Not for publication Appendix for "Understanding Movements in Aggregate and Product-Level Real Exchange Rates" by Ariel Burstein and Nir Jaimovich

Deriving expression (3.3) in Section 3

Expression (3.3) is derived as follow:

$$
\begin{aligned}
& \operatorname{Var}\left(\Delta Q_{nijrr't}\right) \\
= \; & \operatorname{Var}\left(\Delta P_{nirt} - \Delta P_{njr't}\right) \\
= \; & \operatorname{Var}\left(\Delta P_{nirt}\right) + \operatorname{Var}\left(\Delta P_{njr't}\right) - 2\operatorname{Cov}\left(\Delta P_{nirt}, \Delta P_{njr't}\right) \\
= \; & \operatorname{Var}\left(\Delta P_{nirt}\right) + \operatorname{Var}\left(\Delta P_{njr't}\right) - 2\left[\operatorname{Var}\left(\Delta P_{nirt}\right)\right]^{0.5}\left[\operatorname{Var}\left(\Delta P_{njr't}\right)\right]^{0.5}\operatorname{Correl}\left(\Delta P_{nirt}, \Delta P_{njr't}\right) \\
= \; & \left[\operatorname{Var}\left(\Delta P_{nirt}\right) + \operatorname{Var}\left(\Delta P_{njr't}\right)\right]\left[1 - 2\frac{\left[\operatorname{Var}\left(\Delta P_{nirt}\right)\right]^{0.5}\left[\operatorname{Var}\left(\Delta P_{njr't}\right)\right]^{0.5}}{\operatorname{Var}\left(\Delta P_{nirt}\right) + \operatorname{Var}\left(\Delta P_{njr't}\right)}\operatorname{Correl}\left(\Delta P_{nirt}, \Delta P_{njr't}\right)\right]
\end{aligned}
$$

Additional details of proof to Lemma 1

Here we provide analytic expressions in terms of the model's parameters for the sets of matched exported products and measures of latent competitors, defined in Appendix 1 of the paper.

Characterizing sets of matched products Recall that the set of matched products that are supplied in countries 1 and 2 by the same producer located in country 1 is given by:

$$N_{x1} = \left\{ n \in N \text{ s.t. } D\min\left\{\bar{z}_{kn}\right\}_{k=1}^{K_1} \leq \min\left\{\left\{\bar{z}'_{kn}\right\}_{k=1}^{K_1} \cup \left\{\bar{z}_{kn}\right\}_{k=K_1+1}^{K_1+K_2} \cup \left\{D^*\bar{z}_{kn}\right\}_{k=K_1+K_2+1}^{K}\right\}\right\}.$$

The mass of N_{x1} is also equal to the mass of exporters from country 1 to country 2, m_1. Using $\bar{z} = \left(\min\left\{\bar{u}, \bar{u}'\right\}\right)^{\theta}$, and the fact that $\bar{u}'_{kn}$ is never used to export but only for MP, we have

$$m_1 = \operatorname{Prob}\left(D^{1/\theta}\min\left\{\bar{u}_{kn}\right\}_{k=1}^{K_1} \leq \min\left\{\left\{\bar{u}'_{kn}\right\}_{k=1}^{K_1} \cup \left\{\bar{u}_{kn}, \bar{u}'_{kn}\right\}_{k=K_1+1}^{K_1+K_2} \cup D^{*1/\theta} \times \left\{\bar{u}_{kn}, \bar{u}'_{kn}\right\}_{k=K_1+K_2+1}^{K}\right\}\right).$$

Using the assumption that $\bar{u}_{kn}$ and $\bar{u}'_{kn}$ are exponentially distributed,[27] and that producers that are indifferent between exporting or engaging in MP choose to export, m_1 is:

$$m_1 = \begin{cases} \frac{K_1 D^{-1/\theta}}{K_1\left(D^{-1/\theta}+\lambda\right)+K_2(1+\lambda)+K_3 D^{*-1/\theta}(1+\lambda)} & \text{if } D > 1 \\ \frac{K_1(1+\lambda)}{K_1(1+\lambda)+K_2(1+\lambda)+K_3 D^{*-1/\theta}(1+\lambda)} & \text{if } D = 1 \end{cases}.$$

[27] Specifically, we use the following properties of exponential distributions. Suppose $u \sim \exp(\mu)$ and $u' \sim \exp(\lambda)$ are independent, and $d > 0$, then (i) $du \sim \exp(\mu/d)$, (ii) $\min\{u, u'\} \sim \exp(\mu+\lambda)$, and (iii) $\operatorname{Prob}(u \leq u') = \mu/(\mu+\lambda)$.

