Online Appendix: Measured Aggregate Gains from International Trade

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In this online appendix we derive additional results discussed in the paper. In the first section, we include two extensions of the model with perfect competition of Section 3. First, we incorporate intermediate inputs into the production function. Second, we consider the case in which one country operates the trading technologies of all countries. For each of these two extensions, we derive real GDP from the production side and from the expenditure side, and show the equivalence between real GDP and real consumption at the world level. In deriving our second extension, we consider the case (discussed in Footnote 16) where the trading technology is operated by intermediaries rather than by the producers themselves.

In the second section we show that our assumption in equation (19) of the paper that aggregate profits represent a constant share of total revenues ($\Pi_{it} = \kappa_i Y_{it}$, where $Y_{it} = \sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}$), is satisfied in the remaining two special cases of our model described in Section 4. In deriving these results, we show that under the assumptions of Colloraries 2 and 3, in response to changes in variable trade costs the mass of entering firms is constant across steady states.

1 General Production Technologies with Perfect Competition

1.1 Allowing for intermediate inputs

In this section we extend our model in Section 3 to allow for intermediate inputs. We allow for a general specification in which each good z can be used both for consumption and as an intermediate input. We denote by $q^c_{ni}(z)$ and $q^m_{ni}(z)$ the quantities of good z from country n used in country i for consumption and as intermediate inputs. Market clearing implies $q_{ni}(z){=}q^c_{ni}(z)+q^m_{ni}(z)$.

1.1.1 Real GDP, production side

We start by deriving expression (8) in the paper. The profit maximization problem is given by:

$$\Pi_{it} = \max_{\{y_{in}(z), x_i(z)\}} \int_{\Omega_i} \sum_n p_{int}(z)\, y_{in}(z)/\tau_{int} - W_{it} x_i(z)\,\mathrm{d}z - \sum_n \int_{\Omega_n} d_{nit} p_{nit}(z)\, q^m_{nit}(z)\,\mathrm{d}z \tag{1}$$

subject to $\left\{\sum y_{in}(z), q^m_{nit}(z), x_i(z)\right\} \in Y_i(z)$ for all z.

Current-dollar GDP is given by:

$$GDP_{it} = \sum_n \int_{\Omega_i} p_{int}(z)\, y_{int}(z)/\tau_{int}\mathrm{d}z - \sum_n \int_{\Omega_n} p_{nit}(z)\, q^m_{nit}(z)\,\mathrm{d}z.$$

Real GDP from the production side is calculated using the double deflation method, which consist in deflating output and intermediate inputs using different price indexes. In particular, we calculate real GDP as:

$$RGDP_{it} = \frac{\sum_n \int_{\Omega_i} p_{int}(z)\, y_{int}(z)/\tau_{int}\mathrm{d}z}{PPI_{it}} - \frac{\sum_n \int_{\Omega_n} p_{nit}(z)\, q^m_{nit}(z)\,\mathrm{d}z}{MPI_{it}}.$$

Where MPI_{it} denotes the input price index in country i. To a first order approximation, the change in RGDP is given by:

$$\begin{aligned} dlogRGDP_{it} &= \sum_n \int_{\Omega_i} \frac{p_{int_0}(z)\, y_{int_0}(z)/\tau_{int_0}}{GDP_{it_0}} \left(\mathrm{dlog}\left(p_{int}(z)\, y_{int}(z)/\tau_{int}\right) - \mathrm{dlog}PPI_{it}\right)\mathrm{d}z \\ &- \sum_n \int_{\Omega_n} \frac{p_{nit_0}(z)\, q^m_{nit_0}(z)}{GDP_{it_0}} \left(p_{nit_0}(z)\, q^m_{nit_0}(z) - \mathrm{dlog}MPI_{it}\right)\mathrm{d}z. \end{aligned} \tag{2}$$

