Aggregate Implications of Innovation Policy

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Intro

- Firm's investments in innovation
  - large relative to GDP
  - likely important factor in accounting for growth over time

- To what extent can we change path of macroeconomic growth over medium and long term by inducing firms to increase their investments in innovation?

- What is the optimal level of these investments?
This Paper

- Examine these questions using model of growth through innovative investments by firms that nests
  - Neo-Schumpeterian
  - Expanding Varieties
  - Innovative investment by entrant and incumbent firms

- Key features determining models’ quantitative implications
  - Intertemporal Knowledge Spillovers
  - Social Depreciation of Innovation Expenditures

- Results helpful for understanding more complex models
Related literature

- Extended Klette-Kortum models good fit firm-level data

- Misallocation of innovation across firms

- Increase in innovation intensity and long-term trends

- Knowledge spillovers
  - Jones (2005), Bloom, Schankerman, and Van Reenen (2013)

- Measured productivity and intangible capital

- Sufficient statistics
  - Arkolakis, Costinot, Rodriguez-Clare (2012)
Production by Intermediate good firms

- Abstract model of firms to nest wide class of models

- Firm type $j = 1, 2, 3, \ldots$,
  - $n(j)$ products
  - $z_1(j), z_2(j), \ldots, z_{n(j)}(j)$ productivities
  - $\mu_1(j), \mu_2(j), \ldots, \mu_{n(j)}(j)$ markups
  - $\theta(j)$ heterogeneity in innovation technologies

- Production of intermediate good $k$ by firm of type $j$

\[ y_{kt}(j) = \exp(z_k(j)) k_{kt}(j)^\alpha l_{kt}(j)^{1-\alpha} \]
Aggregate Productivity

- State: measures of incumbents $\{N_t(j)\}_{j \geq 1}$

- Final good, $Y_t = \left( \sum_{j \geq 1} \sum_{k=1}^{n(j)} y_{kt}(j)^{(\rho-1)/\rho} N_t(j) \right)^{\rho/(\rho-1)}$

  $$Y_t = Z_t L_{pt}^{1-\alpha} K_t^\alpha = C_t + K_{t+1} - (1 - d_k) K_t$$

- $Z_t \equiv Z(\{N_t(j)\}_{j \geq 1})$

  - example with constant markups

    $$Z_t \equiv Z(\{N_t(j)\}_{j \geq 1}) = \left( \sum_{j \geq 1} \sum_{k=1}^{n(j)} \exp((\rho - 1) z_k(j)) N_t(j) \right)^{1/(\rho-1)}$$
Innovation by Intermediate Good Firms

- Measure of incumbents \( \{N_t(j)\}_{j \geq 1} \)
  - \( \{y_{rt}(j)\}_{j \geq 1} \) innovative investment
- Measure of entrants \( N_t(0) \)
  - \( \bar{y}_r(0) \) entry cost parameter
- Transition Law

\[
\{N_{t+1}(j)\}_{j \geq 1} = T \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
\]
Innovation by Intermediate Good Firms

- **Measure of incumbents** \( \{ N_t (j) \}_{j \geq 1} \)
  - \( \{ y_{rt} (j) \}_{j \geq 1} \) innovative investment

- **Measure of entrants** \( N_t(0) \)
  - \( \bar{y}_r(0) \) entry cost parameter

- **Transition Law**
  \[
  \{ N_{t+1}(j) \}_{j \geq 1} = T \left( \{ y_{rt}(j) \}_{j \geq 1}, N_t(0); \{ N_t(j) \}_{j \geq 1} \right)
  \]

- **Functions** \( Z \) and \( T \) give aggregate productivity growth
  \[
  g_{zt} \equiv \log Z_{t+1} - \log Z_t = G \left( \{ y_{rt}(j) \}_{j \geq 1}, N_t(0); \{ N_t(j) \}_{j \geq 1} \right)
  \]

- **Social Depreciation of Innovation Expenditures**
  \[
  G_t^0 = G \left( \{ 0 \}_{j \geq 1}, 0; \{ N_t(j) \}_{j \geq 1} \right)
  \]
Aggregate Innovation Technology

- Aggregate productivity growth

\[ \log Z_{t+1} - \log Z_t = G \left( \{ y_{rt}(j) \}_{j \geq 1}, N_t(0); \{ N_t(j) \}_{j \geq 1} \right) \]

- Research good used as input for innovation by firms

\[ \sum_{j \geq 1} y_{rt}(j)N_t(j) + \bar{y}_r(0)N_t(0) = Y_{rt} = A_{rt}Z_t^{\gamma - 1}L_{rt} \]

- \( A_{rt} \) freely-available scientific knowledge
- \( \gamma \leq 1 \) intertemporal knowledge spillovers
- Price of research good \( P_{rt} \equiv A_{rt}^{-1}W_tZ_t^{1-\gamma} \)
Aggregate Innovation Technology

