Aggregate Implications of Innovation Policy

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Firm’s investments in innovation

- large relative to GDP
- likely important factor in accounting for growth over time

1. To what extent can we change the path of macroeconomic growth over medium and long term by inducing firms to increase their investments in innovation?
Intro

- Firm’s investments in innovation
  - large relative to GDP
  - likely important factor in accounting for growth over time

1. To what extent can we change the path of macroeconomic growth over medium and long term by inducing firms to increase their investments in innovation?

2. What is the optimal level of these investments?

3. What policies can implement these investments?

- Model of growth through investments by firms to examine 1., 2. and (only partially) 3.
This Paper

- Model of growth through innovative investments by firms nests
  - Neo-Schumpeterian (quality ladders)
  - Expanding Varieties
  - Recent models of innovative investment by entrant and incumbent firms
    - Facts on firm-level innovation and firm dynamics
    - Tractable aggregation
    - Private and social returns differ

- Baseline assumptions: Analytic approximate dynamics of aggregate productivity and GDP to policy-driven change in innovation intensity of the economy

- Helpful for understanding more complex models
Key Features Shaping Aggregate Quantitative Implications

- **Social Depreciation of Innovation Expenditures**
  - Important in shaping medium term (e.g. 20 years) implications if model calibrated to low baseline growth rate

- **Intertemporal Knowledge Spillovers**
  - Important for long run implications and optimal innovation intensity, less so for medium term if model calibrated to low baseline growth rate and low social depreciation

- **Average cost of innovation incumbents versus entering firms**
  - Shapes medium term implications, key statistic determining difference in models with and w/o incumbents innovation

- **Provide simple relation between change in uniform innovation subsidy, fiscal expenditures, innovation intensity in BGP**
Related literature

- Increase in innovation intensity and long-term trends

- Extended Klette-Kortum models fit firm-level data

- Knowledge spillovers
  - Jones (2005), Bloom, Schankerman, and Van Reenen (2013)

- Misallocation of innovation across firms

- Sufficient statistics
Outline

- General model
- Analytical characterization of transition dynamics under three key assumptions
- Implications for socially optimal level of innovative investments
- Quantitative examples
- Change in uniform innovation subsidies, fiscal expenditures, innovation intensity in BGP
- Relation to more complex recent models
  - e.g. gains from reallocating given innovation resources across heterogeneous firms
Production by Intermediate good firms

- Abstract model of firms to nest wide class of models

- Firm type $j = 1, 2, 3, \ldots$
  - $n(j)$ products
  - $z_1(j), z_2(j), \ldots, z_{n(j)}(j)$ productivities
  - $\mu_1(j), \mu_2(j), \ldots, \mu_{n(j)}(j)$ markups
  - $\theta(j)$ heterogeneity in innovation technologies

- Production of intermediate good $m$ by firm of type $j$

  $$y_{mt}(j) = \exp(z_m(j))k_{mt}(j)^{\alpha}l_{mt}(j)^{1-\alpha}$$
Aggregate Productivity

- State: measures of incumbents \( \{N_t(j)\}_{j \geq 1} \)

- Final good: \( Y_t = \left( \sum_{j \geq 1} \sum_{m=1}^{n(j)} y_{mt}(j)^{(\rho-1)/\rho} N_t(j) \right)^{\rho/(\rho-1)} \)

\[
Y_t = Z_t L^{1-\alpha} K^\alpha = C_t + K_{t+1} - (1 - d_k) K_t
\]

- \( Z_t \equiv Z(\{N_t(j)\}_{j \geq 1}) \)

  - example with constant markups

\[
Z_t \equiv Z(\{N_t(j)\}_{j \geq 1}) = \left( \sum_{j \geq 1} \sum_{m=1}^{n(j)} \exp(z_{m}(j))^{(\rho-1)} N_t(j) \right)^{1/(\rho-1)}
\]
Innovation by Intermediate Good Firms

- Measure of incumbents \( \{N_t(j)\}_{j \geq 1} \)
  - \( \{y_{rt}(j)\}_{j \geq 1} \) innovative investment

- Measure of entrants \( N_t(0) \)
  - \( \bar{y}_r(0) \) entry cost parameter

- Transition Law

\[
\{N_{t+1}(j)\}_{j \geq 1} = T \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
\]
Innovation by Intermediate Good Firms

- Measure of incumbents \( \{N_t(j)\}_{j \geq 1} \)
  - \( \{y_{rt}(j)\}_{j \geq 1} \) innovative investment

- Measure of entrants \( N_t(0) \)
  - \( \bar{y}_r(0) \) entry cost parameter

- Transition Law
  \[
  \{N_{t+1}(j)\}_{j \geq 1} = T \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
  \]

- Functions \( Z \) and \( T \) give aggregate productivity growth
  \[
  g_{zt} \equiv \log Z_{t+1} - \log Z_t = G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
  \]

- Social Depreciation of Innovation Expenditures
  \[
  G^0_t \equiv G \left( \{0\}_{j \geq 1}, 0; \{N_t(j)\}_{j \geq 1} \right)
  \]
Example 1: Simple Quality Ladders