Analogously, the mass of the set N_{x2} is equal to m_2 and is given by:

$$m_2 = \begin{cases} \frac{K_2 D^{-1/\theta}}{K_1(1+\lambda)+K_2\left(D^{-1/\theta}+\lambda\right)+K_3(1+\lambda)D^{*-1/\theta}} & \text{if } D > 1 \\ \frac{K_2(1+\lambda)}{K_1(1+\lambda)+K_2(1+\lambda)+K_3(1+\lambda)D^{*-1/\theta}} & \text{if } D = 1. \end{cases}$$

The set of matched products that are supplied by country 3 producers in both countries 1 and 2, N_{x3}, is given by:

$$N_{x3} = \left\{ n \in N \text{ s.t. } D^* \min \left\{ \bar{z}_{kn} \right\}_{k=K_1+K_2+1}^{K} \leq \min \left\{ \left\{ \bar{z}_{kn} \right\}_{k=1}^{K_1} \cup \left\{ \bar{z}_{kn} \right\}_{k=K_1+1}^{K_1+K_2} \right\} \right\}.$$

Using a similar logic as above, we have:

$$m_3 = \frac{K_3 (1+\lambda) D^{*-1/\theta}}{K_1 (1+\lambda) + K_2 (1+\lambda) + K_3 (1+\lambda) D^{*-1/\theta}}.$$

Note that products that are exported from country 3 to country 1 are not necessarily exported to country 2 (and viceversa). Even though country 3 producers have the same cost of supplying both countries, country 1 and country 2 producers have different supply costs if $D > 1$. To see this, note that the measures of exporters from country 3 to country 1 and country 2, are given by:

$$m_{31} = \frac{K_3 (1+\lambda) D^{*-1/\theta}}{K_1 (1+\lambda) + K_2 \left(D^{-1/\theta} + \lambda\right) + K_3 (1+\lambda) D^{*-1/\theta}} \text{ , and}$$

$$m_{32} = \frac{K_3 D^{*-1/\theta}}{K_1 \left(D^{-1/\theta} + \lambda\right) + K_2 (1+\lambda) + K_3 (1+\lambda) D^{*-1/\theta}}$$

are both higher or equal than m_3.

Characterizing measures of latent competitors Recall that the mass of country 1 exporters facing a latent competitor from country 1 when selling in country 2, s_{12}^1, is:

$$s_{12}^1 = Pr\left(D \min_2 \left\{ \bar{z}_{kn} \right\}_{k=1}^{K_1} \leq \min \left\{ \left\{ \bar{z}'_{kn} \right\}_{k=1}^{K_1} \cup \left\{ \bar{z}_{kn} \right\}_{k=K_1+1}^{K_1+K_2} \cup D^* \left\{ \bar{z}_{kn} \right\}_{k=K_1+K_2+1}^{K} \right\} \right).$$

We now derive a closed form solution for this expression. To do so, we first introduce the following notation:

$$\begin{aligned} d &= D^{1/\theta} \text{ ; } d^* = (D^*)^{1/\theta} \text{ ; } k_3 = (1+\lambda) K_3 / d^* \\ \bar{u} &\sim \exp(1) \text{ ; } \bar{u}' \sim \exp(\lambda) \\ \bar{v}_1 &= \min \left\{ \bar{u}_k \right\}_{k=1}^{K_1} \text{ ; } \bar{v}_2 = \min \left\{ \bar{u}_k \right\}_{k=K_1+1}^{K_1+K_2} \text{ ; } \bar{v}_3 = \frac{1}{d^*} \min \left\{ \bar{u}_k, \bar{u}'_k \right\}_{k=K_1+K_2+1}^{K} \\ \bar{v}'_1 &= \min \left\{ \bar{u}'_k \right\}_{k=1}^{K_1} \text{ ; } \bar{v}'_2 = \min \left\{ \bar{u}'_k \right\}_{k=K_1+1}^{K_1+K_2} \\ \bar{v}_1^{2nd} &= \min_2 \left\{ \bar{u}_k \right\}_{k=1}^{K_1} \text{ ; } \bar{v}_2^{2nd} = \min_2 \left\{ \bar{u}_k \right\}_{k=K_1+1}^{K_1+K_2} \end{aligned}$$

