Transition dynamics in aggregate models of innovative investment

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Abstract

What quantitative impact do changes in economic policies and other changes in the economic environment have on the dynamics of aggregate productivity in the short, medium, and long run, through their effect on firms’ incentives to invest in innovation? We present a unifying model that nests a number of canonical models in the literature and characterize the transition dynamics of aggregate productivity following a variety of changes in policies and the economic environment in terms of two sufficient statistics. We review the extent to which these two sufficient statistics can be disciplined by data and discuss some uncertainties that arise with policy changes to which our sufficient statistics approach does not apply.

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1 Introduction

Much of the growth in aggregate productivity in our economy is driven by the investments by entering and incumbent firms in innovation. What is the quantitative impact of changes in tax policies and/or changes in the economic environment such as an increase in monopoly power or a decrease in the population growth rate on the growth of aggregate productivity in the short, medium, and long term through their impact on innovative investments by firms. Despite several decades of important work on models of the impact of innovative investment by firms on aggregate productivity,\(^1\) there are no widely accepted set of assumptions to use in specifying these models nor any widely accepted procedure for disciplining the parameters of these models with data.\(^2\) Moreover, the transition dynamics implied by these models have not received much attention.

In this paper, we build on our work in Atkeson and Burstein (2018) (henceforth AB2018) to present a model that nests many of the canonical models of the interaction of firms’ innovative investments and aggregate productivity and output. We look to “inspect the mechanism” underlying our model’s positive implications for the impact of a range of changes in the economic environment, including changes in uniform subsidies to innovative investment, changes in equilibrium markups for incumbent firms, changes in corporate profits taxes, and changes in the growth rate of population.\(^3\) To do so, we develop an approximate analytical solution to the transition dynamics of aggregate productivity implied by these models.\(^4\) We use this approximate solution to highlight two sufficient statistics that are key for determining the model’s quantitative implications for the short, medium, and long run transition dynamics of aggregate productivity in response to a variety of changes in the economic environment and we discuss what data might be useful in disciplining these two key statistics.

\(^1\)See the excellent reviews in Jones (2005), Acemoglu (2009), Aghion et al. (2014), Aghion et al. (2015), and Akcigit (2017).

\(^2\)The Bureau of Economic Analysis has begun to incorporate measures of firms’ investments in intangible capital into the National Income and Product Accounts starting in 2013. The framework they use for doing so (see e.g. Corrado et al. 2009) is an extension of the standard neo-classical growth model that abstracts from knowledge spillovers from innovation, which are central to the class of models that we consider in this paper.

\(^3\)Important contributions in the Public Finance Literature on the impact of tax policy on business entry (entrepreneurship) include Gentry and Hubbard (2005), Cullen and Gordon (2006) and Giroud and Rauh (2015). For examples of analyses of the impact of tax policies on aggregate productivity through their impact on firms’ investments in innovation in Schumpeterian models, see Peretto (2007), Jaimovich and Rebelo (2017), and Ferraro et al. (2017). Karahan et al. (2016) study the impact of changes in the growth rate of the labor force on firm entry, and De Loecker and Eeckhout (2017) and Edmond et al. (2018) study the macroeconomic implications of changes in markups.

\(^4\)Our approach is similar to that used in Campbell (1994) to study the implications of the stochastic growth model.
Our model, which extends the model of firm dynamics in Garcia-Macia et al. (2016), features innovative investments by entering and incumbent firms that may be directed either at acquiring products new to that firm or at improving the productivity with which a firm produces one of its current products. Entering and incumbent firms acquire new products either by inventing products that are new to society as a whole (as in models of growth through expanding varieties) or by inventing a better technology for producing a product currently produced by some other incumbent firm (as in Schumpeterian models of growth through business stealing). We nest a wider class of models of the spillovers from innovative investment in the literature than in AB2018. Specifically, in addition to the Semi-Endogenous Growth framework of Jones (2002), we nest models that use what is termed Second Generation Endogenous Growth technologies governing the extent to which past discoveries impact the productivity of labor currently allocated to research.\footnote{See, for example, Peretto (1998), Segerstrom (1998), Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999). As discussed in the surveys by Jones (2005), Ha and Howitt (2007), and Akcigit (2017), these Second Generation Endogenous Growth models embed a mechanism through which, in equilibrium, the invention of new varieties has an independent effect of increasing the amount of labor devoted to research required to maintain a given aggregate productivity growth rate.}

To develop our sufficient statistics, we solve analytically for a first order approximation to the model implied path of aggregate productivity following a one-time, exogenous, perturbation to the aggregate allocation of labor to innovative investment. The first of our sufficient statistics, which we term the impact elasticity of the model, is the elasticity of the growth of aggregate productivity between this period and the next with respect to a given change in the log of the labor input allocated to research. The second of our sufficient statistics is the persistence of this impulse to aggregate productivity, i.e. it is the rate at which the level of aggregate productivity returns to its baseline growth path.