- Innovation only by entrants, $N_t(0)$

- $\sigma N_t(0) \leq 1$ products: $z' = z + \Delta z$

  \[
  Z_{t+1}^{\rho-1} = Z_t^{\rho-1} + \sigma N_t(0) (\exp(\Delta z)^{\rho-1} - 1) Z_t^{\rho-1}
  \]

- $G$ function:

  \[
  G = \log \left( \frac{Z_{t+1}}{Z_t} \right) = \frac{1}{\rho - 1} \log \left( 1 + \sigma N_t(0) (\exp(\Delta z)^{\rho-1} - 1) \right)
  \]

- No social depreciation $G^0 = 0$
Example 2: Simple Expanding Varieties

- Innovation only by entrants, $N_t(0)$
- Entrants $z = 0$, incumbents exit $\delta_f$

\[
(Z_{t+1})^{\rho-1} = (1 - \delta_f)(Z_t)^{\rho-1} + N_t(0)
\]

- Add: Entrants average $\exp(z)^{\rho-1} = \sigma Z_t^{\rho-1}$, incumbents exogenous growth $\Delta_z$

\[
Z_{t+1}^{\rho-1} = (1 - \delta_f) \exp(\Delta_z)^{\rho-1} Z_t^{\rho-1} + N_t(0) \sigma Z_t^{\rho-1}
\]

- $G$ function

\[
G = \log \frac{Z_{t+1}}{Z_t} = \frac{1}{\rho - 1} \log \left( (1 - \delta_f) \exp(\Delta_z)^{\rho-1} + \sigma N_t(0) \right)
\]

- $G^0 < 0$ if $(1 - \delta_f) \exp(\Delta_z)^{\rho-1} < 1$, linked to employment share of incumbents
Example 3: Simple Klette-Kortum

- All incumbents have equal investment per product, \( y_{rt}(1) \)
- \( \sigma(N_t(0) + d(y_{rt}(1))) \) products: \( z' = z + \Delta z \), \( d(.) \) increasing and concave
- Aggregate productivity

\[
Z_{t+1}^{\rho-1} = Z_t^{\rho-1} + \sigma(N_t(0) + d(y_{rt}(1))) \left( \exp(\Delta z)^{\rho-1} - 1 \right) Z_t^{\rho-1}
\]

- G function

\[
G = \frac{1}{\rho - 1} \log \left( 1 + \sigma(N_t(0) + d(y_{rt}(1))) \left( \exp(\Delta z)^{\rho-1} - 1 \right) \right)
\]

- No social depreciation, \( G^0 = 0 \)
Example 4: Simple Atkeson and Burstein (2010)

- Expanding varieties with endogenous growth by incumbents
- Incumbent with productivity $z$ invests $y_{rt} (1) \frac{\exp(z)^{\rho-1}}{Z_t^{\rho-1}}$,

$$z' = z + \frac{1}{\rho-1} \log d (y_{rt} (1))$$

- Entrants average $\exp(z)^{\rho-1} = \sigma Z_t^{\rho-1}$

$$Z_{t+1}^{\rho-1} = (1 - \delta_f) d (y_{rt} (1)) Z_t^{\rho-1} + N_t (0) \sigma Z_t^{\rho-1}$$

- $d (.)$ increasing and concave
- $G$ function

$$G = \frac{1}{\rho - 1} \log ((1 - \delta_f) d (y_{rt} (1)) + \sigma N_t (0))$$

- $G^0 < 0 \text{ if } (1 - \delta_f) d (0) < 1$

- Can add $\delta_f (i_{rt})$ decreasing in $i_{rt}$ as in Luttmer (Restud 2011)
Aggregate Innovation Technology

- Aggregate productivity growth

\[
\log Z_{t+1} - \log Z_t = G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
\]

- Research good used as input for innovation by firms

\[
\sum_{j \geq 1} y_{rt}(j) N_t(j) + \bar{y}_r(0) N_t(0) = Y_{rt} = A_{rt} Z_t^{\gamma-1} L_{rt}
\]

- $A_{rt}$ freely-available scientific knowledge
- $\gamma \leq 1$ intertemporal knowledge spillovers
- Price of research good $P_{rt} \equiv A_{rt}^{-1} W_t Z_t^{1-\gamma}$
Aggregate Innovation Technology

- Aggregate productivity growth

$$\log Z_{t+1} - \log Z_t = G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)$$

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- Innovation subsidies $\tau_t(j)P_{rt}y_{rt}(j)$ financed by lump sum taxes
Aggregate Innovation Technology

- Aggregate productivity growth

\[ \log Z_{t+1} - \log Z_t = G \left( \{ y_{rt}(j) \}_{j \geq 1}, N_t(0); \{ N_t(j) \}_{j \geq 1} \right) \]

- Research good used as input for innovation by firms

\[ \sum_{j \geq 1} y_{rt}(j) N_t(j) + \bar{y}_{r}(0) N_t(0) = Y_{rt} = A_{rt} Z_t^{\gamma - 1} L_{rt} \]

- \( A_{rt} \) freely-available scientific knowledge
- \( \gamma \leq 1 \) intertemporal knowledge spillovers
- Price of research good \( P_{rt} \equiv A_{rt}^{-1} W_t Z_t^{1-\gamma} \)

