The Measurement Procedure of AB2017 in a Simplified Version of McGrattan 2017

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1 Introduction

In this document we derive the main results Atkeson Burstein (Aggregate Implications of Innovation Policy, 2017) (AB2017) using a model of growth with intangible capital along the lines of a one-sector version of McGrattan (Intangible Capital and Measured Productivity, 2017) (M2017). Our specific goal in AB2017 is to measure how much a policy induced change in investment in intangible capital impacts measured aggregate productivity and output in the near term. We summarize this effect through what we call the impact elasticity of aggregate productivity with respect to a change in aggregate investment in intangible capital. We show in this note that if AB2017 and the parameters of the simplified version of M2017 presented here are calibrated to the same data on the discounted present value of firm dividends to intangible capital relative to output, firms' investment in intangible capital relative to output, the same real interest rate and growth rate of real output on the BGP, and the parameters governing the extent of increasing returns to scale in the two models are set to be equivalent, then the models have the same quantitative implication for this impact elasticity.

The similarities between the model in AB2017 and M2017 are not immediately apparent in part because there is no obvious analog to an aggregate stock of intangible capital in the model in AB2017 and the models it builds on. A significant portion of the complication in AB2017 is due to the assumptions required to aggregate investment in intangible capital across firms. One important simplification in M2017 relative to AB2017 is that aggregation across firms is immediate in the former paper while it takes some work in the second paper. Specifically, given the decentralization we discuss below with households owning the tangible capital stock, the value of a firm in M2017 is directly proportional to its stock of intangible capital and, due to constant returns in production at the firm level, the allocation of investment in intangible capital across firms does not impact aggregate variables. Much of the complexity of the model in AB2017 (and the literature on which it builds) is there to deliver the same results that there is an aggregate state variable (Z in AB2017 and K_I in M2017) that summarizes the contribution of intangible capital to the productive capacity of the economy and that the dynamics of that aggregate state variable is a function only of aggregated investment in intangible capital.

There are four principal differences between the model presented in these notes and a one-sector version of the model of M2017.

First, in the model presented here, we allow for increasing returns in production at the aggregate level. In AB2017, this is due to love of variety. In these notes, we introduce increasing returns through a production externality arising from the aggregate stock of intangible capital in the same way as was done in the earliest endogenous growth models.

Second, we focus on the fact that the depreciation rate for intangible capital is not measured. This issue is also raised in footnote 18 of M2017. There is a procedure for measuring this depreciation rate of intangible capital in the specific version of the model of M2017 that we discuss below. Specifically, if one assumes Cobb Douglas production at the aggregate level, observes real investment in intangible capital, the relative price of intangible capital, the share of compensation of aggregate capital in aggregate output, and the real interest rate, one can infer the depreciation rate of intangible capital from a standard user cost formula. This approach will not work in the model presented here because of the third difference between the model presented here and M2017.

Third, we assume that real investment in intangible capital is produced in a second sector with intertemporal knowledge spillovers. This means that the relative price of intangible capital and consumption is not observed. This implies that the approach mentioned above for measuring the depreciation rate does not work in the model presented here because we assume that one does not observe the relative price of intangible capital and hence cannot distinguish between depreciation and changes in the relative price in the standard user cost formula.

Fourth, we introduce in an extension of our model the possibility of business stealing. Here we model business stealing as the direct theft of the intangible capital held by another firm. The consideration of business stealing has the same impact on measurement as discussed in AB2017.

2 Model

Time is discrete and denoted by $t = 0, 1, 2, \ldots$. There are two final goods in this economy. The first is a final good Y used for consumption C, investment X_T in tangible capital K_T, and as an input into production of the research good X_I. We call this good the consumption good, and we normalize its price to one. The second is a final good Y_r used for investment in intangible capital K_I. We call this good the research good.

There is a continuum of measure one of final goods producing firms and a single firm that owns and rents physical capital. At each date t, each final goods producing firm is indexed by its stock of intangible capital k_{It}. We let K_{It} denote the aggregate stock of intangible capital and we refer to

$$s_t(k_I) = \frac{k_I}{K_{It}}$$

as the *size* of a firm with intangible capital k_I at date t. We will examine symmetric equilibria, so the aggregate stock of intangible capital in equilibrium is given by $K_{It} = k_{It}$.

The firm that owns and rents tangible capital is indexed by its stock of tangible capital K_{Tt}.