The change in the producer price index is given by equation (7) in the paper. The change in the input price index is given by:

$$dlogMPI_{it} \equiv \sum_n \int_{\Omega_n} \lambda^m_{int}(z)\, dlogp_{nit}(z)\,\mathrm{d}z.$$

where $\lambda^m_{int_0}(z) = \frac{p_{nit_0}(z) q^m_{nit_0}(z)}{\sum_n \int_{\Omega_n} p_{nit_0}(z) q^m_{nit_0}(z)}$. Substituting dlog$PPI_{it}$ and dlogMPI_{it} in expression (2), and following the steps used in Proposition 1 in the paper, we can write the change in aggregate productivity as:

$$\begin{aligned} \text{dlog} A_{it} &= \text{dlog} RGDP_{it} - \frac{W_{it_0} X_{it_0}}{GDP_{it_0}} \text{dlog} X_{it} \\ &= \sum_n \lambda_{int_0} \int_{\Omega_i} \lambda_{int_0}(z) \left(\text{dlog} p_{int}(z) - \text{dlog} \tau_{int} - \text{dlog} \bar{p}_{int}(z) \right) \text{d}z \qquad (3) \\ &+ \sum \left[d_{nit_0} - 1 \right]_n \int_{\Omega_n} \frac{p_{nit_0}(z) q^m_{nit_0}(z)}{GDP_{it_0}} \text{dlog} q^m_{nit_0}(z) \text{d}z. \end{aligned}$$

Note that in the absence of tariffs on intermediate inputs, equation (3) coincides with expression (8) in the paper.

1.1.2 Real GDP, expenditure side

We now derive the equivalence between real GDP from the production and the expenditure side. To a first order approximation, the change in real GDP from the expenditure side is given by,

$$\begin{aligned} GDP^E_{it_0} dlog RGDP^E_{it} = &\sum_n \int_{\Omega_i} d_{nit_0} p_{nit_0}(z) q^c_{nit_0}(z) \left(\text{dlog} \left(d_{nit} p_{nit}(z) q^c_{nit}(z) \right) - \text{dlog} CPI_{it} \right) \text{d}z \\ &+ \sum_{n \neq i} \int_{\Omega_n} p_{int_0}(z) q_{int_0}(z) \left(\text{dlog} \left(p_{int}(z) y_{int}(z) / dlog \tau_{int} \right) - \text{dlog} EPI_{it} \right) \text{d}z \\ &- \sum_{n \neq i} \int_{\Omega_n} p_{nit_0}(z) q_{nit_0}(z) \left(\text{dlog} \left(p_{nit}(z) q_{nit}(z) \right) - dlog IPI_{it} \right) \text{d}z. \qquad (4) \end{aligned}$$

The term in the first line is the change in nominal expenditures, deflated by the CPI. The second and third lines denote exports deflated by the EPI and imports deflated by the IPI respectively.

We can write the consumer price index as:

$$E_{it_0} d \log CPI_{it} = \sum_n \int_{\Omega_i} d_{nit_0} p_{nit_0}(z) q^c_{nit_0}(z) \left(\text{dlog} d_{nit}(z) + \text{dlog} p_{nit}(z) \right) \text{d}z \qquad (5)$$

The log change in the export price index (EPI) can be written as,

$$\sum_{n \neq i} \int_{\Omega_n} p_{int_0}(z) q_{int_0}(z) \, \text{dlog} EPI_{it} = \sum_{n \neq i} \int_{\Omega_n} p_{int_0}(z) q_{int_0}(z) \, \text{dlog} \bar{p}_{int}(z) \, \text{d}z, \qquad (6)$$

and the log change in the import price index (IPI) can be written as:

$$\sum_{n\neq i}\int_{\Omega_n} p_{nit_0}(z)\, q_{nit_0}(z)\, \mathrm{dlog} IPI_{it} = \sum_{n\neq i}\int_{\Omega_n} p_{nit_0}(z)\, q_{nit_0}(z)\, \mathrm{dlog} p_{nit}(z)\, \mathrm{d}z. \tag{7}$$

