilibrium, between innovative investment, product size, firm value, and aggregate TFP growth.

The first of these key implications of the model is that the marginal impact on aggregate TFP growth of innovative investment by firms either to improve existing products, steal products from other firms, and create new products is directly proportional to the marginal impact on product size net of creative destruction that results from such investment, with the factor of proportionality determined by the elasticity of substitution between products in production of the final consumption good. To be specific, in the model, investments by firms either to improve their own products or steal products from each other result in a net increase in the size of the existing product being innovated on due to an increase in the productivity with which it can be produced, while investments by firms to create new products result in the gross addition of a new product of a given size which is determined by the productivity with which that new product can be produced. Thus, to measure the elasticity of aggregate TFP growth with respect to a change in innovative investment by firms, we must measure the impact, at the margin, of each type of innovative investment on product size.

To measure the marginal impact of innovative investment on product size, we use the second key implication of the model — its implication regarding the relationship between firm size and firm value. In the equilibrium of the model, the value of a product to a firm is directly proportional to its size. In the equilibrium of the model, if firms choose innovative investment to maximize profits, the rate of return on innovative investments should be equal to the equilibrium rate of return of the economy. Since the rate of return on each type of innovative investment is directly proportional to the impact, at the margin, of that type of innovative investments on product size, we can use this equilibrium condition of the model to measure that marginal impact of investment on product size that we need to calculate the marginal impact of such investments on aggregate TFP growth.

To be clear, we are not imposing that the private and social returns to innovative investment by firms are equal. These returns differ due to markups, spillovers from successful innovation by one firm on a product on the costs of benefits of innovative investment by other firms and due to failure of firms to internalize the impact of business stealing on other firms. Instead, we bring together data on firm dynamics and firm value interpreted through the equilibrium conditions of the model to conduct a measurement of the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment which we interpret as a measure of the social return to such investments. We see one of the main contributions of our paper to be the development of simple formulas relating data on firm dynamics, firm value, firms' innovative investment expenditures, and the equilibrium rate of return to the elasticities of aggregate TFP growth with respect to a change in aggregate innovative investment allocated in any way across the three categories of innovative investment that we consider.

Our specification of the technologies for how innovative investments by firms translate into improved productivities of products have three central implications that keep our extended GHK model tractable and allow for aggregation of data on investment, size, and value across firms. First, as in Klette and Kortum (2004), Atkeson and Burstein (2010), and many other papers in the literature, the costs and benefits of innovative investment per existing product scale with product size. This implies that, in equilibrium, innovative investments of each firm are directly proportional to firm size. Second, as in many papers in the literature, our assumptions on innovation technologies imply that if innovative investments per existing product are proportional to product size, then the dynamics of product size are consistent with a strong form of Gibrat's Law. With this strong form of Gibrat's Law at the level of products, we can characterize the growth rate of aggregate productivity as a simple function of aggregate real innovative investment in each of the three categories of innovative investment, as in GHK.[2] Third, our assumptions on spillovers impacting the costs and benefits of innovative investments (which extend those in Luttmer (2007)) ensure that, on a balanced growth path with growth in both the average productivity of products and in the total measure of products, aggregate innovative investment in each of the three categories we consider is constant over time.

This paper builds on our work in Atkeson and Burstein (2015). In that previous paper, using a more general model, we showed how, under certain assumptions, the quantitative implications of that model for the change in the dynamics in aggregate TFP that arise from a change in expenditures on innovative investment can be summarized by two key statistics — the impact elasticity of aggregate TFP growth from one year to the next with respect to a change in aggregate innovative investments and the intertemporal spillovers of knowledge. We derived an upper bound on the impact elasticity equal to the difference between the model's calibrated baseline TFP growth rate and the social depreciation of innovative investments. We showed how these two key model statistics also shape the model's implications for the change in welfare that result from a change in aggregate innovative investment. Finally, we derived results regarding the fiscal cost of subsidies required to boost the innovation intensity of the economy on a balanced growth path.

The model we consider in this paper consolidates the five example economies that we considered in Atkeson and Burstein (2015). Under certain parameter restrictions, the model in this paper satisfies the three main assumptions we use in Atkeson and Burstein (2015). Thus, under these restrictions, our earlier results apply directly to the model we use in this paper.

Our primary contribution in this paper relative to our earlier work is to draw on the GHK model's implications for the relationship between product size and product value to measure the impact elasticity of aggregate TFP growth from one year to the next with respect to a change in aggregate innovative investment, and hence to provide a bound on the social depreciation of innovative investments.

In section 2, we lay out the GHK model together with the technologies for innovative

[2] Akcigit and Kerr (2017) provide an alternative specification of the innovative investment technologies available to firms that imply that expenditures by firms on acquiring new products does not scale with firm size. It is possible to extend our measurement methods to a variation of our model in which innovation investments by incumbents firms to acquiring new products scale up with their number of products but not with firm size.

investment that we add in our extension of that model. We also describe the prominent existing models that are nested by this new model. In section 3, we define the elasticities of aggregate TFP growth with respect to changes in innovative investment that we seek to measure. In this section, we also present our first proposition regarding our model's implications for these elasticities under the assumption of no social depreciation of innovative investment characteristic of Neo-Schumpeterian models. In section 4, we present our model's implications for firm dynamics and firm value. In section 5, we use those implications of our model to derive formulas for the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment. In section 6 we use these formulas to conduct our measurement and present our results. We then conclude. Further details on the model and on our measurement procedure are presented in the Appendix.

2 Model and Equilibrium Properties:

In this section, we first present our extension of the GHK model to include specifications of the technologies for innovative investment by firms. We then derive the reduced-form equilibrium relationship between the growth rate of aggregate TFP and the levels of real innovative investment in each of the three categories of investment that we consider: investment by incumbent firms in improving their own products, investment by incumbent firms in acquiring products that are new to that incumbent firm, and investment by entering firms in acquiring products that are new to that entering firm. We finally discuss the set of models that are nested in our extension of the GHK models.

2.1 Aggregate Output and Total Factor Productivity

There is a final good, used for consumption and investment in physical capital, produced from a continuum of intermediate products through a CES aggregator

$$Y_t = \left[\sum_z y_t(z)^{(\rho-1)/\rho} M_t(z) \right]^{\rho/(\rho-1)}$$

where Y_t denotes the output o the final consumption good and $M_t(z)$ is the measure of intermediate products with index z at time t. The total measure of intermediate products at t is given by $M_t = \sum_z M_t(z)$.

These intermediate products are produced at each date t according to production technologies

$$y_t(z) = \exp(z) k_t(z)^{\alpha} l_{pt}(z)^{1-\alpha}$$

where z indexes the position of the marginal cost curve for the producer of the intermediate good with this index, and $k_t(z)$ and $l_{pt}(z)$ denote the quantities of physical capital and labor used in production of the intermediate good with index z at date t. To simplify our notation, we assume that the support of z is a countable grid with $z_n = n\Delta$ for the integers n.[3]

We assume that, in equilibrium, producers of the these intermediate products sell their output to producers of the final good at a constant markup $\mu > 1$ over their marginal cost. This markup may be the monopoly markup $\rho/(\rho - 1)$ determined by the elasticity ρ in the CES aggregator for final good production or a smaller markup determined by Bertrand competition by the equilibrium producer of that intermediate product with a latent competitor with marginal cost that is μ times the marginal cost of the producer of the intermediate product.

Assuming constant markups $\mu > 1$ of prices over marginal costs across intermediate products, and that factor prices are such that capital/labor ratios are equal across products, in equilibrium, aggregate output of the final good is given by

$$Y_t = Z_t K_t^{\alpha} L_{pt}^{1-\alpha}$$

where K_t is the aggregate stock of physical capital, L_{pt} is the aggregate quantity of labor used in production of this final good, and Z_t is total factor productivity given by

$$Z_t = \left[\sum_z \exp((\rho - 1)z) M_t(z) \right]^{1/(\rho-1)}. \tag{1}$$

2.1.1 Product Size

In equilibrium we have that the shares of output and inputs accounted for by an intermediate product with index z at time t is given by

$$s_t(z) = \frac{\exp((\rho - 1)z)}{Z_t^{\rho-1}} = \frac{y_t(z)}{Y_t} = \frac{k_t(z)}{K_t} = \frac{l_{pt}(z)}{L_{pt}}.$$

[3]Under the assumption of a CES aggregator, productivity z can be reinterpreted as a measure of product quality (so that firms innovate to improve the quality of products rather than to increase their productivity), without changing the results in this paper.

Hence, we refer to $s_t(z)$ as the *size* of a product. In data, this can be measured in terms of value added or profits or physical capital or production labor. This measure is also additive, so we can use it to refer to the *size* of categories of products as we do below.

2.1.2 Products and Firms

As in Klette and Kortum (2004), firms in this economy produce a number of products n, where n is a natural number. Entering firms enter with a single product, so $n = 1$. Firms exit when the number of products that they produce drops to zero. We say that a firm is an *incumbent firm* at t if it also produced products at $t-1$. Otherwise, firms at t are *entering firms*. We say that a product is an *existing product* at t if it was also produced at $t-1$. Otherwise, products at t are *new products*. New products are new to society.

Not all products that are new to a firm at t are new to society. Some products that are new to a firm at t are existing products that were produced by some other firm at $t-1$. We refer to existing products that are produced by a different firm at t than at $t-1$ as *stolen products*. We refer to existing products at t that are produced by the same firm at t as at $t-1$ as *continuing products*. Note that, by definition, continuing products are produced by incumbent firms.

With this terminology, we decompose aggregate productivity at t into three components:

$$Z_t^{\rho-1} = Z_{ct}^{\rho-1} + Z_{mt}^{\rho-1} + Z_{et}^{\rho-1}.$$

First, we define the contribution to aggregate productivity at t from continuing products at incumbent firms as

$$Z_{ct} = \left[\sum_z \exp((\rho-1)z) M_{ct}(z)\right]^{1/(\rho-1)}$$

where $M_{ct}(z)$ is the measure of continuing products with index z produced by incumbent firms at date t. Second, we define the gross contribution to aggregate productivity at t from products that are new to incumbent firms (either new products for the society or stolen from other firms), Z_{mt}, in the analogous manner. Third, we define the gross contribution to aggregate productivity at t from products produced by entering firms (either new products for the society or stolen from other firms), Z_{et}, in the analogous manner.

These contributions of different product categories to aggregate productivity at t are directly proportional to the aggregate size of each of these product categories at t. Specifically, let $S_{ct} = Z_{ct}^{\rho-1}/Z_t^{\rho-1}$ denote the aggregate size of continuing products at incumbent firms. Likewise, let $S_{mt} = Z_{mt}^{\rho-1}/Z_t^{\rho-1}$ denote the size of those products that are new to incumbent firms at t (including both new products to society and products stolen from other incumbent firms), and $S_{et} = Z_{et}^{\rho-1}/Z_t^{\rho-1}$ denote the size of those products that are new to entering firms at t.

Similarly, we decompose the total measure of products as

$$M_t = M_{ct} + M_{mt} + M_{et}.$$

We also find it useful to develop notation for the fraction of products in each of these product categories. To that end, let $F_{ct} = M_{ct}/M_t$ denote the fraction of products that are continuing products at incumbent firms. Likewise, let $F_{mt} = M_{mt}/M_t$ denote the fraction of products that are new to incumbent firms at t and $F_{et} = M_{et}/M_t$ denote the fraction of products that are new to entering firms at t.

We define the *average size* of products in a category to be the ratio S/F for any given category. Given this definition, the average size of all products at each date t is equal to one. Hence, our measure of average size of products in a category is a measure of the average absolute size of a category of products relative the average absolute size of all products.

2.2 Technology for Innovative Investment:

We assume that firms make three types of innovative investments: incumbent firms invest to improve their continuing products, incumbent firms invest to acquire new products to the firm either through the creation of a new product or acquisition of a stolen product, and entering firms invest to acquire new products to the entering firm either through the creation of a new product or acquisition of a stolen product.

Innovative investment is undertaken using a second final good, which we term the *research good*, as an input. The aggregate amount of this research good produced at time t is

$$Y_{rt} = A_{rt} Z_t^{\phi-1} L_{rt} \tag{2}$$

where $L_{rt} = L_t - L_{pt}$ is the quantity of labor devoted to production of the research good (the total quantity of labor L_t grows at an exogenous rate $\bar{g}_L$), A_{rt} is the level of exogenous scientific progress (which grows at an exogenous rate $\bar{g}_{A_r}$), and the term $Z_t^{\phi-1}$ for $\phi \leq 1$ reflects intertemporal knowledge spillovers in the production of the research good as in the model of Jones (2002). We let P_{rt} denote the relative price of the research good and the final consumption good at time t.

In specifying the production function (2) for the research good, we are following Bloom et al. (2017) in choosing units for the inputs into innovative investment such that it is possible to maintain a constant growth rate of aggregate TFP by investing a constant real amount Y_r of the research good.[4] Using the language of that paper, L_{rt} denotes the quantity of labor devoted to "research" and $A_{rt}Z_t^{\phi-1}$ denotes the productivity with which that research labor translates into a real flow of "ideas" Y_r available to be applied to the three categories of innovative investment examined in our model. We have assumed that exogenous scientific progress in terms of increasing A_r, by itself, drives up research productivity over time. If we assume $\phi < 1$, then increases in the level of aggregate productivity reduce research productivity in the sense that "ideas become harder to find". As we discuss below, with $\phi < 1$, the growth rate of our model economy on a balanced growth path is driven by the growth of scientific progress and population growth consistent with production of a constant amount of the research good Y_{rt} when research labor L_{rt} grows with the population.

We denote the aggregate quantity of the research good that incumbent firms invest at t in improving z for continuing products at $t+1$ by x_{ct}. We denote the aggregate quantity of the research good that incumbent firms invest at t in acquiring products new to that firm at $t+1$ by x_{mt}. We denote the aggregate quantity of the research good that entering firms invest at t in acquiring products new to that firm at $t+1$ by x_{et}. The resource constraint for the research good is

$$x_{ct} + x_{mt} + x_{et} = Y_{rt}. \tag{3}$$

2.2.1 Overview of assumptions on innovative investment technologies

We now specify the three technologies available to firms for innovative investment. We begin with a brief overview of the key assumptions used in our specification of these technologies and then we provide the details of each of the three technologies.

[4] Our parameter ϕ corresponds to $1 - \beta$ in equation (17) of Bloom et al. (2017).

We specify the technologies for innovative investment used by firms so that, in equilibrium, the allocation of investment across firms satisfies three properties.

First, we follow Klette and Kortum (2004) and Atkeson and Burstein (2010) in choosing innovative investment technologies for incumbent firms so that, in equilibrium, the innovation intensity of each firm is directly proportional to firm size. Specifically, we assume as in Klette and Kortum (2004), that the research capacity of an incumbent firm for improving its products and for acquiring new products is determined by the current number of products that it produces. We extend the model of the firm's research capacity used in that paper so that equilibrium investment by incumbent firms either in improving each of their continuing products or in obtaining new products is directly proportional to the size of the product for each product that they produce.

Second, we extend the spillovers impacting the costs and benefits of innovative investments assumed in Luttmer (2007) to ensure that, on a balanced growth path with growth in both the average productivity of products $Z_t^{\rho-1}/M_t$ and in the total measure of products M_t, aggregate innovative investment in each of the three categories we consider is constant over time.

