Measured Aggregate Gains from International Trade

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Abstract

We examine the implications of workhorse trade models for how aggregate productivity, real GDP and real consumption, as measured by statistical agencies, respond to changes in trade costs. In a range of models, changes in measured productivity are equal to the inverse of an export-share weighted average of changes in variable trade costs incurred domestically. Under certain conditions, despite the multiple biases in the CPI, measured real consumption captures the first-order effects of changes in variable trade costs on welfare. Through the lens of these results, we interpret some of the empirical work on measured gains from trade.

There is a large literature that uses structural models to evaluate welfare and productivity gains from reductions in trade costs (see e.g. Costinot and Rodriguez-Clare 2014, Melitz and Redding 2014, and references therein). There is also an extensive empirical literature estimating the link between trade and aggregate measures of economic activity such as real GDP, aggregate productivity, and real consumption (see e.g. Pavcnik 2002, Romer and Frankel 1999, Feyrer 2009, and Broda and Weinstein 2006). The connection between theoretical and measured gains from trade is not immediate. For example, it is well known that the consumer price index (and hence real consumption) does not reflect changes in the number, composition, and quality of available varieties (see e.g. Boskin Comission 1996 and Hausman 2003), which constitute an important source of welfare gains in recent trade models. Similarly, as we discuss below, GDP deflators (and hence aggregate productivity) may not reflect the reallocation of production across heterogeneous producers emphasized in these models. In this paper we ask whether the gains

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from reductions in trade costs implied by workhorse trade models are captured in aggregate measures of economic activity as calculated by statistical agencies.

We base our analysis first on neoclassical trade models with fairly general production technologies in which the quality and set of consumed goods is constant. We then consider models with monopolistic competition and constant markups,[1] endogenous entry and exit, and endogenous product quality or productivity (including the models in Krugman 1980, Melitz 2003, and Arkolakis, Costinot and Rodriguez-Clare 2012). We calculate our models' implications for how changes in iceberg variable trade costs and tariffs impact real GDP, aggregate productivity, and real consumption, when these are calculated following as closely as possible the procedures used in the United States National Income and Product Accounts (NIPA). For many industries and components of GDP, comprehensive measures of physical quantities are difficult to obtain in practice. In such cases, real quantities are typically calculated by deflating current dollar measures of output or consumption with price indices, e.g. in most cases the producer price index (PPI) for output and the consumer price index (CPI) for consumption.

We provide three key theoretical results in this context. First, in response to reductions in trade costs, measured aggregate productivity increases insofar as prices used to construct output deflators reflect these changes in trade costs. Conditional on an export-share (for continuing goods) weighted average of changes in variable trade costs incurred domestically, changes in aggregate productivity are equal in models with and without firm heterogeneity and with or without endogenous exit and export participation.

Second, under a number of widely-used assumptions, consumption deflators as measured in the data capture the first-order effects of changes in variable trade costs on welfare-based price indices. Specifically, building on results in Atkeson and Burstein (2010) we show that, up to a first order approximation, the multiple biases in the CPI arising from endogenous changes in quality and the number of varieties cancel-out at the world level or, under stronger assumptions (similar to those in Arkolakis, Costinot and Rodriguez-Clare 2012), country-by-country. Under these conditions, measured real consumption is a good approximation of welfare-based consumption.

Third, we relate changes in real GDP to changes in real consumption and welfare. In an open economy, output and consumption deflators can differ due to movements in the price of exports relative to the price of imports (the terms of trade). Hence, even if trade is balanced and there are no other sources of final demand, changes in real GDP need not be equal to changes in real consumption (or welfare). The world as a whole, however, is

[1] We discuss briefly extensions with variable markups (e.g. Arkolakis et al. 2012 and Edmond, Midrigan and Xu 2012).

a closed economy: terms of trade improvements in one country are associated to terms of trade worsenings in another country. This implies that changes in GDP deflators at the world level (e.g. a weighted average of changes in deflators of individual countries) are equal, up to a first-order approximation, to changes in consumption deflators (and hence welfare-based price indices, under the assumptions of the second result).

Based on these theoretical results, we consider three quantitative applications using a two-country calibrated Melitz-style model with endogenous quality. First, we assess numerically the equivalence between real consumption and welfare, country-by-country, in response to large reductions in variable trade costs that double the volume of trade. We show that the trade-related biases in the CPI discussed above are small in comparison to the substitution bias. Fisher-type measures of real consumption, which reduce the substitution bias by using information on initial and final expenditure shares, are a good proxy for welfare following uniform reductions in variable trade costs.

Second, we calculate changes in real consumption using price indices that adjust for changes in quality and varieties. Similar adjustments have been applied to import price indices, given data availability, by e.g. Feenstra (1994), Broda and Weinstein (2006), and Feenstra and Romalis (2012). We show, using data generated by our model, that adjusting import price indices for changes in quality and variety without simultaneously adjusting domestic price indices may result in a significant overstatement of the welfare gains from uniform reductions in variable trade costs.

Finally, we consider a standard productivity decomposition (similar to those used in Doms and Bartelsman 2000, Foster, Haltiwanger and Krizan 2001, among many others) to quantify the role of factor reallocation in aggregate productivity. A consistent finding in the empirical literature is that factor reallocation from small, unproductive producers to large, productive producers contributes significantly to the observed rise in aggregate productivity in episodes of trade liberalization (see e.g. Bernard and Bradford Jensen 1999 for the U.S., Trefler 2004 for Canada, Pavcnik 2002 for Chile). We apply this decomposition to data generated by our model under alternative specifications (in terms of the extent of firm heterogeneity, endogenous entry and exit, and endogenous quality), given initial trade shares and changes in variable trade costs. We show that, while alternative specifications generate very different degrees of factor reallocation across producers, the increase in aggregate productivity is roughly invariant across models. Hence, through the lens of these models, one cannot conclude from the large degree of factor reallocation observed in the trade liberalization data that the rise in aggregate productivity would have been smaller absent this factor reallocation.

Our paper is related to Kehoe et al. (2008), who ask whether the increase in welfare following a trade liberalization translates into an increase in real GDP as measured in NIPA. They conclude, as summarized in Kehoe and Ruhl (2010), that "...standard trade models do not imply that opening to trade increases productivity or real GDP, but that it increases welfare". The central differences of our theoretical results relative to those in Kehoe et al. (2008) are as follows. First, Kehoe et al. (2008) focus on cases in which productivity is unchanged to trade liberalization, either because trade costs are incurred abroad or because countries are in autarky before trade liberalization -in which case price indices of exported goods, as measured by the Bureau of Labor Statistics (BLS), do not reflect changes in trade costs since there are no continuing exported goods. We show, however, that starting with positive trade levels, any reduction in trade costs that shifts the domestic production possibility set and is reflected in price deflators does result in an increase in aggregate productivity.[2] Second we study the implications of reductions in trade costs for measured real consumption, providing conditions under which measured gains from trade equal welfare gains from trade.

The paper is organized as follows. Section I overviews the basic measurement procedures that we apply in our models. Section II uses a neoclassical trade model with a fixed set of consumed goods to establish some basic results on how measured productivity, real GDP and real consumption respond to changes in trade costs. Section III extends these results to a Melitz-style model in which the quality and set of goods consumed is endogenous. Section IV considers three quantitative applications using a calibrated version of our model. Section V concludes.

I Aggregate measurement: overview

In this section we provide a brief overview of the procedures that we use to calculate changes in aggregate quantities in the models below. For expositional purposes, we defer to the following sections (in the context of our specific models) some of the formulas for the measures we discuss.

We follow as closely as possible the procedures outlined by the Bureau of Economic Analysis in the United States to construct the NIPA.[3] These procedures are broadly consistent with the recommendations by the United Nations in their System of National Ac-

[2] The result in the case in which trade costs are incurred abroad is related to the result in Kohli (2004) and Kehoe and Ruhl (2008) that in the absence of other distortions the value of production at constant prices does not respond, up to a first-order approximation, to changes in the terms of trade that leave the domestic production possibility frontier unchanged. Feenstra et al. (2013), Gopinath and Neiman (2011), Kehoe and Ruhl (2008) and Kim (2014) show how, in the presence of other distortions, changes in trade costs or in the terms of trade can result in changes in measured aggregate productivity.

[3] See, e.g. Concepts and Methods of the U.S. National Income and Product Accounts (2012).

counts. Note that, to the extent that the measurement procedures in individual countries differ from those carried out in the United States and recommended by the United Nations, our results should be treated with caution.

Current-dollar GDP in period t is given by $GDP_t = \sum_k p_t^k q_t^k$, where super-script k denotes each detailed component of GDP (e.g. industries, sectors, or groups of narrowly defined goods), and p_t^k and q_t^k denote prices and quantities, respectively, of each detailed component of GDP. To calculate real measures of output (such as real GDP) or expenditures (such as real consumption) we use a Fisher index, which is a geometric average of a Laspeyres and a Paasche quantity index. For example, real GDP in period t relative to period $t-1$ is given by

$$\frac{RGDP_t}{RGDP_{t-1}} = \left(\frac{\sum_k p_{t-1}^k q_t^k}{\sum_k p_{t-1}^k q_{t-1}^k} \right)^{0.5} \times \left(\frac{\sum_k p_t^k q_t^k}{\sum_k p_t^k q_{t-1}^k} \right)^{0.5}. \tag{1}$$

The terms $p_{t-1}^k q_t^k$ and $p_t^k q_{t-1}^k$ represent "real" quantities of any given GDP component evaluated at constant prices . The first term in expression (1) is a Laspeyres quantity index (based on $t-1$ prices), while the second term is a Paasche quantity index (based on t prices). Real GDP in period T relative to period 0 is given by

$$\frac{RGDP_T}{RGDP_0} = \prod_{t=1}^{T} \frac{RGDP_t}{RGDP_{t-1}}. \tag{2}$$

While estimates of the current-dollar value of production, $p_t^k q_t^k$, are typically available for each individual component of GDP, data on physical quantities, q_t^k, are often not. For those components of GDP for which data on physical output are available, real quantities are computed using either the direct valuation method (sum of quantities evaluated at constant prices) or the quantity extrapolation method (using a variety of quantity indicators). For those components of GDP for which estimates of physical quantities are not available, real quantities are estimated using the deflation method, dividing current-dollar values by appropriate price indices.[4] In particular, for any component of GDP,

[4]The direct valuation method is used, for example, to calculate real output of autos and light trucks, while quantity extrapolation is used to calculate real output of housing and utilities services. The majority of the other sub-components of GDP are calculated using the deflation method since physical output is not recorded across producers (see Summary of NIPA Methodologies, 2009, page 12, for a description of the method used to estimate each subcomponent of GDP). In our model below, we consider two extreme cases for how goods are grouped into components of GDP. In our baseline results, we interpret our model as that of a representative sector or industry. We aggregate production by all producers to all destinations into a single component and apply the deflation method. In the other extreme we treat each individual good and destination as a separate component, and apply the direct valuation method. Up to a first order approximation, both procedures give the same results.

$p^k_{t-1}q^k_t = \left(p^k_t q^k_t\right) / \left(\mathcal{P}^k_t / \mathcal{P}^k_{t-1}\right)$ and $p^k_t q^k_{t-1} = \left(p^k_{t-1}q^k_{t-1}\right) \times \left(\mathcal{P}^k_t / \mathcal{P}^k_{t-1}\right)$, where $\mathcal{P}^k_t / \mathcal{P}^k_{t-1}$ denotes the change in the price index of component k between periods $t-1$ and t.

To calculate real GDP from the production side using the deflation method, we deflate the current-dollar value added of production using the producer price index (PPI).[5] To construct the PPI, the BLS collects prices for a sample of items that can be priced consistently through time. Price indices are then constructed by averaging price changes of individual items weighted by the the value of production in some base period. In particular, the change in the PPI between two time periods is a fixed-weighted average of price changes across goods and services that are produced (in both time periods) domestically to sell at home or to export abroad (we present explicit formulas in the next sections after introducing the full notation).[6] Export prices in the PPI and in the export price index (EPI) are typically measured at fob (i.e. free-on-board) values, and hence exclude any shipping services incurred abroad. In practice, for a small number of goods in the PPI (and in the consumer price index), the BLS uses a number of methods to incorporate quality changes into observed price changes.[7] In Section III, we consider two extreme cases: either all or none of the goods' prices are adjusted for quality changes.

A critical assumption determining the impact of changes in trade costs on measured real GDP is whether changes in trade costs are at least partially reflected in measured prices in the PPI. Trade costs include not only shipping services, but also any production, marketing, regulatory, and information costs that apply differentially to exported products.[8] In our baseline specification, we assume that the activities required to sell goods abroad are performed in the home country, and hence changes in the variable component of these trade costs are reflected in the home PPI. In an alternative specification, we assume that all export costs are incurred abroad, in which case changes in trade costs are not reflected in the PPI. Alternatively, this second specification can reflect the case in which trade related activities are performed in the home country but they are not measured in prices in the PPI.