We show how these sufficient statistics shape the model’s quantitative implications for the transition dynamics of aggregate productivity following long-term uniform changes in innovation subsidies, long-term changes in markups, and long-term changes in corporate taxes with equal expensing of innovative investment by entering and incumbent firms. We also show that these two sufficient statistics shape the long response of the aggregate productivity growth to permanent changes in population growth. Furthermore, we show that these sufficient statistics also shape our model’s normative implications for the welfare consequences of these changes in the economic environment.

What data can be used to discipline these two sufficient statistics? The impact elasticity of the model is disciplined by data on firms dynamics. Our second sufficient statistic, the persistence of the response of aggregate productivity, is much harder to measure. This second statistic is shaped by assumptions made about intertemporal knowl-
edge spillovers in research that were central to the debate in the literature twenty years ago about scale effects in endogenous growth models.\(^6\) This sufficient statistic is much harder to pin down with data because it requires one to identify whether the half-life of a shock to aggregate productivity is a decade, a century, or a millenia. We show that this model uncertainty is less important for positive quantitative analysis of the medium term dynamics (e.g. 20 years) of aggregate productivity in response to the changes in the economic environment that we consider. In contrast, this uncertainty regarding intertemporal knowledge spillovers is much more important for the normative implications of our model since these normative implications are driven by the longer term responses of aggregate productivity.

We finish the paper with analysis of the impact of a change in corporate profit taxes when the expensing of innovative investment for tax purposes is not the same for incumbent and entering firms. We discuss how additional effects beyond those summarized by our two sufficient statistics impact the transition dynamics of aggregate productivity. These additional effects arise from a policy-induced reallocation of innovative investment between entering and incumbent firms that may raise or lower the transition path of aggregate productivity relative to that implied by our sufficient statistics, depending on the extent of business stealing and the specific form of intertemporal knowledge spillovers.

The remainder of the paper is organized as follows. In Section 2 we present our model and discuss the models in the literature that are nested in our framework. In Section 3, we derive results regarding the growth rates and levels of variables on the balanced growth path of the model as a function of policies and the growth rate of population. In Section 4, we compute analytically the transition dynamics of aggregate productivity implied by the model to a first order approximation in the case in which we take the dynamics of the allocation of labor between production and research as given. We use these formulas for the model’s transition dynamics to highlight which features of data on firm dynamics and the aggregate economy are critical for identifying the parameters of the model that are key to its dynamic implications. In Section 5, we describe the counterfactual experiments that we conduct with the model and the results we obtain regarding the short and long term responses of aggregate productivity and welfare induced by these various experiments. We present proofs of our propositions, the calibration of the model parameters, and a wide range of supporting technical material in a supplemental appendix.

\(^6\)While interest in this debate in the literature has diminished in the last decade, we find that the issues raised there are still central to understanding the normative implications of the models we consider. See Bloom et al. (2017) for a recent empirical contribution to the measurement of intertemporal knowledge spillovers.
2 The Model

Our model of the impact of innovative investment by heterogeneous firms on aggregate productivity nests several of the canonical models in the literature. The model is specified so as to allow for sufficient aggregation to make analysis of its transition dynamics tractable. We present the key elements of the model here. Further details are provided in the Appendix.

We present the model in discrete time. There are three types of goods: a final good used for consumption and investment in physical capital, a final good that we term the research good that is the input into innovative investment by firms, and differentiated intermediate goods produced by innovating firms.

The Final Consumption Good We let $Y_t$ and $C_t$ denote output and consumption of the final consumption good, and let $K_t$ denote the stock of physical capital available for production in period $t$. The resource constraint for the final consumption good is

$$Y_t = C_t + K_{t+1} - \exp(-\delta_k)K_t$$

where $(1 - \exp(-\delta_k))$ represents the depreciation rate of physical capital. The representative agent has standard preferences given by

$$\sum_{t=0}^{\infty} \beta^t L_t \left( C_t / L_t \right)^{1-\gamma}$$

with $\beta < 1$ and $\gamma > 0$. Population $L_t$ grows at the exogenous rate $\exp(g_{L_t}) = L_{t+1} / L_t$.