Third, as in Klette and Kortum (2004) and Atkeson and Burstein (2010) and many other papers in the literature, we assume that innovative investments result in equilibrium dynamics for the size of existing products consistent with a strong form of Gibrat's Law. With this strong form of Gibrat's Law, we can characterize the growth rate of aggregate productivity as a simple function of aggregate real innovative investment in each of the three categories of investment.

We now describe in more detail the assumptions on the technologies for innovative investment at the firm level and the implied dynamics of the measures and contribution to aggregate productivity of each product category.

2.2.2 Investment in Entry

An entrant at time t spends $1/M_t$ units of the research good to launch a new firm at $t+1$ with one product.[5] With probability δ that product is stolen from an incumbent firm. With probability $1-\delta$ that product is new to society.

We assume that the productivity index for stolen products in new firms is drawn in a

[5]Our results are unchanged if we introduce an additional parameter y_e such that entry costs are given by y_e/M_t (we have imposed $y_e = 1$).

manner similar to that in Klette and Kortum (2004) and other standard Quality Ladders type models. Specifically, we assume that stolen products in new firms at $t+1$ have a productivity index z' that is drawn from a distribution such that the expected value of the term $\exp((\rho-1)z')$ is equal to $\mathbb{E}\exp((\rho-1)z') = \eta_{es} Z_t^{\rho-1}/M_t$.[6] Recall that $Z_t^{\rho-1}/M_t$ is the expected value of $\exp((\rho-1)z)$ across all products produced at t.

We assume that the productivity index for new products in new firms is drawn in a manner similar to that in Luttmer (2007). Specifically, we assume that new products in new firms at $t+1$ have a productivity index z' drawn from a distribution such that the expected value of the term $\exp((\rho-1)z')$ is equal to $\mathbb{E}\exp((\rho-1)z') = \eta_{en} Z_t^{\rho-1}/M_t$, with $\eta_{en} > 0$.

With these assumptions, given a total investment x_{et} of the research good, there are $M_{et+1} = x_{et} M_t$ entering firms at time $t+1$ (a fraction δ of these entering firms produce products that are stolen from other firms and a fraction $1-\delta$ produce products that are new to society). The gross contribution of all products produced in entering firms to aggregate productivity at $t+1$ is given by

$$Z_{et+1}^{\rho-1} = \eta_e Z_t^{\rho-1} x_{et},$$

where we define the parameter η_e as

$$\eta_e = \delta \eta_{es} + (1-\delta)\eta_{en}.$$

2.2.3 Investment in New Products by Incumbent Firms

Associated with each product that an incumbent firm produces is a technology for acquiring new products that depends on the productivity index z of that product. Specifically, if an incumbent firm at t invests $x_{mt}(z)$ units of the research good into the technology for acquiring new products associated with its current product with index z, then it has probability

$$h\left(x_{mt}(z)\frac{Z_t^{\rho-1}}{\exp((\rho-1)z)}\right)$$

of acquiring a new product (new to the firm) at $t+1$. Here, $h(\cdot)$ is a strictly increasing and concave function with $h(0) = 0$ and $h(x) < 1$ for all x. Since we focus on local elasticities,

[6] In standard Quality Ladder models, $\eta_{es} = \exp((\rho-1)\Delta_s)$, where Δ_s denotes the percentage improvement in productivity of stolen products. We allow the step size to be stochastic and independent of the productivity of the stolen product, as in GHK.

we do not make further assumptions on the shape of this function or the function $\zeta(\cdot)$ defined below.

With probability δ, that product is stolen from an incumbent firm that was producing a product with index z at t and it has productivity index z' at $t+1$ drawn from a distribution such that the expected value of the term $\exp((\rho-1)z')$ is equal to $\mathbb{E}\exp((\rho-1)z') = \eta_{ms}\exp((\rho-1)z)$.[7]

With probability $1-\delta$ that product is new to society. We assume that the productivity index z' for new products in incumbent firms acquired using the research technology associated with an existing product with index z is drawn from a distribution such that the expected value of the term $\exp((\rho-1)z')$ is equal to $\mathbb{E}\exp((\rho-1)z') = \eta_{mn}\exp((\rho-1)z)$ with $\eta_{mn} > 0$.

In appendix 8 we show that in an equilibrium with constant markups and uniform subsidies for investment in new products by incumbent firms, the scale of this investment per product with index z is directly proportional to the size of the product. That is,

$$x_{mt}(z) = x_{mt}\frac{\exp((\rho-1)z)}{Z_t^{\rho-1}},$$

where the factor of proportionality x_{mt} is also the aggregate amount of such investment at t ($\sum_z x_{mt}(z)M_t(z)$). With such investment by incumbents in acquiring new products directly proportional to size, the total measure of products that are new to incumbent firms at $t+1$ is

$$M_{mt+1} = (\delta + 1 - \delta)h(x_{mt})M_t = h(x_{mt})M_t,$$

where a fraction δ of these products are stolen from other firms and a fraction $1-\delta$ are new to society.

Likewise, the gross contribution to aggregate productivity at $t+1$ of products that are new to incumbent firms is

$$Z_{mt+1}^{\rho-1} = \sum_z \left(\delta\eta_{ms} + (1-\delta)\eta_{mn}\right) h(x_{mt}) \exp((\rho-1)z)M_t(z) = \eta_m Z_t^{\rho-1},$$

where we define

$$\eta_m = \delta\eta_{ms} + (1-\delta)\eta_{mn}.$$

[7] Our results are unchanged if we allow for differences between entrants and incumbents in the fraction of new products that are stolen, δ, as in GHK (we consider this extension when we calibrate our model using GHK's estimated parameters).

2.2.4 Investment in Continuing Products by Incumbent Firms

Incumbent firms producing M_t products at t lose those products for three reasons. A fraction δ_0 of those product exit exogenously. A fraction $\delta M_{et+1}/M_t = \delta x_{et}$ of these products are lost due to business stealing by entering firms. A fraction $\delta h(x_{mt})$ of these products are lost due to business stealing by incumbent firms. Thus the measure of continuing products in incumbent firms at time $t+1$ is

$$M_{ct+1} = (1 - \delta_{ct})M_t$$

where δ_{ct} denotes the exit rate of incumbent products, given by

$$\delta_{ct} \equiv \delta_0 + \delta\left(h(x_{mt}) + x_{et}\right). \tag{4}$$

Incumbent firms have research capacity associated with each product that they produce that allows them to invest to improve the index z of that product. We follow Atkeson and Burstein (2010) in describing the technology incumbent firms use to improve continuing products. We assume that if an incumbent firm with a product with productivity z at t spends $x_{ct}(z)$ of the research good on improving that product, it draws a new productivity index z' at $t+1$, conditional on continuing, from a distribution such that the expectation of $\exp((\rho-1)z')$ is

$$\mathbb{E}\exp((\rho-1)z') = \zeta\left(x_{ct}(z)\frac{Z_t^{\rho-1}}{\exp((\rho-1)z)}\right)\exp((\rho-1)z).$$

We assume that $\zeta(\cdot)$ is a strictly increasing and concave function.

In appendix 8 we show that in equilibrium with constant markups and uniform subsidies for this category of innovative investment, each incumbent firm chooses the same investment per unit size $x_{ct}(z) = x_{ct}\frac{\exp((\rho-1)z)}{Z_t^{\rho-1}}$. This implies that the contribution of continuing products in incumbent firms to aggregate productivity at $t+1$ is given by

$$Z_{ct+1}^{\rho-1} = (1-\delta_{ct})\,\zeta(x_{ct})Z_t^{\rho-1}. \tag{5}$$

We assume that $\eta_{es} \geq \zeta(x_{ct})$ and $\eta_{ms} \geq \zeta(x_{ct})$. These inequalities correspond to the requirement that a product that is stolen from incumbent firms is, in expectation, produced with a higher z' at $t+1$ in its new firm than it would have had as a continuing product in the firm that previously produced it. Equivalently, stolen products have larger average size than continuing products in incumbent firms. This assumption is justified if the time period in the model is short enough.[8]

2.3 Dynamics of the number of products and aggregate TFP

These assumptions imply dynamics for the total measure of products given by

$$\frac{M_{t+1}}{M_t} = (1 - \delta_{ct}) + h(x_{mt}) + x_{et}. \tag{6}$$

These assumptions also imply dynamics for aggregate productivity

$$\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho-1} = (1 - \delta_0 - \delta(h(x_{mt}) + x_{et}))\,\zeta(x_{ct}) + \eta_m h(x_{mt}) + \eta_e x_{et}. \tag{7}$$

2.4 Balanced growth path

We focus on balanced growth path (BGP), in which the division of labor between production and research, production of the research good, and its division over the three forms of investments are constant over time, $\bar{L}_p/\bar{L}_r$, $\bar{Y}_r$, $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$. On a BGP, aggregate productivity, the measure of products, and output per worker grow at constant (log) rates given by

$$\bar{g}_Z = \frac{\bar{g}_L + \bar{g}_{A_r}}{1 - \phi},$$

$$\bar{g}_{Y/L} = \frac{\bar{g}_Z}{1 - \alpha},$$

$$\bar{g}_M = \log\left(1 - \delta_0 + (1 - \delta)(h(\bar{x}_m) + \bar{x}_e)\right).$$

2.5 Nested Models

This model nest five commonly used models in the literature: three types of Expanding Varieties models and two types of Neo-Schumpeterian models.

If $\delta = 0$, then there is no business stealing and hence all new products acquired by incumbent and entering firms are new products for society, expanding the measure of

[8]Since δ_{ct} corresponds to an exit rate per unit time of existing products in incumbent firms, we do require that δ_0, $h(x_{mt})$ and x_{et} all shrink to zero in proportion to the length of a time period as the time interval between periods t and $t+1$ shrinks to zero. Thus, if we choose a short enough time period, we do not have to be concerned about the possibility that $\delta_{ct} > 1$. Likewise, since $\log(\zeta(x_c))$ corresponds to the expected growth rate of z for continuing products in incumbent firms, we require that $\log(\zeta(x_c))$ converges to zero in proportion to the length of a time period as the time interval between periods t and $t+1$ shrinks to zero. On the other hand, the expected increments to size, η_{es} and η_{ms}, are both independent of the length of the time period.

products M_t. This is the assumption typically made in an Expanding Varieties model. Luttmer (2007) is an example of an expanding varieties model in which there is only innovative investment in entry. (Note that we do not consider the endogenous exit of products due to fixed operating costs featured in that paper and in GHK). Atkeson and Burstein (2010) is an example of an expanding varieties model in which there is innovative investment in entry and by incumbent firms in continuing products. Luttmer (2011) is an example of an expanding varieties model in which there is innovative investment in entry and in the acquisition of new products by incumbent firms.

Neo-Schumpeterian models based on the Quality-Ladders framework typically assume $\delta = 1$ and $\delta_0 = 0$. The simplest versions of these models do not accommodate growth in the measure of varieties M_t. Grossman and Helpman (1991) and Aghion and Howitt (1992) are examples of Neo-Schumpeterian models in which there is only innovative investment in entry. Klette and Kortum (2004) is an example of a Neo-Schumpeterian model in which there is innovative investment in entry and by incumbent firms in acquiring new products (new to the firm, not to society).

3 Elasticities of TFP growth with respect to innovative investment

From equation (7), we can write the growth rate of TFP as a function of innovative investments as

$$g_{Zt} = \log(Z_{t+1}) - \log(Z_t) = G(x_{ct}, x_{mt}, x_{et})$$

where

$$G(x_c, x_m, x_e) = \frac{1}{\rho - 1} \log\left(\left(1 - \delta_0 - \delta(h(x_m) + x_e)\right) \zeta(x_c) + \eta_m h(x_m) + \eta_e x_e \right)$$

We now consider the elasticities of TFP growth with respect to the three types of innovative investment. To do so, we evaluate derivatives of G at a point ($\bar{x}_c$, $\bar{x}_m$, $\bar{x}_e$) and $\bar{Y}_r$ that satisfies equation (3) — recall that x_{ct}, x_{mt}, x_{et}, and Y_{rt} are all constant on a BGP. Then, to a first order approximation, we have

$$\hat{g}_Z = G_c \bar{x}_{ct} \hat{x}_{ct} + G_m \bar{x}_m \hat{x}_{mt} + G_e \bar{x}_e \hat{x}_{et} \tag{8}$$

where

$$G_c \bar{x}_c = \frac{1}{\rho - 1} \frac{\left(1 - \delta_0 - \delta(h(\bar{x}_m) + \bar{x}_e)\right) \zeta(\bar{x}_c)}{\exp((\rho - 1)\bar{g}_Z)} \frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)} \tag{9}$$

$$G_m \bar{x}_m = \frac{1}{\rho - 1} \frac{(\eta_m - \delta\zeta(\bar{x}_c))\, h(\bar{x}_m)}{\exp((\rho-1)\bar{g}_Z)} \frac{h'(\bar{x}_m)\bar{x}_m}{h(\bar{x}_m)} \tag{10}$$

and

$$G_e \bar{x}_e = \frac{1}{\rho - 1} \frac{(\eta_e - \delta\zeta(\bar{x}_c))\, \bar{x}_e}{\exp((\rho-1)\bar{g}_Z)}, \tag{11}$$

where for any variable in levels x, $\hat{x}_t$ denotes the log deviation from its BGP level, $\hat{x}_t = \log(x_t) - \log(\bar{x})$, and for any growth rate g, $\hat{g}_t$ denotes the difference from the BGP growth rate, $\hat{g}_t = g_t - \bar{g}$.

In general, the elasticity of the growth rate of aggregate TFP with respect to a change in aggregate real innovative investment $\hat{Y}_r$ depends on how that change in aggregate innovative investment is allocated across the three different types of investment subject to

$$\frac{\bar{x}_c}{\bar{Y}_r}\hat{x}_{ct} + \frac{\bar{x}_m}{\bar{Y}_r}\hat{x}_{mt} + \frac{\bar{x}_e}{\bar{Y}_r}\hat{x}_{et} = \hat{Y}_{rt}. \tag{12}$$

We consider two types of perturbations to innovative investment, $\hat{x}_{ct}$, $\hat{x}_{mt}$, $\hat{x}_{et}$. The first type is a proportional change in all categories of innovative investment

$$\hat{x}_{ct} = \hat{x}_{mt} = \hat{x}_{et} = \hat{Y}_{rt}$$

so that the elasticity of TFP growth with respect to total innovative expenditure is equal to the sum of three individual elasticities:

$$\hat{g}_Z = (G_c\bar{x}_c + G_m\bar{x}_m + G_e\bar{x}_e)\,\hat{Y}_{rt}.$$

The second type of perturbation is concentrated on a single form of innovative investment

$$\hat{x}_{ct} = \frac{\bar{Y}_r}{\bar{x}_c}\hat{Y}_{rt} \quad \text{or} \quad \hat{x}_{mt} = \frac{\bar{Y}_r}{\bar{x}_m}\hat{Y}_{rt} \quad \text{or} \quad \hat{x}_{et} = \frac{\bar{Y}_r}{\bar{x}_e}\hat{Y}_{rt}.$$

so that the elasticity of aggregate TFP growth with respect to changes in aggregate innovative investment is given by

$$G_c\bar{x}_c\frac{\bar{Y}_r}{\bar{x}_c} \quad \text{or} \quad G_m\bar{x}_m\frac{\bar{Y}_r}{\bar{x}_m} \quad \text{or} \quad G_e\bar{x}_e\frac{\bar{Y}_r}{\bar{x}_e}. \tag{13}$$

In the event that $G_c = G_m = G_e$, then these two different types of perturbations (and all other feasible perturbations) deliver the same elasticity of aggregate TFP growth with respect to a change in aggregate real innovative investment. We are interested in measuring the extent to which this aggregate elasticity differs depending on the type of perturbation, so we aim to measure the six terms $G_c\bar{x}_c$, $G_m\bar{x}_m$, $G_e\bar{x}_e$, and $\bar{Y}_r/\bar{x}_c$, $\bar{Y}_r/\bar{x}_m$, and $\bar{Y}_r/\bar{x}_e$ separately.