We also calculate GDP from the expenditure side, defined as current-dollar expen-

[5]This is the procedure used in the "GDP by Industry Accounts" published by the Bureau of Economic Analysis (BEA). If intermediate inputs are used in production, real value added is calculated using the double deflation method. This consists of first deflating gross output and inputs separately (using their respective PPIs), and then computing real value added as the difference between real gross output and real intermediate inputs.

[6]For more details on the construction of producer price indices and international price indices in the US, see Chapters 14 and 15 of the BLS Handbook of Methods (2008).

[7]See the section on "Product change and quality adjustment" in Chapter 14 of the BLS Handbook of Methods (2008).

[8]Anderson and van Wincoop (2004) argue that these additional costs are at least as important as transportation costs.

ditures (which in our model is equal to current-dollar consumption), plus exports less imports. Real consumption is calculated analogously to real GDP (using expressions 1 and 2), but deflating each component of nominal consumption (when physical quantities are not available) by its consumer price index (CPI) instead of the PPI. The CPI is a fixed-weighted average of consumer price changes of domestic and imported goods consumed in both time periods. In the presence of import tariffs, current-dollar GDP from the expenditure side (defined as the sum of final expenditures including tariffs) is not equal to current-dollar GDP from the production side (defined as the sum of firm value added excluding tariffs). In order to reconcile estimates of GDP from the production and expenditure sides, the BEA adds import taxes to factor payments when computing value added by industry.[9] In deflating consumption expenditures, the CPI is constructed using prices inclusive of tariffs. In deflating imports, the import price index (IPI) is constructed using prices exclusive of import tariffs.

Finally, we measure aggregate productivity as the Solow residual. In particular, the logarithmic change in aggregate productivity between $t-1$ and t is given by:

$$\Delta \log A_t = \Delta \log RGDP_t - \alpha \Delta \log X_t, \tag{3}$$

where X_t denotes the aggregate supply of production factors at time t and α denotes the share of payments to these factors in GDP. In the case in which there is more than one production factor, α and X_t are given by vectors, and $\alpha \Delta \log X_t$ is a scalar product.

II General production technologies with perfect competition

In this section we calculate changes in aggregate productivity and real consumption, as defined in Section I, following reductions in trade costs and tariffs in a class of models with perfect competition and fairly general production technologies.

The world economy is composed of I countries (denoted by i), each of which is endowed with a continuum of production technologies, indexed by $z \in \Omega_i$.[10] A technology z produces output using labor, capital, and other factors that we do not specify.[11] Feasible

[9] In particular, in the "GDP by Industry Accounts" computed by the BEA, value added is defined as the sum of: "Compensation of employees", "Taxes on production and imports less subsidies" and "Gross operating surplus".

[10] In this section, in an abuse of notation, we sometimes refer to technology z as a good or as a producer. In Section III we associate each technology z to a product produced by an individual producer.

[11] To simplify the presentation, in the body of the paper we abstract from the use of intermediate goods

combinations of output, $y_{it}(z)$, and inputs, $x_{it}(z)$ (possibly a vector) are represented by a set $Y_i(z)$, which is unchanged over time. The aggregate supply of inputs in country i at time t is $X_{it} = \int_{\Omega_i} x_{it}(z)\,\mathrm{d}z$.

Goods can be traded internationally subject to standard iceberg trade costs. Specifically, this technology (which we refer to as *trading technology*) transforms one unit of output produced in country i into $1/\tau_{int}$ units available in country n, with $\tau_{iit} = 1$ and $\tau_{int} \geq 1$. Hence, if $y_{int}(z)$ units of output are produced in country i bound to destination n, only $q_{int}(z) = y_{int}(z)/\tau_{int}$ are available to consume in country n. Total output produced in country i is equal to the sum of output bound to all destinations, $y_{it}(z) = \sum_n y_{int}(z)$. We consider separately the case in which for all goods produced and exported by country i, the trading technology is operated in country i (either by the producer or by some intermediary) and the case in which this technology is operated by an intermediary in some other country $j \neq i$.

Utility of the representative consumer is defined over theoretical (welfare-based) consumption, C_{nt}, which is an aggregate over individual goods consumed domestically given by $C_{nt} = C\left(\{q_{int}(z)\}_{i,z}\right)$.[12] For our results on real consumption, we assume that $C(.)$ is homogenous of degree one and fixed over time. We relax the latter assumption in Section III when we allow for endogenous changes in quality.

World prices are all expressed in a common currency, which we refer to as dollars. We let $p_{int}(z)$ be the price in country n, exclusive of import tariffs, of a good produced in country i using technology z. Denoting by $d_{int} \geq 1$ the gross tariff set by country n at time t for imports from country i (with $d_{iit} = 1$), final prices in country n, inclusive of tariffs, are $d_{int} p_{int}(z)$. Tariff revenues are rebated back to consumers in each country.

Under standard assumptions on the production set $Y_i(z)$, the sum of profits obtained by each profit-maximizing production unit that takes prices as given is the same as the sum of profits that would be obtained if, given these prices, they were to coordinate their actions in a joint profit maximization decision (Proposition 5.E.1 in Mas-Colell, Whinston and Green 1995). Therefore, given prices, the equilibrium allocations at any time t are

(domestic or imported) in production. In the online Appendix we extend our results to allow for intermediate inputs, which requires additional notation. In the absence of tariffs, the results on aggregate productivity are unchanged relative to our baseline model. Tariffs induce distortions in production that generate first-order effects in aggregate productivity from changes in the use of intermediate inputs, proportional to the level of tariffs (see Kehoe and Ruhl 2008 and Kim 2014).

[12] Our results in this section cover cases with variable labor supply in which leisure is a component of utility. Since leisure is typically not measured in NIPA, we focus on comparing measured consumption and theoretical consumption. See Jones and Klenow (2010) for an analysis of the inclusion of leisure in aggregate measures of economic activity.

those that maximize aggregate profits, Π_{it}, defined as

$$\Pi_{it} = \max_{\{y_{in}(z), x_i(z)\}} \int_{\Omega_i} \sum_n p_{int}(z) y_{in}(z) / \tau_{int} - W_{it} x_i(z) \, \mathrm{d}z \tag{4}$$
$$\text{subject to} \left\{ \sum y_{in}(z), x_i(z) \right\} \in Y_i(z) \text{ for all } z,$$

where W_{it} denotes factor prices (possibly a vector) faced by producers in country i. To understand the expression for revenues in (4), note that when the trading technology is operated in country i, producers receive a price of $p_{int}(z)$ for each of the $y_{int}(z) / \tau_{int}$ units of output arriving in country n. When the trading technology is operated abroad, country i producers receive a price of $p_{int}(z) / \tau_{int}$ for each of the $y_{int}(z)$ units of output they produce that are bound to country n.

A Real GDP and aggregate productivity

We now use this framework to calculate changes in real GDP and aggregate productivity in response to changes in trade costs and tariffs, following the procedures discussed in Section I. We first construct real GDP and productivity from the production side (excluding tariffs) and then from the expenditure side (including tariffs).

Recall from Section I that current-dollar GDP from the production side (exclusive of tariffs) is equal to the sum of value added across all production units, and also to the sum of factor payments and aggregate profits:

$$GDP_{it} = \int_{\Omega_i} \sum_n p_{int}(z) y_{int}(z) / \tau_{int} \mathrm{d}z = W_{it} X_{it} + \Pi_{it}. \tag{5}$$

Suppose that there is a change in prices, trade costs and import tariffs. Up to a first order approximation around the equilibrium at time t_0, the change in current dollar GDP from the production side is

$$\Delta \log GDP_{it} = \sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) \left(\Delta \log p_{int}(z) - \Delta \log \tau_{int} \right) \mathrm{d}z + \frac{W_{it_0} X_{it_0}}{GDP_{it_0}} \Delta \log X_{it}. \tag{6}$$

Here, λ_{int} denotes the share of sales from country i to n in country i's total sales at time t, and $\lambda_{int}(z)$ denotes the share of sales of producer z in country i's total sales to country n.[13] In deriving expression (6), we differentiated $W_{it} X_{it} + \Pi_{it}$ from expression (5) and

[13] In particular, $\lambda_{int_0} = \left[\int_{\Omega_i} p_{int_0}(z) y_{int_0}(z) / \tau_{int_0} \mathrm{d}z \right] / GDP_{it_0}$ and $\lambda_{int_0}(z) = \left[p_{int_0}(z) y_{int_0}(z) / \tau_{int_0} \right] / \left[\lambda_{int_0} GDP_{it_0} \right]$.

used the envelope theorem associated to (4).

To calculate the change in real GDP from the production side using the deflation method (defined in Section I) we deflate the change in current dollar GDP by the PPI. The log change in the PPI_{it} is a weighted average (with base period t_0) of changes in producer prices, $\bar{p}_{int}(z)$:

$$\Delta \log PPI_{it} = \sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) \Delta \log \bar{p}_{int}(z) \, dz. \tag{7}$$

The log change in real GDP is $\Delta \log RGDP_{it} = \Delta \log GDP_{it} - \Delta \log PPI_{it}$, and the log change in aggregate measured productivity (defined in equation 3) is

$$\begin{aligned} \Delta \log A_{it} &= \Delta \log RGDP_{it} - \frac{W_{it_0} X_{it_0}}{GDP_{it_0}} \Delta \log X_{it} \\ &= \sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) \left(\Delta \log p_{int}(z) - \Delta \log \tau_{int} - \Delta \log \bar{p}_{int}(z) \right) dz. \end{aligned} \tag{8}$$

Note that, from the envelope condition in problem (4), changes in input choices by heterogeneous production units have no first order effects on measured aggregate productivity.

Suppose first that for each good the trading technology is operated by producers in country i.[14] If producer prices in the PPI incorporate changes in trade costs, then $\Delta \log \bar{p}_{int}(z) = \Delta \log p_{int}(z)$ and, from expression (8), changes in aggregate productivity are, up to a first approximation, given by

$$\Delta \log A_{it} = -\sum_n \lambda_{int_0} \Delta \log \tau_{int}. \tag{9}$$

Changes in aggregate productivity are equal to an export-share weighted average of changes in bilateral trade costs. Suppose instead that for each good the trading technology is operated abroad (or that the trading technology is operated at home but producer prices in the PPI exclude trade costs). In this case, $\Delta \log \bar{p}_{int}(z) = \Delta \log p_{int}(z) - \Delta \log \tau_{int}$, and aggregate productivity is unchanged with trade costs: $\Delta \log A_{it} = 0$.[15]

In sum, up to a first order, changes in aggregate productivity only capture the direct effect of changes in technologies operated domestically. Changes in trade costs incurred abroad that leave the production set unchanged have no first order effects on measured

[14] In the online Appendix we show that the results are unchanged if the trading technology is operated by intermediaries rather than by the producers themselves.

[15] It is straightforward to extend our results to the case in which τ_{int} varies with z, in which case, expression (9) becomes $\Delta \log A_{it} = -\sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) \Delta \log \tau_{int}(z) \, dz$.

aggregate productivity.[16] We summarize these results in the following proposition:

Proposition 1. *In response to changes in trade costs and tariffs, up to a first order approximation, aggregate productivity based on real GDP from the production side (excluding tariffs) is unchanged if the trading technology is operated abroad (or if producer prices in the PPI exclude trade costs), and is given by expression* (9) *if the trading technology is operated at home.*

In the Appendix we show that the results in Proposition 1 are unchanged if, instead of calculating real GDP using the deflation method, we use the direct valuation method (as the value of production at constant, t_0, prices).

GDP inclusive of tariffs: We now calculate changes in real GDP from the expenditure side which, in contrast to the measure of GDP from the production side calculated above, includes changes in tariff revenues. In the Appendix, we show that in response to changes in trade costs and tariffs, up to a first-order approximation, the change in real GDP measured from the expenditure side is

$$\Delta\log RGDP_{it}^{E} = \frac{GDP_{it_0}}{GDP_{it_0}^{E}}\Delta\log RGDP_{it} + \frac{\sum_{n\neq i}\int_{\Omega_i}\left(d_{nit_0}-1\right)p_{nit_0}(z)\,q_{nit_0}(z)\,\Delta\log q_{nit}(z)\,\mathrm{d}z}{GDP_{it_0}^{E}}. \tag{10}$$

Expression (10) states that the change in real GDP from the expenditure side equals the change in real GDP from the production side plus a term that captures the change in tariff revenues, evaluated at constant prices. In response to reductions in trade costs or tariffs, this term is always positive if imported physical quantities rise.