Define a random variable $\tilde{w} \sim \exp\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)$. Then,

$$\begin{aligned} s_{12}^1 &= \Pr\left(d\bar{v}_1 \text{ and } d\bar{v}_1^{2nd} \ < \min\left\{\bar{v}_1', \bar{v}_2, \bar{v}_2', \bar{v}_3\right\}\right) = \Pr\left\{\bar{v}_1 < \tilde{w}/d\ ,\ \bar{v}_1^{2nd} < \tilde{w}/d\ \right\} \\ &= \left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right) \times \\ &\quad \int_0^\infty \Pr\left\{\bar{v}_1 < \tilde{w}/d\ ,\ \bar{v}_1^{2nd} < \tilde{w}/d\ \right\} \exp\left(-\tilde{w}\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)\right) \mathrm{d}\tilde{w} \\ &= 1 - \frac{\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)}{K_1/d + K_2 + k_3 + \lambda\left(K_1 + K_2\right)} \\ &\quad - \frac{K_1\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)}{\left(K_1 - 1\right)/d + K_2 + k_3 + \lambda\left(K_1 + K_2\right)} + \frac{K_1\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)}{K_1/d + K_2 + k_3 + \lambda\left(K_1 + K_2\right)}. \end{aligned}$$

The mass of country 1 exporters facing a latent competitor from country 2 when selling in country 1, s_{11}^2, is:

$$s_{11}^2 = Pr\left(\begin{array}{c} D\min\left\{\bar{z}_{kn}\right\}_{k=1}^{K_1} \ \le \min\left\{\left\{\bar{z}_{kn}'\right\}_{k=1}^{K_1} \cup \left\{\bar{z}_{kn}\right\}_{k=K_1+1}^{K_1+K_2} \cup \left\{D^*\bar{z}_{kn}\right\}_{k=K_1+K_2+1}^{K}\right\} \\ \&\ D\min\left\{\bar{z}_{kn}\right\}_{k=K_1+1}^{K_1+K_2} \le \min\left\{\min_2\left\{\bar{z}_{kn}\right\}_{k=1}^{K_1} \cup \left\{\bar{z}_{kn}'\right\}_{k=K_1+1}^{K_1+K_2} \cup \left\{D^*\bar{z}_{kn}\right\}_{k=K_1+K_2+1}^{K}\right\} \end{array}\right).$$

To provide an analytic expression for s_{11}^2, define $\tilde{w} \sim \exp\left(k_3 + \lambda\left(K_1 + K_2\right)\right)$. Then,

$$\begin{aligned} s_{11}^2 &= \Pr\left\{\bar{v}_1 < \bar{v}_2/d\ ,\ \bar{v}_2 < \tilde{w}/d,\ \bar{v}_1^{2nd} > d\bar{v}_2\ \right\} \\ &= K_1K_2\left(k_3 + \lambda\left(K_1 + K_2\right)\right)\ \times \\ &\quad \int_0^\infty \int_0^{\tilde{w}/d} \left(1 - \exp\left(-\bar{v}_2/d\right)\right) \exp\left(-\bar{v}_2 d\left(K_1 - 1\right) - \bar{v}_2 K_2\right) \mathrm{d}\bar{v}_2 \exp\left(-\tilde{w}\left(k_3 + \lambda\left(K_1 + K_2\right)\right)\right) \mathrm{d}\tilde{w} \\ &= K_1K_2\left(k_3 + \lambda\left(K_1 + K_2\right)\right)\ \times \\ &\quad \left[\begin{array}{c} \frac{1}{((K_1-1)d+K_2)(k_3+\lambda(K_1+K_2))} - \frac{1}{((K_1-1)d+K_2)((K_1-1)+K_2/d+k_3+\lambda(K_1+K_2))} \\ -\frac{1}{(1/d+(K_1-1)d+K_2)(k_3+\lambda(K_1+K_2))} + \frac{1}{(1/d+(K_1-1)d+K_2)(1/d^2+K_1-1+K_2/d+k_3+\lambda(K_1+K_2))} \end{array}\right] \end{aligned}$$