In what follows, we show how one can use data on firm dynamics and firm value together with the condition that firms' innovative investments are optimally chosen to measure the elasticities of aggregate TFP growth with respect to perturbations of aggregate innovative investment allocated in various ways across the three categories of investment. Before doing so, we first derive a bound on the elasticity of aggregate TFP growth with respect to a proportional change in all three categories of innovative investment implied by our model once it is calibrated to match a given TFP growth rate on the BGP, $\bar{g}_Z$, that is independent of such data.

3.1 Bounding the elasticity with respect to a proportional change in all innovative investment

Following Atkeson and Burstein (2015), we are able to bound the elasticity of TFP growth with respect to a proportional change in all innovative investments as follows.

Proposition 1. *If $\hat{x}_{ct} = \hat{x}_{mt} = \hat{x}_{et} = \hat{Y}_{rt}$, then the elasticity of TFP growth with respect to innovative investment is bounded by the difference between the baseline growth rate of TFP and the counterfactual growth rate of TFP when all investment is zero, i.e.*

$$\hat{g}_Z \leq (\bar{g}_Z - G(0,0,0))\,\hat{Y}_{rt}$$

Proof. The proof follows from the concavity of the function $H(a)$ defined as

$$H(a) \equiv G(a\bar{x}_c, a\bar{x}_m, a\bar{x}_e)$$

Specifically, if $\hat{x}_{ct} = \hat{x}_{mt} = \hat{x}_{et} = \hat{Y}_{rt}$, then

$$\hat{g}_Z = H'(1)\hat{Y}_{rt}.$$

The result follows from the fact that for concave functions $H'(1) \leq H(1) - H(0)$. We prove that $H(a)$ as defined above is concave in appendix 9, $\square$

We refer to the growth rate of TFP that would arise if all innovative investment were set to zero, $G(0,0,0)$, as the rate of *social depreciation of innovative investments.*

Note that the bound on the elasticity $\hat{g}_Z/\hat{Y}_r$ established in Proposition 1 is independent of parameters outside of those that determine the model's implications for $\bar{g}_Z$ and

$G(0,0,0)$. We have shown above how one can calibrate our model's implications for $\bar{g}_Z$, given a population growth rate $\bar{g}_L$, by choosing the growth rate of scientific progress $\bar{g}_{Ar}$ and the parameter ϕ governing intertemporal knowledge spillovers.

In both Klette and Kortum (2004) and GHK, it is assumed that the rate of social depreciation of innovation $G(0,0,0) = 0$. As a result, the elasticity $\hat{g}_Z/\hat{Y}_r$ corresponding to a proportional change in all three categories of innovative investment is bounded above by $\bar{g}_Z$. This proposition thus imposes a tight bound for advanced economies with low baseline $\bar{g}_Z$ in the GHK model as specified in GHK.

In our implementation of the GHK model, we do not make this assumption that there is no social depreciation of innovation. Because we allow for exogenous exit of existing varieties, denoted by δ_0, and for deterioration of the index z of continuing varieties in incumbent firms, denoted by $\bar{\zeta}(0) \leq 1$, we have social depreciation of innovation given by

$$G(0,0,0) = \frac{1}{\rho - 1} \log \left((1 - \delta_0) \zeta(0) \right) \tag{14}$$

(recall that since $h(x_m)$ denotes a rate at which incumbent firms acquire new products, we impose that $h(0) = 0$). Thus, our version of the GHK model potentially allows for a higher elasticity $\hat{g}_Z/\hat{Y}_r > \bar{g}_Z$ if we allow for social depreciation by calibrating $\delta_0 > 0$ and/or $\zeta(0) < 1$.

Once we allow for the possibility that $G(0,0,0) < 0$, the model admits for a large value of the elasticity $\hat{g}_Z/\hat{Y}_r$ to proportional changes in innovative investment, even in an advanced economy. Moreover, the elasticity of TFP growth with respect to changes in single forms of investment will differ from that of a proportional change in all investment forms if the derivatives G_c, G_m, and G_e defined in (9), (10), and (11) differ from each other.

4 Implications for Firm Dynamics and Firm Value

In this section, we discuss what data one can use to measure the terms needed to compute the derivatives (9), (10), and (11) and the equilibrium allocation of innovative investment $\bar{x}_c/\bar{Y}_r$, $\bar{x}_m/\bar{Y}_r$, and $\bar{x}_e/\bar{Y}_r$ needed to compute the elasticities in (13).

We first show how the values of $\bar{x}_e$, $\zeta(\bar{x}_c)$, $h(\bar{x}_m)$, η_e, and η_m can be inferred using data on product level dynamics. We then show that the baseline decomposition of investment $\bar{x}_c$, $\bar{x}_m$ and $\bar{x}_e$ relative to total investment $\bar{Y}_r$ can be inferred using the model's implications on the value of firms, and that the values of the derivatives $\zeta'(\bar{x}_c)$ and $h'(\bar{x}_m)$ can be inferred using the condition that firms choose investment to maximize their private value.

The parameters δ_0 and δ governing the share of products new to incumbent and entering firms that are stolen from other incumbent firms are not pinned down on a BGP by the data on firm dynamics or firm values without further structural assumptions such as those pursued in GHK. Rather than attempt to identify these parameters, we put bounds on the terms we wish to measure by considering the minimum and maximum permissible values of the parameter δ.

4.1 Implications of Data on Product Level Dynamics

We consider an economist who has data on the growth rate of the measure of products $g_{Mt} = \log(M_{t+1}/M_t)$ as well as data on the fraction of products that are continuing products in incumbent firms F_{ct+1}, the fraction of products that are new to incumbent firms measured as the sum of those that are new to society and stolen F_{mt+1}, and the fraction of products that are produced in entering firms measured as the sum of those that are new to society and stolen F_{et+1}.

We also assume that this economist has data on the growth rate of aggregate TFP $g_{Zt} = \log(Z_{t+1}/Z_t)$ and data on the aggregate size of continuing products in incumbent firms S_{ct+1}, the aggregate size of products that are new to incumbent firms measured as the sum of those that are new products and those that are stolen S_{mt+1}, and the aggregate size of products that are new to entering firms measured as the sum of those that are new products and those that are stolen S_{et+1}.

Data on the dynamics of the measure of products allow one to identify the values of the following parameters on a balanced growth path (variables in a BGP are denoted with a bar):

$$(1 - \bar{\delta}_c) = \bar{F}_c \exp(\bar{g}_M)$$

$$h(\bar{x}_m) = \bar{F}_m \exp(\bar{g}_M)$$

$$\bar{x}_e = \bar{F}_e \exp(\bar{g}_M).$$

Data on size and aggregate TFP growth, together with the data on the measures of products discussed above, allow one to identify the following model parameters. The parameters η_e and η_m are identified from data on the average size of products that are new to entering and incumbent firms on a balanced growth path,

$$\eta_e = \frac{\bar{S}_e}{\bar{F}_e}\frac{\exp((\rho-1)\bar{g}_Z)}{\exp(\bar{g}_M)} \quad \text{and} \quad \eta_m = \frac{\bar{S}_m}{\bar{F}_m}\frac{\exp((\rho-1)\bar{g}_Z)}{\exp(\bar{g}_M)}$$

The value of $\zeta(\bar{x}_c)$ is identified from data on the average size of continuing products in incumbent firms

$$\zeta(\bar{x}_c) = \frac{\bar{S}_c}{\bar{F}_c}\frac{\exp((\rho-1)\bar{g}_Z)}{\exp(\bar{g}_M)}$$

We can thus re-write the derivatives in equations (9), (10), and (11) as

$$G_c\bar{x}_c = \frac{1}{\rho-1}\bar{S}_c\frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)} \tag{15}$$

$$G_m\bar{x}_m = \frac{1}{\rho-1}\left(\bar{S}_m - \delta\frac{\bar{S}_c}{\bar{F}_c}\bar{F}_m\right)\frac{h'(\bar{x}_m)\bar{x}_m}{h(\bar{x}_m)} \tag{16}$$

and

$$G_e\bar{x}_e = \frac{1}{\rho-1}\left(\bar{S}_e - \delta\frac{\bar{S}_c}{\bar{F}_c}\bar{F}_e\right). \tag{17}$$

These formulas indicate that if we do not make the assumption that social depreciation of innovative investment is zero ($G(0,0,0) = 0$), then data on product-level dynamics alone do not place a tight bound on the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment.

Consider, for example, the elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment allocated entirely to increased investment in entry. From equation (17), this is given by

$$G_e\bar{x}_e\frac{Y_r}{\bar{x}_e} = \frac{1}{\rho-1}\left(\bar{S}_e - \delta\frac{\bar{S}_c}{\bar{F}_c}\bar{F}_e\right)\frac{Y_r}{\bar{x}_e}$$

Here the term $\bar{S}_e - \delta\frac{\bar{S}_c}{\bar{F}_c}\bar{F}_e$ is the contribution of entry to product size at $t+1$ net of business stealing of existing products and $\bar{x}_e/\bar{Y}_r$ is the fraction of innovative investment undertaken by entering firms. In a pure expanding varieties model in which there is no business stealing (so $\delta = 0$), this elasticity is then given by $1/(\rho-1)$ times the ratio of the share of employment (and production and physical capital) in entering firms to the share of innovative investment undertaken by entering firms. Thus, if the ratio of the share of employment (and production and physical capital) in entering firms to the share of innovative investment undertaken by entering firms were close to one, then, with a standard value of $\rho = 4$, this elasticity would be close to $1/3$, which is much larger than the level of TFP growth $\bar{g}_Z$ observed in advanced economies.

Likewise, equation (15) implies

$$G_c \bar{x}_c \frac{Y_r}{\bar{x}_c} = \frac{1}{\rho - 1} \bar{S}_c \frac{\bar{Y}_r}{\bar{x}_c} \frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)}$$

where the term $\bar{S}_c \frac{\bar{Y}_r}{\bar{x}_c}$ is the ratio of the size of continuing products in incumbent firms to the share of innovative investment carried out by incumbents aimed at improving continuing products, and the term $\frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)}$ is a measure of the concavity of their innovative investment technology. Again, if the ratio of the size of continuing products in incumbent firms to the share of innovative investment carried out by incumbents aimed at improving continuing products is close to one and the investment technology is not too concave, then this elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment concentrated on continuing products might be quite large.

Accordingly, we turn below to the question of how we might use data on firm value and the condition that firms choose innovative investment to maximize firm value to measure innovative investment shares $\bar{x}_c/\bar{Y}_r$, $\bar{x}_m/\bar{Y}_r$ and $\bar{x}_e/\bar{Y}_r$ and the derivatives

$$\frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)} \quad \text{and} \quad \frac{h'(\bar{x}_m)\bar{x}_m}{h(\bar{x}_m)}. \tag{18}$$

The parameter δ governing the extent of business stealing also has an important impact on the magnitude of the elasticities (16) and (17). We are not able to identify this parameter using the data on firm dynamics and firm value that we consider. We are able, however, to place bounds on this parameter as we next discuss.

4.1.1 Bounds on δ_0 and δ

With data on firm dynamics, we have only identified the exit rate of exiting products indexed by $\bar{\delta}_c$ as defined in equation (4) and the average size of products that are new to incumbent and entering firms indexed by η_m and η_e defined in equation (7). While our data on firm dynamics does not identify the parameters δ_0 and δ individually, our model does impose bounds on these parameters as follows.

Both of these parameters δ_0 and δ must be non-negative. Moreover, we must have $\delta_0 \leq \bar{\delta}_c = 1 - \bar{F}_c \exp(\bar{g}_M)$ since $\delta \geq 0$.

We must also have δ below the minimum of four upper bounds. The first of these is $\delta \leq 1$. The second of these corresponds to the value of δ implied by equation (4) with $\delta_0 = 0$ (since δ_0 cannot be negative) and the data on the exit rate of incumbent products and the fraction of new products in incumbent and entering firms. Specifically

$$\delta \leq \frac{\bar{\delta}_c}{h(\bar{x}_m) + \bar{x}_e} = \frac{1 - \bar{F}_c \exp(\bar{g}_M)}{(1 - \bar{F}_c) \exp(\bar{g}_M)}$$

This bound is lower than 1 if the measure of products grows over time (i.e. $\bar{g}_M > 0$). The third and fourth bound correspond to the requirement that new products (new to society) have non-negative average size ($\eta_{mn} > 0$ and $\eta_{en} > 0$), combined with the restriction above that, in expectation, stolen products be larger than those that they replace ($\eta_{ms} \geq \zeta(\bar{x}_c)$ and $\eta_{es} \geq \zeta(\bar{x}_c)$). Using the definitions of η_m and η_e above, we have

$$\delta \leq \frac{\eta_m}{\zeta(\bar{x}_c)} = \frac{\bar{S}_m}{\bar{F}_m} \Big/ \frac{\bar{S}_c}{\bar{F}_c}$$

and

$$\delta \leq \frac{\eta_e}{\zeta(\bar{x}_c)} = \frac{\bar{S}_e}{\bar{F}_e} \Big/ \frac{\bar{S}_c}{\bar{F}_c}.$$

The minimum of these bounds binds when new products in incumbent or entering firms are smaller than continuing products on average in the data.

We now turn to our model's implications for the relationship between firm value and firm innovative investments.

4.2 Firm Value and Innovative Investment

We assume that in addition to the data on firm dynamics discussed above, the economist has data on the share of output Y_t paid to owners of incumbent firms as profits over and above rental payments for physical capital, the share of output that incumbent firms spend on innovative investments and the rate of return at which owners of firms discount profits. We first discuss how we use these data to infer the value of the intangible capital of incumbent firms and the share of output that entering firms spend on innovative investment. We then discuss how we put bounds on the division of incumbent firms' innovative investments into investment in continuing products and investment in acquiring new products and how we measure the derivatives (18). We conclude this section with a proposition summarizing our results.

To simplify the presentation, we assume that there are no taxes or subsidies in the observed equilibrium. In the Appendix and in the quantitative analysis we take into account the impact of corporate profit taxes, dividend distribution taxes, and R&D subsidies on measures of firm value and on the optimality conditions for innovative investments.

4.2.1 The Value of Incumbent Firms

In our model, the value of incumbent firms is equal to the value of the physical capital owned by the firm plus the expected discounted present value of dividends to innovative investment earned by incumbent firms. The aggregate flow of dividends paid to owners of incumbent firms as compensation for their innovative investments is equal to the profits earned by incumbent firms in excess of their costs of innovative investments

$$D_{vt} = \left[\frac{\mu - 1}{\mu} - S_{irt}\right] Y_t. \tag{19}$$

Here $(\mu - 1)/\mu Y_t$ is the flow of profits earned by incumbent firms in excess of the rental cost of their physical capital and the term $S_{irt} = P_{rt}(x_{ct} + x_{mt})/Y_t$ is the innovative investment undertaken by these firms relative to output of the final consumption good. We denote the innovation intensity of the economy inclusive of innovative investment by entrants by $S_{rt} = P_{rt}Y_{rt}/Y_t$, which is equal to $S_{irt} + P_{rt}x_{et}/Y_{rt}$.