B Real consumption and welfare-based consumption

The theoretical (welfare-based) price index, P_{nt}, is given by

$$P_{nt} = \min_{\{q_{in}(z)\}_{i,z}} \sum_i \int_{\Omega_i} d_{int}p_{int}(z)\,q_{int}(z)\,\mathrm{d}z \tag{11}$$
$$\text{subject to } C\left(\{q_{in}(z)\}_{i,z}\right) = 1.$$

Recall from Section I that the log change in real consumption is equal to the log change in current-dollar consumption expenditures deflated by the change in the consumer price

[16] In the online Appendix we also consider the case in which country i_s operates the trading technology of all other countries. This requires additional notation without changing the core of the results: changes in any country's bilateral trade costs are reflected in country i_s's productivity (since country i_s operates these technologies).

index: $\Delta\log RC_{nt} = \Delta\log E_{nt} - \Delta\log CPI_{nt}$. The change in the CPI (with base t_0 weights) is given by

$$\Delta \log CPI_{nt} = \sum_i \Lambda_{int_0} \int_{\Omega_i} \Lambda_{int_0}(z) \left[\Delta \log p_{int}(z) + \Delta\log d_{int}\right] \mathrm{d}z, \tag{12}$$

where Λ_{int} and $\Lambda_{int}(z)$ denote expenditure shares in country n at time t, inclusive of tariffs.[17]

Consider a marginal change in final prices faced by consumers in country n. By only considering marginal changes in prices we rule out changes in the set of goods that are available for consumption since those would be associated to infinite sized price changes. Log-differentiating P_{nt} in (11) and using the envelope theorem, it is straightforward to show that the change in the theoretical price index is equal to the change in the CPI, up to a first order. The substitution bias is of second order. The next proposition follows immediately.

Proposition 2. *Up to a first order approximation, in each country changes in real consumption are equal to changes in welfare-based consumption.*

C Real consumption and real GDP

Country by country, changes in real GDP and in real consumption do not need to be equal because of trade imbalances and differences between consumer prices and producer prices (e.g. movements in terms of trade). The following proposition states that at the world level these two factors cancel-out, up to a first order approximation.

Proposition 3. *In response to changes in trade costs or tariffs, up to a first order approximation, changes in world real GDP are equal to changes in world real consumption, and given by*

$$\begin{aligned} \Delta logRC^E_{wt} = \Delta logRGDP^E_{wt} = &-\frac{1}{E_{wt_0}} \sum_i \sum_n Exports_{int_0} \times \Delta log\tau_{int} \\ &+ \frac{1}{E_{wt_0}} \sum_i W_{it_0} X_{it_0} \Delta log X_{it} \\ &+ \frac{1}{E_{wt_0}} \sum_i \sum_{n\neq i} \int_{\Omega_i} \left(d_{nit_0} - 1\right) p_{nit_0}(z)\, q_{nit_0}(z)\, \Delta log q_{nit}(z)\, \mathrm{d}z, \end{aligned} \tag{13}$$

where $\Delta logRC_{wt} = \frac{1}{E_{wt_0}} \sum_i E_{it_0} \Delta logRC_{it}$, $\Delta logRGDP^E_{wt} = \frac{1}{E_{wt_0}} \sum_i GDP^E_{it_0} \Delta logRGDP^E_{it}$, *and* $E_{wt} = \sum_i GDP^E_{it} = \sum_i E_{it}$.

[17] Specifically, $\Lambda_{int} = \int_{\Omega_i} d_{int} p_{int}(z)\, q_{int}(z)\, dz / E_{nt}$ and $\Lambda_{int}(z) = d_{int} p_{int}(z)\, q_{int}(z) / (\Lambda_{int} E_{nt})$.

The second equality in (13) follows immediately from (10). Consider now the first equality. For presentation simplicity, we focus on the case in which producers in each country operate the trading technology of its exports. In the Online Appendix we show that Proposition 3 holds more generally when some countries operate the trading technology of other countries. Changes in real GDP from the expenditure side are, up to a first order approximation:

$$\Delta \log RGDP_{it}^{E} = \frac{E_{it_0}}{GDP_{it_0}^{E}} \Delta \log RC_{it} + \frac{GDP_{it_0}}{GDP_{it_0}^{E}} \sum_{n \neq i} \int_{\Omega_i} \lambda_{int_0}(z) \left[\Delta \log p_{int} q_{int}(z) - \Delta \log EPI_{it}\right] \mathrm{d}z$$
$$- \sum_{n \neq i} \frac{GDP_{nt_0}}{GDP_{it_0}^{E}} \int_{\Omega_i} \lambda_{nit_0}(z) \left[\Delta \log p_{nit} q_{nit}(z) - \Delta \log IPI_{it}\right] \mathrm{d}z.$$

Adding over all countries, the second and third terms cancel-out because, as we show in the Appendix, the world's export price index is equal to the world's import price index.

According to expression (13), changes in real consumption at the world level, $\Delta \log RC_{wt}$, and changes in world real GDP from the expenditure side, $\Delta \log RGDP_{wt}^{E}$, are both equal to the sum of export-weighted changes in bilateral trade costs, changes in factor supplies, and changes in import tariff revenues at base-period prices. By Proposition 2, changes in world welfare-based consumption, $\Delta \log C_{wt} = \frac{1}{E_{wt_0}} \sum_i E_{it_0} \Delta \log C_{it}$, are also given by expression (13).

To sum up, in this section we have shown that in the baseline models of perfect competition that we consider, changes in measured productivity accurately reflect changes in the production possibility frontier (due to changes in trade costs), and changes in real consumption are equal to changes in welfare-based consumption, up to a first-order approximation. In the following section we investigate how these results extend to an alternative setting with monopolistic competition and endogenous changes in product quality and in the set of available goods.

III Monopolistic competition, heterogeneous firms and endogenous product quality

In this section, we re-examine the measured aggregate gains from reductions in trade costs in an environment with monopolistic competition and heterogeneous firms in which the set of goods and their quality can change in response to changes in trade costs. This results in additional considerations (relative to the model with perfect competition of Section II) when comparing real GDP and real consumption with welfare-based consumption. To develop closed-form results, we are more specific than in Section II in terms of the technology restrictions. To focus on the new margins implied by these models, we do not include a long taxonomy over the different specification of trade costs we reviewed in Section II. Following the most common assumptions in the literature, we abstract from tariffs and assume that trade costs are incurred in the exporting country and are included in the PPI.

Theoretical consumption is a CES aggregator over a continuum of differentiated goods indexed by ω:[18]

$$C_{nt} = \left[\int_{\Omega_{nt}} a_{nt}\left(\omega\right)^{\frac{1}{\rho}} \left(\omega\right) q_{nt}\left(\omega\right)^{\frac{\rho-1}{\rho}} \mathrm{d}\omega \right]^{\frac{\rho}{\rho-1}}, \text{ with } \rho > 1. \tag{14}$$

Here, $q_{nt}\left(\omega\right)$ and $a_{nt}\left(\omega\right)$ denote the consumption and the quality of good ω, and Ω_{nt} denotes the set of goods that are available for consumption in country n at time t. Demand for each good in country n is $q_{nt}\left(\omega\right) = a_{nt}\left(\omega\right)\left(p_{int}\left(\omega\right)/P_{nt}\right)^{-\rho} C_{nt}$, and the theoretical price index is given by

$$P_{nt} = \left[\int_{\Omega_{nt}} a_{nt}\left(\omega\right) p_{nt}\left(\omega\right)^{1-\rho} \mathrm{d}\omega \right]^{\frac{1}{1-\rho}}. \tag{15}$$

We assume that consumption is not a physically traded good, and hence C_{nt} and P_{nt} cannot be directly observed by the statistical agency.

Each producer specializes in the production of a single differentiated good. Production uses only labor according to the production function $y = zl$, where y and l denote output and labor of a producer with productivity z. We denote by L_{it} the supply of labor in country i, and by $M_{it}\left(z\right)$ the distribution of producers with productivity z in country i at time t. Given the symmetry of goods in the production function of the final good (14), we interchangeably index goods by ω or by their productivity z and source country i.

The trading technology, which transforms one unit of output in country i into $1/\tau_{int}$ units in country n, is operated by the producer in its home country.[19] Hence, final prices paid by consumers in country n are equal to producer prices in country i, $\bar{p}_{int}\left(z\right) = p_{int}\left(z\right)$,

[18] To interpret the statistics we calculate below, we view our model as that of a representative sector or industry composed of differentiated goods which aggregate according to expression (14).

[19] An alternative interpretation of this technology is that each unit delivered in country n requires $1/z$ production labor and $\left(\tau_{int}-1\right)/z$ trade-cost-related labor in country i. Trade-cost-related labor can be incurred by the producer or by a domestic intermediary that is vertically integrated with the producer (if producers and intermediaries did not maximize joint profits, producers would not face a constant elasticity of demand and markups would vary over time). As discussed in footnote 15, τ_{int} could vary across heterogeneous producers without substantially changing our results.

and given by a constant markup over marginal cost,

$$p_{int}(z) = \frac{\rho}{\rho - 1}\frac{\tau_{int} W_{it}}{z}, \tag{16}$$

where W_{it} denotes the wage in country i. We discuss below how our results can be extended to the case in which measured prices in the PPI or the CPI are adjusted for quality. This is isomorphic to the case in which, instead of choosing quality, firms choose productivity (which is reflected in prices).

There are two additional sources of operating costs. First, producers from country i are subject to fixed labor costs f_{int} when selling any positive amount in country n. Unless stated otherwise, we assume that these fixed costs employ labor in the exporter's country. We denote by Ω_{int} the set of producers from country i that sell a positive quantity to country n at time t.

Second, producers incur quality-related costs. A producer from country i with productivity z must employ $h(z, a_{int})$ units of labor at time t to set quality a_{int} for sales in country n. $h(z,.)$ is increasing and convex in a.[20] The more convex is $h(z,.)$, the smaller is the response of a_{int} to changes in trade costs. Unless stated otherwise, we assume that these fixed costs employ labor in the exporter's country. Following procedures in the NIPA accounts, we assume that expenditures on quality innovation are expensed (counted as negative profits) and hence they are not included in GDP.

Every period there is an unbounded mass of potential entrants that can pay a fixed cost f_{Ei} to enter and produce a differentiated good. A measure M_{Eit} of new producers enter with a productivity level z that is drawn from the distribution $G_i(z)$. For some results, we assume that $G_i(z)$ is Pareto. Every period, producers die with probability $\delta > 0$. The free-entry condition implies that expected discounted profits at entry (including the fixed cost of entry) are non-positive. We assume that parameters are such that each period the mass of entrants is positive, $M_{Eit} > 0$, so that expected discounted profits at entry are equal to zero. The distribution of producers in country i, $M_{it}(z)$, is determined by the mass of entrants, exit decisions, and the death rate.

Aggregate profits Π_{it} are equal to aggregate revenues by country i producers across

[20] We assume throughout that $h(z,a)$ is such that a is positive and bounded for active producers. All our theoretical results (with the exception of Corollary 2 and Proposition 6) hold if for each producer a_{int} is constrained to be equal across destination countries. This assumption matters little for all of our quantitative results.

all destinations net of production labor, fixed labor, quality-related labor, and entry costs:

$$\Pi_{it} = \sum_n \int_{\Omega_{int}} \left[p_{int}(z) q_{int}(z) - W_{it} \left(l_{int}(z) + f_{int}(z) + h_{int}(z, a(z)) \right) \right] \mathrm{d}M_{it}(z) - W_{it} f_{Ei} M_{Eit}. \tag{17}$$

In what follows, we consider trade liberalization of the following form. The economy is in a steady-state at $t = 0$. Between $t = 0$ and $t = 1$, there is a permanent, unexpected change in variable and/or fixed trade costs. At $t \geq T'$, the economy reaches a new steady state such that all variables remain constant over time.

We assume that in the initial steady-state ($t = 0$) and in at least one period after the trade-liberalization ($t = T \geq 1$), aggregate profits in country i represent a constant share κ_i of aggregate revenues by country i producers. That is,

$$\Pi_{it} = \kappa_i \sum_n \int_{\Omega_{int}} p_{int}(z) q_{int}(z) \mathrm{d}M_{it}(z) \text{ , for } t = 0 \text{ and } t = T \geq 1. \tag{18}$$

This assumption is similar to assumption R2 in Arkolakis, Costinot and Rodriguez-Clare (2012).

There are three special cases, derived in the Online Appendix, in which condition (18) is satisfied in the steady state of our model. First, as the interest rate approaches zero, aggregate profits are zero in steady-state (from the free-entry condition) so $\kappa_i = 0$ in steady-state. Second, if there are no fixed costs of supplying individual markets ($f_{int} = 0$) and $h(z, a)$ takes the form

$$h(z, a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^{\gamma}, \tag{19}$$

with $\gamma \geq 1$. Third, if the productivity distribution of entering producers is Pareto and $h(z, a)$ takes the form (19). As $\gamma \to \infty$, $a_{int}(z)$ is invariant to changes in trade costs, and hence the model behaves as one with exogenous quality.

In the second and third special cases the mass of entrants M_{Eit} does not respond to permanent changes in variable or fixed trade costs. Hence, there are no transition dynamics in response to permanent trade liberalization, and condition (18) holds for any time period $T \geq 1$.[21] In all other cases with aggregate transition dynamics between steady-states, the share of profits in revenues κ_i need not be constant along the transition paths. In these cases, our results hold across steady-states.[22]

Current-dollar GDP at time $t = 0$ and any time period $t = T$ in which condition (18)

[21] In the second and third special cases, condition (18) also applies if we assume that entry is restricted so that the mass of firms is exogenously fixed (see e.g. Chaney 2008).