A firm with intangible capital k_{It} at t produces the consumption good using tangible capital k_{Tt} and labor l_t as additional inputs. We assume that there is a spillover from the aggregate stock of intangible capital K_{It} that impacts the productivity of all of these firms such that output of the consumption good from a firm that employs k_{It}, k_{Tt} and l_t when the aggregate stock of intangible capital is K_{It} is

$$y_t = K_{It}^{\rho} k_{It}^{\gamma} k_{Tt}^{\alpha} l_t^{1-\gamma-\alpha} \tag{1}$$

Aggregate output of the final consumption good is given by $Y_t = y_t$.

This firm retains x_{It} of its output of the final consumption good to use as an input to produce the research good to invest in its stock of intangible capital. The amount of the research good that can be produced from one unit of the final consumption good is determined by the current stock of exogenous scientific knowledge A_{rt} (which grows at the exogenous rate $\bar{g}_{Ar}$) and the aggregate stock of intangible capital K_{It} raised to a power $\phi - 1$ with $\phi \leq 1$ reflecting that accumulation of intangible capital gets more expensive at the margin the more such capital there is (as in the growth models of Chad Jones). Specifically, we assume that this firm produces research good

$$y_{rt} = A_{rt} K_{It}^{\phi-1} x_{It}, \tag{2}$$

and the stock of intangible capital in that firm grows according to

$$k_{It+1} = (1 - \delta_I)k_{It} + y_{rt}. \tag{3}$$

Aggregate expenditure on investment in intangible capital is given by $X_{It} = x_{It}$.

We assume that these firms rent tangible capital from the firm that owns that stock of capital. This firm that owns the stock of tangible capital augments its stock of tangible capital by investing units of the final consumption good, with the stock of tangible capital evolves over time according to

$$K_{Tt+1} = (1 - \delta_T)K_{Tt} + X_{Tt} \tag{4}$$

where X_{Tt} is households' investment in tangible capital. Factor market clearing requires that k_{Tt} and l_{Tt} chosen by final goods producing firms equals K_{Tt} and L_{Tt}.

The resource constraint for the final consumption good is

$$C_t + X_{Tt} + X_{It} = Y_t \tag{5}$$

Households have standard preferences over consumption

$$\sum_{t=0}^{\infty} \beta^t \log(C_t) \tag{6}$$

and supply labor L_t inelastically, where L_t growing at the constant rate $\bar{g}_L$.

3 Equilibrium with policies

We now describe equilibrium. Intertemporal prices for the final consumption good are denoted by $\{q_t\}$ with $q_0 = 1$. The rental rates for physical capital and the wage rate for labor are denoted by $\{r_{Kt}, w_t\}$. Firms' investments in intangible capital are subsidized at rates $\{\tau_t\}$. These subsidies are financed by lump sum taxes T_t on households. Define the price of the research good relative to the final consumption good.

$$P_{rt} = \frac{K_{It}^{1-\phi}}{A_{rt}}.$$

In defining equilibrium, we rewrite equations (2) and (3) as

$$k_{It+1} = (1 - \delta_I)k_{It} + \frac{1}{P_{rt}} x_{It}. \tag{7}$$

Final goods producing firms start with initial stock of intangible capital k_{I0} and, taking the prices and subsides above as given as well as the path for aggregate intangible capital $\{K_{It}\}$ and scientific progress $\{A_{rt}\}$ as given, choose rentals of physical capital and hiring of labor $\{k_{Tt}, l_t\}$, output and investment in intangible capital $\{y_t, x_{It}\}$ and dividends $\{d_t\}$ with

$$d_{It} = y_t - w_t l_t - r_{Kt} k_{Tt} - (1 - \tau_t) x_{It} \tag{8}$$

to maximize the discounted present value of dividends

$$\sum_{t=0}^{\infty} q_t d_{It} \tag{9}$$

subject to constraints (1), (7), and (8).

The firm that owns the physical capital stock starts with capital K_{Tt} and, taking prices and subsidies as given chooses tangible investment $\{X_{Tt}\}$ and dividends

$$D_{Tt} = r_{Kt} K_{Tt} - X_{Tt}$$

to maximize the discounted present value of dividends

$$\sum_{t=0}^{\infty} q_t d_{Tt} \tag{10}$$

subject to (4).