- Innovation subsidies \( \tau_t(j) P_{rt} y_{rt}(j) \) financed by lump sum taxes
- Labor allocation and innovation intensity of economy

\[ s_{rt} \equiv \frac{P_{rt} Y_{rt}}{Y_t} = \kappa \frac{L_{rt}}{L_{pt}} \]
Positive implications of model

- Baseline allocations in a BGP: $\bar{g}_z$, $\bar{s}_r$, $\{\bar{y}_{rt}(j)\}$, $\{\bar{N}_t(j)\}$

- At $t = 0$, change in innovation subsidies from $\tau(j)$ to $\tau'(j)$

- New path: $s'_{rt}$, $\{y'_{rt}(j)\}_{j \geq 1}$, $\{N'_t(j)\}_{j \geq 0}$

- Approximate implied path $\left\{Z'_t\right\}_t$, $\left\{GDP'_t\right\}_t$
Two Key Equations

- Innovation intensity to research output

\[
(\log Y'_{rt} - \log \bar{Y}_r) = \bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - (1 - \gamma) \left( \log Z'_t - \log \bar{Z}_t \right)
\]
Two Key Equations

- **Innovation intensity to research output**

  \[
  (\log Y_{rt}' - \log \bar{Y}_r) = \bar{L}_p (\log s_{rt}' - \log \bar{s}_r) - (1 - \gamma) (\log Z_t' - \log \bar{Z}_t)
  \]

- **Research output to productivity growth**

  \[
  g_{zt}' - \bar{g}_z \approx \mathcal{E}_{gt} (\log Y_{rt}' - \log \bar{Y}_rt) + \sum_{j \geq 1} \frac{\partial G}{\partial N(j)} (N_t'(j) - \bar{N}_t(j)).
  \]

- **Impact elasticity with respect to change in \( Y_{rt} \)**

  \[
  \mathcal{E}_{gt} = \sum_{j \geq 1} \frac{\partial G}{\partial y_r(j)} \bar{y}_{rt}(j) \frac{d \log y_{rt}(j)}{d \log Y_{rt}} + \frac{\partial G}{\partial N(0)} \bar{N}_t(0) \frac{d \log N_t(0)}{d \log Y_{rt}}
  \]

- **Characterize dynamics with three key assumptions**
A1: Concavity of G function

- If innovation ($\{y_{rt}(j)\}_{j \geq 1}, N_t(0)$) increases proportionally for all firms, then growth rate (in logs) increases less than proportionally.

- Assume $H_t(a)$ weakly concave in $a$

$$H_t(a) = G\left(a \ast (\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0)); \{\bar{N}_t(j)\}_{j \geq 1} + (1 - a) \ast 0\right)$$
A1: Concavity of G function

- If innovation \( \{y_{rt}(j)\}_{j \geq 1}, N_t(0) \) increases proportionally for all firms, then growth rate (in logs) increases less than proportionally

- Assume \( H_t(a) \) weakly concave in \( a \)

\[
H_t(a) = G(a \ast (\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0)); \{\bar{N}_t(j)\}_{j \geq 1} + (1 - a) \ast 0)
\]

- Suppose innovation changes proportionately for all firms:

\[
d \log y_{rt}(j) = d \log N_t(0) = d \log Y_{rt}
\]

- Impact elasticity bounded by

\[
\mathcal{E}_{gt} \leq \bar{g}_{zt} - G_t^0
\]

- Proof: \( \mathcal{E}_{gt} = H'(1) \leq \frac{H(1) - H(0)}{1 - 0} \)
Intuition: Impact elasticity $\mathcal{E}_{gt} \leq \bar{g}_{zt} - G^0_t$

- Moving from $Y_r = 0$ to $\bar{Y}_r$ increases growth from $G^0_t$ to $\bar{g}_{zt}$, concavity implies marginal increase < average increase
Example 1: Simple Quality Ladders

- **G function concave**

  \[
  G = \frac{1}{\rho - 1} \log \left( 1 + \sigma N_t (0) \left( \exp(\Delta_z)^{\rho-1} - 1 \right) \right)
  \]

- **No social Depreciation** \( G^0 = 0 \)

- **Exact Impact Elasticity**

  \[
  \mathcal{E}_{QL}^t = \frac{1}{\rho - 1} \frac{\exp((\rho - 1) \bar{g}_z) - 1}{\exp((\rho - 1) \bar{g}_z)} \leq \bar{g}_z
  \]

  - approaches \( \bar{g}_z \) as \( \rho \to 1 \)
  - approaches 0 as \( \rho \to \infty \) (most output is produced by highest \( z \))

- **Impact Elasticity tightly bounded if model applied to low growth economies**
Example 2: Simple Expanding Varieties

- G function concave

\[ G = \frac{1}{\rho - 1} \log \left( (1 - \delta_f) \exp(\Delta_z)\rho^{-1} + \sigma N_t(0) \right) \]