Substituting (5), (6) and (7) into equation (4), and using $q_{ni}(z){=}q^c_{ni}(z)+q^m_{ni}(z)$ we obtain:

$$\begin{aligned} GDP^E_{it_0} dlogRGDP^E_{it} = &\sum_{n\neq i}\int_{\Omega_i}\left[d_{nit_0}-1\right] p_{nit_0}(z)\, q^c_{nit_0}(z)\, \mathrm{dlog} q^c_{nit}(z) \\ &+\sum_{n}\int_{\Omega_n} p_{int_0}(z)\, q_{int_0}(z)\, \mathrm{dlog} y_{int}(z) - \sum_{n}\int_{\Omega_n} p_{nit_0}(z)\, q^m_{nit_0}(z)\, \mathrm{dlog} q^m_{nit}(z)\, \mathrm{d}z \\ &+\sum_{n\neq i}\int_{\Omega_n} p_{int_0}(z)\, q_{int_0}(z)\left(\mathrm{dlog} p_{int}(z) - \mathrm{dlog}\tau_{int} - \mathrm{dlog}\bar{p}_{nit}(z)\right)\mathrm{d}z. \end{aligned}$$

This can be written as:

$$dlogRGDP^E_{it} = \frac{1}{GDP^E_{it_0}}\mathrm{dlog} RGDP_{it} + \frac{1}{GDP^E_{it_0}}\sum_{n\neq i}\int_{\Omega_i}\left[d_{nit_0}-1\right] p_{nit_0}(z)\, q^c_{nit_0}(z)\, \mathrm{dlog} q^c_{nit}(z).$$

1.1.3 World-level equivalence between real consumption and Real GDP

From the proof of Proposition 4 in the paper, the equivalence between world real GDP and world real consumption holds if the world import price index equals the world export price index. This follows directly from equations (6) and (7) for the case in which each country operates its own trading technology. The case in which one country operates the trading technologies of all the other countries is presented below.

1.2 Trading technologies concentrated in one country

We now consider the case in which there is a sector in country i_s that operates the trading technologies of every country. The section also shows how to extend our results when the trading technology is operated by intermediaries instead of the good producers themselves.

1.2.1 Real GDP, production side

Current-dollar GDP in country i_s is given by:

$$\begin{aligned} GDP_{i_st} = & \sum_n \int_{\Omega_{i_s}} p_{i_snt}(z)/\tau_{i_snt} y_{i_snt}(z)\,\mathrm{d}z \\ & + \sum_i \sum_{n \neq i} \int_{\Omega_i} p_{int}(z)\, y_{int}(z)/\tau_{int} \mathrm{d}z - \sum_i \sum_{n \neq i} \int_{\Omega_i} p_{int}(z)/\tau_{int} y_{int}(z)\,\mathrm{d}z. \end{aligned} \tag{8}$$

The term in the first line denotes value added of goods producers from country i_s. The two terms in the second line capture value added in the sector that operates the trading technology. This sector purchases the production from each source country i that is bound to foreign countries $n \neq i$ at prices $p_{int}(z)/\tau_{int}$, and resells a fraction $1/\tau_{int}$ of this production in each destination country n at prices $p_{int}(z)$.

Computing real GDP from the production side implies using the double deflation method to deflate value added the sector operating the trading technology. That is:

$$\begin{aligned} \frac{RGDP_{i_st}}{RGDP_{i_st-1}} = & \Bigg(\frac{\sum_n \int_{\Omega_{i_s}} p_{i_snt}(z)/\tau_{i_snt} y_{i_snt}(z)\,\mathrm{d}z}{PPI^g_{i_st}} + \frac{\sum_i \sum_{n \neq i} \int_{\Omega_i} p_{int}(z)\, y_{int}(z)/\tau_{int} \mathrm{d}z}{PPI^s_{i_st}} \\ & - \frac{\sum_i \sum_{n \neq i} \int_{\Omega_i} p_{int}(z)/\tau_{int} y_{int}(z)\,\mathrm{d}z}{MPI^s_{i_st}}\Bigg)/GDP_{i_st-1}. \end{aligned}$$