The final consumption good $Y_t$ is produced as a constant elasticity of substitution (CES) aggregate of the output of a continuum of differentiated intermediate goods. At each date $t$, the technology with which any particular intermediate good can be produced is summarized by its productivity index $z$. Production of an intermediate good with productivity index $z$ is carried out with physical capital, $k$, and labor, $l$, according to

$$y_t(z) = zk_t(z)^{1-\alpha}l_t(z)^{\alpha}$$

where $0 < \alpha < 1$. To simplify our notation, we assume that the support of $z$ is a grid with countable elements $z_n$ for integers $n$ with $\log z_{n+1} - \log z_n$ equally spaced. For each intermediate good that can be produced at time $t$, we refer to the technology with the highest value of $z$ on this grid available for producing this good as the frontier technology for this good. An intermediate goods producing firm is an organization that owns the
exclusive rights to use the frontier technology for producing one or more intermediate goods.

The productive capacity of the economy is determined by its population, $L_t$, its current stock of physical capital $K_t$, and the measure $M_t(z)$ which denotes the measure of intermediate goods with frontier technology indexed by $z$ at time $t$. Aggregate output of the final consumption good is then given by the CES aggregator

$$Y_t = \left[ \sum_z y_t(z)^{(\rho - 1)/\rho} M_t(z) \right]^{\rho/(\rho - 1)},\quad (4)$$

with $\rho > 1$ and output levels $y_t(z)$ of the intermediate goods given in equation (3).

Total labor hours employed in production of intermediate goods is denoted by $l_{pt} L_t$, with $l_{pt} \in [0, 1]$ representing the fraction of the population engaged in current production. The constraints on production labor and physical capital require that $l_{pt} L_t = \sum_z l_t(z) M_t(z)$ and $K_t = \sum_z k_t(z) M_t(z)$.

In each period, physical capital and labor are freely mobile across intermediate goods producing firms and the markup $\mu \geq 1$ of price over marginal cost charged by intermediate goods producers is constant across intermediate goods and over time.\(^7\) As is standard, cost minimization by intermediate goods producers implies that $k_t(z)/l_t(z) = K_t/(l_{pt} L_t)$ for all intermediate goods. Factor market clearing and equation (4) then imply that, in equilibrium, aggregate output can be written as

$$Y_t = Z_t \left( K_t \right)^{\alpha} \left( L_{pt} \right)^{1-\alpha},\quad (5)$$

where $Z_t$ is given by

$$Z_t = \left[ \sum_z z^{\rho - 1} M_t(z) \right]^{1/(\rho - 1)}.\quad (6)$$

Throughout this paper, we refer to $Z_t$ as aggregate productivity at time $t$.\(^8\)

\(^7\)As is standard, with Bertrand competition and limit pricing, the gross markup $\mu$ charged by the incumbent producer of each product is the minimum of the monopoly markup, $\rho / (\rho - 1)$, and the technology gap between the frontier technology with productivity index $z$ and any potential second most productive producer of the good, with productivity index $z/\bar{\mu}$ (with $\bar{\mu} > 1$), which potentially depends on the patent system. That is $\mu = \min \left\{ \frac{\rho}{\rho - 1}, \bar{\mu} \right\}$. Changes in the patent system can result in changes in the equilibrium markup.

\(^8\)This model-based measure of aggregate productivity, $Z_t$, does not correspond to measured total factor productivity (TFP), which is given by $\text{TFP}_t = \frac{\text{GDP}_t}{\left( k_t^{1/\alpha} L_t^{1-\alpha} \right)}$, where the definition of GDP depends on the measurement standard for expenditures on innovative investment being used (e.g., the definition of output of the final consumption good $Y_t$ in equation (1) corresponds to the Bureau of Economic Analysis’
We refer to $M_t = \sum_z M_t(z)$ as the total measure of products available, and to the ratio $Z_t^{\phi-1} / M_t$ as the average productivity index of existing intermediate goods (specifically, the average of $z^{\phi-1}$ across intermediate goods). We refer to $s_t(z) = z^{\phi-1} / Z_t^{\phi-1}$ as the size of a product with frontier technology $z$. This is because the revenue and employment at $t$ associated with this product in equilibrium is proportional to $s_t(z)$.