The mass of country 1 exporters facing the same latent competitor from country 3 when selling in countries 1 and 2 is:

$$s_1^3 = \Pr\left(\begin{array}{c} D\min\left\{\bar{z}_{kn}\right\}_{k=1}^{K_1} \ \le \min\left\{\left\{\bar{z}_{kn}'\right\}_{k=1}^{K_1} \cup \left\{\bar{z}_{kn}\right\}_{k=K_1+1}^{K_1+K_2} \cup \left\{D^*\bar{z}_{kn}\right\}_{k=K_1+K_2+1}^{K}\right\} \\ \&\ \min\left\{D^*\bar{z}_{kn}\right\}_{k=K_1+K_2+1}^{K} \le \min\left\{\min_2\left\{\bar{z}_{kn}\right\}_{k=1}^{K_1} \cup \left\{\bar{z}_{kn}\right\}_{k=K_1+1}^{K_1+K_2}\right\} \end{array}\right)$$

Define the random variable $\tilde{w} \sim \exp\left(K_2 + \lambda\left(K_1 + K_2\right)\right)$. Then,

$$\begin{aligned} s_1^3 &= \Pr\left\{\bar{v}_3 < \tilde{w},\ \bar{v}_1 < \bar{v}_3/d,\ \bar{v}_1^{2nd} > \bar{v}_3\ \right\} \\ &= K_1\left(K_2 + \lambda\left(K_1 + K_2\right)\right) k_3 \times \\ &\quad \int_0^\infty \int_0^{\tilde{w}} \left(1 - \exp\left(-\bar{v}_3/d\right)\right) \exp\left(-\bar{v}_3\left(K_1 - 1\right)\right) \exp\left(-\bar{v}_3 k_3\right) \exp\left(-\tilde{w}\left(K_2 + \lambda\left(K_1 + K_2\right)\right)\right) \mathrm{d}\bar{v}_3 \mathrm{d}\tilde{w} \\ &= K_1\left(K_2 + \lambda\left(K_1 + K_2\right)\right) k_3 \times \\ &\quad \left[\begin{array}{c} \frac{1}{(K_2+\lambda(K_1+K_2))(K_1-1+k_3)} - \frac{1}{(K_1-1+k_3)(K_2+\lambda(K_1+K_2)+K_1-1+k_3)} \\ -\frac{1}{(K_2+\lambda(K_1+K_2))(1/d+K_1-1+k_3)} + \frac{1}{(1/d+K_1-1+k_3)(K_2+\lambda(K_1+K_2)+1/d+K_1-1+k_3)} \end{array}\right]. \end{aligned}$$

The mass of country 1 exporters facing a latent competitor from country 3 when selling in country 2 is:

$$s_{12}^3 = \Pr \left(\begin{array}{c} D \min \{\bar{z}_{kn}\}_{k=1}^{K_1} \leq \min \left\{ \{\bar{z}'_{kn}\}_{k=1}^{K_1} \cup \{\bar{z}_{kn}\}_{k=K_1+1}^{K_1+K_2} \cup D^* \{\bar{z}_{kn}\}_{k=K_1+K_2+1}^{K} \right\} \\ \& \ \min \{D^* \bar{z}_{kn}\}_{k=K_1+K_2+1}^{K} \leq \min \left\{ D \min_2 \{\bar{z}_{kn}\}_{k=1}^{K_1} \cup \{\bar{z}'_{kn}\}_{k=1}^{K_1} \cup \{\bar{z}_{kn}\}_{k=K_1+1}^{K_1+K_2} \right\} \end{array} \right).$$

Define the random variable $\tilde{w} \sim \exp(K_2 + \lambda(K_1 + K_2))$. Then,

$$\begin{aligned} s_{12}^3 &= \Pr \left\{ \bar{v}_3 < \tilde{w}, \, \bar{v}_1 < \bar{v}_3/d \, , \, \bar{v}_1^{2nd} > \bar{v}_3/d \, \right\} \\ &= K_1 (K_2 + \lambda (K_1 + K_2)) \, k_3 \ \times \int_0^\infty \\ & \int_0^{\tilde{w}} (1 - \exp(-\bar{v}_3/d)) \exp(-\bar{v}_3/d \, (K_1 - 1)) \exp(-\bar{v}_3 k_3) \, \mathrm{d}\bar{v}_3 \exp(-\tilde{w} (K_2 + \lambda (K_1 + K_2))) \, \mathrm{d}\tilde{w} \\ &= K_1 (K_2 + \lambda (K_1 + K_2)) \, k_3 \times \\ & \left[\begin{array}{c} \frac{1}{((K_1-1)/d+k_3)((K_2+\lambda(K_1+K_2)))} - \frac{1}{((K_1-1)/d+k_3)(K_2+\lambda(K_1+K_2)+(K_1-1)/d+k_3)} \\ - \frac{1}{(K_1/d+k_3)(K_2+\lambda(K_1+K_2))} + - \frac{1}{(K_1/d+k_3)(K_2+\lambda(K_1+K_2)+K_1/d+k_3)} \end{array} \right]. \end{aligned}$$