Here, $PPI^g_{i_st}$ denotes the producer price index of the goods sector in country i, while $PPI^s_{i_st}$ and $MPI^s_{i_st}$ denote the producer and the input price index of the sector operating the trading technology. Taking a first order approximation around time $t = t_0$ and using the fact that all deflators are normalized to equal 1 at date t_0 we obtain:

$$\begin{aligned} dlogRGDP_{i_st} = & \sum_n \lambda_{i_snt_0} \int_{\Omega_{i_s}} \lambda_{i_snt_0}(z) \left(\mathrm{dlog} p_{i_snt}(z) - \mathrm{dlog}\tau_{i_snt} + \mathrm{dlog} y_{i_snt}(z) - \mathrm{dlog} PPI^g_{i_st}\right) \mathrm{d}z \\ & + \frac{\sum_i \sum_{n \neq i} \int_{\Omega_i} p_{int_0}(z)\, y_{int_0}(z)/\tau_{int_0} \mathrm{d}z}{GDP_{i_st_0}} \left(\mathrm{dlog} MPI^s_{i_st} - \mathrm{dlog} PPI^s_{i_st}\right). \end{aligned} \tag{9}$$

Following the steps used in Proposition 1 in the paper, the first line equals $\frac{W_{i_st_0} X_{i_st_0}}{GDP_{i_st_0}} dlogX_{i_st}$, since prices in $PPI^g_{i_st}$ do not include changes in trade costs τ_{i_snt}. The second line captures the difference between input prices and final prices in the operating the trading technology. Noting that final prices in the $PPI^s_{i_st}$ equal $p_{int}(z)$, and that input prices in the MPI^s_{it}

equal $p_{int}(z)/\tau_{int}$, we obtain:

$$\text{dlog}MPI^{s}_{i_st} - \text{dlog}PPI^{s}_{ist} = \frac{-\sum_i \sum_{n\neq i} \int_{\Omega_i} p_{int_0}(z)\, q_{int_0}(z)\, \text{dlog}\tau_{int}\text{d}z}{\sum_i \sum_n \int_{\Omega_i} p_{int_0}(z)\, q_{int_0}(z)}. \tag{10}$$

Substituting into equation (8) in the paper we can write the change in aggregate productivity as:

$$\text{dlog}A_{i_st} = -\sum_i \frac{GDP_{it_0}}{GDP_{i_st_0}} \sum_{n\neq i} \lambda_{int_0} \text{dlog}\tau_{int}.$$

1.2.2 Real GDP, expenditure side

We now show the relation in Proposition 2 between changes in real GDP from the production and the expenditure side. We assume that tariffs d_{ni_st} only apply to goods that imported to be consumed in country i_s (and not to goods that are imported by intermediaries in country i_s to be resold in other countries). GDP from the expenditure side for country i_s is given by:

$$\begin{aligned} GDP^{E}_{i_st} = & \sum_n \int_{\Omega_n} d_{ni_st} p_{ni_st}(z)\, q_{ni_st}(z)\, \text{d}z + \sum_i \sum_{n\neq i, n\neq i_s} \int_{\Omega_i} p_{int}(z)\, y_{int}(z)\, /\tau_{int}\text{d}z. \\ & - \sum_{i\neq i_s} \sum_{n\neq i} \int_{\Omega_n} p_{int}(z)\, /\tau_{int} y_{int}(z)\, \text{d}z. \end{aligned}$$

The first terms in this expression denotes current-dollar expenditures in country i_s, inclusive of tariffs. The second term denotes exports from country i_s, which equal the foreign sales from each source country i, except those bound to country i_s. The last term denotes imports of i_s, which equal expenditures in foreign goods in every country n, excluding expenditure in goods produced in county i_s. To a first-order approximation, the change in real GDP from the expenditure side is:

$$\begin{aligned} GDP^{E}_{i_st_0} dlogRGDP^{E}_{i_st} = & \sum_n \int_{\Omega_n} d_{ni_st_0} p_{ni_st_0}(z)\, q_{nit_{s0}}(z) \left(\text{dlog}\left(d_{ni_st} p_{ni_st}(z)\, q_{ni_st}(z)\right) - \text{dlog}CPI_{i_st}\right) \text{d}z \\ & + \sum_i \sum_{n\neq i, n\neq i_s} \int_{\Omega_i} p_{int_0}(z)\, q_{int_0}(z) \left(\text{dlog}\left(p_{int}(z)\, q_{int}(z)\right) - \text{dlog}EPI_{i_st}\right) \text{d}z \\ & - \sum_{i\neq i_s} \sum_{n\neq i} \int_{\Omega_n} p_{int_0}(z)\, q_{int_0}(z) \left(\text{dlog}\left(p_{int}(z)\, q_{int}(z)\right) - \text{dlog}IPI_{i_st}\right) \text{d}z. \end{aligned} \tag{11}$$