The Research Good  The second final good in this economy, which we call the research good, is the input used for innovative investment by firms. Production of the research good is carried out using research labor hours $l_{rt}L_t$, with $l_{pt} + l_{rt} = 1$. Output of the research good is given by

$$Y_{rt} = A_{rt}Z_t^{\phi-1}l_{rt}L_t.$$  \hfill (7)

Here, $A_{rt}$ represents the stock of freely available scientific progress, which grows at an exogenous rate $g_{At} = \bar{g}_A$. The term $Z_t^{\phi-1}$ with $\phi \leq 1$ reflects intertemporal knowledge spillovers in the production of the research good, as in the model of Jones (2002). Using the language of Bloom et al. (2017), $A_{rt}Z_t^{\phi-1}$ denotes the productivity with which research labor $L_{rt}$ translates into a real flow of “ideas” $Y_{rt}$ available to be applied to innovative investment. Exogenous scientific progress (growth in $A_{rt}$) drives up research productivity over time. If $\phi < 1$, then increases in the level of aggregate productivity $Z_t$ reduce research productivity in the sense that “ideas become harder to find.” Because the impact of advances in $Z_t$ on research productivity is external to any particular firm, we call it a “spillover.” The parameter $\phi$ indexes the extent of this spillover.

Innovative investment is undertaken by intermediate goods producing firms. Aggregate productivity grows as a result of innovations by intermediate goods producing firms that either increase the average productivity index $z$ of frontier technologies available for existing intermediate goods or increase the total measure of intermediate goods available. These innovations arrive at rates determined by the investments in innovation undertaken by these firms. We refer to those firms producing intermediate goods at $t$ that pre-2013 measurement of GDP, which did not include expenditures on innovative investment), and $1 - \tilde{\alpha}$ denotes the share of labor compensation in measured GDP. Our analytic comparative statics can be used to construct alternative measures of TFP and GDP.

9Some papers in the theoretical literature on economic growth with innovating firms assume that all productivity growth is driven entirely by firms’ expenditures on R&D (Griliches 1979, p. 93). As noted in Corrado et al. (2011) and Akcigit (2017) this view ignores the productivity-enhancing effects of investments by actors other than business firms. We capture all of these other productivity-enhancing effects with $A_r$. Akcigit et al. (2013) consider a growth model that distinguishes between basic and applied research and introduce a public research sector. As we discuss below, the only role served by the exogenous growth of scientific progress $A_{rt}$ in our analysis is that, by adjusting the parameter $g_{A_r}$, we can target a given baseline growth rate of output in the balanced growth path as we vary the parameter $\phi$ and $\psi$ (for a given growth rate of population, $\bar{g}_L$).
also produced at $t - 1$ as incumbent firms. We refer to those firms at $t$ that are new (and hence did not produce intermediate goods at $t - 1$) as entering firms.

We now describe the technologies for innovative investment by entering and incumbent firms.

**Innovative investment by entering firms**

Let $x_{et}M_t$ denote the measure of entering firms at $t$, where $x_{et}$ denotes the measure of entering firms relative to the measure of existing products at $t$. Each of these entering firms invests units of the research good to acquire the frontier technology $z'$ to produce an intermediate good new to that firm at the start of period $t + 1$.

This newly acquired frontier technology may apply to an intermediate good that was previously produced by an incumbent firm or may apply to a good that is new to society. Specifically, with probability $\delta_e$, this productivity index $z'$ drawn by the entrant at $t + 1$ is associated with an intermediate good that was already being produced by an incumbent firm at $t$, but with a lower productivity index. Since identical intermediate goods are perfect substitutes in the production of the final consumption good, competition in the product market between the entering firm and the previous incumbent producer of this intermediate good implies that the previous incumbent producer ceases production of the good. In this case, the innovative investment by the entering firm does not result in a net increase in the total measure of products available $M_{t+1}$. Instead, it only results in a positive increment to the average productivity index across existing products. As is common in the literature, we say that this intermediate good that is new to the entering firm represents business stealing from an incumbent firm.

With the complementary probability $1 - \delta_e$, this newly acquired frontier technology allows this entering firm to produce an intermediate good that is new to society as a whole. In this case, the innovative investment by the entering firm results in a net increase in the total measure of products available $M_{t+1}$. As is common in the literature, we say that this intermediate good that is new to this entering firm represents a contribution to productivity through expanding varieties.

The productivity index $z'$ for stolen products in entering firms is drawn in a manner similar to that in Klette and Kortum (2004) and other standard quality ladder models. The productivity index $z'$ for products that are new to society in entering firms is drawn in a manner similar to that in Luttmer (2007). The average value of $z^{\rho-1}$ across all products obtained by entering firms at $t + 1$ is given by $\mathbb{E}z^{\rho-1} = \eta_e Z_t^{\rho-1} / M_t$. As we discuss below, the parameter $\eta_e$ controls the average size (in terms of sales and employment) of products produced by entering firms relative to the average size of all products.
Each entering firm at \( t \) must invest \( M_t^{1-\psi} \) units of the research good. Thus, the total use of the research good by entering firms is thus \( x_{et}M_t^{1-\psi} \), with \( \psi \leq 1 \). As we discuss below, in the literature, two values of \( \psi \) are typically considered. The first value is \( \psi = 1 \). In this case, the resources required to create one new product fall with the number of existing products. The second value is \( \psi = 0 \). In this case, the investment of the research good required to invest in a new product is independent of the number of existing products.