[22] See Burstein and Melitz (2012) for an extended discussion of aggregate transition dynamics in this class of models.

holds is given by

$$GDP_{it} = W_{it}L_{it} + \Pi_{it} = \frac{W_{it}L_{it}}{1-\kappa_i}. \tag{20}$$

We now calculate changes in real GDP, aggregate productivity, and real consumption between $t=0$ and $t=T$.

A Real GDP and aggregate productivity

To calculate real GDP via the deflation method, we define the PPI in country i in period t relative to period $t-1$, with base period t_0, as

$$\frac{PPI_{it}}{PPI_{it-1}} = \sum_n \bar{\lambda}_{int} \int_{\Omega^c_{int}} \bar{\lambda}_{int}(z) \left(\frac{p_{int}(z)}{p_{int-1}(z)}\right) \mathrm{d}M_{it_0}(z) = \sum_n \bar{\lambda}_{int} \frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}. \tag{21}$$

Here, Ω^c_{int} is the set of goods sold from country i to country n with positive sales at time t_0, $t-1$, and t; $\bar{\lambda}_{int}(z)$ is the time t_0 share of producer z's sales to country n in total sales from i to n by continuing producers at time t; and $\bar{\lambda}_{int}$ is the time t_0 share of country i's sales to country n in total sales by continuing producers.[23] If the set of active producers and exporters does not change over time, then $\bar{\lambda}_{int}(z) = \lambda_{int_0}(z)$ and $\bar{\lambda}_{int} = \lambda_{int_0}$. Note that using base-period weights $t_0 = t$, then expression (21) is equivalent to (7), up to a first order approximation.

Real GDP in period T relative to period 0 is:[24]

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \prod_{t=1}^{T} \left(\frac{GDP_{it}/GDP_{it-1}}{PPI_{it}/PPI_{it-1}}\right) = \frac{GDP_{iT}}{GDP_{i0}} \prod_{t=1}^{T} \left(\frac{1}{PPI_{it}/PPI_{it-1}}\right) = \frac{L_{iT}}{L_{i0}} \prod_{t=1}^{T} \left(\frac{1}{\sum_n \bar{\lambda}_{int} \frac{\tau_{int}}{\tau_{int-1}}}\right).$$

If trade costs change permanently between $t=0$ and $t=1$, so that $\tau_{int}/\tau_{int-1} = 1$ for $t>1$, then

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{L_{iT}}{L_{i0}} \frac{1}{\sum_n \bar{\lambda}_{in1} \frac{\tau_{in1}}{\tau_{in0}}}, \tag{22}$$

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[23] Specifically, $\Omega^c_{int} = \Omega_{int_0} \cap \Omega_{int-1} \cap \Omega_{int}$, $\bar{\lambda}_{int}(z) = \frac{p_{int_0}(z) q_{int_0}(z)}{\int_{\Omega^c_{int}} p_{int_0}(z) q_{int_0}(z) \mathrm{d}M_{it_0}(z)}$, and $\bar{\lambda}_{int} = \frac{\int_{\Omega^c_{int}} p_{int_0}(z) q_{int_0}(z) \mathrm{d}M_{it_0}(z)}{\sum \int_{n\Omega^c_{int}} p_{int_0}(z) q_{int_0}(z) \mathrm{d}M_{it_0}(z)}$. Note that, if countries i,n do not trade at time t_0, $t-1$ or t, then $\Omega^c_{int} = \varnothing$ and the PPI excludes price changes from this pair of countries. Hence, if a country is in autarky at time t_0, $t-1$ or t then the PPI only takes into account changes in domestic prices.

[24] To derive this expression, we use expression (1) with a single aggregate component. With a single aggregate component, the Laspeyres quantity index, $\frac{GDP_{it}/(PPI_{it}/PPI_{it-1})}{GDP_{it-1}}$ is equal to the Paasche quantity index, $\frac{GDP_{it}}{GDP_{it-1} \times (PPI_{it}/PPI_{it-1})}$. Note that the PPI does depend on the choice of base period t_0.

and the change in aggregate productivity between periods 0 and T is

$$\frac{A_{iT}}{A_{i0}} = \frac{1}{\sum_n \bar{\lambda}_{in1} \frac{\tau_{in1}}{\tau_{in0}}}. \tag{23}$$

Expression (23) is equal, up to a first order approximation, to expression (9) in the model of Section II.[25] In the Appendix we calculate changes in aggregate productivity based on the direct valuation method (using product-level prices and quantities). The resulting expression coincides with expression (23), up to a first order approximation. To a first order approximation, the change in aggregate productivity is given by expression (9).

According to expression (23), changes in aggregate measured productivity depend only on the two following statistics: export shares (in the base period t_0) for continuing goods, and changes in variable trade costs. Note that if variable trade costs are not measured in prices in the PPI, then aggregate productivity is unchanged in response to changes in variable trade costs, consistent with the result in Proposition 1.

To derive expressions (22) and (23), we exploited three features of our framework. First, the fraction of variable trade costs in final prices is the same for all producers from country i selling in country n.[26] Second, markups are constant. Third, current-dollar GDP is a constant fraction of the wage bill (after a certain time period $T \geq 1$). In models satisfying these restrictions, changes in aggregate measured productivity depend on the same set of sufficient statistics stated above. This result is summarized in the following Proposition:

Proposition 4. *Under the three assumptions described above, in response to changes in variable or fixed trade costs, the change in aggregate productivity between $t = 0$ and $t = T$ is given by expression* (23)*, which up to a first order approximation is equal to expression* (9)*.*

Note that, in models with entry and exit into production and exporting, export shares for continuing producers at time t_0, $\bar{\lambda}_{int}$, differ from export shares for all producers, λ_{int_0}. If, for example, there is exit by non-exporters and entry into exporting in response to reductions in trade costs, then export shares for continuing producers are larger than export

[25] Whereas expression (8) only holds up to a first order approximation, expression (23) holds globally. This is because the model in this section imposes more restrictive assumptions on technology (e.g. all goods are produced with identical, constant returns to scale production functions).

[26] Suppose that prices are given by $p_{int}(z) = \frac{\rho}{\rho-1} W_{it} \left(\frac{1}{z} + \tau_{int} - 1 \right)$, so that the share of gross trade costs in final prices, $s_{int}(z) = \frac{\tau_{int}}{\frac{1}{z} + \tau_{int} - 1}$, varies with z. In our baseline model, prices are given by (16), so $s_{int}(z) = 1$. Up to a first order approximation, $\Delta \log p_{int}(z) = \Delta \log W_{it} + s_{int}(z) \Delta \log \tau_{int}$ and $\Delta \log A_{it} = -\sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) s_{int}(z) \Delta \log \tau_{int} dz$. Hence, this alternative formulation is equivalent to assuming producer-specific changes in trade costs, $\Delta \log \tau_{int}(z) = s_{int}(z) \Delta \log \tau_{int}$, as discussed in footnote 15.

shares for all producers. Therefore, for given trade shares at time t_0 and given reductions in variable trade costs, exit by non-exporters and entry into exporting contribute to a larger increase in real GDP and aggregate productivity.[27]

Consider now changes in fixed trade costs (or, similarly, in the size of foreign countries). If variable costs are unchanged, then the PPI at time t relative to $t-1$ is equal to W_{it}/W_{it-1}. Aggregate productivity, given by expression (23), is unchanged. Intuitively, fixed costs are expensed and hence do not show-up in GDP or in producer prices.

In our baseline results we assumed that prices in the PPI (and in the CPI) do not reflect changes in quality. Recall from Section I that for some goods and services in the price indices the statistical agency attempts to adjust prices to reflect quality changes. To examine the implications of this practice, suppose that measured prices are quality-adjusted (or, similarly, that firms choose productivity instead of quality). In, particular, we assume that price changes used to compute the PPI are given by

$$\frac{p^{qa}_{int}(z)}{p^{qa}_{int-1}(z)} = \frac{p_{int}(z)}{p_{int-1}(z)} \left(\frac{a_{int}(z)}{a_{int-1}(z)} \right)^{\frac{1}{1-\rho}}. \tag{24}$$

Typically, in response to a reduction in trade costs, quality falls for domestic sales (which increases the PPI) and rises for export sales (which reduces the PPI). In general, the overall effect on real GDP is ambiguous. Under some special assumptions, these two effects cancel-out in response to changes in variable trade costs, and changes in real GDP and aggregate productivity are unaffected by the endogenous response of measured quality. These conditions are summarized in the following corollary, which is proved in the Appendix.

Corollary 1. *Suppose that all prices in the PPI are adjusted for quality as in expression* (24). *If fixed costs and quality-related costs use domestic labor, firm productivities are Pareto distributed, and $h(z,a)$ takes the form* (19)*, then in response to changes in variable trade costs the change in real GDP and measured aggregate productivity are, up to a first order approximation, given by expressions* (22) *and* (23)*, respectively.*

We now relax the assumption of constant markups and constant share of aggregate profits in GDP. Following the previous steps, it is straightforward to show that the change in measured aggregate productivity between any two time periods is given by

[27] Standard Ricardian trade models with perfect competition, such as those in Dornbusch, Fischer and Samuelson (1977) and Eaton and Kortum (2002), also satisfy the three properties described above. These models feature endogenous specialization in production (so, in general $\bar{\lambda}_{int}$ differs from λ_{int_0}) and a fixed set of varieties consumed with constant quality (hence, Proposition 2 applies).

$$\frac{A_{it}}{A_{it-1}} = \left(\frac{1-\kappa_{it-1}}{1-\kappa_{it}}\right)\left(\frac{1}{\sum_n \bar{\lambda}_{int} \frac{\bar{\mu}_{int}}{\bar{\mu}_{int-1}} \frac{\tau_{int}}{\tau_{int-1}}}\right),$$

where $\bar{\mu}_{int}$ denotes the revenue-weighted average markup of continuing producers from country i selling to country n.

Variable markups result in two additional sources of movements in measured aggregate productivity. First, reductions in the share of aggregate profits in GDP, κ_{it}, reduce measured aggregate productivity. Second, reductions in average markups increase measured aggregate productivity. In models covered in Bernard et al. (2003) and Arkolakis et al. (2012), the share of profits in GDP and the distribution of markups by destination are invariant to changes in trade costs. In those models, as well as in the model of Edmond, Midrigan and Xu (2012), markups typically fall for continuing domestic producers and increase for continuing exporters in response to reductions in trade costs. The overall effect of a trade liberalization on aggregate measured productivity will depend on the strength of these two effects.

More generally, interactions between changes in trade costs and producer-level distortions of the type considered in e.g. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) can result in additional sources of changes in measured aggregate productivity that are not captured in expression (23).

B Real GDP, real consumption, and welfare-based consumption

To calculate real consumption, current-dollar expenditures are deflated by the CPI, a weighted average of changes in final prices of continuing goods. Without adjusting for changes in quality, the CPI between periods $t-1$ and t is

$$\frac{CPI_{nt}}{CPI_{nt-1}} = \sum_i \bar{\Lambda}_{int} \int_{\Omega^c_{int}} \bar{\lambda}_{int}(z) \left(\frac{p_{int}(z)}{p_{int-1}(z)}\right) \mathrm{d}M_{it_0}(z) = \sum_i \bar{\Lambda}_{int} \left(\frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}\right), \quad (25)$$

where $\bar{\Lambda}_{int}$ is the time t_0 share of country n's expenditure from country i in total expenditures in continuing goods.[28] Note that using base-period weights $t_0 = t$, then expression (25) is equivalent to (12), up to a first order approximation.

Given that the PPI and the CPI in this section coincide, up to a first order approximation, with those in the model of Section II, it follows that Proposition 3 still holds. Specifically, changes in world real consumption and in world real GDP are, up to a first-order

[28]Specifically, $\bar{\Lambda}_{int} = \frac{\int_{\Omega^c_{int}} p_{int}(z) q_{int}(z) \mathrm{d}M_{it_0(z)}}{\sum_i \int_{i\Omega^c_{int}} p_{int}(z) q_{int}(z) \mathrm{d}M_{it_0(z)}}$.

approximation, both equal to a world export-weighted average of changes in variable trade costs, as indicated in expression (13).[29]

We now turn to the comparison between real and welfare-based consumption, first at the world-level and then (under stronger conditions) country-by-country. Proposition 2, establishing the equality between changes in real consumption and welfare-based consumption country-by-country, was based on the equality, up to a first order, between the CPI and the welfare-based price index. In the model in this section, however, marginal changes in trade costs can induce changes in the set of products available for consumption and in their quality. These changes are captured in the welfare-based prices index defined in equation (15), but not in the CPI. However, in response to marginal changes in variable trade costs, these biases cancel-out at the world level, so that changes in world real consumption equal changes in world theoretical consumption. This result, which builds on results in Atkeson and Burstein (2010), is summarized in the following proposition, proved in the Appendix.[30]

Proposition 5. *In response to changes in variable trade costs, steady-state changes in world real consumption and welfare-based consumption are equal, up to a first-order approximation, and both are equal to changes in world real GDP, given by expression* (13)*.*

Since, according to Proposition 5, changes in world real GDP are equal (up to a first-order approximation) to changes in world welfare-based consumption, then for the world as a whole measured aggregate productivity captures the first-order effects of changes in variable trade costs on welfare-based consumption.