- Aggregate productivity growth

\[
\log Z_{t+1} - \log Z_t = G \left( \{ y_{rt}(j) \}_{j \geq 1}, N_t(0); \{ N_t(j) \}_{j \geq 1} \right)
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- Labor allocation and innovation intensity of economy

\[
s_{rt} \equiv \frac{P_{rt} Y_{rt}}{Y_t} = \kappa \frac{L_{rt}}{L_{pt}}
\]
Positive implications of model

- Baseline allocations in a BGP: $\bar{g}_z$, $\bar{s}_r$, $\{\bar{y}_{rt}(j)\}$, $\{\bar{N}_t(j)\}$

- At $t = 0$, policy-induced change to $s'_r$, $\{y'_{rt}(j)\}_{j \geq 1}$, $\{N'_t(j)\}_{j \geq 0}$
  - e.g. changes in subsidies to use of research good

- Approximate implied path $\left\{Z'_t\right\}_t^\infty$, $\left\{GDP'_t\right\}_t^\infty$
Two Key Equations

- **Innovation intensity to Research output**

\[
(\log Y_{rt}' - \log \bar{Y}_r) = \bar{L}_p (\log s_{rt}' - \log \bar{s}_r) - (1 - \gamma) (\log Z_t' - \log \bar{Z}_t)
\]
Two Key Equations

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\]

- **Research output to productivity growth**

\[
g'_z - \bar{g}_z \approx E_{gt} \left( \log Y'_{rt} - \log \bar{Y}_rt \right) + \sum_{j \geq 1} \frac{\partial G}{\partial N(j)} \left( N'_t(j) - \bar{N}_t(j) \right).
\]

- **Impact elasticity with respect to change in \( Y_{rt} \)**

\[
E_{gt} = \sum_{j \geq 1} \frac{\partial G}{\partial y_r(j)} \bar{y}_{rt}(j) \frac{d \log y_{rt}(j)}{d \log Y_{rt}} + \frac{\partial G}{\partial N(0)} \bar{N}_t(0) \frac{d \log N_t(0)}{d \log Y_{rt}}
\]

- **Characterize dynamics with three key assumptions**
First Key Assumption: Concavity

- $G$ is concave from the origin
  - If innovation $(\{y_{rt}(j)\}_{j \geq 1}, N_t(0))$ increases proportionally for all firms, then growth rate (in logs) increases less than proportionally
First Key Assumption: Concavity

- $G$ is concave from the origin
  - If innovation ($\{y_{rt}(j)\}_{j \geq 1}, N_t(0)$) increases proportionally for all firms, then growth rate (in logs) increases less than proportionally

- Suppose innovation changes proportionately for all firms:
  
  \[ d \log y_{rt}(j) = d \log N_t(0) = d \log Y_{rt} \]
First Key Assumption: Concavity

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- Suppose innovation changes proportionately for all firms:
  \[
  d \log y_{rt}(j) = d \log N_t(0) = d \log Y_{rt}
  \]

- Then impact elasticity is bounded by
  \[
  \epsilon_{gt} \leq \bar{g}_{zt} - G_t^0
  \]

- Intuition:
  - Going from $Y_r = 0$ to $\bar{Y}_r$ increases growth from $G_t^0$ to $\bar{g}_{zt}$
  - Concavity: marginal increase smaller than average effect
Example 1: Simple Quality Ladders

- Quality Ladders Model.
  - Innovation only by entrants

\[
G = \frac{1}{\rho - 1} \log (\sigma N_t(0) (\exp(\Delta_z)^{\rho^{-1}} - 1) + 1)
\]

- Social Depreciation \( G^0 = 0 \)

- Exact Impact Elasticity

\[
\mathcal{E}_{QL} = \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)} \leq \bar{g}_z
\]

  - approaches \( \bar{g}_z \) as \( \rho \to 1 \)
  - approaches 0 as \( \rho \to \infty \)

- Impact Elasticity tightly bounded if model applied to advanced economies
Example 2: Simple Expanding Varieties

- Expanding Varieties Model
  - Innovation only by entrants
  - Entrants imitate fraction $\lambda$ of $Z_t^{\rho-1}$
  - Exogenous growth $\Delta_z$ and exit $\delta_f$ of incumbents

$$
G = \frac{1}{\rho - 1} \log ((1 - \delta_f) \exp ((\rho - 1) \Delta_z) + \lambda N_t(0))
$$

- $G^0 < 0$ linked to employment share of incumbents

- Exact impact elasticity

$$
\varepsilon_{gt}^{EV} = \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - \exp((\rho - 1)G^0)}{\exp((\rho - 1)\bar{g}_z)} \leq \bar{g}_z - G^0
$$