Aggregate dividends in the economy are given in equilibrium by

$$D_t = D_{Tt} + D_{It} = Y_t - w_t - (1 - \tau_t) X_{It} - X_{Tt} \tag{11}$$

Households are endowed with their labor and ownership of firms. They take as given the wage rate for labor as well as firm dividends and lump sum taxes. They choose a sequence for consumption l to maximize their utility (6) subject to the budget constraint

$$\sum_{t=0}^{\infty} q_t \left[C_t - w_t L_t - D_t - T_t\right] \leq 0.$$

The government budget constraint is

$$T_t = \tau X_{It} \tag{12}$$

An equilibrium consists of prices and quantities such that firms are maximizing profits (9) subject to their constraints, households are maximizing utility (6) subject to their constraints, the government budget constraint (12) holds, markets clear (all lower case quantities are equal to their upper case counterparts), and the resource constraint for the final good (5) is satisfied.

4 Balanced Growth Path

We say that an equilibrium is on a balanced growth path (BGP) if output Y_t, consumption C_t, both categories of investment X_{Tt} and X_{It}, and capital stock K_{Tt} grow at a common constant rate $\bar{g}_Y$. The growth rate of the intangible capital stock K_{It} and output of the research good Y_{rt} is denoted $\bar{g}_{KI}$. The ratio of intertemporal prices is constant over time and equal to $q_t/q_{t+1} = \exp(\bar{g}_Y)\beta^{-1} = 1 + \bar{R}$.

From equation (2), after imposing market clearing (lower case quantities equal to their upper case counterparts), we have that on a BGP,

$$(2-\phi)\bar{g}_{KI} = \bar{g}_{Ar} + \bar{g}_Y.$$

From equation (1), after imposing market clearing , we have that on a BGP,

$$(1-\alpha)\bar{g}_Y = (\rho+\gamma)\bar{g}_{KI} + (1-\gamma-\alpha)\bar{g}_L.$$

These equations determine the BGP growth rates of output and the intangible capital stock. If $(1-\alpha)(2-\phi) > \rho+\gamma$ we have

$$\bar{g}_{KI} = \frac{(1-\alpha)\bar{g}_{Ar} + (1-\gamma-\alpha)\bar{g}_L}{(1-\alpha)(2-\phi)-(\rho+\gamma)}$$

$$\bar{g}_Y = \frac{(\rho+\gamma)\bar{g}_{Ar} + (1-\phi)(1-\gamma-\alpha)\bar{g}_L}{(1-\alpha)(2-\phi)-(\rho+\gamma)}$$

If $(1-\alpha)(2-\phi) = \rho+\gamma$, the growth rate is jointly determined with the other equilibrium conditions. We consider as a special case $\phi = 1$ and $\bar{g}_{Ar} = 0$ so that $\bar{g}_{KI} = \bar{g}_Y$.

Note that on a BGP, the price of the research good relative to the final good falls at rate

$$\log P_{rt+1} - \log P_{rt} = -\bar{g}_{Ar} + (1-\phi)\bar{g}_{KI}$$

5 The Elasticity of Measured TFP growth with respect to intangible investment

We are interested in measuring the contribution of intangible capital to the growth in output. Define the series:[1]

$$\log Z_t \equiv \log Y_t - \alpha \log K_{Tt} - (1-\gamma-\alpha)\log L_t.$$

[1] This series is not the standard measure of TFP because the coefficients on capital and labor do not add up to one. If one could truly measure changes in the stock of intangible capital, then measured aggregate TFP would be $\rho \log(K_{It})$. Given the uncertainty over measurement of depreciation of intangible capital, and hence the stock of intangible capital, we have chosen to focus on the dynamics of $\log Z_t$ instead

The contribution of changes in the stock of intangible capital to growth in output is given by

$$g_{Zt} \equiv \log Z_{t+1} - \log Z_t = (\rho + \gamma)\left(\log K_{It+1} - \log K_{It}\right).$$

To make this equation parallel to AB2017, we write it as

$$g_{Zt} \equiv \log Z_{t+1} - \log Z_t = G\left(\frac{Y_{rt}}{K_{It}}\right) \tag{13}$$

where

$$G\left(\frac{Y_{rt}}{K_{It}}\right) \equiv (\rho + \gamma)\log\left(1 - \delta_I + \frac{Y_{rt}}{K_{It}}\right) \tag{14}$$

This equation is the direct analog of the equation (11) for G of AB2017.