Finally, the mass of country 1 exporters facing a latent competitor from country 3 when selling in country 1 is:

$$s_{11}^3 = \Pr \left(\begin{array}{c} D \min \{\bar{z}_{kn}\}_{k=1}^{K_1} \leq \min \left\{ \{\bar{z}'_{kn}\}_{k=1}^{K_1} \cup \{\bar{z}_{kn}\}_{k=K_1+1}^{K_1+K_2} \cup \{D^* \bar{z}_{kn}\}_{k=K_1+K_2+1}^{K} \right\} \\ \& \ \min \{D^* \bar{z}_{kn}\}_{k=K_1+K_2+1}^{K} \leq \min \left\{ \min_2 \{\bar{z}_{kn}\}_{k=1}^{K_1} \cup D \{\bar{z}_{kn}\}_{k=K_1+1}^{K_1+K_2} \cup \{\bar{z}'_{kn}\}_{k=K_1+1}^{K_1+K_2} \right\} \end{array} \right).$$

Define the random variable $\tilde{w} \sim \exp(\lambda(K_1 + K_2))$. Then,

$$\begin{aligned} s_{11}^3 &= \Pr \left\{ \bar{v}_1 < \bar{v}_2/d \, , \, \bar{v}_1 < \bar{v}_3/d, \, \bar{v}_1 < \tilde{w}/d, \, \bar{v}_3 < d\bar{v}_2, \, \bar{v}_3 < \tilde{w}, \, \bar{v}_1^{2nd} > \bar{v}_3 \, \right\} \\ &= \Pr \left\{ \bar{v}_3 < \bar{v}_2, \, \bar{v}_3 < \tilde{w}, \, \bar{v}_1 < \bar{v}_3/d, \, \bar{v}_1^{2nd} > \bar{v}_3 \, \right\} \\ & \quad + \Pr \left\{ \bar{v}_2 < \bar{v}_3 < d\bar{v}_2 \, , \, \bar{v}_3 < \tilde{w}, \, \bar{v}_1 < \bar{v}_2/d \, , \, \bar{v}_1^{2nd} > \bar{v}_3 \, \right\} \\ &= s_1^3 + \Pr \left\{ \bar{v}_2 < \bar{v}_3 < d\bar{v}_2 \, , \, \bar{v}_3 < \tilde{w}, \, \bar{v}_1 < \bar{v}_2/d \, , \, \bar{v}_1^{2nd} > \bar{v}_3 \, \right\} \end{aligned}$$

so using the definition of s_1^3,

$$\begin{aligned} s_{11}^3 &= s_1^3 + \Pr \left\{ \bar{v}_2 < \bar{v}_3 < d\bar{v}_2 \, , \, \bar{v}_3 < \tilde{w}, \, \bar{v}_1 < \bar{v}_2/d \, , \, \bar{v}_1^{2nd} > \bar{v}_3 \, \right\} \\ &= s_1^3 + K_2 k_3 \times \\ & \quad \int_0^\infty \int_{\bar{v}_2}^{d\bar{v}_2} K_1 (1 - \exp(-\bar{v}_2/d)) \exp(-\bar{v}_3 (K_1 - 1)) \ \times \\ & \quad \exp(-\bar{v}_3 \lambda (K_1 + K_2)) \exp(-\bar{v}_3 k_3) \exp(-\bar{v}_2 K_2) \, \mathrm{d}\bar{v}_3 \mathrm{d}\bar{v}_2 \\ &= s_1^3 + \frac{K_1 K_2 k_3}{\bar{K}} \left[\frac{1}{K_2 + \bar{K}} - \frac{1}{K_2 + d\bar{K}} - \frac{1}{1/d + K_2 + \bar{K}} + \frac{1}{1/d + K_2 + d\bar{K}} \right]. \end{aligned}$$

where $\bar{K} = K_1 - 1 + \lambda(K_1 + K_2) + k_3$. Note that $s_1^3 \leq s_{11}^3$ and $s_1^3 \leq s_{12}^3$. We obtain symmetric expressions for the shares of latent competitors from each country faced by country 2 exporters.