We now consider our model's value of the intangible capital of an individual incumbent firm. The intangible capital of each incumbent firm at time t is indexed by the number of products it has, n, and the productivity indices for those products, z_k for $k = 1, \ldots, n$ (both of which vary across firms). Define the size of the firm at t as

$$s_t = \sum_{k=1}^{n} s_t(z_k) = \sum_{k=1}^{n} \frac{\exp((\rho - 1)z_k)}{Z_t^{\rho-1}}$$

Given the assumptions we have made about the technologies for innovative investment, the size of an incumbent firm is a summary statistic of its intangible capital. The owners of an incumbent firm of size s at time t collect a dividend as compensation for their innovative investments of $D_{vt}s$ composed of the difference between the variable profits earned by the firm $[(\mu - 1)/\mu]\, Y_t s$ less expenditures $P_{rt}x_{ct}s$ and $P_{rt}x_{mt}s$ on innovative investment in improving its own products and in acquiring new products for the firm respectively.

Because the risk of gaining losing a product is idiosyncratic and since the increment to size for continuing products is also idiosyncratic, we can value future dividends according to expected size. In expectation, this incumbent firm will have size s' at $t + 1$ given by

$$\mathbb{E}_t\left[s'|s\right] = \left[(1 - \delta_{ct})\,\zeta(x_{ct}) + \eta_m h(x_{mt})\right] \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} s. \tag{20}$$

We then have that the intangible capital value an incumbent firm of size s at time t (including its profits earned at t) satisfies the following recursion

$$V_t s = D_{vt} s + \frac{1}{1+R_t} V_{t+1} \left[(1-\delta_{ct}) \zeta(x_{ct}) + \eta_m h(x_{mt}) \right] \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} s,$$

where R_t denotes the consumption interest rate between t and $t+1$.

Using the notation $v_t \equiv \frac{V_t}{Y_t}$, $d_{vt} \equiv \frac{D_{vt}}{Y_t}$, $g_{Yt} \equiv \log(\frac{Y_{t+1}}{Y_t})$ and our expressions for S_{ct+1} and S_{mt+1} above, we have

$$v_t = d_{vt} + \frac{\exp(g_{Yt})}{1+R_t} v_{t+1} \left(S_{ct+1} + S_{mt+1} \right). \tag{21}$$

Hence, on a BGP, the value of firms' intangible capital is

$$\bar{v} = \left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e) \right]^{-1} \bar{d}_v, \tag{22}$$

where $\bar{d}_v$ is the *share of compensation to intangible capital in output* and

$$\left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e) \right]^{-1} \tag{23}$$

is the *multiple* at which this compensation of intangible capital is valued.

Equation (22) is equivalent to

$$\bar{S}_{ir} = \frac{\mu-1}{\mu} - \left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e) \right] \bar{v}. \tag{24}$$

We now use this equation together with the zero profit condition for entering firms to infer the research intensity of the economy on the BGP, $\bar{S}_r$.

4.2.2 The value of a new firm

The zero proft condition for entering firms is given by

$$\frac{P_{rt}}{M_t} = \frac{1}{1+R_t} V_{t+1} \frac{\eta_e}{M_t} \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}. \tag{25}$$

Multiplying this equation by $x_{et} M_t$ and dividing by Y_t gives

$$\frac{P_{rt} x_{et}}{Y_t} = \frac{\exp(g_{Yt})}{1+R_t} v_{t+1} S_{et+1}.$$

In a BGP, this equation becomes

$$\frac{\bar{P}_r \bar{x}_e}{\bar{Y}} = \frac{\exp(\bar{g}_Y)}{1+\bar{R}} \bar{v} \bar{S}_e. \tag{26}$$

Equations (24) and (26) together imply that $\bar{S}_r$ can be inferred using

$$\bar{S}_r = \frac{\mu-1}{\mu} - \left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} \right] \bar{v}. \tag{27}$$

4.2.3 Implications for the allocation of innovative investment by incumbents

We are not able to precisely identify the division of incumbent firms' innovative investments on the BGP, $\bar{S}_{ir}$, into the two separate categories $\bar{P}_r\bar{x}_c/\bar{Y}$ and $\bar{P}_r\bar{x}_m/\bar{Y}$. We are, however, able to establish bounds on those investment levels as follows.

The contribution of investment in acquiring products each period to firm value must be non-negative. That is, on a BGP, we must have $\bar{v}$ at least as large as the value that the firm would obtain if it were to set investment into acquiring new products equal to zero in every period. Given the assumption that $h(0) = 0$, this alternative value of incumbent firms on a BGP is given by

$$\tilde{v} = \left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_c\right]^{-1}\left[\frac{\mu-1}{\mu} - \frac{\bar{P}_r\bar{x}_c}{\bar{Y}}\right].$$

The requirement that $\bar{v} \geq \tilde{v}$ implies that

$$\frac{\left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}(1-\bar{S}_e)\right]}{\left[1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_c\right]}\left[\frac{\mu-1}{\mu} - \frac{\bar{P}_r\bar{x}_c}{\bar{Y}}\right] \leq \left[\frac{\mu-1}{\mu} - \bar{S}_{ir}\right]$$

or, equivalently, we have that the research expenditures of incumbents on improving continuing products relative to value added must lie between the bounds

$$\bar{S}_{ir} \geq \frac{\bar{P}_r\bar{x}_c}{\bar{Y}} \geq \frac{1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_c}{1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}(1-\bar{S}_e)}\bar{S}_{ir} - \frac{\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_m}{1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}}(1-\bar{S}_e)}\frac{\mu-1}{\mu}. \tag{28}$$

We now summarize the model's implications for the division of innovative investment into components

$$\bar{S}_r = \frac{\bar{P}_r\bar{x}_c}{\bar{Y}} + \frac{\bar{P}_r\bar{x}_m}{\bar{Y}} + \frac{\bar{P}_r\bar{x}_e}{\bar{Y}}.$$

The innovation expenditure share of entrants is given by equation (26). We are not able to precisely identify the division of expenditures $\bar{S}_{ir}$ between $\frac{\bar{P}_r\bar{x}_c}{\bar{Y}}$ and $\frac{\bar{P}_r\bar{x}_m}{\bar{Y}}$. Our calibration of $\frac{\bar{P}_r\bar{x}_c}{\bar{Y}}$ however is constrained by the bounds (28).

We now turn to the final step of our measurement procedure in which we measure the derivatives in (18) using the first-order conditions characterizing firms' optimal choices of innovative investment.

4.2.4 The implications of optimal investment

When an incumbent firm chooses investment $x_{ct}(z) > 0$ optimally to innovate on one of its products with index z at t, its choice satisfies the first order condition

$$P_{rt} = \frac{1}{1+R_t} V_{t+1}(1-\delta_{ct})\zeta'(x_{ct}) \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}, \tag{29}$$

which, on a BGP is equivalent to

$$\frac{\bar{P}_r \bar{x}_c}{\bar{Y}} = \bar{v} \frac{\exp(\bar{g}_Y)}{1+\bar{R}} \bar{S}_c \frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)},$$

or

$$\frac{\zeta'(\bar{x}_c)\bar{x}_c}{\zeta(\bar{x}_c)} = \left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}} \bar{S}_c\right]^{-1} \left[\frac{\bar{P}_r \bar{x}_c / \bar{Y}}{\bar{v}}\right]. \tag{30}$$

Likewise, when an incumbent firm chooses investment $x_m(z) > 0$ optimally, its choice satisfies the first order condition

$$P_{rt} = \frac{1}{1+R_t} V_{t+1}\eta_m h'(x_{mt}) \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}$$

which, in a BGP is equivalent to

$$\frac{\bar{P}_r \bar{x}_m}{\bar{Y}} = \bar{v} \frac{\exp(\bar{g}_Y)}{1+\bar{R}} \bar{S}_m \frac{h'(\bar{x}_m)\bar{x}_m}{\bar{h}(\bar{x}_m)}.$$

or

$$\frac{h'(\bar{x}_m)\bar{x}_m}{\bar{h}(\bar{x}_m)} = \left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}} \bar{S}_m\right]^{-1} \left[\frac{P_r \bar{x}_m / Y}{\bar{v}}\right]. \tag{31}$$

5 Measured Elasticities

We now are able to measure the implications of our model for the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment as follows. Recall that we assume that the economist has access to data on the markup μ and the ratio of incumbent firms' innovative investments to value added $\bar{S}_{ir}$ (or equivalently, to data on $\bar{S}_{ir}$ and the share of value added paid to intangible capital, $\frac{\mu-1}{\mu} - \bar{S}_{ir}$), data on the ratio of the BGP growth rate to the consumption interest rate $\exp(\bar{g}_Y)/(1+\bar{R})$, and to the data on firm dynamics in step 1 of our measurement procedure. The parameter determining the extent of business stealing, δ, and the division of innovation expenditures, $\bar{S}_{ir}$, between $\bar{P}_r\bar{x}_c/\bar{Y}$ and $\bar{P}_r\bar{x}_m/\bar{Y}$ cannot be directly inferred from these data and hence must be assigned by the economist, subject to the bounds defined in Section 4.1.1 and equation (28), respectively. The elasticities are summarized in the following proposition.

Proposition 2. *Our model's implications for the relationship between firm value and firm innovative investments give that the elasticities (9) - (13) can be measured as*

$$G_c\bar{x}_c = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[\frac{\bar{P}_r\bar{x}_c/\bar{Y}}{\bar{v}}\right] \tag{32}$$

$$G_c\bar{x}_c\frac{\bar{Y}_r}{\bar{x}_c} = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[\frac{\bar{S}_r}{\bar{v}}\right] \tag{33}$$

$$G_m\bar{x}_m = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[1-\delta\frac{\bar{S}_c}{\bar{F}_c}\Big/\frac{\bar{S}_m}{\bar{F}_m}\right]\left[\frac{\bar{P}_r\bar{x}_m/\bar{Y}}{\bar{v}}\right] \tag{34}$$

$$G_m\bar{x}_m\frac{\bar{Y}_r}{\bar{x}_m} = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[1-\delta\frac{\bar{S}_c}{\bar{F}_c}\Big/\frac{\bar{S}_m}{\bar{F}_m}\right]\left[\frac{\bar{S}_r}{\bar{v}}\right] \tag{35}$$

$$G_e\bar{x}_e = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[1-\delta\frac{\bar{S}_c}{\bar{F}_c}\Big/\frac{\bar{S}_e}{\bar{F}_e}\right]\left[\frac{\bar{P}_r\bar{x}_e/\bar{Y}}{\bar{v}}\right] \tag{36}$$

and

$$G_e\bar{x}_e\frac{\bar{Y}_r}{\bar{x}_e} = \frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\left[1-\delta\frac{\bar{S}_c}{\bar{F}_c}\Big/\frac{\bar{S}_e}{\bar{F}_e}\right]\left[\frac{\bar{S}_r}{\bar{v}}\right] \tag{37}$$

where the value $\bar{v}$ is given in equation (22). The results imply that the elasticity of aggregate TFP growth with respect to a proportional change in all three categories of innovative investment is given by

$$G_c\bar{x}_c + G_m\bar{x}_m + G_e\bar{x}_e = \tag{38}$$

$$\frac{1}{\rho-1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\frac{1}{\bar{v}}\left(\bar{S}_r - \delta\frac{\bar{S}_c}{\bar{F}_c}\left[\frac{\bar{P}_r\bar{x}_m/\bar{Y}}{\frac{\bar{S}_m}{\bar{F}_m}} + \frac{\bar{P}_r\bar{x}_e/\bar{Y}}{\frac{\bar{S}_e}{\bar{F}_e}}\right)\right].$$

The values of δ, $P_r\bar{x}_c/Y$, and $P_r\bar{x}_m/Y$ are constrained by bounds defined in Section 4.1.1 and expression (28).

Proof. Equation (32) follows directly from equations (15) and (30). We have

$$\frac{\bar{Y}_r}{\bar{x}_c} = \bar{S}_r\frac{\bar{Y}}{\bar{P}_r\bar{x}_c}.$$

With equation (32), we then have the result (33). Equation (34) follows from equations (16) and (31). Equations (35) and (37) follow from the derived allocation of investment. □

To develop intuition for the economics of these elasticity formulas (32) - (37), it is useful to consider a second approach to deriving the same results that emphasizes the role of the marginal impact of investment on product size in shaping both the social products of innovative investment G_c, G_m and G_e and the private returns to such investment.

To start, consider the impact at the margin of a change in the level of investment at t by a single incumbent firm in improving an existing product with index z and size s_t at t on aggregate TFP at $t+1$ From equations (1) and (5), this derivative is given by

$$\frac{\partial}{\partial x_{ct}(z)} \log(Z_{t+1}) = \frac{1}{\rho-1} \frac{(1-\delta_{ct})\zeta'(x_{ct}) Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} = \frac{1}{\rho-1} \frac{\partial}{\partial x_{ct}(z)} \mathbb{E}\left[s_{t+1}|s_t\right],$$

where the second equality follows from (20). Thus, we have that the marginal contribution to the logarithm of aggregate productivity at $t+1$ of an incumbent firms' investment at t in improving one of its products is directly proportional to the marginal contribution of that investment to the expected size of the product at $t+1$, with the factor of proportionality given by $1/(\rho-1)$.

Now we have also seen that firms earn the market rate of return in equilibrium on their investments in increasing the expected size of their continuing products. This observation allows us to identify the marginal contribution of an incumbent firm's investment in a continuing product on the expected size of that product. Specifically, from equation (29), we have that, if this level of investment is chosen optimally, then

$$\frac{\partial}{\partial x_{ct}(z)} \mathbb{E}\left[s_{t+1}|s_t\right] = \frac{(1-\delta_{ct})\zeta'(x_{ct}) Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} = (1+R_t)\frac{P_{rt}}{V_{t+1}}$$

These equations then give us that

$$\frac{\partial}{\partial x_{ct}(z)} \log(Z_{t+1}) = \frac{1}{\rho-1}(1+R_t)\frac{P_{rt}}{V_{t+1}}$$

Our assumptions that the costs of firms' investments in continuing products and the incentives to invest are both proportional to product size together with our imposition of a strong form of Gibrat's Law at the product level give us the result that these derivatives are independent of z and hence

$$G_{ct} = \frac{\partial}{\partial x_{ct}} \log(Z_{t+1}) = \frac{1}{\rho-1}(1+R_t)\frac{P_{rt}}{V_{t+1}}$$

Multiplying and dividing this equation by Y_{t+1}/Y_t, multiplying both sides by x_{ct}, and imposing that we are on the balanced growth path then gives equation (32).

To be clear, we are not imposing that the private and social returns to innovative investment by incumbent firms in continuing products are equal. Instead, we rely on the structure of the model with its CES aggregator to measure the marginal social returns to such investment as $1/(\rho-1)$ times the marginal impact of such investment on the size of continuing products in incumbent firms. We then use the implication of the model that firm value is proportional to size to argue that, when innovative investment in continuing products by incumbent firms is chosen optimally, the marginal impact of such investment on the size of continuing products is equal to $(1 + R_t)P_{rt}/V_{t+1}$, where V_{t+1} is the factor of proportionality between private firm value and firm size.