We define the *social rate of depreciation of innovation expenditures* as the growth rate of Z_t that would occur if firms were to invest nothing in innovation. From equation (7) applied to aggregates, this social rate of depreciation of innovation expenditures is given in this model by

$$G(0) = (\rho + \gamma)\log\left(1 - \delta_I\right) \tag{15}$$

Our aim is to calculate the elasticity of Z_{t+1} with respect to policy-induced changes at t in Y_{rt} (we characterize the full dynamics in AB2017).[2] We assume the economy starts on a BGP with growth rate of $\log Z$ given by $\bar{g}_Z$ and a constant ratio $\bar{Y}_r/\bar{K}_I$. Then this elasticity is given by

$$\log Z_{t+1} - \log Z_t - \bar{g}_Z \approx G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I}\left(\log Y_{rt} - \log \bar{Y}_{rt}\right)$$

where

$$G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I} = (\rho + \gamma)\frac{\exp(\bar{g}_Z)^{1/(\rho+\gamma)} - \exp(G(0))^{1/(\rho+\gamma)}}{\exp(\bar{g}_Z)^{1/(\rho+\gamma)}}. \tag{16}$$

By direct calculation, one can show that for $\rho + \gamma \in (0, 1)$

$$G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I} \leq \bar{g}_Z - G(0). \tag{17}$$

Thus, if one writes a model with a value of $G(0)$ close to zero, then the model's implications for this elasticity will be tightly bounded if $\bar{g}_Z$ is small. This result is the direct analog of equation 25 of AB2017.

[2] When considering transition dynamics to a new BGP one has to take into account that, when $\phi < 1$, increases over time in the aggregate stock of intangible capital reduces the productivity of innovative investments X_I.

We make use of the following result on a balanced growth path in our measurement. From equation (14), on a BGP we have

$$\frac{\bar{Y}_{rt}}{\bar{K}_{It}} = \exp(\bar{g}_Z)^{1/(\rho+\gamma)} - \exp(G(0))^{1/(\rho+\gamma)}$$

where

$$\exp(G(0))^{1/(\rho+\gamma)} = (1-\delta_I).$$

Since

$$\exp(\bar{g}_Z)^{1/(\rho+\gamma)} = \exp(\bar{g}_{KI}) = \frac{\bar{K}_{It+1}}{\bar{K}_{It}},$$

multiplying both sides of the previous equation by $\bar{K}_{It}/\bar{K}_{It+1}$ gives

$$\frac{\bar{Y}_{rt}}{\bar{K}_{It+1}} = \frac{\exp(\bar{g}_Z)^{1/(\rho+\gamma)} - \exp(G(0))^{1/(\rho+\gamma)}}{\exp(\bar{g}_Z)^{1/(\rho+\gamma)}} \tag{18}$$

We next discuss how data on firm value can be used to inform us on the magnitude of the depreciation of intangible capital, which is required to calculate the elasticity in (16).

6 How we use Firm Valuation in Measurement

We define the value of a firm to be the discounted present value of its dividends. Define V_t, the value of all firms in the aggregate (including the current dividend) as

$$V_{It} = \sum_{k=0}^{\infty} \frac{q_{t+k}}{q_t} D_{It+k}, \tag{19}$$

where aggregate dividends to intangible capital are given by

$$D_{It} = \gamma Y_t - (1-\tau)X_{It}.$$

The value of a firm with intangible capital stock k_{It} at time t is proportional to its size, that is

$$V_{It}\frac{k_{It}}{K_{It}} = V_{It}s_t(k_I)$$

where $s_t(k_I) = k_I/K_{It}$. We write V_{It} as a recursion as follows:

$$V_{It} = \max_{x_I} \gamma Y_t - (1-\tau)x_I + \frac{q_{t+1}}{q_t}V_{It+1}\left[\frac{K_{It}}{K_{It+1}}(1-\delta_I) + \frac{x_I}{P_{rt}K_{It+1}}\right] \tag{20}$$

From the first order conditions governing the choice of x_{It} at the firm level, we have the optimality condition

$$(1-\tau) = \frac{q_{t+1}}{q_t}V_{It+1}\frac{1}{P_{rt}K_{It+1}}, \tag{21}$$

which implies the standard valuation of the intangible capital stock

$$P_{rt}K_{It+1}(1-\tau) = \frac{q_{t+1}}{q_t}V_{It+1} \tag{22}$$

While this standard valuation of intangible capital holds in our model, the implication of equation (21) that we use in AB2017 is

$$X_{It}(1-\tau) = \frac{q_{t+1}}{q_t}V_{It+1}\frac{Y_{rt}}{K_{It+1}} \tag{23}$$

(where, recall, $X_{It} = P_{rt}Y_{rt}$ denotes aggregate investment in intangible capital) derived by multiplying both sides of (21) by Y_{rt}.

