Online Appendix for "Measured Aggregate Gains from International Trade"

Ariel Burstein (UCLA) and Javier Cravino (UCLA), January 2012

This online appendix derives our Result 6 in the paper for the extensions of the model that allow for endogenous quality and multiple factors of production. The logic to obtain these results is very similar to that used in Appendix E of the paper.

Endogenous quality

We first show our Result 6 for the extension of the model with endogenous quality. We assume that fixed cost of exporting are paid in the destination country and that the productivity distribution of entering firms is Pareto ($G_i(z) = 1 - z^{-\theta}$ for $z \geq 1$). A firm with productivity z choosing quality a_{in} in country n must use $h(z, a_{in}) = \frac{\gamma_0}{\gamma} z^{\mu} a_{in}^{\gamma}$ units of labor in country n. We also assume that trade is balanced every period, taking into account imports of goods, fixed trade costs, and innovation costs incurred in foreign destinations.

We start by deriving the optimal a_{int}. Variable profits are given by:

$$\pi_{int}(z) = \frac{a_{int} z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^{\rho} [\rho-1]^{1-\rho}} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt} \ .$$

The optimality condition for $a_{int}(z)$ is:

$$a_{int}(z) = \left[\frac{1}{\gamma_0} \left[\frac{\rho}{\rho-1} \right]^{1-\rho} \frac{1}{W_{nt}^{\rho}} \tau_{int}^{1-\rho} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt} \right]^{\frac{1}{\gamma-1}} z^{\frac{\rho-1-\mu}{\gamma-1}} \ ,$$

then, we can write variable profits as:

$$\pi_{int}(z) = z^{\frac{(\rho-1)\gamma-\mu}{\gamma-1}} \left[\frac{1}{\gamma_0 W_{nt}} \right]^{\frac{1}{\gamma-1}} \left[\frac{\tau_{int}^{1-\rho} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt}}{\rho^{\rho} [\rho-1]^{1-\rho}} \right]^{\frac{\gamma}{\gamma-1}} \ .$$

Note that:

$$\frac{M_{\mathrm{Ei}\,t}}{\delta} \int_{\Omega_{int}} \pi_{int} \mathrm{d}G_i = \frac{1}{\rho} Y_{int} = \frac{M_{\mathrm{Ei}\,t}}{\delta} \left[\frac{1}{\gamma_0 W_{nt}} \right]^{\frac{1}{\gamma-1}} \left[\frac{\tau_{int}^{1-\rho} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt}}{\rho^{\rho} [\rho-1]^{1-\rho}} \right]^{\frac{\gamma}{\gamma-1}} \bar{Z}_{int} \ , \tag{A1}$$

where we defined:

$$\bar{Z}_{int} = \int_{\Omega_{int}} z^{\frac{(\rho-1)\gamma-\mu}{\gamma-1}} \mathrm{d}G_i = \frac{\theta}{\theta + 1 - \bar{\rho}} \bar{z}_{int}^{\bar{\rho}-1-\theta} \tag{A2}$$

with $\bar{\rho} \equiv \frac{\gamma\rho-1-\mu}{\gamma-1}$.

Innovation costs incurred in country n are given by:

$$\begin{aligned} \frac{M_{\mathrm{Ei}\,t}}{\delta}\int_{\Omega_{in}}\frac{\gamma_0}{\gamma}W_{it}^{\varepsilon}W_{nt}^{1-\varepsilon}\bar{h}\left(z\right)a_{int}^{\gamma}\left(z\right)dG_i\left(z\right) &= \frac{1}{\gamma}\frac{M_{\mathrm{Ei}\,t}}{\delta}\int_{\Omega_{in}}\pi_{int}\left(z\right)dG_i\left(z\right) \\ &= \frac{1}{\gamma\rho}Y_{int}\ , \end{aligned} \tag{A3}$$

The expression that defines the cutoff $\bar{z}_{int}$ is now given by:

$$\frac{\gamma-1}{\gamma}\bar{z}_{int}^{\bar{\rho}-1-\theta}\left[\frac{1}{\gamma_0 W_{nt}}\right]^{\frac{1}{\gamma-1}}\left[\frac{\tau_{int}^{1-\rho}W_{it}^{1-\rho}P_{nt}^{\rho}C_{nt}}{\rho^{\rho}\left[\rho-1\right]^{1-\rho}}\right]^{\frac{\gamma}{\gamma-1}} = W_{nt}f_{int}\bar{z}_{int}^{-\theta}\ . \tag{A4}$$