In the event that there is no business stealing, so δ equals zero, this line of argument we used to derive the derivative G_c gives the following corollary of Proposition 2:

Corollary 3. *If there is no business stealing, so $\delta = 0$, then all three partial derivatives of G are equal to each other, $G_c = G_m = G_e$. This implies that the elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment $\hat{Y}_r$ is independent of how that change in aggregate innovative investment is allocated across the three categories of investment and is given by $\hat{g}_Z$*

$$\frac{\hat{g}_Z}{\hat{Y}_r} = \frac{1}{\rho - 1} \left[\frac{\exp(\bar{g}_Y)}{1 + \bar{R}} \right]^{-1} \left[\frac{\bar{S}_r}{\bar{v}} \right].$$

Proof. The partial derivatives of G are obtained from equations (32), (34), and (36) with $\delta = 0$ after dividing each expression by the corresponding level of investment. The overall elasticity is obtained from equation (38) with $\delta = 0$. □

Now consider the derivation of the elasticities (34) and (36) when there is business stealing, so $\delta > 0$. These elasticities can be derived in a similar manner as with (32) except that one must adjust for the impact of business stealing in these calculations. Specifically, investment by incumbent and entering firms in acquiring new products has a marginal contribution to the logarithm of aggregate productivity at $t + 1$ that depends on the *net* contribution of such investment to the size of products at $t + 1$ (net of business stealing) while it has a contribution to firm value that depends on the gross contribution of such investment to the size of products produced by the firm at $t + 1$. The terms

$$\left[1 - \delta \frac{\bar{S}_c}{\bar{F}_c} \Big/ \frac{\bar{S}_m}{\bar{F}_m} \right] \quad \text{and} \quad \left[1 - \delta \frac{\bar{S}_c}{\bar{F}_c} \Big/ \frac{\bar{S}_e}{\bar{F}_e} \right]$$

in equations (34) and (36) denote the ratio of the net contribution of these innovative investments to the size of products at $t + 1$ to the gross contribution of these investments to the size of products produced by the firm at $t + 1$.

This consideration of business stealing allows us to rank the elasticities considered above as follows.

Corollary 4. *The elasticities (33), (35), and (37) can be ranked as*

$$G_m \bar{x}_m \frac{\bar{Y}_r}{\bar{x}_m} \leq G_c \bar{x}_c \frac{\bar{Y}_r}{\bar{x}_c}$$

$$G_e \bar{x}_e \frac{\bar{Y}_r}{\bar{x}_e} \leq G_c \bar{x}_c \frac{\bar{Y}_r}{\bar{x}_c}.$$

In the event that the average size of new products in incumbent and entering firms is the same, that is $\bar{S}_m/\bar{F}_m = \bar{S}_e/\bar{F}_e$, then

$$G_m \bar{x}_m \frac{\bar{Y}_r}{\bar{x}_m} = G_e \bar{x}_e \frac{\bar{Y}_r}{\bar{x}_e}.$$

Proof. The first inequality follows from the observation that

$$\left[1 - \delta \frac{\bar{S}_c}{\bar{F}_c} \Big/ \frac{\bar{S}_m}{\bar{F}_m}\right] \leq 1$$

and the second from the observation that

$$\left[1 - \delta \frac{\bar{S}_c}{\bar{F}_c} \Big/ \frac{\bar{S}_e}{\bar{F}_e}\right] \leq 1.$$

The final equality follows directly from the assumption on the equality of the average size of new products in incumbent and entering firms. □

5.1 Relationship to Atkeson and Burstein (2015)

In our previous paper, Atkeson and Burstein (2015), we showed that, under certain assumptions, the quantitative implications of that model for the change in the dynamics in aggregate TFP that arise from a change in expenditures on innovative investment can be summarized by two key statistics — the impact elasticity of aggregate TFP growth from one year to the next with respect to a change in aggregate innovative investments and the intertemporal spillovers of knowledge.

The model we consider in this paper consolidates the five example economies that we considered in Atkeson and Burstein (2015). Under certain parameter restrictions, the model in this paper satisfies the three main assumptions we use in Atkeson and Burstein (2015). These parameter restrictions are as follows.

We use Assumption 1 in Atkeson and Burstein (2015) to develop the bound on the elasticity of aggregate TFP growth with respect to a proportional change in all categories of innovative investment that we apply in Proposition 1. For our model here to satisfy assumption 1 in Atkeson and Burstein (2015), it is sufficient for the functions $\zeta(\cdot)$ and $h(\cdot)$ to be concave. The proof of this point is given in Appendix 9.

We use Assumption 2 in Atkeson and Burstein (2015) to give conditions under which the impact elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment does not depend on how that change in investment is allocated across categories of investment. In Corollary 3 we show that this assumption 2 holds in our current model if there is no business stealing. In Corollary 4 we show that this assumption 2 also holds in our current model if there is business stealing, but if there is no investment by incumbent firms in continuing products ($x_c = 0$) and if the average size of new products in incumbent and entering firms is the same.

We use Assumption 3 in Atkeson and Burstein (2015) to derive the dynamic implications of a change in innovation expenditure for the path of aggregate TFP. Given that aggregate TFP growth in this current model can be written as a simple function of three aggregate categories of innovative investment without any further dependence on time, $G(x_{ct}, x_{mt}, x_{et})$, this assumption 3 holds whenever assumption 2 is satisfied. Assumption 3 also holds if the perturbations to categories of innovative investment are done in the same proportions across time periods, i.e. $\hat{x}_{ct}/\hat{Y}_{rt}$ is constant across time and likewise for $\hat{x}_{mt}/\hat{Y}_{rt}$ and $\hat{x}_{et}/\hat{Y}_{rt}$.

6 Quantitative analysis

We use our model to measure the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment given several alternative calibrations of our model. In these various calibrations of our model, we consider alternative measures of the share of innovative investment by incumbent firms in output $\bar{S}_{ir}$, alternative measures of product-level dynamics, and alternative specifications of the business stealing parameter δ. We focus here on giving an overview of the sources used to calibrate the parameters of the model. A detailed presentation of the calibration is given in the appendix.

Throughout, we set the time period in the model to one year. We set the elasticity of substitution $\rho = 4$ in the CES aggregator in equation (1) defining aggregate TFP. Note that this assumption fixes the term $1/(\rho - 1)$ that enters into all of our elasticity formulas. Holding the equilibrium markup μ fixed, changes in this parameter ρ have no impact on any other implication of our model for the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment other than through this term. Hence, it is clear that our measured elasticities will be larger if we choose a value of $1 < \rho < 4$ and smaller if we choose a value of $\rho > 4$.

We fix a BGP consumption interest rate of $\bar{R} = 4.5\%$. This rate of return is close to that estimated by Poterba (1998) (5.1%), Hall (2003) (4.46%), and by McGrattan and Prescott (2005) (4.1% for 1990-2001). We consider the implications of alternative choices for this interest rate at the end of our quantitative analysis. We set the growth rate of the labor force $\bar{g}_L = 0.008$ corresponding to the annual growth rate of employment in the non-financial corporate sector over the period 1990-2014.

Output and Innovative Investment by Incumbent Firms To measure the share of innovative investment by incumbent firms in the output of these firms, we use data from the Integrated Macroeconomic Accounts and Fixed Assets Tables for the US non-financial corporate sector derived from the U.S. National Income and Product Accounts (NIPA). We use averages of the data for the period 1990-2014 to calibrate our model's BGP.

Since 2013, the NIPA data that we use include investments in intellectual property products in measured value added. We map these data on gross value added of the non-financial corporate sector and the expenditures on innovative investments by incumbent firms in this sector to variables in our model as follows. In our model, total expenditures on innovative investments correspond to output of the research good $P_{rt}Y_{rt}$, which equals the compensation of research labor W_tL_{rt}. We presume that what is measured in the data is the innovative investment expenditures of incumbent firms $P_{rt}(x_{ct} + x_{mt})$. We describe below how we infer the innovative investment expenditures of entering firms $P_{rt}x_{et}$, since we assume that entrants are not in the non-financial corporate sector when they make these expenditures. Hence, in mapping measures of output and labor compensation in the data to variables in our model, we must make the several adjustments to the data. We do so as follows.

We consider two measures of aggregate innovative investment by incumbent firms in the data corresponding to the model quantity $P_{rt}(\bar{x}_c + \bar{x}_m)$. We consider first a broad measure of such investment corresponding to measured investment by non-financial corporations in all intellectual property products. These intellectual property products include research and development, software, and artistic and literary originals. We next consider a narrow measure of such investment corresponding to measured investment by non-financial corporations in research and development alone. When we use this narrow measure, we treat investment in software and artistic and literary originals as standard investment in physical capital as is currently done in the NIPA.

Using either of these measures of innovative investment expenditures by incumbent firms, aggregate output of the final consumption good Y_t in our model corresponds to measured Gross Value Added of the non-financial corporate sector less indirect business taxes and either the broad or narrow measure of innovative investment expenditures by incumbent firms. Likewise, compensation of production labor $W_t L_{pt}$ corresponds to compensation of employees in the non-financial corporate sector less either the broad or narrow measure of innovative investment expenditures by incumbent firms.

With these adjustments, we arrive at measures of $\bar{S}_{ir} = 0.06$ or 0.03 using our broad and narrow measures of innovative investment expenditures by incumbent firms respectively. The corresponding BGP growth rate of output of the final consumption good is $\bar{g}_Y = 0.025$ in both cases.

We report on our measures of incumbent firms' innovation expenditures relative to output in Table 1.

Payments to Physical Capital and Profits In the data, Gross Value Added of the non-financial corporate sector is decomposed into indirect business taxes, compensation of employees, and gross operating surplus. In our model, a portion of this gross operating surplus is paid as rental payments to physical capital and a portion is paid to owners of firms as variable profits. To measure variable profits, we impute rental payments to physical capital using a standard user cost formula relating the implicit rental rate to the return required by investors $1 + R$, the corporate profits tax rate, the depreciation rate, and the expected revaluation of physical capital. We use data from the Integrated Macroeconomic Accounts to do this imputation as described in appendix 11. In doing so, we remove intellectual property products from the measures of the replacement value of physical capital and consumption of fixed capital in the data when we use our broad measure of innovative investment. Likewise, we remove research and development from these measures of the replacement value of physical capital and consumption of fixed capital in the data when we use our narrow measure of innovative investment.

With these adjustments, we arrive at measures of profit shares net of corporate profit tax of 0.112 or 0.074 using our broad and narrow measures of innovative investment

Table 1: Innovation Intensity, Profits, Dividends, and Value

Calibration	$\bar{S}_{ir}$	profit share (net of taxes)	dividends $\bar{d}_v$	value $\bar{v}$	$\bar{S}_r$
Broad $\bar{S}_{ir}$, LBD Dynamics	0.06	0.112	0.052	1.14	0.092
Narrow $\bar{S}_{ir}$, LBD Dynamics	0.03	0.074	0.042	1.01	0.057
Broad $\bar{S}_{ir}$, GHK Dynamics	0.06	0.112	0.052	1.14	0.093

expenditures by incumbent firms respectively.

With these profit shares, we measure dividends to intangible capital relative to output after corporate profits taxes and subsidies for innovative investment of $d_{vt} = 0.052$ and 0.042 using our our broad and narrow measures of innovative investment expenditures by incumbent firms, respectively.

With our imputed share of rental payments to physical capital in production costs ($\alpha = 0.266$ or $\alpha = 0.283$ corresponding to our broad and narrow measures of innovative investment expenditures by incumbents, respectively), we construct estimates of the BGP growth rate of total factor productivity $\bar{g}_Z = 0.0123$ and 0.0125 corresponding to each of the two measures of innovative investment expenditures.

We report on our measures of profits and dividends to intangible capital in Table 1.

Product level dynamics:

To measure product-level firm dynamics $\bar{g}_M$, $\bar{S}_c$, $\bar{S}_m$, $\bar{S}_e$, $\bar{F}_c$, $\bar{F}_m$, $\bar{F}_e$, we consider two data sources. First we use data from the Longitudinal Business Database on the dynamics of establishments and firms. Second we use estimates of product level dynamics from GHK.

Consider first the data from the Longitudinal Business Database (LBD). This data set reports on the number and employment of business establishments in the United States. These establishments are matched to the firms that own them. We make the identifying assumption that an establishment in the LBD data corresponds to an intermediate good in the model. We assume that the growth rate in the number of products corresponds to the growth rate in the number of establishments. The fraction of new products in entering firms in the model, $\bar{F}_e$, corresponds to the fraction of establishments in the data that are new and are owned by new firms. The fraction of new products in incumbent firms in the model, $\bar{F}_m$, corresponds to the fraction of establishments that are new and are owned by firms that are not new. The fraction of products that are continuing products in

Table 2: Product Level Dynamics

Statistic	LBD Data	GHK Estimates
$\bar{S}_c$	0.948	0.956
$\bar{S}_m$	0.025	0.016
$\bar{S}_e$	0.027	0.028
$\bar{F}_c$	0.900	0.956
$\bar{F}_m$	0.023	0.020
$\bar{F}_e$	0.078	0.024
δ_m	0.33 (upper bound)	0.58
δ_e	0.33 (upper bound)	0.94

incumbent firms in the model, $\bar{F}_c$, corresponds to the fraction of establishments are are not new. We make the same mapping between model and data for product (establishment) size measured as the share of employment in establishments in each of the three categories $\bar{S}_e$, $\bar{S}_m$, and $\bar{S}_c$. If we assume time periods that are longer than one year, entrants represent a higher share in the total number of establishments and in employment (see appendix 12).

With $\bar{S}_e = 0.027$, in the LBD data, from equation (23), we then have that the value of intangible capital to output $\bar{v}$ is equal to a multiple of 22 times the ratio of dividends to intangible capital to output $\bar{d}_v$. With our broad measure of innovative investment, we measure $\bar{v} = 1.14$, and $\bar{S}_r = 0.092$. With our narrow measure of innovative investment, we measure $\bar{v} = 1$ and $\bar{S}_r = 0.057$. The model's implied Tobin's q (the ratio of the market value of firms – including intangible and physical capital— relative to the value of the replacement cost of physical capital) is 1.29 with our broad measure of innovative investment and 1.23 with our narrow measure.

We also consider the estimates of product level dynamics from GHK. In that paper, the authors estimate product level dynamics over a five-year time horizon. In appendix 12 we describe how we map their estimates to our model. We use their estimates for the 2003-2013 time period in Table 4 of that paper, with innovation rates annualized. With these estimates of product level dynamics, we measure $\bar{v} = 1.1$ and $\bar{S}_r = 0.093$.

We present our measures of product-level firm dynamics in Table 4. We report on our measures of firm value and the total innovation intensity of the economy in Table 1.