On the BGP, V_t grows at a rate $\bar{g}_Y$ and $q_t/q_{t+1} = 1 + \bar{R}$, so by equation (23) we can express the ratio Y_{rt}/K_{It+1} on a BGP as:

$$\frac{\bar{Y}_{rt}}{\bar{K}_{It+1}} = \frac{\left(1+\bar{R}\right)}{\exp\left(\bar{g}_Y\right)}\left[\frac{X_{It}(1-\tau)}{V_{It}}\right]. \tag{24}$$

From equation (18), we then have

$$\frac{\exp(\bar{g}_Z)^{1/(\rho+\gamma)} - \exp(G(0))^{1/(\rho+\gamma)}}{\exp(\bar{g}_Z)^{1/(\rho+\gamma)}} = \frac{\left(1+\bar{R}\right)}{\exp\left(\bar{g}_Y\right)}\left[\frac{X_{It}(1-\tau)}{V_{It}}\right]. \tag{25}$$

Thus, from equation (16), we have that

$$G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I} = (\rho+\gamma)\left(\frac{1+\bar{R}}{\exp(\bar{g}_Y)}\right)\left[\frac{X_{It}(1-\tau)}{V_{It}}\right] \leq \bar{g}_Z - G(0). \tag{26}$$

Equation (26) is the central measurement formula in AB2017 when there is no business stealing. Specifically this formula (26) is the direct analog of equation (82) in the Appendix in AB2017, with $1/(\rho-1)$ from that paper replacing $(\rho+\gamma)$ here.[3]

The intuition for this formula is as follows. Using equation (3) applied to aggregates, the representative firm sees itself as raising its size at period $t+1$ from

$$\frac{1-\delta_I}{\exp(\bar{g}_{KI})}$$

to 1 by investing resources $(1-\tau)X_I$ at t. Thus, the change in size that the firm enjoys from this investment is

$$1 - \frac{1-\delta_I}{\exp(\bar{g}_{KI})} = \frac{\bar{Y}_{rt}}{\bar{K}_{It+1}}$$

[3]Note that X_{It}/V_{It} in this paper is equal to i_{rt}/v_t in AB2017.

and the value of this change in size is

$$\frac{q_{t+1}}{q_t}V_{It+1}\frac{\bar{Y}_{rt}}{\bar{K}_{It+1}}.$$

In equilibrium, this value must be equal to the cost of these investments, $X_{It}(1-\tau)$. A low ratio of size valuation, V_{It+1}, relative to the cost of innovative investment, $(1-\tau)X_{It}$, must be associated with a high ratio $\bar{Y}_r/\bar{K}_I$, which is indicative of a high depreciation rate of innovative investments (and a correspondingly high impact elasticity of innovative investments on Z_t). A similar intuition applies to the formula (82) in AB2017.

7 Measurement

We now discuss how if one follows the same procedure of identifying the quantities in equation (26) and the quantities in the equivalent formula (82) of AB2017, then one will get the same quantitative answer. We make the following specific assumptions about what data is used to discipline the models.

We assume that the BEA measures expenditures on consumption C_t, investment in tangible capital X_{Tt}, and investment on investment in intangible capital $X_{It} = P_{rt}Y_{rt}$. This means that total output Y_t is measured. M2017 uses a broader concept of investment in intangible capital than is currently used in the NIPA, and is able to use additional data on firms' expenditures on intangible investment to construct measures of total output Y_t. AB2017 constructs output produced by the final good by subtracting intangible investment (which is produced by the research sector). Moreover, in the model in AB2017, there is unmeasured investment in intangible capital undertaken by entering firms. That paper outlines a procedure for inferring the quantity of that unmeasured investment. To compare the models precisely, one must be sure to use identical measures of output and investment in intangible capital.