Proposition 5 can be understood as follows. When countries are symmetric, this result states that changes in real consumption equal changes in theoretical consumption in response to marginal changes in variable trade costs. This is because, as shown in Atkeson and Burstein (2010), the indirect effect of a change in trade cost on welfare through its effect on the set of consumed goods (due to changes in the mass of entering firms, changes in exit and export thresholds) and changes in quality is zero, up to a first-order-approximation. Since these margins are not captured in the CPI either, it follows that changes in the theoretical price index are approximately equal to changes in the CPI. With asymmetric countries, changes in relative country sizes alter the country-by-country equivalence between real consumption and theoretical consumption. This effect, however, washes-out across countries (i.e. the gain in one country is a loss for another)

[29] If prices in the PPI and in the CPI are adjusted for quality (as in expression 24), we show in the Appendix that this result holds if firm productivities are Pareto distributed and $h(z,a)$ takes the form (19).

[30] As we show in the Appendix, Proposition 5 holds irrespective of whether fixed costs and quality-related costs employ labor in the exporting country or in the importing country (or a combination of the two).

when comparing steady-state changes in world real consumption and world theoretical consumption.[31]

For the case in which prices in the CPI are quality adjusted, we have the following corollary to Proposition 5, which is proved in the Appendix.

Corollary 2. *Suppose that all prices in the CPI are adjusted for quality as in* (24)*. If firm productivities are Pareto distributed, and $h(z,a)$ takes the form* (19)*, then the result in Proposition 5 holds.*

Finally, the following proposition (proved in the Appendix) establishes, under stronger assumptions, the equality between real consumption and welfare-based consumption, country-by-country, in response to changes in variable trade costs (as in Proposition 2).

Proposition 6. *Suppose that (i) fixed costs and quality-related costs use labor in the importing country, (ii) the distribution of entering firms is Pareto, (iii) $h(z,a)$ takes the form $\frac{\gamma_0}{\gamma} z^{\eta} a^{\gamma}$, and (iv) the ratio of trade balance / GDP in country i is fixed across steady states. Then, in response to changes in variable trade costs, up to a first-order approximation, steady-state changes in real consumption and welfare-based consumption in country i are equal. This result holds irrespective of whether prices in the CPI are quality adjusted or not.*

The assumptions of this proposition are similar to the assumptions in Proposition 2 of Arkolakis, Costinot and Rodriguez-Clare (2012). Arkolakis, Costinot and Rodriguez-Clare (2012) show that, under these assumptions, the model behaves like an Armington model in response to global changes in variable trade costs (even though the number and quality of consumed products may change in each country). Since the welfare-based price index in the Armington model is, up to a first-order approximation, equal to the CPI, the equivalence between real consumption and consumption-based welfare follows.

Note that Proposition 6 holds irrespective of whether prices in the CPI are quality adjusted or not. This implies that the quality and variety biases in the CPI cancel-out margin by margin. This does not require that the mass of consumed varieties is unchanged, nor that the average change in quality of imported goods equals the negative of the average change in quality of domestic goods, since what matters for the welfare-based price index is the expenditure-weighted change in the mass of varieties and quality.[32]

[31] For this result to hold, it is important that fixed and entry costs are denominated in terms of labor. If these costs entail a combination of labor and final good, then changes in the real wage can result in additional indirect effects from changes in the mass of consumed varieties on the welfare-based price index that are not captured in the CPI (see the related discussion for welfare in Atkeson and Burstein 2010 and Arkolakis, Costinot and Rodriguez-Clare 2012).

[32] Arkolakis et al. (2008) show that, under the assumptions of Proposition 6 but abstracting from endogenous quality, the mass of consumed varieties rises in response to reductions in variable trade costs if domestic fixed costs are larger than export fixed costs.

To sum up, in this section we have shown that in models of monopolistic competition with endogenous quality and firm selection, under a number of (widely-used) conditions, the main conclusions from Section II apply: in response to changes in variable trade costs, measured productivity accurately reflects changes in the production possibility frontier, and changes in real consumption are equal to changes in welfare-based consumption, up to a first-order approximation.

IV Quantitative applications

In this section, we conduct a quantitative evaluation of the theoretical results presented above, and relate these results to existing empirical work on measured aggregate gains from trade. To do so, we consider a uniform reduction in variable trade costs in a calibrated two-country version of the model of Section III.

We first assess the equivalence between real consumption and welfare derived in Propositions 5 and 6. The results in Proposition 5 were derived at the world level, for marginal changes in trade costs. Here, we calculate aggregate measured gains in each country in response to large uniform reductions in trade costs that double trade shares. Second, we calculate measured aggregate gains from reductions in trade costs using price indices that adjust for changes in quality and varieties. Third, we perform a productivity decomposition to quantify how factor reallocation across heterogeneous producers contributes to measured changes in aggregate productivity.

A Parametrization

We consider a two-country version of our model with balanced trade in each country and symmetric trade costs ($\tau_{12t} = \tau_{21t} = \tau_t$ and $f_{12t} = f_{21t}$). Fixed export costs and quality related costs are fully incurred in each exporting country. If these costs were incurred in the importing country, the equivalence results between real consumption and welfare, country by country, would be even stronger (in line with Proposition 6). Quality-related costs are given by $h\left(z;a\right) = \frac{\gamma_0}{\gamma} z^{\rho-1} a^{\gamma}$, so that all firms from a given country selling to the same destination choose the same quality a. The parameter γ determines the elasticity of a with respect to changes in trade costs. We choose $\gamma = 2.33$ to obtain a relatively large endogenous response of quality. Specifically, the elasticity of exporters' quality to trade costs is 1.3, so that in response to the large trade cost reduction considered below, a_{12t} rises by roughly 25 percent.

We choose the initial level of variable trade costs and relative country sizes so that the trade share is 15 percent in the small country, and 7 percent in the large country. With balanced trade, the share of the small country in world GDP is $1/3$. We assume that entering firms draw their idiosyncratic productivity from a Pareto distribution. We choose the slope parameter of the Pareto distribution and the elasticity of substitution ρ to target, given γ, the two following moments: (i) trade elasticity of 3.5 (Simonovska and Waugh 2012), and (ii) right tail coefficient of the firm-size distribution of 1.2 (Atkeson and Burstein 2010). The invariant levels of fixed costs do not affect our reported results.[33]

We consider a one-time reduction in variable trade costs so that trade shares roughly double in each country, which requires setting $\tau_1/\tau_0 = 0.8$. We also consider a marginal reduction in trade costs (as in our first-order approximations). For any measure of gains from trade, we report its elasticity with respect to the change in variable trade costs. We report separately the responses in each country and at the world level.

Real GDP and consumption are calculated according to the deflation method, using aggregate price indices (21) to deflate GDP and (25) to deflate consumption. We report results using price indices with weights based on period $t_0 = 0$ and $t_0 = 1$. To isolate the importance of trade related biases, we also report a Fisher quantity index using the direct valuation method to mitigate the substitution bias. Since aggregate employment is assumed constant, the change in aggregate productivity coincides with the change in real GDP.

B Real GDP, real consumption, and theoretical consumption

Table 1 examines the equivalence between changes in real GDP (and aggregate productivity), real consumption and welfare-based consumption. Panel A considers a marginal reduction in variable trade costs. Note first that the change in each aggregate measure is invariant to the base period in price indices, since the substitution bias does not have first order effects. Second, changes in world real consumption, world real GDP, and world welfare are equal, as proved in Proposition 5. Third, this equivalence is quite accurate country-by-country: differences between the elasticities of real GDP, real consumption, and welfare in each country are smaller than 0.01. Changes in real GDP and real consumption are very close because terms of trade movements are small relative to the overall change in price indices. Changes in real consumption and welfare are very close because the two biases in the CPI roughly cancel-out (this result is exact under the assumptions of Proposition 6).

Panel B of Table 1 considers a large reduction in variable trade costs that roughly dou-

[33]The specific parameter values are $\gamma = 2.33$, $\theta = 2$, $\rho = 2.67$, $\tau_0 = 1.81$, $f_E = 0.1$, $f_{11} = 1.2$ and $f_{12} = 1.75$.

Table 1: Real GDP, Real Consumption and Welfare

		Panel A: Marginal reduction in trade costs			Panel B: Large reduction in trade costs		
		Price indices based on $t_0 = 0$ weights	Price indices based on $t_0 = 1$ weights	Fisher quantity index	Price indices based on $t_0 = 0$ weights	Price indices based on $t_0 = 1$ weights	Fisher quantity index
Small Country	RGDP	0.15	0.15	0.15	0.16	0.33	0.26
	Cons.	0.15	0.15	0.15	0.17	0.35	0.27
Large Country	RGDP	0.07	0.07	0.07	0.07	0.16	0.12
	Cons.	0.07	0.07	0.07	0.07	0.15	0.12
World	RGDP	0.10	0.10	0.10	0.10	0.21	0.16
	Cons.	0.10	0.10	0.10	0.10	0.21	0.16
		Welfare-based consumption			Welfare-based consumption		
Small Country		0.16			0.26		
Large Country		0.07			0.11		
World		0.10			0.16		

Note: For any measure Z_t, we report its elasticity with respect to the change in variable trade costs, $\frac{\Delta Z_t}{Z_{t-1}} / \frac{\Delta \tau_t}{\tau_{t-1}}$. World-level elasticities are calculated as weighted averages of country level elasticities. The formulas for the Fisher quantity index for GDP and consumption are provided in the Appendix.

bles trade shares. Now changes in aggregate measures differ substantially depending on the base period in price indices. For example, if the CPI is constructed using date $t_0 = 0$ ($t_0 = 1$) weights, the elasticity of real consumption in the small country underestimates (overestimates) the elasticity of welfare by roughly 9 percent. This large disparity reflects the magnitude of the substitution bias following large changes in prices. On the other hand, real consumption based on a Fisher quantity index (which is designed to mitigate the substitution bias) is very close to welfare. For both countries, the difference in the elasticities of real consumption and welfare is roughly 1 percent. Hence, the substitution bias is significantly more important than the trade related biases discussed above. Moreover, since movements in the terms of trade are relatively small, changes in real GDP are quite close to changes in real consumption (and welfare).

C Adjusting price indices for quality and variety changes

We now consider alternative measures of real GDP and consumption calculated using quality-adjusted price indices (using price changes defined in expression 24), as well as measures of real consumption adjusted for changes in varieties. We conduct a uniform reduction in variable trade cost (marginal and large) described above. We report results for the small country using Fisher quantity indices to abstract from the substitution bias.

Comparing rows 1 and 2 in Table 2, we can observe that adjusting all producer prices for quality movements does not make a substantial difference for the change in real GDP predicted by the model, in line with Corollary 1. For large reductions in trade costs, the elasticity of real GDP is only 2 percentage points higher, even though the quality of individual products changes substantially (e.g. quality rises by roughly 25 percent for exported goods).

Consider now changes in real consumption. Recall from the previous results that changes in real consumption calculated based on CPIs without any adjustments for quality and variety are very close to real consumption based on CPIs that adjust for changes in quality and for changes in varieties consumed from all source countries (e.g. the theoretical price index). Suppose now that all prices in the CPI are adjusted for quality movements. Comparing rows 3 and 4 in Table 2, we can observe that this adjustment does not make a substantial difference (less than 2 percent) for the elasticity of real consumption (in line with Corollary 2, but at the level of individual countries).

Finally, suppose that due to data availability, the CPIs only adjust for changes in quality and/or changes in variety for imported goods without adjusting for the domestic component of the CPI. Row 5 reports changes in real consumption when import prices are adjusted for changes in the number of varieties. Row 6 reports changes in real consumption when import prices are adjusted for both changes in quality and in the number of varieties. We can observe that changes in real consumption based on these adjusted price indices greatly overstate the welfare gains (by a factor larger than 2 if we adjust both for quality and varieties) in response to changes in variable trade costs.

It is important to note that, while the focus in this section has been changes in variable trade costs, there are instances (i.e. specifications of our model and shocks) under which adjusting import price indices for variety changes only may bring real consumption closer to welfare-based consumption. For example, in a Krugman type specification in which all firms export, in response to a change in the size of the foreign country, the mass of consumed varieties increases only due to a rise in the measure of imported varieties.