Business Stealing

We are not able the split of new products to firms between those that are new to society and those that are stolen from other firms, indexed in our model by δ. The fraction δ is bounded below by zero and above as describe in Section 4.1.1. We consider specifications of our model with δ set to its minimum value of zero and its maximum value of 0.329. Note that the LBD data on firm dynamics has the average size of products ordered by

$$\frac{\bar{S}_e}{\bar{F}_e} < \frac{\bar{S}_m}{\bar{F}_m} < \frac{\bar{S}_c}{\bar{F}_c}.$$

Hence, our maximum value of δ_{MAX} satisfies

$$\frac{\bar{S}_e}{\bar{F}_e} = \delta_{MAX} \frac{\bar{S}_c}{\bar{F}_c} \tag{39}$$

and, hence, from equation (37), the elasticity of aggregate TFP growth with respect to a change in investment in entry is equal to zero. At this maximum level of δ, the products stolen by entrants are the same size that they would have been had they not been stolen ($\eta_{es} = \zeta(\bar{x}_c)$) and new products created by entrants have zero size ($\eta_{en} = 0$).

We also consider the values of δ implied by the estimates in GHK (they infer this parameter using firm-level data and further structural assumptions). These authors allow for different values of δ for incumbent firms and for entering firms. We show in Table 3 that GHK's estimates also imply elasticities with respect to changes in investment in entry that are close to zero.

We present our measures of the parameters δ governing the extent of business stealing in Table 4.

Decomposition of incumbents' innovative investments

To measure the elasticities (32), (34), and (38), we must divide incumbent's innovative investments into those aimed at improving continuing products and those aimed at acquiring new products. As discussed in section 4.2.3, we cannot measure precisely the division of incumbents' innovation expenditures into these two categories. The share $P_r\bar{x}_c/Y$ is bounded above by $\bar{S}_{ir}$ and below as indicated in equation (28). In our baseline calibration, we use the middle point between these two bounds.

Note that the elasticities of aggregate TFP growth with respect to a change in aggregate innovative investment concentrated entirely in any of the three categories of innovative investment as given in (33), (35), and (37) do not depend on the assumptions we make about the division of innovative investment by incumbents $\bar{S}_{ir}$ into categories $P_r\bar{x}_c/Y$ and $P_r\bar{x}_m/Y$. Instead, they depend only on the total innovation intensity of the economy $\bar{S}_r$.

6.1 Model Implied Elasticities

We consider the following alternative specifications of our model.

Baseline Specification 0: We consider our broad measure of innovative investments, use use product-level dynamics estimated from the LBD, and we set the business stealing parameter δ equal to zero, so that there is no business stealing. We treat this specification as a baseline against which we compare the alternatives below

Alternative Specification 1: We consider our narrow measure of innovative investments, use use product-level dynamics estimated from the LBD, and we set the business stealing parameter δ equal to zero.

Alternative Specification 2: We consider our broad measure of innovative investments, use use product-level dynamics estimated from the LBD, and we set the business stealing parameter δ equal to its maximum value.

Alternative Specification 3: We consider our broad measure of innovative investments, and use product-level dynamics estimated from GHK including their estimated business stealing parameters.

We present the model implied elasticities of aggregate TFP growth with respect to a change in aggregate innovative investment concentrated on investment by incumbents in continuing products, (33), investment by incumbents in new products, (35), and investment by entrants, (37), as well as this elasticity with the change in investment allocated proportionally across all three categories of investment, (38) in Table 3. We present results for our baseline specification 0 and the three alternative specifications.

Consider first, the implications of our model in the baseline specification 0 reported in the first row in Table 3. In that table we see that all four reported elasticities are identical. Since the business stealing parameter $\delta = 0$, from Proposition 2 and Corollary 3, we have that all four elasticities listed in the table are equal to

$$\frac{1}{\rho - 1}\left[\frac{\exp(\bar{g}_Y)}{1+\bar{R}}\right]^{-1}\frac{\bar{S}_r}{\bar{v}}$$

Note that these elasticities all substantially exceed the bound $\bar{g}_Z$ from Proposition 1 that would obtain if we had assumed no social depreciation of innovative investments as is done in Neo-Schumpeterian specifications of our model. Below, we use our measurement of these elasticities to infer an upper bound on the growth rate of aggregate TFP that would obtain in the absence of all innovative investments $G(0, 0, 0)$ implied by this proposition.

This bound is implied by the result that the ratio of innovative investment $\bar{S}_r$ to the value of the dividends paid to this investment $\bar{v}$ is too high to be consistent with a model in which incumbent firms could have kept the same productivity index z on their continuing products with no investment at all.

Compare the results for our baseline specification to those in our alternative specification 1 listed in the second row in Table 3. Since we also assume that there is no business stealing, we again have that all four entries in this second row are equal to each other. But now they are lower than those for our baseline specification 0. This result follows simply from the fact that we have used a narrow measure of innovative investment. In particular, we have that the ratio of innovative investment $\bar{S}_r$ to the value of the dividends paid to this investment $\bar{v}$ is lower in this specification and this accounts for the lower elasticities found in this first alternative specification.

Now compare the results for our baseline specification to those in our alternative specification 2 listed in the third row in Table 3. In this alternative specification 2, we again use the broad measure of innovative investment, but we now assume the maximum level of business stealing permitted in the model.

In this alternative specification 2, we find that the elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment concentrated on incumbent firms' investment in continuing products listed in the third column of the table ($G_c\bar{x}_c(\bar{Y}_r/\bar{x}_c) = 0.027$) is the same as in our baseline specification. This is because consideration of business stealing does not impact our measurement of the terms for innovative investment $\bar{S}_r$ and firm intangible value $\bar{v}$ that enter into equation (33). But now we find that the other elasticities are lower than in the baseline specification due to business stealing. In particular, the elasticity with respect to entry only is equal to zero. At the maximum level of business stealing, at the margin, entry makes no contribution to aggregate TFP growth given that equation (39) holds at this maximum value of δ. The elasticity of aggregate TFP growth with respect to a change in innovative investment concentrated on investment by incumbent firms in acquiring new products is slightly positive because in our firm level data, the average size of new products in incumbent firms is slightly larger than the average size of new products in new firms.

The overall elasticity of aggregate TFP growth with respect to a proportional change in all three categories of innovative investment is similar to that in the other alternative

Table 3: Elasticities of aggregate TFP growth

Specification	x_e only	x_m only	x_c only	Proportional change
0. Broad Inv, LBD Dyn, $\delta = 0$	0.027	0.027	0.027	0.027
1. Narrow Inv, LBD Dyn, $\delta = 0$	0.019	0.019	0.019	0.019
2. Broad Inv, LBD Dyn, max δ	0.000	0.018	0.027	0.016
3. Broad Invest., GHK Dyn and δ	0.005	0.008	0.028	0.018

specifications. It is certainly the case that it would be possible to increase aggregate productivity growth substantially in this specification if it were possible to find a policy that would reallocate innovative investment away from entry and the acquisition of new products and towards incumbent firm investment in continuing products.

Finally, consider the results for our alternative specification 3 listed in the fourth row of Table 3. In this alternative specification 3, we again use the broad measure of innovative investment, but we now use the estimates of product-level dynamics from GHK and we use their estimates of business stealing.

In this alternative specification 3, we find that the elasticity of aggregate TFP growth with respect to a change in aggregate innovative investment concentrated on incumbent firms' investment in continuing products listed in the third column of the table ($G_c \bar{x}_c(\bar{Y}_r/\bar{x}_c) = 0.028$) is the similar to that in our baseline specification (0.027) because the ratio $\bar{S}_r/\bar{v}$ is similar across the two specifications. Because these authors estimate that business stealing is a large part of firms' acquisition of new products, the elasticities of aggregate productivity growth with respect to changes in aggregate innovative investment concentrated on entry or on the acquisition of new products by incumbent firms are both small. In sum, consideration of the product-level dynamics estimates from GHK do not yield substantially different elasticities than those we found with the LBD data and high levels of business stealing in alternative specification 2.

We also consider the sensitivity of our estimates of these elasticities to our assumed equilibrium rate of return on the BGP $\bar{R}$. We have chosen $\bar{R} = 0.045$ as our baseline.

If we choose a larger value of the equilibrium rate of return, we obtain larger measures of the elasticities in Table 3. For example, if we choose $\bar{R} = 0.055$, in our baseline specification, the elasticities in the first row of the table rise from 0.027 to 0.052. A larger value of $\bar{R}$ impacts our measurement in that it raises our estimate of the share of capital paid to tangible capital (from .236 to .262 and hence lowers our estimate of the share of profits paid to intangible capital in output $(\mu-1)/\mu$ from .11 to .086. This thus lowers our measure of the dividends to intangible capital $\bar{d}_v$ from 0.52 to 0.026. With these changes, our measurement of $\bar{S}_r/\bar{v}$ rises from 0.081 to 0.155.

With a lower value of the equilibrium rate of return, we obtain smaller elasticities. With $\bar{R} = 0.035$, we obtain elasticities of 0.018 in the baseline specification of the model in the first row of Table 3.

6.2 Implications for social depreciation of innovation expenditures

Now consider the implications of our measured elasticities in Table 3 for the social depreciation rate of innovative investments implied by the baseline and alternative specifications of our model. From Proposition 1, we have the following upper bound on the value of $G(0,0,0)$ consistent with the data on product level dynamics and firm value to which the model has been calibrated:

$$G(0,0,0) \leq \bar{g}_Z - (G_c\bar{x}_c + G_m\bar{x}_m + G_e\bar{x}_e)$$

Thus, in our baseline specification of the model, we have $G(0,0,0) \leq 0.0123 - 0.0268 = -0.0144$, and $G(0,0,0)$ bounded above by -0.0062, -0.0041, and -0.0058 in the three alternative specifications of our model.

As indicated in equation (14), the rate of social depreciation of innovation expenditures is determined by parameters ρ, δ_0, and $\zeta(0)$. In each of the specifications of the model we have presented, we have calibrated values of ρ and δ_0. We are not able to identify $\zeta(0)$, which corresponds to the growth rate of continuing products in incumbent firms in the absence of any investment by these firms in these products.

6.3 Dynamics of aggregate productivity

In Table 3, we summarize the implications of our model for the elasticity of aggregate TFP growth over a one year horizon with respect to changes in aggregate innovative investment, either allocated entirely to one of the three categories of innovative investment, $\hat{x}_c$, $\hat{x}_m$, and $\hat{x}_e$ or proportionally to all three categories of investment. We now briefly consider the implications of our results for TFP growth over a medium-term horizon.

It is not typically possible to measure real aggregate innovative investment Y_{rt}. Instead, in the data, it is possible to measure expenditures on innovation by incumbents S_{irt}

(and infer the innovation intensity of the economy S_{rt}) as we have done above, or the allocation of labor between production and research L_{pt} and L_{rt}.[9] Here, we focus on the cumulative impact on aggregate TFP over a 20 year horizon of a permanent change in the allocation of labor away from production and into research such that $\hat{L}_{rt} = \log(L_{rt}) - \log(\bar{L}_r)$. We envision that this change in the allocation of labor between production and research is induced by changes in policies so that the research technologies that we consider remain unchanged.

By equation (8), the logarithm of aggregate TFP (relative to the initial BGP) at time $t+1$, $\hat{Z}_{t+1}$, is

$$\hat{Z}_{t+1} = \hat{Z}_t + \left(G_c \bar{x}_c \hat{x}_{ct} + G_m \bar{x}_m \hat{x}_{mt} + G_e \bar{x}_e \hat{x}_{et}\right),$$

where changes in aggregate investment of each type, $\hat{x}_{ct}$, $\hat{x}_{mt}$, and $\hat{x}_e$, must satisfy the aggregate constraint

$$\frac{\bar{x}_c}{\bar{Y}_r}\hat{x}_{ct} + \frac{\bar{x}_m}{\bar{Y}_r}\hat{x}_{mt} + \frac{\bar{x}_e}{\bar{Y}_r}\hat{x}_{et} = \hat{L}_{rt} + (\phi - 1)\hat{Z}_t. \tag{40}$$

Recall that if $\phi < 1$, increases over time in aggregate productivity reduce the productivity of research labor. Fernald and Jones (2014) and Bloom et al. (2017) discuss aggregate and industry-level data that suggest that ϕ is significantly less than one. We set $\phi = -1.6$ based on estimates in Fernald and Jones (2014), and we show that the change in aggregate productivity at twenty year horizons is not very sensitive to our choice of ϕ.

We consider a permanent 0.1 increase in the logarithm of the fraction of total labor employed in research relative to it's BGP level ($\hat{L}_{rt} = 0.1$). Using our broad measure of innovative investments, this implies that L_{rt}/L_t rises from 0.127 to 0.14, and that the innovation intensity of the economy S_{rt} rises from 0.092 to 0.104. Using our narrow measure of innovative investments, this implies that L_{rt}/L_t rises from 0.081 to 0.089, and that S_{rt} rises from 0.057 to 0.064.

We assume that any change in the aggregate resources available for innovation (relative to the BGP level) are either split proportionally between the three types of innovative investment or fully allocated to investments by incumbents on their own products.

We first consider the specification of our model without business stealing ($\delta = 0$). Recall that, according to Corollary 3, the change in aggregate productivity is independent of how the research good is allocated between the three forms of innovative investment.

[9] The innovation intensity of the economy, S_{rt}, is related to the share of labor employed in research, L_{rt}/L_t, through the equilibrium equation $L_{rt}/L_{pt} = \mu/(1-\alpha)S_{rt}$.

Table 4 displays the log of aggregate TFP relative to its BGP level, one year and twenty years after the increase in research labor. In the first row of this table, we report on results using our baseline specification of the model. Using our broad measure of innovative investments, the logarithm of aggregate TFP rises in the first year by 0.003 relative to BGP (this corresponds to the elasticity reported in Table 3 multiplied by $\hat{L}_r = 0.1$). After 20 years, the cumulative rise in the logarithm of aggregate TFP relative to BGP is 0.029.[10]

In the second row of Table 4 we report on the results using our first alternative specification of the model. Using our narrow measure of innovative investment but with no business stealing, the rise in the logarithm of aggregate TFP is 0.002 and 0.024, after one and twenty years, respectively.

In the third row of Table 4 we report on the results using our second alternative specification of the model. In this specification of the model with the maximum amount of business stealing, the change in aggregate productivity is equal to the one in the specification without business stealing if the change in aggregate innovative investment is fully allocated to investments by incumbents on their own products (0.003 after one year and 0.029 after 20 years). In contrast, the rise in the logarithm of aggregate productivity is smaller in the case of proportional changes in the three types of innovative investment, since $G_m < G_c$ and $G_e < G_c$ (0.002 after one year and 0.022 after 20 years).

Finally, we calculate the gains from reallocating all innovative investments towards own-product innovation by incumbents, while assuming that the allocation of labor between research and production remains unchanged. Specifically, we set $x_{mt}\hat{x}_{mt} = -\bar{x}_m$, $x_{et}\hat{x}_{et} = -\bar{x}_e$ and $x_{ct}\hat{x}_{ct} = \bar{x}_m + \bar{x}_e + \bar{Y}_r(\phi - 1)\hat{Z}_t$. From Corollary 3, without business stealing, aggregate TFP remains unchanged. In contrast, at the maximum level of business stealing, from Corollary 4, since $G_m < G_c$ and $G_e < G_c$, the logarithm of aggregate TFP increases substantially relative to BGP: 0.01 after one year, and 0.113 after 20 years. Thus, in this specification of our model, there is considerable room for improving aggregate productivity by reallocating innovative expenditure from the acquisition of new products by entering and incumbent firms towards the improvement of existing varieties by incumbent firms. Of course, if it was possible to achieve this reallocation of innovative investment, firm dynamics would change considerably. The entry of new firms and creation of new products would stop and thus the range of existing products and number of

[10] If $\phi = 0.99$, very close the the upper bound of 1, the increase in the logarithm of aggregate TFP after 20 years is only slightly higher, 0.053.