Assume that the BEA can measure labor compensation w_tL_t and hence can measure the term $(1-\gamma-\alpha)$ from the share of labor compensation in GDP. The analogous term in AB 2017 is $(1-\alpha)/\mu$. Assume that the BEA can measure the depreciation rate for tangible capital δ_T, and thus use measured investment X_{Tt} to construct the tangible capital stock K_{Tt}. The concept of physical capital is identical in this model as in AB2017.

Assume that the BEA can measure the user cost of tangible capital r_{Kt} (and hence knows the appropriate economywide return on capital R_t and the depreciation rate for tangible capital δ_T)

$$r_{t+1} = R_t + \delta_T$$

and hence can measure the parameter α from the share of rentals of tangible capital $r_{Kt}K_{Tt}$ in GDP. The analogous term in AB2017 is α/μ. The procedure for measuring the user cost of physical capital is the same in the two models.

By imposing constant returns in production at the level of the firm, we then identify γ. The equivalent term in AB2017 is $(\mu - 1)/\mu$.

The parameter ρ governing increasing returns at the aggregate level is not known. We show that the term $1/(\rho - 1)$ is equivalent to $(\rho + \gamma)$ in our key measurement formulas, so, for the models to have the same quantitative implications, we must have these two terms equal.

We now discuss how to construct a measure of valuation of firms' intangible capital relative to innovative investments, $V_{It}/\left((1-\tau)X_{It}\right)$, which we require to apply our measurement formula (26). By equation (19), and using the fact that V_{It}, D_t and X_{It} grow at the same rate as the final good, $\bar{g}_Y$, V_t is given by

$$V_{It} = \left(\frac{1+\bar{R}}{1+\bar{R}-\exp(\bar{g}_Y)}\right)\left(\gamma Y_t - (1-\tau)X_{It}\right).$$

This formula is the direct analog of valuation equation (35) (combined with the equation at the top of Appendix page 24) of AB2017. The term i_r used in that formula in that paper is a measure of all intangible investment, including investment by entrants. Thus the implications of this model and AB2017 for firm value relative to output are the same given identical measures of the innovation intensity of the economy X_I/Y (or, in the notation of AB2017, i_r, which is inclusive of innovative investment by entering firms). Note that the discount factor in equation (35) of AB2017 is adjusted for by the share of employment in entering firms, which has a very small effect on the impact elasticity given plausible magnitudes of this share.

This formula implies that

$$\frac{V_{It}}{(1-\tau)X_{It}} = \left(\frac{1+\bar{R}}{1+\bar{R}-\exp(\bar{g}_Y)}\right)\left(\frac{\gamma}{1-\tau}\frac{Y_t}{X_{It}} - 1\right),$$

where X_{It}/Y_t denotes the ratio of innovative investments relative to output of the final consumption good. From equation (26), we then have

$$G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I} = (\rho+\gamma)\left(\frac{1+\bar{R}-\exp(\bar{g}_Y)}{\exp(\bar{g}_Y)}\right)\left(\frac{\gamma}{1-\tau}\frac{Y_t}{X_{It}} - 1\right)^{-1} \quad (27)$$

as our implementation of measurement equation (26).

The implication of this comparison of the model in AB2017 in the model presented in these notes is that they have the same implied impact elasticities of aggregate productivity with respect to a policy induced change in innovative investment if they are calibrated to the same data on the discounted present value of dividends to intangible capital relative to output, the same factor shares for physical capital and labor, the same real interest rate, the same BGP growth rate of output, and the same policies and taxes as long as the parameters $\rho + \gamma$ here are calibrated to be equal to $1/(\rho - 1)$ in AB2017.

8 Alternative method for measuring δ_I

An alternative method for measuring δ_I would be to infer it from a user cost calculation for intangible capital. In this section, we discuss why this method does not work in the specification of the model presented here.

First, to understand the proposed alternative method for inferring δ_I, consider a specification of this model with $\phi = 1$, $\bar{g}_{Ar} = 0$ and $A_{rt} = 1$ so $P_{rt} = 1$. Under these assumptions, on a balanced growth path, the stock of intangible capital relative to output is constant. This intangible capital to output ratio (using equation 7) and the depreciation rate (applying the user cost calculation to intangible capital) must solve the following two equations

$$\frac{\bar{K}_I}{\bar{Y}} = \frac{1}{\exp(\bar{g}_Y) - (1 - \delta_I)} \frac{\bar{X}_I}{\bar{Y}}$$

and

$$\bar{R} = \gamma \frac{\bar{Y}}{\bar{K}_I} - \delta_I$$

It is straightforward to solve these two equations for δ_I and $\bar{K}_I/\bar{Y}$ given data on $\bar{X}_I/Y$ and a calibration of γ as described above.