Additional details of proof to Lemma 2

Here we prove that r_i, the fraction of exporters from country i facing the same latent competitor when selling in countries 1 and 2, is decreasing in D, for $i = 1$ and 2. We focus on r_1, but given that the expressions for r_2 are symmetric, the same proof applies, and a similar logic applies for r_3. Recall that $r_1 = \left(s_{12}^1 + s_{11}^2 + s_1^3\right)/m_1$. We will show that s_{12}^1/m_1 , s_{11}^2/m_1, and s_1^3/m_1 are decreasing in d, where $d = D^{1/\theta}$. Consider first the term s_{12}^1/m_1. Using the expressions above, we have

$$\begin{aligned}
\frac{s_{12}^1}{m_1} &= 1 - \frac{d\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)\left(K_1\left(1/d + \lambda\right) + K_2\left(1 + \lambda\right) + k_3\right)}{\left(K_1 - 1\right)/d + K_2 + k_3 + \lambda\left(K_1 + K_2\right)} \\
&\quad + \frac{d\left(K_2 + k_3 + \lambda\left(K_1 + K_2\right)\right)\left(K_1\left(1/d + \lambda\right) + K_2\left(1 + \lambda\right) + k_3\right)}{K_1/d + K_2 + k_3 + \lambda\left(K_1 + K_2\right)} \\
&= 1 - \frac{d\left(K_2 + k_3 + \lambda K_1 + \lambda K_2\right)}{K_1 - 1 + d\left(K_2 + k_3 + \lambda K_1 + \lambda K_2\right)},
\end{aligned}$$

which is decreasing in d. Consider now the second term, s_{11}^2/m_1. Defining $k = \left(K_1 + K_2\right)\lambda$, we have from the expressions above that

$$\frac{s_{11}^2}{m_1} = d\left(K_1/d + K_2 + k_3 + k\right)\left[\begin{array}{c} \frac{1}{((K_1-1)d+K_2)(k_3+k)} - \frac{1}{((K_1-1)d+K_2)(K_1-1+K_2/d+k_3+k)} \\ -\frac{1}{(1/d+(K_1-1)d+K_2)(k_3+k)} + \frac{1}{(1/d+(K_1-1)d+K_2)(1/d^2+K_1-1+K_2/d+k_3+k)} \end{array}\right].$$

After some tedious algebra, one can show that $\frac{s_{11}^2}{m_1}$ is decreasing in d. Consider now the third term, s_1^3/m_1. Using the expressions above, we have

$$\frac{s_1^3}{m_1} = k_3 \frac{K_1 + d\left(K_2 + k_3 + \lambda K_1 + \lambda K_2\right)}{\left(K_1 + K_2 + k_3 + \lambda K_1 + \lambda K_2 - 1\right)\left(1 + d\left(K_1 - 1\right) + d\left(K_2 + k_3 + \lambda K_1 + \lambda K_2\right)\right)}.$$

If the term g is decreasing in d, where

$$g = \frac{K_1 + dK_2 + dk_3 + d\lambda\left(K_1 + K_2\right)}{K_1 + K_2 + k_3 + \lambda\left(K_1 + K_2\right) - 1},$$

then s_1^3/m_1 is decreasing in d. Let's define $x = \left(K_2 + k_3 + \lambda K_1 + \lambda K_2\right)$, so

$$g = \frac{K_1 + dx}{1 + d\left(K_1 - 1\right) + dx}$$

Then,

$$\begin{aligned}
\frac{\partial g}{\partial d} &= \frac{x\left[1 + d\left(K_1 - 1\right) + dx\right] - \left(K_1 + dx\right)\left(K_1 - 1 + x\right)}{\left[1 + d\left(K_1 - 1\right) + dx\right]^2} \\
&= \frac{-x\left(K_1 - 1\right) - K_1\left(K_1 - 1\right)}{\left[1 + d\left(K_1 - 1\right) + dx\right]^2} < 0.
\end{aligned}$$

So, the third term is decreasing in d.