- \( G^0 < 0 \) linked to employment share of incumbents

- Exact impact elasticity

\[ \mathcal{E}^{EV}_{gt} = \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - \exp((\rho - 1)G^0)}{\exp((\rho - 1)\bar{g}_z)} \leq \bar{g}_z - G^0 \]

- Upper bound can be several times bigger than in Quality Ladders Model

- Higher \( \rho \), exact elasticity less sensitive to \( G^0 \)
A1': Concavity in Entrants

- $\tilde{H}(a)$ weakly concave: value of $G$ evaluated at convex combination of $\{\bar{y}_{rt}(j)\}_{j \geq 1}$, $\tilde{N}_t(0)$ and $\{\bar{y}_{rt}(j)\}_{j \geq 1}$, $N^0_t(0)$
  - $N^0_t(0)$ implements $G^0_t$ at fixed $\{\bar{y}_{rt}(j)\}_{j \geq 1}$
  - $Y^0_{rt} = \sum_{j \geq 1} y_{rt}(j) N_t(j) + \bar{y}_r(0) N^0_t(0)$
A1': Concavity in Entrants

- $\tilde{H}(a)$ weakly concave: value of $G$ evaluated at convex combination of $(\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0))$ and $(\{\bar{y}_{rt}(j)\}_{j \geq 1}, N^0_t(0))$
  
  - $N^0_t(0)$ implements $G^0_t$ at fixed $\{\bar{y}_{rt}(j)\}_{j \geq 1}$
  - $Y^0_{rt} = \sum_{j \geq 1} y_{rt}(j)N_t(j) + \bar{y}_r(0)N^0_t(0)$

- Suppose only entry varies:
  
  $$d \log y_{rt}(j) = 0 \text{ and } d \log N_t(0) = \frac{\tilde{Y}_{rt}}{\bar{y}_{rt}(0)\bar{N}_t(0)} d \log Y_{rt}$$

- Alternative bound: $E_{gt} \leq (\bar{g}_{zt} - G^0_t) \frac{\tilde{Y}_{rt}}{Y_{rt} - Y^0_{rt}}$
  
  - Proof: $E_{gt} = \tilde{H}'(1) \frac{\tilde{Y}_{rt}}{Y_{rt} - Y^0_t} \leq (\tilde{H}(1) - \tilde{H}(0)) \frac{\tilde{Y}_{rt}}{Y_{rt} - Y^0_t}$

- $Y^0_{rt} < 0$ (tighter bound) when incumbents have lower average cost of innovation

$$\frac{\tilde{Y}_{rt} - 0}{\bar{g}_z - G^0_t} < \frac{\tilde{Y}_{rt} - Y^0_{rt}}{\bar{g}_z - G^0_t}$$
Example 3: Simple Klette-Kortum

- **G function**
  \[ G = \frac{1}{\rho - 1} \log \left( 1 + \sigma (N_t(0) + d(y_{rt}(1))) \left( \exp(\Delta_z)^{\rho - 1} - 1 \right) \right) \]

- No social Depreciation \( G^0 = 0 \)
- \( d(y_r) \) concave implies
  Both concavity assumptions satisfied
  - \( Y_r^0 \leq 0 \), so \( \frac{\bar{Y}_{rt}}{Y_{rt} - Y_r^0} \leq 1 \)
  - Exact elasticity to change in innovation by entrants
    \[ \begin{align*}
    \mathcal{E}_{gt}^{KK} &= \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)} \frac{\bar{Y}_r}{\bar{Y}_r - Y^0}
    \end{align*} \]

- Given \( \rho \) and \( \bar{g}_z \), \( \mathcal{E}_{gt}^{KK} \leq \mathcal{E}_{gt}^{QL} \)
A2: Growth maximized given current research output

- Given $\bar{Y}_{rt}$, then $\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0)$ solves

$$\max G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{\bar{N}_t(j)\}_{j \geq 1} \right) \text{ subject to }$$

$$\sum_{j \geq 1} y_{rt}(j) \bar{N}_t(j) + \bar{y}_r(0) N_t(0) = \bar{Y}_{rt}$$
A2: Growth maximized given current research output

- Given $\bar{Y}_{rt}$, then \( \{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0) \) solves

\[
\max \ G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{\bar{N}_t(j)\}_{j \geq 1} \right) \quad \text{subject to}
\]

\[
\sum_{j \geq 1} y_{rt}(j) \bar{N}_t(j) + \bar{y}_r(0) N_t(0) = \bar{Y}_{rt}
\]

- If baseline allocation satisfies A2, then \( \mathcal{E}_{gt} \) is independent of how the change in the output of the research good is allocated across incumbent and entering firms

\[
\mathcal{E}_{gt} = \lambda_t \bar{Y}_{rt}
\]
A2: Growth maximized given current research output

- Given $\bar{Y}_{rt}$, then $\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0)$ solves

$$
\max G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{\bar{N}_t(j)\}_{j \geq 1} \right) \text{ subject to }