Incumbent firms have the opportunity to invest in acquiring new products to their firm and to improve their existing products. We describe these investment technologies next.

**Innovative investment by incumbent firms to acquire new products**  An incumbent firm that owns the frontier technology \( z \) for producing a particular intermediate good possesses the capacity to acquire the frontier technology on additional goods new to that firm through innovative investment. This investment technology is specified so that to attain any given probability of acquiring a new product at \( t + 1 \), a firm must invest at rate \( x_{mt}(z) \) in proportion to \( s_t(z)M_t^{1-\psi} \) which is the size of its current product with index \( z \) at time \( t \) times \( M_t^{1-\psi} \). When \( \psi = 1 \), then investment to attain a given probability of gaining a new product must be proportional to the size of the product \( s_t(z) \). If \( \psi = 0 \), then investment must be proportional to the ratio of \( z^{\rho-1} \) to \( Z^{\rho-1}/M_t \), which is the average value of \( z^{\rho-1} \) across incumbent products. In equilibrium, incumbent firms invest in proportion to \( s_t(z)M_t^{1-\psi} \). Thus, if \( x_{mt}M_t^{1-\psi} \) is aggregate investment by incumbent firms in obtaining new goods, then the probability that incumbent firms gain a new product per product that they currently produce is given by \( 1 - \exp \left( -h \left( x_{mt} \right) \right) \), where \( h(\cdot) \) is a strictly increasing and concave function with \( h(0) = 0 \) and \( h(x) < 1 \) for all \( x \).

As is the case with entering firms, new products acquired by incumbent firms may be stolen from other incumbent firms (with probability \( \delta_m \)) or new to society (with probability \( 1 - \delta_m \)). The average value of \( (z')^{\rho-1} \) of the new products acquired by an incumbent firm investing based on a current product with frontier productivity \( z \) is \( Ez^{\rho-1} = \eta_m z^{\rho-1} \). This assumption implies that the average value of \( z^{\rho-1} \) across all new products obtained by incumbent firms at \( t + 1 \) is given by \( Ez^{\rho-1} = \eta_m Z_t^{\rho-1}/M_t \).

We next consider investment by incumbent firms in improving continuing products.

**Investment in continuing products by incumbent firms**  An incumbent firm that owns the frontier technology for producing a particular intermediate good with frontier technology \( z \) can also invest to improve the technology on that product. This investment technology is specified so that to attain any given percentage growth in this frontier tech-
nology, the firm must invest at a rate \( x_{ct}(z) \) in proportion to \( s_t(z)M_t^{1-\psi} \). The interpretation of the parameter \( \psi \) is the same as above. In equilibrium, incumbent firms invest in proportion to \( s_t(z)M_t^{1-\psi} \). Thus, if \( x_{mt}M_t^{1-\psi} \) is aggregate investment by incumbent firms in improving their products, the expected growth rate of the frontier technology for products retained by incumbent firms is \( \mathbb{E}z^{\rho-1} = \exp \left( \zeta \left( x_{ct} \right) \right) z^{\rho-1} \). We assume that \( \zeta(\cdot) \) is a strictly increasing and concave function, with \( \zeta(x) > 0 \) for all \( x \geq 0 \).

With these definitions, we can write the resource constraint for the research good as

\[
(x_{ct} + x_{mt} + x_{et}) M_t^{1-\psi} = Y_t \tag{8}
\]

**Dynamics of \( M \) and \( Z \)** We now characterize the dynamics of the measure of intermediate goods \( M_t \) and aggregate productivity \( Z_t \) as functions of innovative investment by entering and incumbent firms. We have specified the model so that, in equilibrium, the aggregation of innovative investment is highly tractable. As a result the equilibrium evolution of the variables \( M_t \) and \( Z_t \) can be described as functions only of aggregate innovative investment of each type per product.