Table 2: Adjusting for quality and variety changes

		Marginal reduction in trade costs	Large reduction in trade costs
(1)	RGDP	0.15	0.26
(2)	RGDP quality adj.	0.15	0.28
(3)	RC	0.15	0.27
(4)	RC quality adj.	0.16	0.29
(5)	RC variety adj. imports	0.20	0.35
(6)	RC quality and variety adj. imports	0.30	0.53
(7)	Welfare-based consumption	0.16	0.26

Note: This table reports results for the small country. "RGDP quality adj." and RC quality adj." refer to real GDP and real consumption calculated after adjusting prices in the PPI and the CPI for quality changes, according to expression (24). "RC variety. adj. imports" refers to real consumption calculated after adjusting import prices for variety changes. "RC quality and variety. adj. imports" refers to real consumption calculated after adjusting import prices for variety and quality changes. All measures are based on Fisher quantity indices. For any measure Z_t, we report its elasticity with respect to the change in variable trade costs, $\frac{\Delta Z_t}{Z_{t-1}} / \frac{\Delta \tau_t}{\tau_{t-1}}$.

D Aggregate productivity decomposition

Finally, we perform a standard productivity decomposition (similar to those used in Doms and Bartelsman 2000, Foster, Haltiwanger and Krizan 2001, among many other) to quantify how factor reallocation across heterogeneous producers contributes to measured changes in aggregate productivity in response to reductions in trade costs.

Aggregate productivity is defined as the Solow residual according to expression (3). The percentage change in aggregate productivity between periods $t-1$ and t is[34]

$$\frac{A_{it}}{A_{it-1}} - 1 = \left(\frac{L_{it-1} - M_{eit-1} f_{eit-1}}{GDP_{it-1}} \right) \left[\sum_{z} \frac{rva_{it}(z)}{l_{it}^{T}(z)} s_{it}(z) - \sum_{z} \frac{rva_{it-1}(z)}{l_{it-1}^{T}(z)} s_{it-1}(z) \right],$$

where $l_{it}^{T}(z)$ denotes total labor used by producer z (including fixed and quality-related costs), $s_{it}(z) = l_{it}^{T}(z) / (L_{it} - M_{eit} f_{eit})$ denotes the labor share of firm z in total labor (exclusive of entry costs so that the shares add up to one), and $rva_{it}(z) = \sum_{n} p_{int}(z) q_{int}(z) / (PPI_{it}/PPI_{it-1})$ denotes real value added of producer z. The ratio $rva_{it}(z) / l_{it}^{T}(z)$ is often referred to as revenue productivity, in contrast to physical productivity which is harder to measure in practice. Note that both fixed costs and endogenous quality can induce variation in revenue productivity across producers even if they all charge the same markup and face the same wage.[35]

Denoting $\Delta Z_t = Z_t - Z_{t-1}$ and $\bar{Z} = \frac{1}{2}(Z_{t-1} + Z_t)$ for any variable Z, the change in productivity can be decomposed as follows:

$$\begin{aligned}
\frac{A_{it}}{A_{it-1}} - 1 \quad = \quad & \left(\frac{L_{it-1} - M_{eit-1} f_{eit-1}}{GDP_{it-1}} \right) \times \qquad (26) \\
& \Bigg[\underbrace{\sum_{z \in \Omega_{cont}} \Delta \left(rva_i(z) / l_i^{T}(z) \right) \times \overline{s_i(z)}}_{\text{Own}} + \underbrace{\sum_{z \in Z_{cont}} \overline{rva_i(z) / l_i^{T}(z)} \times \Delta s_i(z)}_{\text{Reallocation}} \\
& + \underbrace{\sum_{z \in \Omega_{entry}} \left(rva_{it}(z) / l_{it}^{T}(z) \right) s_{it}(z)}_{\text{Entry}} - \underbrace{\sum_{z \in Z_{exit}} \left(rva_{it-1}(z) / l_{it-1}^{T}(z) \right) s_{it-1}(z)}_{\text{Exit}} \Bigg].
\end{aligned}$$

The first component, "Own", indicates increases in aggregate productivity due to a rise

[34] In deriving this equation we have used the result from our model that the ratio of entry costs to total labor depends on a subset of parameters which does not include the level of variable trade costs.

[35] We obtain similar results if aggregate productivity is based on real GDP calculated using the direct valuation method, deflating each firm's revenues by its price rather than by the aggregate PPI. This should come as no surprise since, as we discussed above, both procedures generate expressions for aggregate productivity that coincide, up to a first order approximation.

in measured productivity for continuing firms. The second component, "Reallocation", indicates increases in aggregate productivity due to labor reallocation to producers with higher measured productivity. The third component, "Entry", indicates increases in aggregate productivity due to producer entry. The fourth component, "Exit", indicates reductions in aggregate productivity due to producer exit. Different variations of expression (26) have been used in the literature. For example, if data is observed for continuing firms only, then only the first two components are considered.

We consider three specifications of our model which satisfy the assumptions in Propositions 4 and 5: (i) our baseline Melitz model with endogenous quality, exit, and entry into export, (ii) a variation of our baseline model in which quality, exit, and entry into exporting are fixed at the initial equilibrium levels, and (iii) a Krugman model in which all firms export (and quality and exit decisions are constant over time). All three specifications are calibrated to match the same initial trade shares. Specifications (i) and (ii) assume the same parameter values. In specification (iii), we choose $\rho = 4.5$ so that the trade elasticity is the same as in the baseline specification (i).

Table 3 presents the productivity decomposition between periods 0 and 1 using data generated by each model specification in response to a uniform reduction in trade costs that doubles trade shares. We only report results for the small country. The first two rows in Table 3 display the total change in aggregate productivity based on a Fischer quantity index (Row 1) and based on a PPI with $t_0 = 0$ weights (Row 2). The change in aggregate productivity under our baseline specification (i) coincides with the change in real GDP reported in Table 1, since total employment is constant. Note that the change in aggregate productivity based on a PPI with initial weights is very similar across the three specifications, since initial trade shares and changes in trade costs are the same across specifications (recall expression 23). The specification with endogenous quality, exit, and exporting generates slightly larger productivity gains than the other two specifications because some non-exporting producers exit after the reduction in trade costs. Note also that the change in aggregate productivity based on a Fisher quantity index (which uses information on trade shares before and after the reduction in trade costs) is roughly equal in specifications (i) and (iii), but is lower in specification (ii). This is because, while the trade elasticity is equal in specifications (i) and (iii), it is lower in specification (ii) in which quality, exit, and exporting decisions are fixed at their initial level. Thus, trade shares in period $t = 1$ are lower in specification (ii) and so is the rise in aggregate productivity based on $t_0 = 1$ weights.

We now summarize the results of the productivity decomposition, based on the $t_0 = 0$ measure of productivity (we obtain similar results if we use the Fischer measure of productivity). We observe first that the "Own" term is the most important in all specifications, reflecting the direct effect off the reduction in physical trade cost. The effects of changes in product quality on the "Own" component are small, reflecting the offset between the rise in quality for export sales and the decline in quality for domestic sales.

Second, the three model specifications generate very different degrees of factor reallocation across producers. In particular, the reallocation term is 0 in the Krugman specification in which all firms export, and positive in both Melitz specifications in which only a subset of firms export. It is much larger in the specification with endogenous quality and entry into exporting, as exporters increase quality relative to non-exporters, and new firms start exporting following the reduction in trade costs. The exit term is negative because some firms choose to exit following the reduction in trade costs.

Empirical studies cited in the introduction have consistently shown that factor reallocation from non-exporters to exporters contributes to increases in aggregate productivity following trade liberalization. This evidence provides direct support for the type of factor reallocation embedded in specifications (i) and (to a smaller extent) (ii). However, through the lens of our results, one cannot conclude that the rise in aggregate productivity would have been smaller in the absence of this reallocation. This is because in all models (with and without endogenous exit and export participation), changes in aggregate productivity are only a function of changes in variable trade costs and export shares for continuing producers.[36]

V Conclusions

In this paper we have studied the implications of trade cost reductions for aggregate measures of economic activity in a widely-used class of workhorse models of international trade. We have characterized how aggregate productivity, real GDP and real consumption, as calculated by statistical agencies in the United States, respond to changes in variable trade costs, fixed trade costs, and tariffs.

Our conclusions can be broadly summarized as follows. First, aggregate productivity increases in response to reductions in trade costs insofar as prices used to construct output deflators reflect these changes in trade costs (which shift the domestic production possibility set). Conditional on an export-share (for continuing goods) weighted average of changes in variable trade costs incurred domestically, aggregate productivity is equal

[36]This result is even starker when considering reductions in fixed export costs. Recall that changes in fixed costs generate no change in measured aggregate productivity. However, even in this case, specifications (i) and (ii) produce a positive reallocation component to aggregate productivity.

Table 3: Productivity Decomposition

	(i) Melitz : Endogenous exit, entry and quality		(ii) Melitz : Exogenous exit, entry and quality		(iii) Krugman	
	Change	Share of total	Change	Share of total	Change	Share of total
Agg. Productivity, Fisher quantity index	0.26		0.21		0.26	
Agg. Productivity, $t_0 = 0$ weights	0.16		0.15		0.15	
Own	0.12	0.77	0.14	0.94	0.15	1.00
Reallocation	0.16	1.02	0.01	0.06	0.00	0.00
Entry	0.00	0.00	0.00	0.00	0.00	0.00
Exit	-0.12	-0.79	0.00	0.00	0.00	0.00

Note: This table reports results for the small country. The formula for aggregate productivity based on the Fisher quantity index is provided in the Appendix.

in models with and without firm heterogeneity and without or without endogenous exit and export participation. Hence, through the lens of these models, one cannot conclude from the large degree of factor reallocation observed in data following trade liberalization that the rise in aggregate productivity would have been smaller absent this factor reallocation. Similar arguments could be used to interpret aggregate productivity gains from other sources of globalization such as multinational production.

Second, under certain conditions, consumption deflators as measured in the data capture the first-order effects of changes in variable trade costs on welfare-based price indices. Under these conditions, the multiple biases in the CPI arising from endogenous changes in quality and the number of varieties cancel-out (at the world level or, under stronger assumptions, country-by-country) and end up being less important than the substitution bias. Adjusting import price indices for changes in quality and variety without simultaneously adjusting domestic price indices can result in a significant overstatement of the welfare gains from reductions in variable trade costs.

Our results imply that consumption deflators are a better proxy for welfare-based price indices than output deflators. Given the high correlation between output and consumption deflators in the data,[37] we conjecture that the findings in the empirical literature documenting the link between trade and real GDP (e.g. Romer and Frankel 1999, Rodriguez and Rodrik 2001, Feyrer 2009, and Feyrer 2011) would not be substantially altered if, for a given current-dollar measure of economic activity, consumption deflators were used instead of output deflators.

One has to be cautious in using our results on the equivalence between real consumption and welfare to interpret in a welfare sense the observed relation between real consumption and trade in the data. Many of the restrictions underlying our results may not be met in practice. For example, changes in trade shares are not exclusively driven by changes in variable trade costs.[38] We view these results as establishing a theoretical benchmark under which real consumption is a good measure of welfare in response to trade liberalization of the form of changes in variable trade cost

[37]For OECD countries, the correlation between annual changes in the GDP deflator and the expenditure deflator between 1960 and 2012 is 0.98. Between 1993 and 1998, following the introduction of NAFTA, the difference between these two deflators relative to an average of their change is 17 percent in Canada, 1 percent in Mexico, and 8 percent in the US. Similarly, between 1999 and 2004, following the introduction of the Euro, this statistic is 7 percent for the median Euro country.

[38]While other forces certainly have played an important role in shaping the increase in trade over the last few decades, a number of papers highlight the role of improvements in the efficiency of the trade technology. For example, Bernhofen, El-Sahli and Kneller (2013) show that the expansion in the use of containers in international freight transport accounts for a substantial portion of the rise in bilateral trade between 1962 and 1990 (more so than bilateral free-trade agreements). Feyrer (2009) shows that improvements in aircraft technology increase bilateral trade for pairs of countries with short air routes relative to sea routes.

Appendix A Proofs and derivations in section II

A Direct valuation method

The results in Proposition 1 are unchanged if, instead of calculating real GDP using the deflation method, we use the direct valuation method (as the value of production at constant, t_0, prices). Suppose first that the trading technology is operated abroad. Real GDP at time t relative to time t_0 is

$$\frac{RGDP_{it}}{RGDP_{it_0}} = \frac{\int_{\Omega_i} \sum_n p_{int_0}(z) / \tau_{int_0} y_{int}(z) \, \mathrm{d}z}{\int_{\Omega_i} \sum_n p_{int_0}(z) / \tau_{int_0} y_{int_0}(z) \, \mathrm{d}z}.$$

To understand how aggregate productivity responds to changes in trade costs, it is useful to define aggregate profits at constant prices, $R\Pi_{it} = \int_{\Omega_i} \sum_n p_{int_0}(z) / \tau_{int_0} y_{int}(z) - W_{it_0} X_{it}$. From revealed production choices, $R\Pi_{it} \leq R\Pi_{it-1}$ if $t_0 = t-1$ and $R\Pi_{it-1} \leq R\Pi_{it}$ if $t_0 = t$. Changes in aggregate productivity between periods $t-1$ and t are given by:

$$\frac{A_{it} - A_{it-1}}{A_{it-1}} = \frac{RGDP_{it} - RGDP_{it-1}}{RGDP_{it_0}} - \frac{W_{it_0} X_{it_0}}{GDP_{it_0}} \left(\frac{X_{it} - X_{it-1}}{X_{it_0}} \right) = \frac{R\Pi_{it} - R\Pi_{it-1}}{GDP_{it0}}$$

In response to changes in trade costs, aggregate productivity falls when evaluated at $t_0 = t-1$ base prices, and rises when evaluated at $t_0 = t$ base prices. Up to a first order approximation, aggregate productivity is unchanged.