- Impact elasticity can be several times bigger than in Quality Ladders Model
Second Key Assumption: Conditional Efficiency

- Given $Y_{rt}$, then $\{y_{rt}(j)\}_{j \geq 1}, N_t(0)$ solves

$$
\max G \left( \{y_{rt}(j)\}_{j \geq 1}, N_t(0); \{N_t(j)\}_{j \geq 1} \right)
$$

subject to

$$
\sum_{j \geq 1} y_{rt}(j) N_t(j) + \bar{y}_r(0) N_t(0) = Y_{rt}
$$

- If baseline allocation is conditionally efficient, then $\mathcal{E}_{gt}$ is independent of how the change in the output of the research good is allocated across incumbent and entering firms

$$
\mathcal{E}_{gt} = \lambda_t \bar{Y}_{rt}
$$

- Previous bound holds
Alternative first key assumption: Concavity in Entrants

- $G$ concave with respect to entry
- Alternative bound on impact elasticity

$$
\epsilon_{gt} \leq (\bar{g}_{zt} - G^0_t) \frac{\bar{Y}_{rt}}{\bar{Y}_{rt} - Y_{rt}^0}
$$

- $Y_{rt}^0$ aggregate use of research good when entry low enough to implement growth $G^0_t$
- $Y_{rt}^0 < 0$ (tighter bound) when incumbents have lower average cost of innovation
Alternative first key assumption: Concavity in Entrants

- $G$ concave with respect to entry
- Alternative bound on impact elasticity

$$
\mathcal{C}_{gt} \leq (\bar{g}_{zt} - G_t^0) \frac{\bar{Y}_{rt}}{\bar{Y}_{rt} - Y_{rt}^0}
$$

- $Y_{rt}^0$ aggregate use of research good when entry low enough to implement growth $G_t^0$
- $Y_{rt}^0 < 0$ (tighter bound) when incumbents have lower average cost of innovation

- Paper: illustrate with Klette-Kortum, Atkeson-Burstein models
  - Given $\bar{g}_z$ and $\rho$, $\mathcal{C}_{gt}^{K\&K} \leq \mathcal{C}_{gt}^{QL}$
Example 3: Klette-Kortum

- Quality ladders model
  - Innovation by entrants and incumbents
  - Simple version in which all incumbents invest same per product

\[
G = \frac{1}{\rho - 1} \log \left( \sigma(d(y_{rt}(1)) + N_t(0)) \left( \exp(\Delta_z)\rho^{-1} - 1 \right) + 1 \right)
\]

- Social Depreciation \(G^0 = 0\)
- Conditional efficiency if \(d'(\bar{y}_r(1)) = 1/\bar{y}_r(0)\) – equil: entrants and incumbents subsidized at same rate
- \(d(y_r)\) concave implies
  - Both concavity assumptions satisfied
  - \(Y^0_r < 0\), so \(\frac{\bar{y}_{rt}}{\bar{y}_{rt} - Y^0_t} \leq 1\)

- Exact elasticity

\[
\mathcal{E}^{KK}_{gt} = \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)} \frac{\bar{Y}_r}{\bar{Y}_r - Y^0}
\]

- Given \(\rho\) and \(\bar{g}_z\), \(\mathcal{E}^{KK}_{gt} \leq \mathcal{E}^{QL}_{gt}\)
Results so far

- Innovation intensity to Research output

\[
(\log Y_{rt}^\prime - \log \bar{Y}_r) = \bar{L}_p (\log s_{rt}^\prime - \log \bar{s}_r) - (1 - \gamma) (\log Z_t^\prime - \log \bar{Z}_t)
\]

- Research output to Productivity growth
  
  - Conditional Efficiency implies allocation of innovative investment does not matter

\[
g_{zt}^\prime - \bar{g}_z \approx \mathcal{E}_{gt} (\log Y_{rt}^\prime - \log \bar{Y}_r) + \sum_{j \geq 1} \frac{\partial G}{\partial N(j)} (N_t^\prime(j) - \bar{N}_t(j))
\]

  - Concavity bounds impact elasticity, \( \mathcal{E}_{gt} \leq (\bar{g}_{zt} - G_t^0) \)
    
    - Or even tighter bound if incumbents have lower average cost of innovation

- To characterize dynamics, assume \( \frac{\partial G}{\partial N(j)} = 0 \)

- Key assumption 3 satisfied by our three examples
Putting these results together

\[
\log Z'_{t+1} - \log \bar{Z}_{t+1} = \sum_{k=0}^{t-1} \Gamma_k \left( \log s'_{rt-k} - \log \bar{s}_r \right)
\]

**Impact effect:**  \( \Gamma_0 = \bar{L}_p \epsilon_0 g \)

**Decay:**  \( \Gamma_{k+1} = [1 - (1 - \gamma) \epsilon g_0] \Gamma_k \)

**Endogenous growth:**

\[
\log Z'_{t+1} - \log Z'_t - \bar{g}_z = \Gamma_0 \left( \log s'_{rt} - \log \bar{s}_r \right)
\]