We re-write the log-change in the consumer price index in country i_s given by equation

(13) in the paper:

$$d\log CPI_{i_st}=\sum_n\int_{\Omega_{i_s}}\frac{d_{ni_st_0}p_{ni_st_0}\left(z\right)q_{ni_st_0}\left(z\right)}{E_{i_st_0}}\left(dlogd_{ni_st}+dlogp_{ni_st}\left(z\right)\right)\mathrm{d}z. \tag{12}$$

All export prices for country i_s include trade costs. The log change in the EPI is:

$$dlogEPI_{i_st}=\frac{\sum_i\sum_{n\neq i,n\neq i_s}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\mathrm{dlog}p_{int}\left(z\right)\mathrm{d}z}{\sum_i\sum_{n\neq i,n\neq i_s}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\mathrm{d}z}. \tag{13}$$

In contrast, import prices do not include trade costs. The log change in the IPI is,

$$d\log IPI_{it}=\frac{\sum_{i\neq i_s}\sum_{n\neq i}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\left(\mathrm{dlog}p_{int}\left(z\right)-\mathrm{dlog}\tau_{int}\right)\mathrm{d}z}{\sum_{i\neq i_s}\sum_{n\neq i}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\mathrm{d}z}. \tag{14}$$

Substituting (8), (12), (13) and (14) in expression (11) we can write:

$$\begin{aligned}GDP^E_{i_st_0}\mathrm{dlog}RGDP^E_{i_st}&=\sum_n\int_{\Omega_n}d_{ni_st_0}p_{ni_st_0}\left(z\right)q_{ni_st_0}\left(z\right)\mathrm{dlog}q_{ni_st}\left(z\right)\mathrm{d}z\\&+\sum_i\sum_{n\neq i,n\neq i_s}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\mathrm{dlog}q_{int}\left(z\right)\mathrm{d}z\\&-\sum_{i\neq i_s}\sum_{n\neq i}\int_{\Omega_n}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\left(\mathrm{dlog}q_{int}\left(z\right)+\mathrm{dlog}\tau_{int}\right)\mathrm{d}z.\end{aligned}$$

Adding and subtracting $\sum_{n\neq i_s}\int_{\Omega_n}p_{ni_st_0}\left(z\right)q_{nit_{s0}}\left(z\right)dlogq_{ni_st}\left(z\right)\mathrm{d}z/GDP^E_{i_st_0}$ and $\sum_{n\neq i_s}\int_{\Omega_i}p_{i_snt_0}\left(z\right)q_{int_{s0}}\left(z\right)dlogy_{i_snt}\left(z\right)\mathrm{d}z/GDP^E_{i_st_0}$ we obtain:

$$\begin{aligned}GDP^E_{i_st_0}dlogRGDP^E_{i_st}&=\sum_n\int_{\Omega_i}p_{i_snt_0}\left(z\right)q_{int_{s0}}\left(z\right)\mathrm{dlog}y_{i_snt}\left(z\right)\mathrm{d}z\\&+\sum_{n\neq i_s}\int_{\Omega_n}\left[d_{ni_st_0}-1\right]p_{ni_st_0}\left(z\right)q_{ni_st_0}\left(z\right)\mathrm{dlog}q_{ni_st}\left(z\right)\mathrm{d}z\\&+\sum_i\sum_{n\neq i}\int_{\Omega_i}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\mathrm{dlog}q_{int}\left(z\right)\mathrm{d}z\\&-\sum_i\sum_{n\neq i}\int_{\Omega_n}p_{int_0}\left(z\right)q_{int_0}\left(z\right)\left(\mathrm{dlog}q_{int}\left(z\right)+\mathrm{dlog}\tau_{int}\right)\mathrm{d}z.\end{aligned}$$