\sum_{j \geq 1} y_{rt}(j) \bar{N}_t(j) + \bar{y}_r(0) N_t(0) = \bar{Y}_{rt}
$$

- If baseline allocation satisfies A2, then $\mathcal{E}_{gt}$ is independent of how the change in the output of the research good is allocated across incumbent and entering firms

$$
\mathcal{E}_{gt} = \lambda_t \bar{Y}_{rt}
$$

- Previous bounds hold
- A2 holds in equilibrium of KK model if $\tau(j) = \tau$ in initial BGP
- A2 does not imply necessarily imply $\{\bar{y}_{rt}(j)\}_{j \geq 1}, \bar{N}_t(0)$ maximizes welfare (given $\bar{Y}_{rt}$) unless $\frac{\partial G}{\partial \bar{N}(j)} = 0$ for $j \geq 1$
Failure of A2

- Growth maximized subject to current research output, implies:

\[ \mathcal{E}_{gt} = \lambda_t \tilde{Y}_{rt} \]

- Suppose instead that:

\[ \frac{\partial G}{\partial y_{rt}(j)} / \tilde{N}_t(j) = \lambda_t \tau_t(j) \]

\[ \frac{\partial G}{\partial N(0)} / \tilde{y}_r(0) = \lambda_t \]

- Impact elasticity:

\[ \mathcal{E}_{gt} = \lambda_t \left[ \sum_{j \geq 1} (\tau_t(j) - 1) \tilde{y}_{rt}(j) \tilde{N}_t(j) \frac{d \log y_{rt}(j)}{d \log Y_{rt}} + \tilde{Y}_{rt} \right] \]
Results so far

▶ Innovation intensity to research output

\[(\log Y'_{rt} - \log \bar{Y}_r) = \bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - (1 - \gamma) (\log Z'_t - \log \bar{Z}_t)\]

▶ Research output to productivity growth

▶ A2 implies allocation of innovative investment does not matter

\[g'_{zt} - \bar{g}_z \approx \mathcal{E}_{gt} (\log Y'_{rt} - \log \bar{Y}_r) + \sum_{j \geq 1} \frac{\partial G}{\partial N(j)} (N'_t(j) - \bar{N}_t(j))\]

▶ Concavity A1 bounds impact elasticity, \(\mathcal{E}_{gt} \leq (\bar{g}_{zt} - G_{t}^0)\)

▶ Or even tighter bound if A1’ and incumbents have lower average cost of innovation
Results so far

- **Innovation intensity to research output**

\[
\log Y'_{rt} - \log \bar{Y}_r = \bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - (1 - \gamma) (\log Z'_t - \log \bar{Z}_t)
\]

- **Research output to productivity growth**
  - A2 implies allocation of innovative investment does not matter

\[
g'_zt - \bar{g}_z \approx \varepsilon_{gt} (\log Y'_{rt} - \log \bar{Y}_r) + \sum_{j \geq 1} \frac{\partial G}{\partial N(j)} (N'_t(j) - \bar{N}_t(j))
\]

- Concavity A1 bounds impact elasticity, \( \varepsilon_{gt} \leq (\bar{g}_zt - G^0_t) \)
  - Or even tighter bound if A1' and incumbents have lower average cost of innovation

- To characterize dynamics analytically
  - A3: \( \frac{\partial G}{\partial N(j)} = 0 \) for \( j \geq 1 \) (generalizes to \( G \) depending on \( Z_t \))
  - Satisfied by our three examples
Putting these results together

\[
\log Z'_{t+1} - \log \tilde{Z}_{t+1} = \sum_{k=0}^{t-1} \Gamma_k \left( \log s'_{rt-k} - \log \bar{s}_r \right)
\]

**Impact effect:** \( \Gamma_0 = \bar{L}_p \mathcal{E}_0 \gamma \)

**Decay:** \( \Gamma_{k+1} = [1 - (1 - \gamma)\mathcal{E}_0 \gamma] \Gamma_k \)

**Endogenous growth:**

\[
\log Z'_{t+1} - \log Z'_t - \bar{g}_z = \Gamma_0 \left( \log s'_{rt} - \log \bar{s}_r \right)
\]

- **Higher** \( \mathcal{E}_0 \gamma \), **larger response at every finite horizon**

**Long-term impact permanent change in** \( s_r \): \( \bar{L}_p / (1 - \gamma) \)
Using these results: numerical example

- Baseline annual growth $\bar{g}_z = 0.0125$
- Social Depreciation $G^0 = 0$
- $\mathcal{E}_g = 0.0122$ close to upper bound $\bar{g}_z - G^0$
- $\Gamma_0 = \mathcal{E}_g \bar{L}_p = .01$
- Vary spillover $\gamma = -2, \gamma = 0, \gamma \to 1$
  - $Z$ long-run elasticity ranges from $1/4$ to infinity
- Innovation subsidy raises $s_{rt}$ permanently starting at date $t = 0$ from $11\%$ to $14\%$
  - $\Delta \log s_r = \log s_{rt} - \log \bar{s}_r = 0.24$
- Analytic impulse response (constant $K/Y$).
Aggregate productivity: 100 years
Aggregate productivity: 20 years
GDP dynamics

- Suppose $K/Y$ fixed

$$
\log \frac{GDP'_t - G\bar{DP}_t}{1 - \alpha} = (\log Z'_t - \log \bar{Z}_t) - \bar{L}_r (\log s'_{rt} - \log \bar{s}_r)
$$