**Long-term impact permanent change in \( s_r \):**  \( \bar{L}_p / (1 - \gamma) \)
GDP dynamics

- Suppose $K/Y$ fixed

$$\log GDP'_t - \log G\bar{D}P_t = \frac{1}{1 - \alpha} (\log Z'_t - \log \bar{Z}_t) - \bar{L}_r (\log s'_r - \log \bar{s}_r)$$

- Tradeoff: GDP $\uparrow$ with productivity, $\downarrow$ with innovation investment
Using these results: numerical example

- Baseline annual growth $\bar{g}_z = 0.0125$
- Social Depreciation $G^0 = 0$
- $\varepsilon_g = 0.0122$ close to upper bound $\bar{g}_z - G^0$
- $\Gamma_0 = \varepsilon_g \bar{L}_p = .01$
- Vary spillover $\gamma = -2, \gamma = 0, \gamma \to 1$
  - $Z$ long-run elasticity ranges from $1/4$ to infinity
- Innovation subsidy raises $s_{rt}$ permanently starting at date $t = 0$ from $11\%$ to $14\%$
  - $\Delta \log s_r = \log s_{rt} - \log \bar{s}_r = 0.24$
- Analytic impulse response (constant $K/Y$).
Aggregate productivity: 100 years
Aggregate productivity: 20 years
GDP: 100 years

GDP (excluding innovation expenditures), relative to BGP, 100 years

- Dashed red line: low spillovers
- Solid black line: medium spillovers
- Dashed blue line: high spillovers

Y-axis: GDP (relative to baseline)
X-axis: Years (0 to 100)

The graph shows the relative GDP over 100 years with different spillover scenarios.
2.25% social depreciation

- Expanding varieties: $\rho = 4$ and 10% employment by entering products
- Impact elasticity $E_g$ increases from 0.0122 to 0.033
- Impact effect $\Gamma_0$ increases from 0.01 to 0.028
2.25% social depreciation: Aggregate Productivity 20 years
2.25% social depreciation: GDP 20 years
Welfare: Optimal Innovation Intensity

- Optimality perturb $\log s'_{r0}$ only, $\Delta$ welfare $= 0$

$$\sum_{k=0}^{\infty} \hat{\beta}^{1+k} \frac{\Gamma_k}{1 - \alpha} - L_r \left(\log s'_0 - \log \bar{s}_r\right) = 0$$

- Gap between BGP interest and growth rates $\hat{\beta}$

- Optimal BGP allocation satisfies

$$s^*_r = (1 - \alpha) \frac{L^*_r}{L^*_p} = \frac{\hat{\beta} \mathcal{E}^*_g}{1 - \hat{\beta} \left[1 - (1 - \gamma) \mathcal{E}^*_g\right]}$$

- Huge range of implications depending on $\hat{\beta}$ and $\gamma$

- If small $\mathcal{E}^*_g$, then disconnect between $s^*_r$ and 20 year response
Example 3: Klette-Kortum Failure of Conditional Efficiency

- Quality Ladders Model
  - Innovation by entrants and incumbents
- Conditional Efficiency $d'(\bar{y}_r(1)) = 1/\bar{y}_r(0)$
- If $d'(\bar{y}_r(1)) \neq 1/\bar{y}_r(0)$ then additional welfare gain
  - can achieve same growth rate $\bar{g}_z$ with permanently lower $Y_r$
  - and higher consumption by reallocating to
    $$d'(\bar{y}_r(1)) = 1/\bar{y}_r(0)$$
  - need to know details of innovation technology $G$ and baseline allocation to implement this gain with industrial policies
  - in numerical examples, can get big effects
Lentz and Mortensen 2014

- Estimated Klette-Kortum model
- Incumbent types have large and small innovation step sizes
  - Equilibrium is not conditionally efficient (assumption 2)
  - Distribution of incumbents impacts growth rate (assumption 3)
- Endogenous growth $\gamma = 1$
- No social depreciation
- Estimated elasticity of productivity growth w.r.t. $Y_r$ moving from equilibrium to social optimum

$$\frac{g^*_z - \bar{g}_z}{\log Y^*_r - \log \bar{Y}_r} = 0.0125 < \bar{g}_z$$

- Smaller elasticity than from baseline quality-ladders model
Conclusion

- Wide class of growth models
  - Simple approximation to transition dynamics
- Two key sufficient statistics:
  - Impact elasticity
    - baseline growth rate
    - social depreciation of innovation expenditures
    - incumbents’ average cost of innovation
  - Intertemporal knowledge spillovers
    - price deflator for research output
- Useful benchmark for evaluating quantitative implications of richer new growth models