The evolution of the total measure of products \( M_t \) is determined by the rate at which incumbent firms lose the frontier technologies for products that they produced at \( t \) and the rates at which incumbent and entering firms gain the frontier technologies for products new to these firms. Consider first the rate at which incumbent firms lose products. For each product that they produce at \( t \), incumbent firms can lose its frontier technology at \( t+1 \) either due to exogenous exit (with probability \( (1 - \exp (-\delta_0)) \)), or due to business stealing. Let \( \exp (-\delta_{ct}) \) denote the probability that a product remains in the same incumbent firm at \( t+1 \), where \( \delta_{ct} = \delta_c(x_{mt}, x_{et}) \) is defined by the equation

\[
\exp (-\delta_c(x_{mt}, x_{et})) = \exp (-\delta_0) - \delta_m (1 - \exp (-h(x_{mt}))) - \delta_e x_{et}. \tag{9}
\]

The corresponding evolution of the total measure of intermediate products \( M_t \) is given by \( \log M_{t+1} - \log M_t = H(x_{mt}, x_{et}) \) where

\[
H(x_{mt}, x_{et}) \equiv \log \left( \exp (-\delta_{ct}) + 1 - \exp (-h(x_{mt})) + x_{et} \right). \tag{10}
\]

Using a similar logic, the evolution of aggregate productivity is given by \( \log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et}) \) where

\[
G(x_{ct}, x_{mt}, x_{et}) \equiv \frac{1}{\rho - 1} \log \left( \exp (-\delta_{ct}) \exp \left( \zeta \left( x_{ct} \right) \right) + \eta_m (1 - \exp (-h(x_{mt}))) + \eta_e x_{et} \right). \tag{11}
\]
Equilibrium

We now describe the market structure, policies, and equilibrium in our model economy.

Policies and Firm Profit Maximization Problems  We now describe the profit maximization problems of the various types of firms in this economy. We start with the firms that purchase intermediate goods to produce the final consumption good using the technology in equation (4). These firms are competitive. They choose output and inputs to maximize profits taking the price of the final consumption good (which we normalize to one) and the prices of intermediate goods as given, and receive a production subsidy $\tau_y$ per unit sold.\(^{10}\) The prices of intermediate goods are denoted by $p_t(z)$. Profit maximization by these firms implies standard CES input demands for each intermediate good with demand elasticities determined by $\rho$. Because the technology in equation (4) is constant returns to scale, these final consumption good producing firms have no profits in equilibrium.

The physical capital stock is managed by competitive firms that rent out physical capital to intermediate goods producing firms and invest in physical capital. Each firm takes the price of the final consumption good, the rental rate for physical capital $R_{kt}$, and intertemporal prices for the final consumption good $\{Q_t\}$ as given, and are subject to a corporate profits tax $\tau_{corp}$ with expensing for investment in physical capital of $\lambda_k$. These firms seek to maximize the discounted present value of after tax dividends with dividends given by

$$D_{kt} = (1 - \tau_{corp}) R_{kt} K_t - (1 - \tau_{corp} \lambda_k) (K_{t+1} - \exp(-\delta_k) K_t)$$

Intermediate goods producing firms rent physical capital at rate $R_{kt}$ and hire production labor at wage $W_t$ to produce intermediate goods according to the technology in equation (3). They set prices $p_t(z)$ at a markup of $\mu > 1$ over their marginal cost of production. Standard arguments give that, in equilibrium, these firms expend a fraction $(1 - \alpha) / \mu$ of revenue $p_t(z) y_t(z)$ on wages for production labor and a fraction $\alpha / \mu$ of revenue on rental payments for physical capital. A fraction $(\mu - 1) / \mu$ of revenue is left over as variable profits. These same arguments imply that aggregate output inclusive of the production subsidy is split into wage payments to production labor, rental payments for physical capital, and other variable profits.\(^{10}\)

\(^{10}\) We introduce this production subsidy to allow us to undo the distortion in incentives for physical capital accumulation arising from markups and the corporate profits tax in some of our counterfactual experiments. This will allow us to focus attention on the impact of policies on welfare through their impact on the dynamics of aggregate productivity and not through their impact on the incentives to accumulate capital relative to an initially distorted equilibrium.
capital, and aggregate variable profits in the same manner.

The research good is produced by competitive firms (or in-house by intermediate good producing firms) using the technology in equation (7). These firms take the productivity of research labor as determined by $A_{rt}$ and $Z_t^{\phi-1}$ as given. They hire research labor at wage $W_t$ and sell the research good at price $P_{rt}$. In equilibrium, $P_{rt} = W_t/A_{rt}Z_t^{\phi-1}$. Because the technology in equation (7) is constant returns to scale, the wage bill for research exhausts revenues, so these research good producing firms have no taxable earnings. That is, $P_{rt}Y_{rt} = W_tA_{rt}Z_t^{\phi-1}$ in all periods $t$.