Log differentiating $(A4)$, and $(A2)$ we obtain:

$$d\log\bar{Z}_{nit} = \frac{\left(\bar{\rho}-1-\theta\right)\gamma}{\gamma-1}\frac{1-\rho}{1-\bar{\rho}}\left[d\log\tau_{nit}+d\log W_{nt}/W_{it}+d\log C_{it}\right]\ . \tag{A5}$$

The condition of balanced trade in country i is:

$$\begin{aligned} &\sum_{n\neq i}Y_{int}+\sum_{n\neq i}\frac{M_{Ent}}{\delta}W_{it}\left[1-G_{it}\left(\bar{z}_{nit}\right)\right]f_{nit}+\frac{1}{\gamma\rho}\sum_{n\neq i}Y_{nit} \\ = &\sum_{n\neq i}Y_{nit}+\sum_{n\neq i}\frac{M_{Eit}}{\delta}W_{nt}\left[1-G_{it}\left(\bar{z}_{int}\right)\right]f_{int}+\frac{1}{\gamma\rho}\sum_{n\neq i}Y_{nit}\ , \end{aligned} \tag{A6}$$

where we used equation $(A3)$ in the appendix to write the innovation costs incurred abroad as $\frac{1}{\gamma\rho}\sum_{n\neq i}Y_{nit}$. From equations $(A3)$ and $(A12)$, we obtain $(A19)$. Hence, balanced trade in any country implies balanced trade in fixed export cost services, innovation costs services and in goods in that country.

Balanced trade in fixed costs and innovation services implies $Y_{it} = GDP_{it}$, so equation$(A20)$ holds. Using $\frac{W_{it}}{P_{it}} = \frac{C_{it}}{\bar{L}_i}\left(1-\kappa_i\right)$ from (30) in the paper and balanced trade in goods we can re-express $(A1)$ as:

$$Y_{int} = \hat{\varphi}W_{it}\left[\frac{W_{it}}{W_{nt}}\right]^{\frac{1}{\gamma-1}}\left[\tau_{int}^{1-\rho}S_{int}C_{it}^{1-\rho}\right]^{\frac{\gamma}{\gamma-1}}\bar{Z}_{int}\ ,$$

where $\hat{\varphi} \equiv \left[\frac{1}{\gamma_0}\right]^{\frac{1}{\gamma-1}} \left[\left(\frac{\bar{L}_i}{1-\kappa_i}\right) / \left[\rho^{\rho}\left[\rho-1\right]^{1-\rho}\right]\right]^{\frac{\gamma}{\gamma-1}}$ is a constant. Log-differentiating we obtain:

$$\begin{aligned} d\log Y_{int}/W_{it} &= \frac{\gamma}{\gamma-1}\left[(1-\rho)\, d\log \tau_{int} + d\log S_{int} + (1-\rho)\, d\log C_{it}\right] \\ &\quad -\frac{1}{\gamma-1} d\log \frac{W_{nt}}{W_{it}} + d\log \bar{Z}_{int}\ , \end{aligned} \tag{A7}$$

substituting into $(A20)$ yields:

$$d\log C_{it} = -\sum_n \frac{Y_{int}}{Y_{it}} \left[\left[d\log \tau_{int} + \frac{d\log S_{int}}{1-\rho}\right] - \frac{1}{\gamma(1-\rho)} d\log \frac{W_{nt}}{W_{it}} + \frac{\gamma-1}{\gamma}\frac{d\log \bar{Z}_{int}}{1-\rho}\right] .$$

Log differentiating $(A19)$, substituting $(A7)$, and some algebra gives:

$$\begin{aligned} &\sum_{n\neq i} Y_{int}\left[\left[d\log \tau_{int} + d\log S_{int}\right] - \frac{1}{\gamma(1-\rho)} d\log \frac{W_{nt}}{W_{it}} + \frac{\gamma-1}{\gamma}\frac{d\log \bar{Z}_{int}}{1-\rho}\right] \\ = &\sum_{n\neq i} Y_{nit}\left[d\log \tau_{nit} + d\log \frac{W_{nt}}{W_{it}} + \frac{\gamma-1}{\gamma}\frac{d\log \bar{Z}_{nit}}{1-\rho}\right] \end{aligned}$$

then:

$$d\log C_{it} = -\sum_n \frac{Y_{nit}}{Y_{it}}\left[d\log \tau_{nit} + d\log \frac{W_{nt}}{W_{it}} + \frac{\gamma-1}{\gamma}\frac{d\log \bar{Z}_{nit}}{1-\rho}\right] . \tag{A8}$$

Substituting $(A5)$ into $(A8)$ and using balanced trade in goods:

$$d\log C_{it} = -\sum_n \frac{E_{nit}}{E_{it}}\left[d\log \tau_{nit} + d\log W_{nt}/W_{it}\right]\ ,$$

where $E_{int} = Y_{int}$. Which coincides with the change in real consumption when prices indices do not adjust for quality changes, as derived in equation $(A25)$ in the paper.