- Tradeoff: GDP $\uparrow$ with productivity, $\downarrow$ with innovation investment
GDP: 100 years

GDP (excluding innovation expenditures), relative to BGP, 100 years

- Red dotted line: low spillovers
- Black line: medium spillovers
- Blue dashed line: high spillovers

Y-axis: GDP relative to BGP
X-axis: Years (0 to 100)

The graph shows the growth of GDP relative to baseline growth point (BGP) over 100 years, with different spillover scenarios.
GDP: 20 years + productivity shocks
GDP including innovation: 20 years + productivity shocks

Low spillovers
High spillovers
2.25% social depreciation

- Expanding varieties: $\rho = 4$ and 10% employment by entering products
- Impact elasticity $E_g$ increases from 0.0122 to 0.033
- Impact effect $\Gamma_0$ increases from 0.01 to 0.028
2.25% social depreciation: Aggregate productivity 20 years
2.25% social depreciation: GDP 20 years

GDP (excluding innovation expenditures), relative to BGP, 20 years

- no depr, low spillovers
- no depr, high spillovers
- 2.5% depr, low spillovers
- 2.5% depr, high spillovers

years [0, 20]
Welfare: Optimal Innovation Intensity

- Welfare $\sum_{t=0}^{\infty} \frac{\beta^t}{1-\xi} (C_t/L_t)^{1-\xi}$

- Perturb log $s_{r0}'$ only, $\Delta$ welfare = zero

\[
\left[ \sum_{k=0}^{\infty} \tilde{\beta}^{1+k} \frac{\Gamma_k}{1-\alpha} - \bar{L}_r \right] (\log s_0' - \log \bar{s}_r) = 0
\]

- Gap between BGP interest and growth rates
  \[\tilde{\beta} = \beta \exp((1-\xi)\bar{g}_y)\]

- Optimal BGP allocation satisfies

\[
s_{r}^* = (1 - \alpha) \frac{L_r^*}{L_p^*} = \frac{\tilde{\beta}E_{g}^*}{1 - \tilde{\beta} [1 - (1-\gamma)E_{g}^*]}
\]
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\[
s_r^* = (1 - \alpha) \frac{L_r^*}{L_p^*} = \frac{\tilde{\beta} \mathcal{E}_g^*}{1 - \tilde{\beta} \left[ 1 - (1 - \gamma) \mathcal{E}_g^* \right]}
\]

- Huge range of implications depending on \( \tilde{\beta} \) and \( \gamma \)
  - \( \tilde{\beta} = .99, \gamma = .99 \implies s_r^* = 1.21; \)
  - \( \tilde{\beta} = .96, \gamma = -2 \implies s_r^* = 0.156; \)

- If small \( \mathcal{E}_g^* \), then disconnect between \( s_r^* \) and 20 year response

- Level of optimal subsidy depends on additional, model-specific parameters e.g. markup, rate of production destruction, etc.
Change in innovation subsidies and innovation intensity

- Up to here: aggregate impact of policy induced increase in $s_{rt}$
- Now: What is the impact of uniform subsidies on $s_{rt}$?
Change in innovation subsidies and innovation intensity

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- Assumptions (satisfied in our model examples)
  - Static profits, $\Pi_t (j) = B_t H(j)$, in BGP $\bar{B}_t = \kappa \bar{Y}_t$
  - Growth, interest rate unchanged between BGPs ($\gamma < 1$)
  - Positive entry in BGP, $\bar{N}(0) > 0$
  - Given $y_r (j)$, unique invariant distribution $N(j)/N(0)$, in BGP
  - Uniform innovation policies, $\tau(j) = \tau$

- Across old and new BGP:

  $$\bar{s}_r (1 - \tau) = \bar{s}'_r (1 - \tau')$$

  $$\frac{\bar{E}'}{Y'} - \frac{\bar{E}}{\bar{Y}} = \bar{s}'_r - \bar{s}_r$$

- Given initial $\tau$, $s_r$, can solve for change in $\tau$ to implement $s^*_r$
Sketch of proof

- Equilibrium value function:

\[
V_t(j) = \max_{y_{rt}(j)} B_t H(j) - (1 - \tau_t) P_{rt} y_{rt}(j) + \frac{\mathbb{E}_{tjj'}|y_{rt}(j) V_{t+1}(j')} {1 + R_t}
\]

- Entrants:

\[
V_t(0) = - (1 - \tau_t) \bar{y}_r(0) P_{rt} + \frac{\mathbb{E}_{t0j'} V_{t+1}(j')} {1 + R_t}
\]

- Define \( v_t(j) = \frac{V_t(j)}{(1 - \tau_t) P_{rt}} \). In BGP with entry

\[
\bar{v}_t(j) = \max_{y_{rt}(j)} \frac{\kappa \bar{Y}_t} {P_{rt} (1 - \tau)} H(j) - y_{rt}(j) + \frac{\mathbb{E}_{tjj'}|y_{rt}(j) \bar{v}_{t+1}(j')} {1 + \bar{r}_t}
\]

\[
0 = - \bar{y}_r(0) + \frac{\mathbb{E}_{t0j'} \bar{v}_{t+1}(j')} {1 + \bar{r}_t}
\]