These results regarding factor shares imply a simple relationship between the innovation intensity of the economy as measured by the ratio of spending on the research good to output inclusive of the production subsidy, $i_{rt} \equiv P_{rt}Y_{rt}/(1+\tau_y)Y_t$, and the allocation of labor between research and current production given by

$$\frac{l_{rt}}{l_{pt}} = \frac{\mu}{1-\alpha} i_{rt}. \quad (13)$$

Intermediate goods firms also choose innovative investment for each product that they manage. The dividend at time $t$ per unit of time associated with a product with frontier technology $z$ is given by $D_t(z) = D_t s_t(z)$, where

$$D_t = (1 - \tau_{corp}) \frac{\mu - 1}{\mu} (1 + \tau_y) Y_t - (1 - \tau_{corp} \lambda_I) P_{rt} M_t^{1-\psi} [(1 - \tau_c) x_{ct} + (1 - \tau_m) x_{mt}], \quad (14)$$

where $\tau_c$ and $\tau_m$ are rates at which incumbent firms’ innovative investments are subsidized and $\lambda_I$ is the rate at which incumbent firms can expense their innovative investment for tax purposes. Each existing product remains in the same incumbent firm at $t+1$ with probability $\exp(-\delta_{ct})$ and has expected size conditional on survival in the same firm equal to $\exp(\zeta(x_{ct})) s_t(z) Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$. In addition, this firm anticipates acquiring a new product with expected size of $\eta_m s_t(z) Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$ with probability $1 - \exp(-h(x_{mt}))$. Thus, the expected discounted present value of dividends associated with a product of size $s_t(z)$ at $t$ inclusive of the dividend at $t$ is directly proportional to the size of the product, i.e. it can be written as $V_t s_t(z)$, where the factors of proportionality $\{V_t\}$ satisfy the recursion

$$V_t = D_t + \exp(-R_t) V_{t+1} \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \left[ \exp(-\delta_{ct}) \exp(\zeta(x_{ct})) + \eta_m (1 - \exp(-h(x_{mt}))) \right] \quad (15)$$

where the interest rate $R_t$ is defined by $\exp(-R_t) \equiv Q_{t+1}/Q_t.$
In equilibrium, entering firms must earn non-positive profits, so we must have

\[
(1 - \tau_{\text{corp}} \lambda_E) (1 - \tau_e) P_{rt} M_{t}^{1-\psi} \geq \exp (-R_t) \frac{Z_{t}^{\rho-1}}{Z_{t+1}^{\rho-1}} \eta_e
\]

where this expression is an equality if there is positive investment in entry in period \( t \).

In any period \( t \) with positive entry, we can combine the first order condition for the optimal choice of \( x_{mt} \) to maximize the right hand side of equation (15) and (16) to obtain a static equation determining \( x_{mt} \) given by

\[
\frac{(1 - \tau_{\text{corp}} \lambda_I) (1 - \tau_m) \eta_e}{(1 - \tau_{\text{corp}} \lambda_E) (1 - \tau_e) \eta_m} = \exp (-h (x_{mt})) h' (x_{mt}) .
\]

This condition implies that \( x_{mt} \) is constant in any periods in which entry is positive. Likewise, in any period \( t \) with positive entry, we can the first order condition for the optimal choice of \( x_{ct} \) to maximize the right hand side of (15) and (16) to obtain a static equation relating \( x_{et}, x_{mt} \) and \( x_{ct} \) given by

\[
\frac{(1 - \tau_{\text{corp}} \lambda_I) (1 - \tau_c) \eta_e}{(1 - \tau_{\text{corp}} \lambda_E) (1 - \tau_e) \eta_m} = \exp (-\delta_c (x_{mt}, x_{et})) \exp \left( \xi \left( x_{ct} \right) \right) \zeta' \left( x_{ct} \right)
\]

Since \( x_{mt} \) is constant in all periods \( t \) in which entry is positive, equation (18) defines an implicit function \( x_c \left( x_{et} \right) \) that determines \( x_{ct} \) as a function of \( x_{et} \) in every period in which entry is positive. In the Appendix, we show that the derivative \( dx_{c} / dx_{e} \) approaches zero in the limit as the length of a time period in the model approaches zero. In this case, in the continuous time limit, equation (18) implies that \( x_{ct} \) is also constant in any periods in which entry is positive.

**Household and government budget constraints** Each period, the household collects wage payments \( W_t L_t \), dividends from the physical capital firms, \( D_{kt} \), and dividends from the incumbent intermediate goods firms, \( D_t \). The household also finances investment in entry net of subsidies and taxes \((1 - \tau_{\text{corp}} \lambda_E)(1 - \tau_e)P_{rt} M_{t}^{1-\psi}x_{et} \) and pays lump sum taxes \( T_t \). The government sets lump sum taxes to finance the gap between corporate tax receipts and expenditures on subsidies for output and innovative investment.