When the trading technology is operated domestically, real GDP at time t relative to t_0 is

$$\frac{RGDP_{it}}{RGDP_{it_0}} = \frac{\int_{\Omega_i} \sum_n p_{int_0}(z) y_{int}(z) / \tau_{int} \mathrm{d}z}{\int_{\Omega_i} \sum_n p_{int_0}(z) y_{int_0}(z) / \tau_{int_0} \mathrm{d}z}.$$

Following similar steps as above, we can show that the first order effects of changes in trade costs on aggregate productivity are given by expression (9).

B GDP inclusive of tariffs and Proposition 3

We now calculate changes in real GDP from the expenditure side, which are affected by changes in tariff revenues. Current-dollar absorption (which is equal to current-dollar consumption, given that we abstract from other sources of final demand such as investment and government expenditures) is given by

$$E_{it} = \sum_n \int_{\Omega_n} d_{nit} p_{nit}(z) q_{nit}(z) \, \mathrm{d}z.$$

Current-dollar GDP from the expenditure side is equal to consumption expenditures plus exports minus imports (exclusive of tariffs), which is also equal to GDP from the production side plus tariff revenues:

$$\begin{aligned} GDP^E_{it} &= E_{it} + \sum_{n\neq i}\int_{\Omega_i} p_{int}\left(z\right) y_{int}\left(z\right)/\tau_{int}\mathrm{d}z - \sum_{n\neq i}\int_{\Omega_n} p_{nit}\left(z\right) q_{nit}\left(z\right)\mathrm{d}z \\ &= GDP_{it} + \sum_{n}\int_{\Omega_n}\left(d_{nit}-1\right) p_{nit}\left(z\right) q_{nit}\left(z\right)\mathrm{d}z. \end{aligned} \tag{A.1}$$

Changes in real GDP from the expenditure side are calculated by deflating the change in current-dollar consumption expenditures by the consumer price index (CPI), nominal exports by the export price index (EPI), and nominal imports by the import price index (IPI). Up to a first-order approximation,

$$\begin{aligned} \Delta\log RGDP^E_{it} = &\frac{GDP_{it_0}}{GDP^E_{it_0}}\Delta\log GDP_{it} - \frac{E_{it_0}}{GDP^E_{it_0}}\Delta\log CPI_{it} - \frac{GDP_{it_0}\left(1-\lambda_{iit_0}\right)}{GDP^E_{it_0}}\Delta\log EPI_{it} \\ &+ \frac{\sum_{n\neq i}\int_{\Omega_n} d_{nit}\left(z\right) p_{nit}\left(z\right) q_{nit}\left(z\right)\left(\Delta\log d_{nit}\left(z\right) + \Delta\log\left(p_{nit}\left(z\right) q_{nit}\left(z\right)\right)\right)\mathrm{d}z}{GDP^E_{it_0}} \\ &- \frac{\sum_{n\neq i}\int_{\Omega_n} p_{nit_0}\left(z\right) q_{nit_0}\left(z\right)\left(\Delta\log\left(p_{nit}\left(z\right) q_{nit}\left(z\right)\right) - \Delta\log IPI_{it}\right)\mathrm{d}z}{GDP^E_{it_0}}. \end{aligned} \tag{A.2}$$

The log change in the export price index (EPI) is

$$\Delta logEPI_{it} = \sum_{n\neq i}\lambda_{int_0}\frac{\int_{\Omega_i}\lambda_{int_0}\left(z\right)\Delta\log\bar{p}_{int}\left(z\right)\mathrm{d}z}{\sum_{n\neq i}\lambda_{int_0}\int_{\Omega_i}\lambda_{int_0}\left(z\right)\mathrm{d}z}, \tag{A.3}$$

and the log change in the import price index (IPI) is:

$$\Delta logIPI_{it} = \sum_{n\neq i}\lambda_{nit_0}\frac{\int_{\Omega_n}\lambda_{nit_0}\left(z\right)\Delta\log\bar{p}_{nit}\left(z\right)\mathrm{d}z}{\sum_{n\neq i}\lambda_{nit_0}\int_{\Omega_i}\lambda_{nit_0}\left(z\right)\mathrm{d}z}. \tag{A.4}$$

Given these definitions, we can write the log change in the CPI in expression (12) as

$$\begin{aligned} E_{it_0}\Delta\log CPI_{it} = &GDP_{it_0}\left(\Delta\log PPI_{it} - \left(1-\lambda_{iit_0}\right)\Delta\log EPI_{it}\right) \\ &+ \sum_{n\neq i}\Lambda_{int_0}E_{nt_0}\int_{\Omega_i}\Lambda_{int_0}\left(z\right)\left(\Delta\log d_{nit}\left(z\right) + \Delta\log p_{nit}\left(z\right)\right)\mathrm{d}z. \end{aligned} \tag{A.5}$$

Substituting (A.3), (A.4) and (A.5) into (A.2), we obtain (10). Note also that expressions

(A.3) and (A.4) imply

$$\sum_i GDPI_{it_0} \Delta \log EPI_{it} = \sum_i GDPI_{it_0} \Delta \log IPI_{it},$$

which was used in the sketch of the proof of Proposition 3.

Appendix B Proofs and derivations in section III

A Calculating aggregate productivity using the direct valuation method

We derive the formula for real GDP and aggregate productivity using the direct valuation method (e.g. as the value of production at constant, t_0, prices). The ratio of real GDP in period T to period 0, evaluated at t_0 prices, is

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{\sum_n \int_{\Omega_{in1}} \frac{p_{int_0}(z)}{p_{in1}(z)} p_{in1}(z) y_{in1}(z) / \tau_{in1} \mathrm{dM_{it0}}(z)}{\sum_n \int_{\Omega_{in0}} \frac{p_{int_0}(z)}{p_{in0}(z)} p_{in0}(z) y_{in0}(z) / \tau_{in0} \mathrm{dM_{it0}}(z)}.$$

To use this method, we must observe price changes by destination at the level of individual goods. For goods sold from i to n in periods t_0 and t, the change in price is given by $\frac{p_{int}(z)}{p_{int_0}(z)} = \frac{\tau_{int} W_{it}}{\tau_{int_0} W_{it_0}}$. For goods that are not sold in one of the two periods, we assume that the statistical agency imputes a price change equal to the average price change for goods being sold into that destination, that is $\frac{p_{int}(z)}{p_{int_0}(z)} = \frac{\tau_{int} W_{it}}{\tau_{int_0} W_{it_0}}$. We obtain the same expressions, up to a first-order approximation, if we assume that for these goods that $\frac{p_{int}(z)}{p_{int_0}(z)} = \frac{PPI_{it}}{PPI_{it_0}}$ or $\frac{p_{int}(z)}{p_{int_0}(z)} = \frac{p_{iit}(z)}{p_{iit_0}(z)}$.[39] Using expression (20), we have

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{\sum_n \frac{\tau_{int_0}}{\tau_{in1}} \lambda_{in1}}{\sum_n \frac{\tau_{int_0}}{\tau_{in0}} \lambda_{in0}} \frac{L_{iT}}{L_{i0}}.$$

The Fisher quantity index for real GDP is a geometric average of real GDP based on date $t_0 = 0$ and $t_0 = 1$ prices (recall that $\tau_{int}/\tau_{int-1} = 1$ for $t > 1$):

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{RGDP_{i1}}{RGDP_{i0}} = \left(\frac{\sum_n \frac{\tau_{in0}}{\tau_{in1}} \lambda_{in1}}{\sum_n \frac{\tau_{in1}}{\tau_{in0}} \lambda_{in0}} \right)^{0.5} \frac{L_{iT}}{L_{i0}}. \tag{B.1}$$

[39] For large changes in trade costs, as considered in Table 1, the elasticity of real GDP in the small country is 0.24 instead of 0.26 under either of these two alternative assumptions.

The change in aggregate productivity is $A_{iT}/A_{i0} = \left(\frac{RGDP_{iT}}{RGDP_{i0}}\right) / \left(\frac{L_{iT}}{L_{i0}}\right)$. Up to a first order approximation, these expressions are equivalent to those based on the deflation method in the body of the paper. Following similar steps, the Fisher quantity index for consumption is given by

$$\frac{RC_{iT}}{RC_{i0}} = \left(\frac{\sum_n \Lambda_{ni1} \frac{\tau_{ni0}}{\tau_{ni1}} \frac{W_{n0}}{W_{n1}}}{\sum_n \Lambda_{ni0} \frac{\tau_{ni1}}{\tau_{ni0}} \frac{W_{n1}}{W_{n0}}}\right)^{0.5} \frac{P_{i1}C_i}{P_{i0}C_{i0}}.$$

B Preliminary derivations for Proposition 5, 6, Corollaries 2, 3

We first derive some equilibrium conditions that we use throughout the proofs. Variable profits of firm z selling from country i into destination n are:

$$\pi_{int}(z) = \frac{1}{\rho} p_{int}(z) q_{int}(z) = \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^{\rho} (\rho-1)^{1-\rho}} W_{it}^{1-\rho} a_{int}(z) P_{nt}^{\rho} C_{nt}. \tag{B.2}$$

We focus in interior equilibria in which only firms with $z \geq \bar{z}_{int}$ operate in destination n (so that $\Omega_{int} = \{z : z \geq \bar{z}_{int}\}$):

$$\pi_{int}(\bar{z}_{int}) - W_{it}^{1-\psi} W_{nt}^{\psi} h(\bar{z}_{int}, a_{int}(\bar{z}_{int})) = W_{it}^{1-\phi} W_{nt}^{\phi} f_{int}\,. \tag{B.3}$$

In expression (B.3) we have nested the case in which fixed costs and quality-related costs are incurred in the exporting country ($\phi = 0$, $\psi = 0$) and the case in which they are incurred in the importing country ($\phi = 1$, $\psi = 1$).

Firms choose destination-specific quality to maximize $\pi_{int}(z,a) - W_{it}^{1-\psi} W_{nt}^{\psi} h(z,a)$. Using the functional form for quality-adjusted costs $h(z,a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^{\gamma}$, the optimal quality choice is given by

$$\pi_{int}(z,a) = W_{it}^{1-\psi} W_{nt}^{\psi} a h'(z,a) = W_{it}^{1-\psi} W_{nt}^{\psi} \gamma h(z,a), \tag{B.4}$$

which implies

$$a_{int}(z) = \left(\frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^{\rho} (\rho-1)^{1-\rho}} \frac{W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt}}{W_{it}^{1-\psi} W_{nt}^{\psi} \gamma_0 \bar{h}(z)}\right)^{\frac{1}{\gamma-1}}. \tag{B.5}$$

The free entry condition in an equilibrium with positive entry is, using (B.2), given by

$$\sum_n \frac{\tau_{int}^{1-\rho} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt} Z_{int}}{\rho^{\rho} (\rho-1)^{1-\rho}} - \sum_n \int_{\bar{z}_{int}} \left(W_{it}^{1-\phi} W_{nt}^{\phi} f_{int} + W_{it}^{1-\psi} W_{nt}^{\psi} h_{int}(z, a_{int}(z))\right) \mathrm{d}G_i(z) \tag{B.6}$$
$$= (r+\delta) W_{it} f_{Ei},$$

where r is the net-interest rate in steady-state and $Z_{int} = \int_{\bar{z}_{int}} a_{int}(z) z^{\rho-1} \mathrm{d}G_i(z)$.

When fixed costs and quality-related costs are partly incurred in the importing country, the value of these services must be included in the importing country's GDP. Given these considerations, current-dollar GDP in this more general case is

$$GDP_{it} = \sum_n Y_{int} + \phi \sum_n \left[W_{nt}^{1-\phi} W_{it}^{\phi} f_{nit} \int_{\bar{z}_{int}} dM_{nt}(z) - W_{it}^{1-\phi} W_{nt}^{\phi} f_{int} \int_{\bar{z}_{int}} dM_{it}(z) \right] \tag{B.7}$$
$$+ \psi \sum_n \left[\int_{\bar{z}_{nit}} W_{nt}^{1-\psi} W_{it}^{\psi} h_{nit}(z, a_{nit}(z)) \, \mathrm{d}M_{nt}(z) - \int_{\bar{z}_{int}} W_{it}^{1-\psi} W_{nt}^{\psi} h_{int}(z, a_{int}(z)) \, \mathrm{d}M_{it}(z) \right].$$

C Proof of Proposition 5

We derive the steady-state change in world theoretical consumption in response to marginal changes in variable trade costs. We allow for fixed costs and quality-related costs to be incurred in the exporter's country or in the importer's country (or a combination of the two).