- \( \bar{r}, \frac{\bar{Y}_t} {P_{rt}(1 - \tau)}, \bar{y}_r(j), \frac{N_t(j)} {N_t(0)}, \bar{Y}_{rt} \) unchanged

- Using \( s_{rt} = P_{rt} Y_{rt} / Y_t \) and \( E_t = \tau_t s_{rt} \) implies result
Failure of A2, Simple Klette-Kortum

- Technological inefficiency
- If \( d'(\bar{y}_r(1)) \neq 1/\bar{y}_r(0) \) then additional welfare gain
  - can achieve same growth rate \( \bar{g}_z \) with permanently lower \( Y_r \)
    and higher consumption by reallocating to
    \[
    d'(\bar{y}_r(1)) = 1/\bar{y}_r(0)
    \]
  - need to know details of innovation technology \( G \) and baseline allocation
- Define misallocation “wedge” \( \tau = \bar{y}_r(0)/d'(\bar{y}_r(1)) \)
- Permanent percentage increase in production labor:
  - 0% if \( \tau = 1 \), 0.1% if \( \tau = 1.1 \), 1.4% if \( \tau = 1.3 \), 6.8% if \( \tau = 1.5 \)
Quality ladders, $\rho \to 1$, own-product innovation by incumbents increases markup

Aggregate productivity $Z_t = V_t M_t$,

$$\log V_{t+1} - \log V_t = \sigma (d(y_{rt}(1)) + N_t(0)) \Delta z$$

- $V$ implies linear $G$, but $G$ not only driven by technology

$$M_t = \frac{\exp \int [\log (\mu(j)^{-1})] dN_t(j)}{\int \mu(j)^{-1} dN_t(j)}$$

Rise in entry, reduction in markup dispersion, increase in $M_t$

- $G$ can violate A1 and A1' (also violate A2 and A3)
Gains from reallocating innovation across firms

- A number of models violate A2 and/or A3
  - Distribution $N(j)$ enters $G$
  - Starting in equilibrium BGP, there exist reallocation of innovation investment across firms, given $\bar{Y}_{rt}$, that increases current growth rate
  - Can imply dynamic gains from favoring innovation of one type of firms versus another, changing firm distribution in long run
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- A4: Dynamic conditional efficiency
  - Given $\{\bar{L}_{rt}\}_{t=0}^{\infty}$, allocation maximizes welfare
  - A2 + A3 $\rightarrow$ A4, not the converse
Gains from reallocating innovation across firms

- **A4: Dynamic conditional efficiency**
  - Given $\{L_{rt}\}_{t=0}^{\infty}$, allocation maximizes welfare
  - $A2 + A3 \rightarrow A4$, not the converse

- **Implications of A4:**
  - Starting from dynamic conditional efficient BGP, no first-order welfare gains from reallocating $Y_r$ across firms
  - Can approximate transition dynamics of change in $\{s_{rt}\}$ (and $\{L_{rt}\}$) by considering any variation of innovation investment by incumbents or entrants, e.g. vary only entry, without having to re-solve dynamic optimization
  - Different variations can imply different aggregate dynamics, same first-order change in welfare
Gains from reallocating innovation across firms

- Model examples that do not satisfy A2 and A3, but equilibrium dynamically conditional efficient:

  1. Luttmer (2011): expanding varieties with heterogeneity in innovation technology

  2. variation of Lentz and Mortensen (2014): Klette-Kortum with heterogeneity in innovation technology, separate innovation capacity from product leadership, constant markups

  3. variation of Acemoglu et. al. (2013): 2. + endogenous product exit, spillovers from both active and inactive products

- No dynamic welfare gains, up to a first-order, from reallocating innovation activities or changing exit threshold

- Potential gains only from changing $s_r$
Conclusion

- Wide class of growth models
  - Simple approximation to transition dynamics

- Two key sufficient statistics:
  - Impact elasticity
    - baseline growth rate
    - social depreciation of innovation expenditures (how big is it?)
    - incumbents’ average cost of innovation
  - Intertemporal knowledge spillovers
    - price deflator for research output

- Useful benchmark for evaluating quantitative implications of richer new growth models

- Challenge: find reliable measures of deviations from conditional efficiency
Social planner

$$\max \sum_{t=0}^{\infty} \beta^t u (Z_t \left( 1 - \bar{L}_{rt} \right)) \quad \text{subject to}$$

$$Z_t^{\rho-1} = \sum_{j \geq 1} \phi_j N_t(j)$$

$$N_{t+1}(j) = \phi_{0j} N_t(0) + \sum_{j'} \phi_{jj'} \left( 1 - \delta^f (j') + g_{j'} (y_{rt}(j')) \right) N_t(j')$$

$$\sum_{j \geq 1} y_{rt}(j) N_t(j) + \bar{y}_r(0) N_t(0) = A_{rt} Z_t^{\gamma-1} \bar{L}_{rt}$$