Re-arranging, using equations (9) and (10), we can write:

$$dlogRGDP^{E}_{i_s t} = \frac{GDP_{i_s t_0}}{GDP^{E}_{i_s t_0}} \text{dlog} RGDP_{i_s t} + \frac{\sum_{n \neq i_s} \int_{\Omega_n} \left[d_{n i_s t_0} - 1 \right] p_{n i_s t_0} \left(z \right) q_{n i_s t_0} \left(z \right) \text{dlog} q_{n i_s t} \left(z \right) \text{d}z}{GDP^{E}_{i_s t_0}}. \tag{15}$$

which coincides with expression (11) in the paper.

1.2.3 World level real consumption and real GDP

We now show the equivalence between real consumption and real GDP from the expenditure side at the world level. As in Proposition 4 in the paper, we need to show that, to a first approximation, the change in the world's import price index equals the world export price index. The change in the world import price index is given by:

$$\begin{aligned} \sum_i Imports_{it_0} dlogIPI_{it} &= \sum_{i \neq i_s} Imports_{it_0} \text{dlog} IPI_{it} + Imports_{i_s t_0} \text{dlog} IPI_{i_s t} \\ &= \sum_n \sum_{i \neq i_s, i \neq n} \int_{\Omega_n} p_{nit} \left(z \right) y_{nit} \left(z \right) / \tau_{nit} \text{dlog} p_{nit} \left(z \right) \text{d}z \\ &+ \sum_{n \neq i_s} \sum_{n \neq i} \int_{\Omega_n} p_{nit} \left(z \right) y_{nit} \left(z \right) / \tau_{nit} \left(\text{dlog} p_{nit} \left(z \right) - \text{dlog} \tau_{nit} \right) \text{d}z. \end{aligned} \tag{16}$$

The change in the export price index is given by:

$$\begin{aligned} \sum_i Exports_{it_0} \text{dlog} EPI_{it} &= \sum_{i \neq i_s} Exports_{it_0} \text{dlog} EPI_{it} + Exports_{i_s t_0} \text{dlog} EPI_{i_s t} \\ &= \sum_{n \neq i_s} \sum_{, n \neq i} \int_{\Omega_n} p_{nit_0} \left(z \right) y_{nit_0} \left(z \right) / \tau_{nit} \left(\text{dlog} p_{nit} \left(z \right) - \text{dlog} \tau_{nit} \right) dz \\ &+ \sum_n \sum_{i \neq n, i \neq i_s} \int_{\Omega_n} p_{nit_0} \left(z \right) y_{nit_0} \left(z \right) / \tau_{nit} \text{dlog} p_{nit} \left(z \right) dz. \end{aligned} \tag{17}$$

Expressions (16) and (17) coincide. From expression equation (11) in the paper and (15), the change in world GDP is given by:

$$\begin{aligned}\mathrm{d}\log RGDP_{wt}^{E} &= \frac{-1}{E_{wt_0}}\sum_i\sum_n Exports_{int_0}\mathrm{d}\log\tau_{int}dz \\ &+ \frac{1}{E_{wt_0}}\sum_i W_{it_0}X_{it_0}\mathrm{d}\log X_{it} \\ &+ \frac{1}{E_{wt_0}}\sum_i\sum_{n\neq i}\int_{\Omega_i}\left[d_{nit_0}-1\right]p_{nit_0}(z)\,q_{nit_0}(z)\,\mathrm{d}\log q_{nit}(z)\,dz.\end{aligned}$$

which coincides with expression (14) in the paper.