Table 4: Change in aggregate TFP over time

Specification	10% increase in L_r Proportional change	10% increase in L_r x_c only	No change in L_r Reallocation to x_c
0. Broad Inv, LBD Dyn, $\delta = 0$			
Year 1	0.003	0.003	0
Year 20	0.029	0.029	0
1. Narrow Inv, LBD Dyn, $\delta = 0$			
Year 1	0.002	0.002	0
Year 20	0.024	0.024	0
2. Broad Inv, LBD Dyn, max δ			
Year 1	0.002	0.003	0.010
Year 20	0.022	0.029	0.113

existing firms would both stop growing.

7 Conclusion

In this paper, we have extended the model of Garcia-Macia et al. (2016) so that it nests some of the canonical models of firms' innovative investments, firm dynamics, and aggregate productivity growth. We have shown how to use data on firm dynamics and firm value together with the equilibrium conditions of the model to measure the elasticities of aggregate TFP growth with respect to changes in aggregate innovative investment allocated either to investment in improving existing varieties within an incumbent firm or to the acquisition of new varieties by incumbent and entering firms. We found elasticities modestly larger than would be consistent with those implied by a model with no social depreciation of innovative investments. Our results thus imply modest levels of social depreciation of these investments. In the specification of our model with the maximum amount of business stealing, we find large potential gains from reallocating innovative investment away from investments by incumbent and entering firms in acquiring new products and towards investments by incumbent firms in improving their existing products. It is clear that it is important in future research to find methods to improve measurement of the extent of business stealing.

8 Value functions

Conjecture that the equilibrium value for an incumbent firm of a product with productivity index z at time t is $V_t(z) = V_t s_t(z)$, where $s_t(z)$ was defined above as the size of a product with productivity $\exp(z)$. $V_t(z)$ must satisfy

$$V_t(z) = \frac{\mu - 1}{\mu} Y_t s_t(z) - (x_{mt}(z) + x_{ct}(z))\, P_{rt} +$$

$$+\frac{1-\delta_{ct}}{1+R_t} V_{t+1}\left[\zeta\left(\frac{x_{ct}(z)}{s_t(z)}\right) + \eta_m h\left(\frac{x_{mt}(z)}{s_t(z)}\right)\right]\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} s_t(z),$$

where τ_{sRD} denotes the subsidy rate on innovative spending. Taking the first order conditions with respect to x_{ct} and x_{mt}, it is evident that $\frac{x_{ct}(z)}{s_t(z)}$ and $\frac{x_{mt}(z)}{s_t(z)}$ are each common across z. This also verifies our conjecture that $V_t(z) = V_t s(z)$.

9 Concavity of H function

Define

$$H(a) = \frac{\log\left(\tilde{H}\,(a)\right)}{\rho - 1},$$

where

$$\tilde{H}\,(a) = (1 - \delta_0 - \delta\,(h\,(a\bar{x}_m) + a\bar{x}_e))\,\zeta\,(a\bar{x}_c) + \eta_m h\,(a\bar{x}_m) + \eta_e a\bar{x}_e$$

A sufficient condition for $H''\,(1) < 0$ is that $\tilde{H}''\,(1) < 0$ (since the logarithm function is concave). We have:

$$\tilde{H}'\,(a) = \zeta'\,(a\bar{x}_c)\,(1 - \delta_0 - \delta\,(h\,(a\bar{x}_m) + a\bar{x}_e))\,\bar{x}_c - \delta\,(\bar{x}_m h'\,(a\bar{x}_m) + \bar{x}_e)\,\zeta\,(a\bar{x}_c) + \eta_m\bar{x}_m h'\,(a\bar{x}_m) + \eta_e\bar{x}_e$$

and

$$\tilde{H}''\,(a) = \zeta''\,(a\bar{x}_c)\,(1 - \delta_0 - \delta\,(h\,(a\bar{x}_m) + a\bar{x}_e))\,\bar{x}_c^2 - \zeta'\,(a\bar{x}_c)\,\delta\,(h'\,(a\bar{x}_m)\,\bar{x}_m + \bar{x}_e)\,\bar{x}_c$$

$$-\delta\,(\bar{x}_m h''\,(a\bar{x}_m)\,\bar{x}_m)\,\zeta\,(a\bar{x}_c) - \delta\,(\bar{x}_m h'\,(a\bar{x}_m) + \bar{x}_e)\,\zeta'\,(a\bar{x}_c)\,\bar{x}_c + \eta_m\bar{x}_m h''\,(a\bar{x}_m)\,\bar{x}_m$$

Evaluated at $a = 1$,

$$\tilde{H}''\,(1) = \zeta''\,(\bar{x}_c)\,(1 - \delta_0 - \delta\,(h\,(\bar{x}_m) + \bar{x}_e))\,\bar{x}_c^2 - \zeta'\,(\bar{x}_c)\,\delta\,(h'\,(\bar{x}_m)\,\bar{x}_m + \bar{x}_e)\,\bar{x}_c$$

$$-\delta\,(\bar{x}_m h'\,(\bar{x}_m) + \bar{x}_e)\,\zeta'\,(\bar{x}_c)\,\bar{x}_c + (\eta_m - \delta\zeta\,(\bar{x}_c))\,h''\,(\bar{x}_m)\,\bar{x}_m^2$$

Given our assumptions that $h'\,(\bar{x}_m) > 0$, $\zeta'\,(\bar{x}_c) > 0$, $\zeta''\,(\bar{x}_c) < 0$, $h''\,(\bar{x}_m) < 0$, and $\eta_m > \delta\zeta\,(\bar{x}_c)$, all terms are negative so $\tilde{H}''\,(1) < 0$. This implies that $H(a)$ is also concave.

10 Adding taxes and subsidies

In this section we extend the analysis in McGrattan and Prescott (2005) to consider the impact of taxes and subsidies on the formulas derived from our model. We consider the following taxes and subsidies: a tax on corporate distributions to households (through the income tax system) τ_{dist}, a tax on corporate profits τ_{corp}, a subsidy for physical investment τ_{sk}, and a subsidy for investment in innovation (which, for simplicity, is equal for incumbent and entering firms) τ_{sRD}.

We assume that firms manage their tangible capital k_t and intangible capital separately. That is, they rent their tangible capital either to themselves or to other firms. This gives us that the value of a particular firm at time t is the sum of two components: the value of its physical capital $q_{kt}k_t$ equal to the discounted present value of the after-tax value of the corporate distributions to households due to rentals of physical capital, and the value of the firm's intangible capital $q_{st}s_t$ equal to the discounted present value of the after-tax value of the corporate distributions to households due to earnings on intangible capital.

We now compute the model's implications for the impact of these taxes and subsidies on the formulas in our model. We augment the model to include a changing relative price of physical capital and consumption goods.

Physical capital value of firms

As in McGrattan and Prescott (2005) equation (4), the contribution of physical capital to the firms' pre-tax payouts is

$$D_{kt} = (1 - \tau_{corp})r_t K_t - (1 - \tau_{sk})P_{kt}x_{kt} + \tau_{corp}\delta_k P_{kt}K_t$$

where τ_{corp} is the effective tax rate on corporate profits, τ_{sk} is the subsidy rate on tangible investment, r_t is the rental rate on physical capital, x_{kt} is the firms' investment in physical capital, P_{kt} is the relative price of physical investment goods and consumption goods, and δ_k is the depreciation rate on physical capital. The firm's stock of physical capital evolves according to (their equation (5))

$$K_{t+1} = (1 - \delta_k)K_t + x_{kt}$$

These two equations give us that

$$D_{kt} = (1 - \tau_{corp})r_t K_t - (1 - \tau_{sk})P_{kt}K_{t+1} + (1 - \tau_{sk})(1 - \delta_k)P_{kt}K_t + \tau_{corp}\delta_k P_{kt}K_t =$$

$$(1-\tau_{corp})(r_t - \delta_k P_{kt})K_t + \tau_{sk}\delta_k P_{kt} K_t - (1-\tau_{sk})P_{kt}(K_{t+1} - K_t)$$

The valuation of physical capital in an incumbent firm in the stock market (ex-dividend) relative to consumption is given by (their equation 5 for physical capital)

$$q_{kt} P_{kt} K_t = \sum_{j=1}^{\infty} \frac{p_{t+j}}{p_t}(1-\tau_{dist}) D_{kt+j}$$

where τ_{dist} is the tax rate on corporate distributions to households and p_t are the intertemporal consumption prices.

The first order condition for consumers trading claims to the physical capital in incumbent firms is

$$\frac{p_t}{p_{t+1}} = \frac{(1-\tau_{dist})D_{kt+1} + q_{kt+1}P_{kt+1}K_{t+1}}{q_{kt}P_{kt}K_t}$$

The first order condition for incumbent firms managing their capital stock (raising x_t and cutting x_{t+1} to keep K_{t+2} unchanged or, equivalently, simply changing K_{t+1}) is

$$\left(\frac{p_t}{p_{t+1}} - 1\right) = \frac{(1-\tau_{corp})(r_{t+1} - \delta_k P_{kt+1}) + \tau_{sk}\delta_k P_{kt+1} + (1-\tau_{sk})\left(P_{kt+1} - P_{kt}\right)}{(1-\tau_{sk})P_{kt}} \tag{41}$$

This first order condition for capital implies that

$$(1-\tau_{corp})(r_{t+1} - \delta_k P_{kt+1}) + \tau_{sk}\delta_k P_{kt+1} + (1-\tau_{sk})P_{kt+1} = (1-\tau_{sk})\frac{p_t}{p_{t+1}}P_{kt}$$

Hence

$$D_{kt+1} = (1-\tau_{sk})\frac{p_t}{p_{t+1}}P_{kt}K_{t+1} - (1-\tau_{sk})P_{kt+1}K_{t+2}$$

This equation, together with the first order condition for consumers, implies that

$$q_{kt}P_{kt}K_t = (1-\tau_{dist})(1-\tau_{sk})P_{kt}K_{t+1} - \frac{p_{t+1}}{p_t}\left((1-\tau_{dist})(1-\tau_{sk})P_{kt+1}K_{t+2} - q_{t+1}P_{kt+1}K_{t+1}\right)$$

The solution to this equation (that satisfies that the value of physical capital equals the discounted present value of dividends) is given by

$$q_{kt}P_{kt}K_t = (1-\tau_{dist})(1-\tau_{sk})P_{kt}K_{t+1}.$$

On a balanced growth path, this equation gives

$$\frac{q_k P_k K}{Y} = (1-\tau_{dist})(1-\tau_{sk})\frac{\exp(g_Y)}{P'_k/P_k}\frac{P_k K}{Y} \tag{42}$$

Note that as we shrink the time period (for example to a quarterly model) then we have in the limit,

$$\frac{q_k P_k K}{Y} = (1-\tau_{dist})(1-\tau_{sk})\frac{P_k K}{Y}. \tag{43}$$

We use this equation (43) in measuring the portion of firm market value attributable to physical capital.

Note that the standard equation for the evolution of the capital stock above gives us the following formula for the decomposition of the change in the balance sheet measure of the replacement value of physical capital from t to $t+1$ into components as follows

$$P_{kt+1}K_{t+1} - P_{kt}K_t = (P_{kt+1} - P_{kt})K_{t+1} + P_{kt}(K_{t+1} - K_t) =$$

$$(P_{kt+1} - P_{kt})K_{t+1} - \delta_k P_{kt}K_t + P_{kt}x_{kt},$$

where $(P_{kt+1} - P_{kt})K_{t+1}$ is the revaluation of capital (relative to consumption goods), $\delta_k P_{kt}K_t$ is the consumption of fixed capital, and $P_{kt}x_{kt}$ is gross fixed capital formation. (We ignore disaster losses in this formula. In the data, we include these in consumption of fixed capital.)

When we compute the share of compensation of intangible capital in Gross Value Added, we will need to impute a rental rate for physical capital r_{t+1} to impute rental payments to physical capital $r_{t+1}K_t/Y_{t+1}$. We do so using equation (41). That equation implies

$$\frac{p_t}{p_{t+1}}(1-\tau_{sk})P_{kt}K_{t+1} = (1-\tau_{corp})(r_{t+1}-\delta_k P_{kt+1})K_{t+1}+\tau_{skt}\delta_k P_{kt+1}k_{t+1}+(1-\tau_{sk})P_{kt+1}K_{t+1}$$

or

$$\left(\frac{r_{t+1}}{P_{kt+1}}\right) = \frac{(1-\tau_{sk})}{(1-\tau_{corp})}R_t - \frac{(1-\tau_{sk})}{(1-\tau_{corp})}(1+R_t)\frac{(P_{kt+1}-P_{kt})}{P_{kt+1}} + \frac{(1-\tau_{corp}-\tau_{sk})}{(1-\tau_{corp})}\delta_k, \quad (44)$$

where r_{t+1}/P_{kt+1} is the rental rate on physical capital measured in units of consumption, and

$$R_t = \left(\frac{p_t}{p_{t+1}} - 1\right)$$

is the net real rate of return in units of consumption on household's holdings of the corporate sector between t and $t+1$.

Intangible capital value of firms

The current dividend paid by a firm of size s at time t (the analog of equation 19 in the model without taxes) is

$$D_{vt}s = (1-\tau_{corp})\frac{\mu-1}{\mu}Y_t s - (1-\tau_{sRD})\left(P_{rt}x_{ct} + P_{rt}x_{mt}\right)s,$$

where τ_{sRD} is the subsidy rate for investments by incumbent firms into intangible capital. Note that we have assumed here that these investments are not deducted from profits for the purposes of corporate taxes. This is based on our use of measured spending on research and development as a measure of investment in intangible capital.

The intangible capital value of the incumbent firm of size s at time t including its profits earned at t satisfies the following recursion

$$V_t s = (1-\tau_{corp})\frac{\mu-1}{\mu}Y_t s - (1-\tau_{sRD})\left(P_{rt}x_{ct} + P_{rt}x_{mt}\right)s + \frac{1}{1+R_t}V_{t+1}(1-S_{et+1})s.$$

On a balanced growth path,

$$\bar{v} \equiv \frac{V}{Y} = \frac{1}{1-\frac{\exp(\bar{g}_Y)}{1+R}(1-\bar{S}_e)}\left[(1-\tau_{corp})\frac{\mu-1}{\mu} - (1-\tau_{sRD})\bar{S}_{ir}\right] \tag{45}$$

A firm of size s looking to choose x_{ct} and x_m to maximize V_t has optimality conditions

$$(1-\tau_{sRD})P_{rt}s = \frac{p_{t+1}}{p_t}V_{t+1}\frac{\partial}{\partial x_{ct}}\mathbb{E}_t\left[s_{t+1}|s_t\right] = \frac{1}{1+R_t}V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}(1-\delta_{ct})\bar{\zeta}'(x_{ct})s$$

Thus

$$(1-\tau_{sRD})\frac{P_{rt}x_{ct}}{Y_t} = \frac{1}{1+R_t}\frac{Y_{t+1}}{Y_t}\frac{V_{t+1}}{Y_{t+1}}S_{ct+1}\frac{\bar{\zeta}'(x_{ct})x_{ct}}{\bar{\zeta}(x_{ct})}$$

and, hence, on a BGP we have

$$(1-\tau_{sRD})\frac{\bar{P}_r\bar{x}_c}{Y} = \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_c\bar{v}\frac{\bar{\zeta}'(\bar{x}_c)\bar{x}_c}{\bar{\zeta}(\bar{x}_c)} \tag{46}$$

A similar argument gives that the optimal choice of x_{mt} satisfies on a BGP

$$(1-\tau_{sRD})\frac{\bar{P}_r\bar{x}_m}{Y} = \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{S}_m\bar{v}\frac{h'(\bar{x}_m)\bar{x}_m}{h(\bar{x}_m)} \tag{47}$$

We have to make an assumption as to whether expenditure by entrants is eligible for research subsidies. To keep the formulas simpler, we will say yes. That gives us free entry condition

$$(1-\tau_{sRD})\frac{P_{rt}x_{et}}{Y_t} = \frac{\exp(g_{Yt})}{1+R_t}v_{t+1}S_{et+1}$$

On a BGP, this equation becomes

$$(1-\tau_{sRD})\frac{\bar{P}_r\bar{x}_e}{\bar{Y}} = \frac{\exp(\bar{g}_Y)}{1+\bar{R}}\bar{v}\bar{S}_e. \tag{48}$$

Given these relationships between firm value and firm innovative investments, the elasticities (9) - (13) are given by expressions (32) - (37) in Proposition 2 where the right hand side is multiplied by $(1-\tau_{sRD})$.