Equilibrium value functions

$$V_t(j) = \max_{y_{rt}(j)} \prod_t \varphi_j - P_{rt} y_{rt}(j) + \ldots$$

$$+ \frac{1 - \delta^f (j) + g_j (y_{rt}(j))}{1 + R_t} \sum_{j'} \phi_{jj'} V_{t+1}(j')$$

$$P_{rt} \bar{y}(0) = \frac{1}{1 + R_t} \sum_j \phi_{0j} V_{t+1}(j)$$

Satisfies A4
Klette-Kortum with heterogeneity in innovation technologies

- Social planner

\[
\max \sum_{t=0}^{\infty} \beta^t u \left( Z_t \left( 1 - \bar{L}_{rt} \right) \right) \quad \text{subject to}
\]

\[
\delta_t = \sigma \left( N_t(0) + \sum_{j \geq 1} d_j(y_{rt}(j)) N_t(j) \right)
\]

\[
\left( \frac{Z_{t+1}}{Z_t} \right)^{\rho-1} = \delta_t \left( \exp(\Delta^\rho z) - 1 \right) + 1
\]

\[N_{t+1}(j) = (1 - \delta_t) N_t(j) + \sigma \left[ \phi_j N_t(0) + d_j(y_{rt}(j)) N_t(j) \right]\]

- Equilibrium value functions:

\[
V_t(z;j) = \Pi_t \exp(z)^{\rho-1} - P_{rt} y_{rt}(j) + \frac{(1 - \delta_t) V_{t+1}(z;j) + \sigma d_j(y_{rt}(j)) \tilde{V}_{t+1}(j)}{1 + R_t}
\]

\[
\tilde{V}_{t+1}(j) = \int V_{t+1}(z + \Delta z;j) dM_t(z)
\]

\[
\tilde{y}_r(0) = \frac{\sigma \sum \phi_j \tilde{V}_{t+1}(j)}{1 + R_t}
\]

- Does not satisfy A4: firms do not consider \( \partial N_{t+1}(j) / \partial y_{rt}(j') \)
Separate research capacity from product market leadership

Social planner

\[
\max \sum_{t=0}^{\infty} \beta^t u \left( Z_t (1 - \bar{L}_{rt}) \right) \quad \text{subject to}
\]

\[
\left( \frac{Z_{t+1}}{Z_t} \right)^{\rho^{-1}} = \sigma \left( \sum_{j \geq 1} d_j (i_{rt} (j)) N_t (j) \right) \left( \exp \left( \Delta_Z^{p^{-1}} \right) - 1 \right) + 1
\]

\[
N_{t+1} (j) = \phi_j N_t (0) + \left( 1 - \delta^f (j) + g_j (y_{rt} (j)) \right) N_t (j)
\]

\[
\sum_{j \geq 1} (y_{rt} (j) + i_{rt} (j)) N_t (j) + \bar{y}_r (0) N_t (0) = A_{rt} Z_t^{\gamma^{-1}} \bar{L}_{rt}
\]

Equilibrium value functions

\[
V_t (j) = -P_{rt} (y_{rt} (j) + i_{rt} (j)) + \ldots + \frac{1 - \delta^f (j) + g_j (y_{rt} (j))}{1 + R_t} V_{t+1} (j) + \frac{\sigma d_j (i_{rt} (j))}{1 + R_t} \bar{V}_{t+1}
\]

\[
P_{rt} \bar{y}_r (0) = \frac{1}{1 + R_t} \sum \phi_j V_{t+1} (j)
\]

Satisfies A4
Quality ladders with fixed costs of product operation

- Social planner

\[
\max \sum_{t=0}^{\infty} \beta^t u \left( \left( \int_{\tilde{z}_t} \exp(z)^{\rho-1} \, dM_t(z) \right)^{\frac{1}{\rho-1}} \left( 1 - \bar{L}_rt - f \int_{\tilde{z}_t} dM_t(z) \right) \right)
\]

\[
\left( \frac{Z_{t+1}}{Z_t} \right)^{\rho-1} = \sigma \left( \sum_{j \geq 1} d_j(y_{rt}(j)) N_t(j) \right) \left( \exp \left( \Delta_z^{\rho-1} \right) - 1 \right) + 1
\]

\[
M_{t+1}(z) = (1 - \chi) \left[ M_t(z) + \sigma \left( \sum_{j \geq 1} d_j(y_{rt}(j)) N_t(j) \right) \left( M_t(z - \Delta_z) - M_t(z) \right) \right]
\]

and for \( z = \log Z_t^{\rho-1} \)

\[
M_{t+1}(z) = \chi + (1 - \chi) \left[ M_t(z) + \sigma \left( \sum_{j \geq 1} d_j(y_{rt}(j)) N_t(j) \right) \left( M_t(z - \Delta_z) - M_t(z) \right) \right]
\]

- Equilibrium exit threshold: \( \Pi_t \exp(\tilde{z}_t)^{\rho-1} = W_t f \), equal to

\[
\frac{1}{\rho - 1} \left( \int_{\tilde{z}_t} \exp(z)^{\rho-1} \, dM_t(z) \right) \left( 1 - \bar{L}_rt - f \int_{\tilde{z}_t} dM_t(z) \right) = f
\]

- Satisfies A4 (not if innovations on active products only)