2 Proofs and Derivations: Monopolistic competition, heterogeneous firms and quality

2.1 Deriving the share of profits in total revenues

We show that our assumption in equation (19) of the paper that aggregate profits represent a constant share of total revenues ($\Pi_{it}=\kappa_i Y_{it}$, where $Y_{it}=\sum_n\int_{\Omega_{int}}p_{int}q_{int}\mathrm{d}M_{it}$), is satisfied in the remaining two special cases of our model described in Section 4. In the first case, there are no fixed costs of selling into individual countries so that all firms sell in each country. In the second case, there are positive fixed costs of selling in individual countries (incurred in either the exporting or importing country) and productivities are Pareto distributed. We derive equation (19) for the general case in which a fraction $1-\phi$ of these fixed costs are incurred in the exporting country and a fraction ϕ of these fixed costs are incurred in the importing country. We also allow for a fraction $1-\psi$ of the innovation costs to be incurred in the exporting country and a fraction ψ to be incurred in the importing country. The baseline model in the body of the paper assumes $\phi=\psi=0$. We also show that, in these two cases, the mass of firms is unchanged following a trade liberalization. Remember that in the third special case described in Section 4, when $r\to 0$, it is straightforward to show that the free entry condition implies that $\kappa_i=0$ in steady-state.

We start by re-writing some preliminary equations of the model to make this section self-contained. First, aggregate profits in country i in period t net of fixed labor costs, innovation costs, and entry costs are:

$$\Pi_{it}=Y_{it}-W_{it}L_{it}^{p}-\sum_n W_{it}^{1-\phi}W_{nt}^{\phi}f_{int}\int_{\Omega_{int}}\mathrm{d}M_{it}-\sum_n\int_{\Omega_{int}}W_{it}^{1-\psi}W_{nt}^{\psi}h_{int}(z)\,\mathrm{d}M_{it}-W_{it}f_{Ei}M_{Eit},$$

Aggregate revenues $Y_{it} = \sum_n Y_{int}$ are proportional to variable labor payments:

$$Y_{it} = \frac{\rho}{\rho - 1} W_{it} L_{it}^{p}. \tag{18}$$

If $h(z;a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^{\gamma}$, aggregate innovation expenditures are a constant fraction of aggregate revenues:

$$\sum_n \int_{\Omega_{int}} W_{it}^{1-\psi} W_{nt}^{\psi} h(z; a_{in}) \, \mathrm{d}M_{it} = \frac{1}{\rho\gamma} Y_{it}. \tag{19}$$

If the trade balance is a constant share $\bar{\kappa}_i$ of GDP, and condition (19) in the paper holds, aggregate revenues can be written as:

$$Y_{it} = \frac{1 - \bar{\bar{\kappa}}_i}{1 - \kappa_i \left(1 - \bar{\bar{\kappa}}_i\right)} W_{it} L_{it}, \tag{20}$$

where $\bar{\bar{\kappa}}_i = \psi \bar{\kappa}_i / (\psi + \rho\gamma)$ and if there are no fixed fixed costs of selling into individual countries, and $\bar{\bar{\kappa}}_i = \bar{\kappa}_i / (1 + (\phi \frac{(\theta+1-\rho)(\gamma-1)}{\rho\theta\gamma} + \frac{\psi}{\rho\gamma})^{-1})$ if there are fixed costs of selling into individual countries and productivities are Pareto distributed.

Suppose we are on a steady-state equilibrium in which aggregate variables are constant. In steady-state, the distribution of firms is given by $M_i(z) = \frac{M_{Ei}}{\delta} G_i(z)$ (we omit time subscripts for the reminder of this section to simplify notation). The aggregate free-entry condition (equation B.6. in the paper) in steady-state is:

$$(r + \delta) W_i f_{Ei} M_{Ei} = Y_i - W_i L_i^{p} - \sum_n W_i^{1-\phi} W_n^{\phi} \left[1 - G_i(\bar{z}_{in})\right] f_{in} - \sum_n \int_{\Omega_{in}} W_i^{1-\psi} W_n^{\psi} h_{in}(z) \, \mathrm{d}M_i. \tag{21}$$

In what follows, we solve for the constant of proportionality $\Pi_i / Y_{it} = \kappa_i = \kappa$ in steady-state under two special cases of our model. We then show that, in these two special cases, the aggregate response to a change in variable or fixed trade costs is immediate (i.e. there are no transition dynamics), so that κ remains constant over time.