Log differentiating the free entry condition (B.6) with respect to variable trade costs, using condition (20), multiplying by M_{Eit}/δ, and using the first-order conditions for $\bar{z}_{int}$ and $a_{int}(z)$, we have

$$\sum_n \int_{\bar{z}_{int}} \left(\phi W_{it}^{1-\phi} W_{nt}^{\phi} f_{int} + \psi W_{it}^{1-\psi} W_{nt}^{\psi} h_{int}(z, a_{int}(z)) \right) \mathrm{d}M_{it}(z) \, \Delta\log\left(\frac{W_{nt}}{W_{it}}\right)$$
$$= \sum_n \frac{Y_{int}}{\rho} \left[(1-\rho) \Delta\log\tau_{int} - \rho \Delta\log W_{it} + (\rho - 1) \Delta\log P_{nt} + \Delta\log P_{nt} C_{nt} \right],$$

where $Y_{int} = \int_{\Omega_{int}} p_{int}(z) q_{int}(z) \, \mathrm{d}M_{it}(z)$. This can be re-written as

$$\frac{1-\rho}{\rho} \sum_n Y_{int} \Delta\log\tau_{int} C_{nt} = \sum_n \int_{\bar{z}_{int}} \left(\phi W_{it}^{1-\phi} W_{nt}^{\phi} f_{int} + \psi W_{it}^{1-\psi} W_{nt}^{\psi} h_{int}(z, a_{int}(z)) \right) \mathrm{d}M_{it}(z) \, \Delta\log\left(\frac{W_{nt}}{W_{it}}\right)$$
$$+ \sum_n Y_{int} \left(\Delta\log W_{it} - \Delta\log P_{nt} C_{nt} \right).$$

Adding across countries, multiplying by $1/E_{wt}$, using the definition of GDP_{it} in (B.7), and re-arranging we obtain

$$\frac{1}{E_{wt}}\sum_n E_{nt}\Delta\log C_{nt} = -\frac{1}{E_{wt}}\sum_i\sum_n \text{Exports}_{int} \times \Delta\log\tau_{int}$$
$$+\frac{\rho}{1-\rho}\frac{1}{E_{wt}}\left(\sum_i GDP_{it}\Delta\log W_{it} - \sum_n E_{nt}\Delta\log P_{nt}C_{nt}\right). \quad \text{(B.8)}$$

By condition (20), in response to changes in variable trade costs, $\Delta\log W_{it} = \Delta\log GDP_{it}$. Therefore, given balanced trade at the world level, the term in the second line is equal to zero, and the change in world theoretical consumption is given by equation (13).

D Proof of Proposition 6

We now derive the equivalence between real consumption and theoretical consumption, country-by-country, under the assumptions listed in Proposition 6. Here we assume that prices are not adjusted for quality. In Section E we consider the case in which prices are adjusted for quality.

We first present some preliminary steps. Under the assumptions of Proposition 6, aggregate fixed cost, quality related costs, aggregate revenues, the trade balance, and current dollar GDP are all proportional to aggregate labor payments. Specifically, with G_i Pareto ($G_i(z) = 1 - z^{-\theta}$ for $z \geq 1$), aggregate fixed costs are

$$\sum_n \frac{M_{Ei}}{\delta} W_{it}^{1-\phi} W_{nt}^{\phi} f_{int} \bar{z}_{int}^{-\theta} = \frac{(\theta+1-\rho)(\gamma-1)}{\rho\theta\gamma}\sum_n Y_{int}, \quad \text{(B.9)}$$

where M_{Ei} is the steady-sate level of entry that is unchanged with variable trade costs. Given the functional form (19), equations (B.2) and (B.4), aggregate quality-related costs are

$$\sum_n \int_{\Omega_{int}} W_{it}^{1-\psi} W_{nt}^{\psi} h(z, a_{int}(z))\, \mathrm{d}M_{it}(z) = \frac{1}{\rho\gamma}\sum_n Y_{int}. \quad \text{(B.10)}$$

If the ratio of trade balance (including trade in goods, and also payments to fixed costs and quality-related costs incurred abroad) to GDP in country i is a constant $\bar{\kappa}_i$, by equations (B.9), (B.10) and (20) we can write the trade balance as

$$(1+\phi\frac{(\theta+1-\rho)(\gamma-1)}{\rho\theta\gamma} + \frac{\psi}{\rho\gamma})(\sum_n Y_{nit} - \sum_n Y_{int}) = \bar{\kappa}_i GDP_{it}. \quad \text{(B.11)}$$

In addition, from equations (B.7), (B.9), (B.10), and (B.11) we can write aggregate revenues as

$$\sum_n Y_{int} = GDP_{it}(1-\bar{\bar{\kappa}}_i) = \frac{1-\bar{\bar{\kappa}}_i}{1-\kappa_i\left(1-\bar{\bar{\kappa}}_i\right)} W_{it} L_{it}, \tag{B.12}$$

where $\bar{\bar{\kappa}}_i = \bar{\kappa}_i/(1+(\phi\frac{(\theta+1-\rho)(\gamma-1)}{\rho\theta\gamma}+\frac{\psi}{\rho\gamma})^{-1})$. The second equality follows from the identity $GDP_{it} = W_{it}L_{it} + \Pi_{it}$ and condition (18). Finally, we note that under the functional form $h(z,a) = \frac{\gamma_0}{\gamma} z^{\eta} a^{\gamma}$, $\bar{Z}_{int} = \frac{\gamma-1}{\gamma}\int_{\bar{z}_{int}} z^{\frac{(\rho-1)\gamma-\eta}{\gamma-1}} \mathrm{d}G_i(z)$.

We now derive the result in Proposition 6. Substituting (B.12) in (B.11) and log-differentiating with respect to changes in variable trade costs (with L_i constant), we obtain

$$\sum_n Y_{nit} \Delta\log\left(Y_{nit}/W_{it}\right) = 0. \tag{B.13}$$

From the definition of Y_{nit}, equations (B.2) and (B.5) with $\psi = 1$, and the fact (derived in the online appendix) that in this specification M_{Ei} is constant in response to changes in variable trade costs, we can write $\Delta log Y_{nit}$ as

$$\Delta\log\left(Y_{nit}/W_{it}\right) = \frac{\gamma}{\gamma-1}\Delta\log\left[\tau_{ni}^{1-\rho}\frac{W_{nt}^{1-\rho}}{W_{it}^{1-\rho}}\frac{P_{it}^{\rho}}{W_{it}^{\rho}}C_{it}\bar{Z}_{nit}\right]. \tag{B.14}$$

Substituting (B.14) into (B.13), using fixed trade balance and equation (B.12), we have

$$\Delta\log C_{it} = -\sum_n \Lambda_{nit}\left(\Delta\log\tau_{nit} + \Delta\log W_{nt}/W_{it} + \frac{\Delta\log\bar{Z}_{nit}}{1-\rho}\right). \tag{B.15}$$

Log differentiating $\bar{Z}_{int}$ and (B.3) with respect to changes in variable trade costs, and setting $\phi = 1$, we obtain

$$\Delta\log\bar{Z}_{nit} = \bar{\gamma}\left(\Delta\log\tau_{nit} + \Delta\log W_{nt}/W_{it} + \Delta\log C_{it}\right), \tag{B.16}$$

where $\bar{\gamma} = \left(\frac{\gamma\rho-\gamma}{\gamma-1} + \frac{\theta(\gamma\rho-\gamma)}{\gamma-\gamma\rho+\eta}\right)$. Substituting (B.16) into (B.15), we obtain

$$\Delta\log C_{it} = -\sum_n \Lambda_{nit}\left(\Delta\log\tau_{nit} + \Delta\log W_{nt}/W_{it}\right). \tag{B.17}$$

This coincides with the change in real consumption when measured prices changes do not adjust for quality, given by the negative of equation (12) in the paper (setting W_{it} as the numeraire).

E Quality adjusted prices: Proofs of Corollary 2, 3 and statement in footnote 29

To a first order approximation, the difference between the change in the quality adjusted and non-quality adjusted PPI is:

$$\Delta \log PPI_{it}^{QA} - \Delta \log PPI_{it} = \sum_n \lambda_{int} \int_{\bar{z}_{int}} \lambda_{int}(z) \frac{1}{1-\rho} \Delta \log a_{int}(z) \, \mathrm{d}M_{it}(z). \tag{B.18}$$

The difference between the quality adjusted and the non-quality adjusted CPI is:

$$\Delta \log CPI_{nt}^{QA} - \Delta \log CPI_{nt} = \sum_i \bar{\Lambda}_{int} \int_{\bar{z}_{int}} \lambda_{int}(z) \frac{1}{1-\rho} \Delta \log a_{int}(z) \, \mathrm{d}M_{it}(z). \tag{B.19}$$

At the world level,

$$\begin{aligned} \sum_i GDP_i \left(\Delta \log PPI_{it}^{QA} - \Delta \log PPI_{it} \right) &= \sum_n E_{nt} \left(\Delta \log CPI_{nt}^{QA} - \Delta \log CPI_{nt} \right) \\ &= \frac{1}{1-\rho} \sum_i \sum_n \int_{\bar{z}_{int}} p_{int}(z) \, q_{int}(z) \, \Delta \log a_{int}(z) \, \mathrm{d}M_{it}(z). \end{aligned} \tag{B.20}$$

Footnote 29: World-level equivalence We first show that if productivities are Pareto distributed and $h(z,a)$ takes the form (19), the third term in expression (B.20) is zero. By equations (B.5) and (20) we can write (B.20) as

$$\begin{aligned} \sum_i \sum_n \int_{\bar{z}_{int}} p_{int}(z) \, q_{int}(z) \, \Delta log a_{int}(z) \, \mathrm{d}M_{it}(z) = & \frac{1-\rho}{\gamma-1} \sum_i \sum_n Y_{int} \left(\Delta \log \tau_{int} + \Delta \log C_{nt} \right) \\ & + \frac{\rho-\psi}{\gamma-1} \sum_n \sum_i Y_{int} \left(\Delta \log P_{nt} C_{nt} - \Delta \log W_{it} \right). \end{aligned}$$

The first term in the right hand side is equal to zero by Proposition (5). The second term in the right hand side is equal to zero by world trade balance.

Corollary 2: Quality-related costs incurred in the exporting country We show that if quality and fixed costs employ labor in the exporting country ($\psi = \phi = 0$), expression (B.18) is equal to zero. By equations (B.10) with $\psi = 0$ and (B.12) we have $\sum_n \int_{\bar{z}_{in}} h(z, a_{int}(z)) \, \mathrm{d}M_{it}(z) = \frac{1}{\rho\gamma} \frac{\bar{\kappa}_i L_{it}}{1-\kappa_i}$. Differentiating this expression, using (B.2), (B.4) and the fact (derived in the online appendix) that the mass of entering firms is fixed in response to changes in variable trade costs, we have that

$$\frac{1}{\gamma\rho}\sum_{n}\int_{\bar{z}_{in}} p_{int}(z)\, q_{int}(z)\, \Delta\log a_{int}(z)\, \mathrm{d}G_i(z) - \sum_{n} W_{it} h\left(\bar{z}_{int}, a_{int}(\bar{z}_{in})\right) \mathrm{d}G(\bar{z}_{int})\, d\bar{z}_{int} = 0.$$

By equations (B.4) and (B.3), the second term is proportional to $\sum_n W_{it} f_{in} \mathrm{d}G(\bar{z}_{int})\, d\bar{z}_{int}$. This term is zero (implying the result of this Corollary). This follows from plugging equation (B.12) into (B.9), which gives $\sum_n \frac{M_{Ei}}{\delta} f_{int} \bar{z}_{int}^{-\theta} = \frac{(\theta+1-\rho)(\gamma-1)}{\rho\theta\gamma} \frac{1-\bar{\bar{\kappa}}_i}{1-\kappa_i(1-\bar{\bar{\kappa}}_i)} L_{it}$ with $\psi = \phi = 0$, and differentiating with respect to changes in variables trade costs.

Corollary 3: Quality cost incurred in the importing country We show that if quality and fixed costs are incurred in the importing country ($\psi = \phi = 1$) expression (B.19) equals zero. We can write (B.19) as

$$\frac{1}{1-\rho}\sum_{i}\int_{\bar{z}_{int}} p_{int}(z)\, q_{int}(z)\, \Delta\log a_{int}(z)\, \mathrm{d}M_{it}(z) = \frac{1}{1-\rho}\sum_{i} Y_{int} \Delta\log\left(\tau_{in}^{1-\rho} \frac{W_{it}^{1-\rho}}{W_{nt}^{1-\rho}} C_{nt}^{1-\rho}\right),$$

where we used (B.5) combined with (20) and constant trade balance relative to GDP. By expression (B.17) (which is valid only when $\psi = \phi = 1$), we obtain the result.

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