The value of a firm of size s, ex-dividend, denoted by $q_{vt}s$ is given by

$$q_{vt}s = (1-\tau_{dist}) \sum_{k=1}^{\infty} \frac{p_{t+k}}{p_t} D_{vt+k} \mathbb{E}_t \left[s_{t+k} | s \right]$$

This is the price that the household pays for the firm. For the consumer purchasing the firm, we must have for all s,

$$q_{vt}s = \frac{p_{t+1}}{p_t} \left[(1-\tau_{dist}) D_{vt+1} + q_{vt+1} \right] \mathbb{E}_t \left[s_{t+1} | s \right].$$

The valuation of all incumbent firms ($s = 1$) is

$$q_{vt} = \frac{p_{t+1}}{p_t} \left[(1-\tau_{dist}) D_{vt+1} + q_{vt+1} \right] (1 - S_{et+1})$$

where we used the fact that $\mathbb{E}_t \left[s_{t+1} | s = 1 \right] = S_{ct+1} + S_{mt+1} = (1 - S_{et+1})$. On a BGP, we then have

$$\frac{\bar{q}_v}{\bar{Y}} = (1-\tau_{dist}) \frac{\frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e)}{1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e)} \frac{D_v}{Y} = \tag{49}$$

$$(1-\tau_{dist}) \frac{\frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e)}{1 - \frac{\exp(g_Y)}{1+\bar{R}} (1-\bar{S}_e)} \left[(1-\tau_{corp}) \frac{\mu - 1}{\mu} - (1-\tau_{sRD}) \bar{S}_{ir} \right]$$

Total firm market value

Total firm market value on a BGP is

$$\frac{\bar{q}_k \bar{P}_k \bar{K}}{\bar{Y}} + \frac{\bar{q}_v}{\bar{Y}} = (1-\tau_{dist})(1-\tau_{sk}) \frac{\exp(\bar{g}_Y)}{\bar{P}_k' / \bar{P}_k} \frac{\bar{P}_k \bar{K}}{\bar{Y}} +$$

$$(1-\tau_{dist}) \frac{\frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e)}{1 - \frac{\exp(\bar{g}_Y)}{1+\bar{R}} (1-\bar{S}_e)} \left[(1-\tau_{corp}) \frac{\mu - 1}{\mu} - (1-\tau_{sRD}) \bar{S}_{ir}. \right]$$

The model's implied Tobin's q is given by the ratio of $\frac{\bar{q}_k \bar{P}_k \bar{K}}{\bar{Y}} + \frac{\bar{q}_v}{\bar{Y}}$ to $\frac{\bar{P}_k \bar{K}}{\bar{Y}}$.

11 Calibration: Additional details

We measure aggregate innovative investment by incumbent firms, $(\bar{x}_c + \bar{x}_m) \bar{P}_r$, in two ways. The narrow measure is given by "gross fixed investment, nonresidential research and development, non-financial corporate business" (identifier FA105013043.A in the Integrated macroeconomic accounts). The broad measure is given by the sum of the narrow

measure, "gross fixed investment, nonresidential software, non-financial corporate business" (FA105013033.A), and "gross fixed investment, nonresidential entertainment, literary, and artistic originals"
(FA105013053.A).

We measure aggregate output of the final consumption good Y_t, as gross value added of the non-financial corporate sector (FA106902501.A) less indirect taxes (taxes on production and imports less subsidies) of the non-financial corporate business (FA106240101) less aggregate innovative investment by incumbent firms (one of the two measures defined above).

In order to measure the growth rate of real output, $\bar{g}_Y$, we deflate Y_t by the "Implicit Price deflator of the non-financial corporations sector" (identifier PRS88003143 on FRED, Federal Reserve Bank of St. Louis). In order to measure the growth rate of the labor force, $\bar{g}_L$, we use "Employment by the Nonfinancial Corporations Sector" (PRS88003013 on FRED).

We measure compensation of production labor, $W_t L_{pt}$, as "Compensation of employees paid by the non-financial corporate business" (FA106025005.A) less aggregate innovative investment by incumbent firms (one of the two measures defined above). We must then separate $Y - W_t L_{pt}$ (equal to, using the definitions above, to gross value added less indirect taxes less total compensation of employees) into rental payments to physical capital and profits.

For a given rental rate to physical capital in units of consumption, r_t/P_{kt}, we impute payments to physical capital as $\left(\frac{r_t}{P_{kt}}\right) P_{kt} K_t$. We measure the replacement value of physical capital, $P_{kt} K_t$, as "Nonfinancial assets, non-financial corporate business" (FL102010005.A) minus the replacement value of intangible capital. We measure the replacement value of intangible capital as either "Nonresidential intellectual property products, non-financial corporate business, current cost basis" (FL105013765.A) when we use the broad measure of innovative investments or as "Nonresidential research and development, non-financial corporate business, current cost basis" (FL105013465.A) when we use the narrow measure of innovative investments.

In order to infer the rental rate to physical capital, we use equation (44). We set the net real rate of return $R_t = 0.045$ and, following McGrattan and Prescott (2005), the subsidy rate to physical capital investment $\tau_{sk} = 0$. We set the revaluation rate of physical capital relative to consumption, $\frac{(P_{kt+1}-P_{kt})}{P_{kt+1}} = -0.005$. This revaluation rate is the dollar value of revaluations of produced assets recorded in the Integrated Macroeconomic Accounts for the non-financial corporate sector over the replacement value of the end of period capital stock all deflated by the change in the personal consumption expenditure deflator. This updates the figure for revaluations of close to -0.01 used in Hall (2003). We measure the corporate profits tax rate, τ_{corp}, as the ratio of "Current taxes on income, wealth, etc. paid, non-financial corporate business" (FA106220001.A) to "Operating surplus, net, nonfinancial corporate business" (FA106402101.A). We measure the depreciation rate δ_k, as the ratio of ["Consumption of fixed capital, structures, equipment, and intellectual property products, including equity REIT residential structures, non-financial corporate business" (FA106300083) minus consumption of intangible capital] to $P_{kt}K_t$. In order to measure consumption of intangible capital we multiply the replacement value of intangible capital (described above) by the depreciation rate of all intellectual property products (when we use the broad measure of innovative investments) or for research and development (when we use the narrow measure of innovative investments). We construct measures of depreciation of intellectual property products and research and development using Tables 2.1 and 2.4 of the BEA's Fixed Asset Tables.

We assume a subsidy rate on innovative investments of $\tau_{sRD} = 0.03$, obtained from Tyson and Linden (2012) (Table 7 in that paper reports estimates of corporate R&D tax credit claims relative to R&D business spending in the U.S.). When calculating the model's implied Tobin q, we follow McGrattan and Prescott (2005) and set the tax rate on corporate distributions to households equal to $\tau_{dist} = 0.17$.

Equipped with these measures, we calculate between 1990 and 2014, dividends to intangible capital relative to output after corporate profits taxes and subsidies for innovative investment

$$\bar{d}_v = (1-\tau_{corp})\frac{\mu - 1}{\mu} - (1-\tau_{sRD})\bar{S}_{ir}.$$

12 GHK Calibration

In this Appendix we describe how we can use the parameter values in the model of GHK to obtain alternative measures of $\bar{g}_Z$, $\bar{g}_M$, $\bar{S}_e$, $\bar{S}_m$, $\bar{F}_e$, $\bar{F}_m$ (and hence $\bar{S}_c = 1 - \bar{S}_e - \bar{S}_m$ and $\bar{F}_c = 1 - \bar{F}_e - \bar{F}_m$) for our model calibration. We then compare the elasticities implied from our calibration to those implied by the calibration of GFK.

We start by summarizing the parameters in the GHK model. δ_0 is the fraction of products that exit due to selection in the presence of per-period fixed costs (as opposed to the exogenous exit in our model). Given that δ_0 is close to zero in their paper, we set it to zero throughout. λ_i is the fraction of continuing products in a given period that receive an own innovation. $\tilde{\delta}_i \equiv \delta_i(1-\lambda_i)$ is the fraction of products in a given period that receive an innovation by an incumbent firm, and $\tilde{\delta}_e \equiv \delta_e(1-\delta_i)(1-\lambda_i)$ is the fraction of products that receive an innovation by an entering firm. κ_e and κ_i are the rate (per existing product) at which new products to the society are created by entering and incumbent firms, respectively. s_κ is the the average value of $\exp((\rho-1)z')/Z_t^{\rho-1}$ for products that are new to society in period t (GHK denote the elasticity of substitution by σ, but we use ρ to be consistent with our notation). $s_q^{\rho-1}$ is the average value of $\exp((\rho-1)z')/Z_t^{\rho-1}$ for products that receive an innovation, whether as a continuing product, as a product stolen by an incumbent firm, or a product stolen by an entering firm.

These parameters lead to the following formulas for the BGP growth rate of the number of products and BGP shares of products in our five categories:

$$
\begin{aligned}
\exp(\bar{g}_M) &= (1+\kappa_e+\kappa_i) \\
\bar{F}_{en} &= \frac{\kappa_e}{\exp(\bar{g}_M)} \\
\bar{F}_{es} &= \frac{\tilde{\delta}_e}{\exp(\bar{g}_M)} \\
\bar{F}_{in} &= \frac{\kappa_i}{\exp(\bar{g}_M)} \\
\bar{F}_{is} &= \frac{\tilde{\delta}_i}{\exp(\bar{g}_M)} \\
\bar{F}_c = 1-\bar{F}_{en}-\bar{F}_{es}-\bar{F}_{in}-\bar{F}_{is} &= \frac{1-\tilde{\delta}_e-\tilde{\delta}_i}{\exp(\bar{g}_M)}
\end{aligned}
$$

Note $\bar{F}_e = \bar{F}_{en}+\bar{F}_{es}$ and $\bar{F}_m = \bar{F}_{in}+\bar{F}_{is}$ can be measured directly in the data given our assumptions linking products to establishments. In our model, $\bar{\delta}$ is equivalent to $\bar{F}_{es}/\bar{F}_e$ and $\bar{F}_{is}/\bar{F}_m$. GHK do not impose the same value for these ratios. So we can define $\bar{\delta}_e = \bar{F}_{es}/\bar{F}_e$ and $\bar{\delta}_i = \bar{F}_{is}/\bar{F}_m$, and use their model implied values of these product shares to compute their model implied values of δ in our setting.

Similarly, their parameters lead to the following growth rate of productivity and size shares of five categories of products

$$
\exp((\rho-1)\bar{g}_Z) = 1+s_\kappa(\kappa_e+\kappa_i)+(s^{\rho-1}-1)\lambda_i+(s^{\rho-1}-1)(\tilde{\delta}_e+\tilde{\delta}_i)
$$

$$\bar{S}_{en} = \frac{s_\kappa \kappa_e}{\exp((\rho-1)\bar{g}_Z)}$$

$$\bar{S}_{es} = \frac{s_q^{\rho-1}\tilde{\delta}_e}{\exp((\rho-1)\bar{g}_Z)}$$

$$\bar{S}_{in} = \frac{s_\kappa \kappa_i}{\exp((\rho-1)\bar{g}_Z)}$$

$$\bar{S}_{is} = \frac{s_q^{\rho-1}\tilde{\delta}_i}{\exp((\rho-1)\bar{g}_Z)}$$

$$\bar{S}_c = 1 - \bar{S}_{en} - \bar{S}_{es} - \bar{S}_{in} - \bar{S}_{is} = \frac{1 + (s^{\rho-1}-1)\lambda_i - \tilde{\delta}_e - \tilde{\delta}_i}{\exp((\rho-1)\bar{g}_Z)}$$

Note that we define $\bar{S}_e = \bar{S}_{en} + \bar{S}_{es}$ and $\bar{S}_m = \bar{S}_{in} + \bar{S}_{is}$, which we observe directly in data. The contribution to growth from improvements in continuing products by incumbents, one of the key variables of interest in GHK, is given by

$$\frac{(s^{\rho-1}-1)\lambda_i}{\exp((\rho-1)\bar{g}_Z)-1} = \frac{\exp((\rho-1)\bar{g}_Z)\bar{S}_c - \exp(\bar{g}_M)\bar{F}_c}{\exp((\rho-1)\bar{g}_Z)-1}$$

Based on the parameter values reported in GHK (they assume 5-year periods) we obtain for the period 1976-86: $\bar{F}_e = 0.179$, $\bar{F}_m = 0.195$, $\bar{F}_c = 0.626$, $\bar{S}_e = 0.201$, $\bar{S}_m = 0.097$, $\bar{S}_c = 0.702$, $\bar{g}_M = 0.126$, $\bar{g}_Z = 0.0512$, $\delta_e = 1$ and $\delta_i = 0.394$, and for the period 2003-2013: $\bar{F}_e = 0.124$, $\bar{F}_m = 0.103$, $\bar{F}_c = 0.774$, $\bar{S}_e = 0.125$, $\bar{S}_m = 0.072$, $\bar{S}_c = 0.803$, $\bar{g}_M = 0.046$, $\bar{g}_Z = 0.072$, $\delta_e = 0.969$ and $\delta_i = 0.60$.[11]

These can be compared with our 5-year moments based on the LBD between 1977 and 2014: $\bar{F}_e$ falls from 0.389 in the first 5-year-period to 0.218 in the last 5-year period, $\bar{F}_m$ rises from 0.638 to 0.0751, $\bar{F}_c$ rises from 0.547 to 0.706, $\bar{S}_e$ falls from 0.176 to 0.092, $\bar{S}_m$ falls from 0.105 to 0.087, and $\bar{S}_c$ rises from 0.718 to 0.821.

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[11] Table 4 reports $\bar{F}_e$, $\bar{F}_m$, $\bar{F}_c$, $\bar{S}_e$, $\bar{S}_m$, $\bar{S}_c$ that we obtain when we annualize the innovation rates estimated in GHK. In order to satisfy the constraint that $\delta_0 > 0$ in our model, we must adjust slightly downward (by a ratio of 0.965) their estimates of δ_m and δ_e.

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