Case 1: No fixed costs

Assume that there are no fixed costs of selling in individual countries, i.e. $f_{ii} = f_{in} = 0$, so that there is no selection. In this case, the aggregate free entry condition (21) is:

$$\frac{W_i f_{Ei} M_{Ei}}{\delta} = \frac{1}{r + \delta} \left[Y_i - W_i L_i^{p} - \sum_n \int_{\Omega_{int}} W_i^{1-\psi} W_n^{\psi} h(z, a_{in}(z)) \, \mathrm{d}M_i(z) \right].$$

and using (18) and (19):

$$W_i f_{Ei} M_{Ei} = \frac{\delta}{r+\delta}\frac{\gamma-1}{\rho\gamma}Y_i. \tag{22}$$

Aggregate profits are:

$$\begin{aligned}\Pi_i &= Y_i - W_i L_i - \sum_n \int_{\Omega_{in}} W_i^{1-\psi} W_n^{\psi} h_{in}(z)\, \mathrm{d}M_i - W_i M_{Ei} f_{Ei} \\ &= \frac{r}{r+\delta}\frac{\gamma-1}{\rho\gamma}Y_i,\end{aligned}$$

so $\kappa = \frac{r}{r+\delta}\frac{\gamma-1}{\rho\gamma}$. Note that if $r > 0$, aggregate cross-sectional profits are positive even though discounted profits at entry are zero.

The steady-state mass of entering firms is given by:

$$M_{Ei} = \frac{\delta}{r+\delta}\frac{\gamma-1}{\rho\gamma}\frac{1-\bar{\bar{\kappa}}_i}{1-\kappa_i\left(1-\bar{\bar{\kappa}}_i\right)}\frac{L_i}{f_{Ei}},$$

where we used (20) and (22). Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state, and $\kappa_{it} = \kappa$.

Finally, aggregate variable profits gross of entry costs are: $\Pi_i + W_i M_{Ei} f_{Ei} = \frac{\gamma-1}{\rho\gamma}Y_i$. Hence, with restricted entry (so that there are no costs incurred in entry), equation (19) in the paper holds with $\kappa = \frac{\gamma-1}{\rho\gamma}$.

Case 2: Pareto distributed productivities

Now assume that there are positive fixed costs of selling in individual countries and that the distribution of entering firms G_i is Pareto with shape parameter θ, i.e. $G_i = 1 - z^{-\theta}$ for $z \geq 1$. We also assume that the productivity cutoffs are interior, $\bar{z}_{in} > 1$. We start by noting that aggregate fixed labor costs are proportional to aggregate revenues.

$$\frac{M_{Ei}}{\delta}\sum_n W_i^{1-\phi} W_n^{\phi} f_{in}\left[1 - G_i\left(\bar{z}_{in}\right)\right] = \frac{\theta+1-\rho}{\theta}\frac{\gamma-1}{\rho\gamma}Y_i. \tag{23}$$

Using (18), (19), and (23), we can write the aggregate free entry condition (21) as:

$$M_{Ei} W_i f_{Ei} = \frac{\delta}{r+\delta}\frac{\rho-1}{\rho\theta}\frac{\gamma-1}{\gamma}Y_i, \tag{24}$$

Finally, combining (18), (23) and (24), aggregate profits are

$$\Pi_i = \frac{r}{r+\delta}\frac{\rho-1}{\theta\rho}\frac{\gamma-1}{\gamma}Y_i$$

so $\kappa_i = \frac{r}{r+\delta}\frac{\rho-1}{\theta\rho}\frac{\gamma-1}{\gamma}$. The steady-state mass of entering firms is given by:

$$M_{Ei} = \frac{\delta}{r+\delta}\frac{\rho-1}{\rho\theta}\frac{\gamma-1}{\gamma}\frac{1-\bar{\bar{\kappa}}_i}{1-\kappa_i\left(1-\bar{\bar{\kappa}}_i\right)}\frac{L_i}{f_{Ei}},$$

where we used (20), and (24). Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state.

Finally, aggregate variable profits gross of entry costs are $\Pi_i + W_i M_{Ei} f_{Ei} = \frac{\rho-1}{\theta\rho}\frac{\gamma-1}{\gamma}Y_i$. Hence, in the model with restricted entry (in which there are no entry costs), equation (19) holds with $\kappa = \frac{\gamma-1}{\gamma}\left(\rho-1\right)/\left(\theta\rho\right)$.