We now examine whether this procedure works in our model if we allow for trends in productivity of the research good (that is, if $\phi \neq 1$ and $\bar{g}_{Ar} \neq 0$) so that $P_{rt} \neq 1$. With $P_{rt} \neq 1$, equation (7) implies

$$\frac{P_{rt} K_{It}}{Y_t} = \frac{\exp(\bar{g}_{Pr})}{\exp(\bar{g}_Y) - (1 - \delta_I)\exp(\bar{g}_{Pr})} \frac{\bar{X}_I}{\bar{Y}}$$

where $\bar{g}_{Pr}$ is the growth rate of the relative price of the research good. The user cost formula for intangible capital with a trend in the price of the intangible capital good implies

$$\frac{(1+\bar{R})-(1-\delta_I)\exp(\bar{g}_{Pr})}{\exp(\bar{g}_{Pr})}\frac{\bar{P}_r\bar{K}_I}{\bar{Y}}=\gamma.$$

Combining these two equations gives us the implication that

$$\frac{(1+\bar{R})-(1-\delta_I)\exp(\bar{g}_{Pr})}{\exp(\bar{g}_Y)-(1-\delta_I)\exp(\bar{g}_{Pr})}\frac{\bar{P}_r\bar{K}_I}{\bar{Y}}=\gamma$$

so we can solve for the term $(1-\delta_I)\exp(\bar{g}_{Pr})$, but we cannot solve for δ_I separately.

Thus, when we have trends in productivity in the production of the research good, we cannot use the simple procedure above to measure depreciation of the intangible capital good (unless we bring additional information to measure $\bar{g}_{Pr}$) but we can use our procedure using valuation information.

9 Business Stealing

Now let us extend the model in these notes to allow for "business stealing". We derive a direct analog to equation (82) in AB2017.

We model business stealing in a very reduced form way. Specifically, we model it as the theft of units of investment in intangible capital. We imagine that this investment in intangible capital is embodied in people who can walk out the door or in books that can be stolen. We assume that firms that have their investment in intangible capital stolen lose the use of that capital.

We assume that firms steal units of intangible capital in proportion κ to their investment in that capital and that they lose units of intangible capital to theft in proportion κ to aggregate investment in intangible capital. Firms receive lump sum inflows of stolen intangible capital.

With these assumptions, we write the transition equation for intangible capital for firms given in equation (7) as

$$k_{It+1}=(1-\delta_I)k_{It}+\frac{1+\kappa}{P_{rt}}x_{It}-\frac{\kappa}{P_{rt}}\frac{k_{It}}{K_{It}}X_{It} \tag{28}$$

Note that the aggregate law of motion for intangible capital (and hence the social depreciation of intangible capital) is unchanged since a loss for one firm is a gain for another one.

With this change, the first order condition for optimal investment in intangible capital in equation (21) becomes

$$(1-\tau) = (1+\kappa)\frac{q_{t+1}}{q_t} V_{It+1} \frac{1}{P_{rt} K_{It+1}} \tag{29}$$

The standard valuation equation (22) then becomes

$$P_{rt} K_{It+1}(1-\tau) = (1+\kappa)\frac{q_{t+1}}{q_t} V_{It+1} \tag{30}$$

which can be re-written as

$$X_{It}(1-\tau) = (1+\kappa)\frac{q_{t+1}}{q_t} V_{It+1} \frac{Y_{rt}}{K_{It+1}} \tag{31}$$

Hence we have a revised version of equation (26)

$$G'\left(\frac{\bar{Y}_r}{\bar{K}_I}\right)\frac{\bar{Y}_r}{\bar{K}_I} = (\rho+\gamma)\left(\frac{1}{1+\kappa}\right)\left(\frac{1+\bar{R}}{\exp(g_Y)}\right)\left[\frac{X_{It}(1-\tau)}{V_{It}}\right] \tag{32}$$

This equation is the direct analog of equation (82) in AB2017 where the term

$$\left(\frac{1}{1+\kappa}\right)$$

correcting for business stealing is equivalent to the term

$$\left[1-\delta_e \frac{av.\bar{s}ize_c}{av.\bar{s}ize_e}\right]$$

correcting for business stealing in AB2017.