Multiple factors of production

We now show that our result 6 extends to the model with multiple factors of production. Note again that with Pareto distributed productivities, aggregate fixed costs of exporting are proportional to aggregate revenues as given in equation $(A13)$ in the paper. Then, balanced trade implies balanced trade both in goods and in services, and equation $(A19)$ holds. Further, from equation $(A28)$, we know that revenues are proportional to GDP and to aggregate labor payments, so equation $(A20)$ holds. From CES production we still have:

$$\frac{M_{\mathrm{Ei}\,t}}{\delta}\int_{\Omega_{int}}\pi_{int}\mathrm{d}G_i=\frac{1}{\rho}Y_{int}\ .$$

Log-differenciating equation $(A31)$ in the paper we get:

$$\begin{aligned}d\log Y_{int}/W_{it} &= (1-\rho)\,d\log\tau_{int}+(1-\rho)\,d\log\left[c_{it}/W_{it}\right]+d\log S_{int}\\ &\quad +d\log Z_{int}+(1-\rho)\,d\log C_{it}\ ,\end{aligned} \tag{A9}$$

substituting into $(A20)$ yields:

$$d\log C_{it}=-\sum_n\frac{Y_{int}}{Y_{it}}\left[d\log\tau_{int}+d\log\left[c_{it}/W_{it}\right]+\frac{d\log S_{int}}{1-\rho}+\frac{d\log Z_{int}}{1-\rho}\right]\ .$$

Log differentiating equation $(A19)$ in the paper, substituting $(A9)$, and some algebra gives:

$$\begin{aligned}&\sum_n Y_{int}\left[d\log\tau_{int}+d\log\left[c_{it}/W_{it}\right]+\frac{d\log S_{int}}{1-\rho}+\frac{d\log Z_{int}}{1-\rho}\right]\\ =&\sum_n Y_{nit}\left[d\log\tau_{nit}+d\log\left[c_{nt}/W_{nt}\right]+\frac{d\log Z_{nit}}{1-\rho}+d\log\frac{W_{nt}}{W_{it}}\right]\ ,\end{aligned}$$

then:

$$d\log C_{it}=-\sum_n\frac{Y_{nit}}{Y_{it}}\left[d\log\tau_{nit}+d\log\left[c_{nt}/W_{nt}\right]+\frac{d\log Z_{nit}}{1-\rho}+d\log\frac{W_{nt}}{W_{it}}\right]\ . \tag{A10}$$

Finally, using the Pareto form for G, we have:

$$Z_{int}=\frac{\theta}{\theta+1-\rho}\bar{z}_{int}^{\rho-1-\theta}\ ,$$

the export cutoff is given by:

$$\frac{z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}\left[\rho-1\right]^{1-\rho}}c_{it}\left[\frac{c_{it}}{P_{nt}}\right]^{-\rho}Q_{nt}=W_{nt}f_{int}\ .$$

Log-differentiating the last two equations and using equation $(A29)$ in the paper we obtain:

$$d\log Z_{nit}=(\rho-1-\theta)\left[d\log\tau_{nit}+d\log\frac{c_{nt}}{W_{nt}}+d\log\frac{W_{nt}}{W_{it}}+d\log C_{it}\right]\ ,$$

substituting into $(A10)$ and using balanced trade in goods:

$$d\log C_{it} = -\sum_{n} \frac{E_{nit}}{E_{it}} \left[d\log\tau_{nit} + d\log c_{nt}/W_{nt} + d\log W_{nt}/W_{it} \right]$$

where $E_{int} = Y_{int}$.

As shown in Appendix D, the change in the CPI is given by $d\log CPI_{it} = \sum_n \frac{E_{nit}}{E_{it}} d\log CPI_{nit}$, so the change in real consumption is given by expression (18) in the paper. With multiple factors of production,

$$d\log CPI_{nit} = d\log\tau_{nit} + d\log c_{nt} \ .$$

From equation $(A28)$ and balanced trade, $d\log E_{it} = d\log W_{it}$. Substituting in equation (18) , we obtain:

$$d\log RC_{it} = -\sum_{n} \frac{E_{nit}}{E_{it}} \left[d\log\tau_{nit} + d\log c_{nt}/W_{nt} + d\log W_{nt}/W_{it} \right] \ ,$$

which coincides with $d\log C_{it}$.