**Definition of Equilibrium** The economy starts with initial conditions for \( K_0, Z_0, \) and \( M_0 \) as given. The paths for \( \{L_t\} \) and \( \{A_{rt}\} \) are given exogenously as well.

An allocation in this model is a sequence of variables \( \{K_{t+1}, l_{pt}, l_{rt}, C_t, Y_t, Y_{rt}, Z_{t+1}, M_{t+1}, \delta_{ct}\} \)
and innovative investments \( \{x_{ct}, x_{mt}, x_{et}\} \). An allocation is feasible if it satisfies equations (1)-(11). The vector of prices and values in this economy are \( \{Q_t, W_t, R_{kt}, P_{rt}, V_t, D_{kt}, D_t\} \) together with a markup of price over marginal cost \( \mu \).

The policies in this economy are the corporate tax rate \( \tau_{\text{corp}} \), subsidy rates for innovative investment \( \tau_c, \tau_m, \) and \( \tau_e \), a subsidy rate for output of the final consumption good \( \tau_y \), and expensing allowances for physical investment \( \lambda_K \), innovative investment by incumbents \( \lambda_I \), and innovative investment in entry \( \lambda_E \).

An equilibrium given policies is a feasible allocation together with a vector of prices and values and a markup such that firms producing the final consumption good, intermediate goods, and the research good, as well as the holding company for physical capital maximize profits, households maximize utility given their budget constraint, the the government budget constraint is satisfied.

### Nested Models

Our model nests several commonly used models in the literature and has two important features.

The first important feature of our model is that it permits sufficient aggregation in equilibrium so that the evolution of aggregate productivity and the total measure of products is a simple function of a few types of aggregate innovative expenditure per product \( (x_{ct}, x_{mt}, x_{et}) \) as in equations (10) and (11). Note that we do not need to record other attributes of the measure of frontier technologies across products \( M_t(z) \) as state variables. This aggregation dramatically simplifies the computation of transitions relative to models in which one must keep track of the full measure \( M_t(z) \) as a state variable.

Five commonly used models in the literature share this aggregation property: three types of expanding varieties models and two types of Neo-Schumpeterian models. We now discuss these models.

If \( \delta_e = \delta_m = 0 \), then there is no business stealing and hence all new products acquired by incumbent and entering firms are new products for society, expanding the measure of products \( M_t \). This is the assumption typically made in an expanding varieties model. Luttmer (2007) is an example of an expanding varieties model in which there is only innovative investment in entry. (Note that we do not consider the endogenous exit of products due to fixed operating costs featured in that paper.) Atkeson and Burstein (2010) is an example of an expanding varieties model in which there is innovative investment in entry and by incumbent firms in continuing products. Luttmer (2011) is an example of an expanding varieties model in which there is innovative investment in entry and in the
acquisition of new products by incumbent firms.

Neo-Schumpeterian models based on the quality ladder framework typically assume \( \delta_e = \delta_m = 1 \) and \( \delta_0 = 0 \). The simplest versions of these models do not accommodate growth in the measure of varieties \( M_t \). Grossman and Helpman (1991) and Aghion and Howitt (1992) are examples of Neo-Schumpeterian models in which there is only innovative investment in entry. Klette and Kortum (2004) is an example of a Neo-Schumpeterian model in which there is innovative investment in entry and by incumbent firms in acquiring new products (new to the firm, not to society). Acemoglu and Cao (2015) is an example of a Neo-Schumpeterian model in which there is innovative investment in entry and by incumbent firms in improving their own products.

The second important feature of our model is that it allows for a simple and flexible reduced form specification of the technology for producing real innovative investment per product (what we call the technology for research) that nests three specifications of this technology that have played an important role in the literature. Specifically, from equations (7) and (8), we can write a single constraint on real innovative investment per product

\[
x_{ct} + x_{mt} + x_{et} = A_{rt}Z_t^{\phi-1}M_t^{\psi-1}l_{rt}L_t.
\]

(19)

We focus on three specifications of this technology for research that have played an important role in the literature (see Jones (2005) and Ha and Howitt (2007) for a more extensive discussion).

The first specification of the technology for research that we consider has \( \phi = 0.96 \) and \( \psi = 1 \). We refer to this first specification as the First Generation Endogenous Growth specification of the technology for research. As we show below and as discussed in Jones (2005), with \( \phi \) close to one and \( \psi = 1 \), the economic implications of our model with this technology for research for the first several centuries of a transition to a new balanced growth path resemble the transition dynamics of a fully endogenous growth model with a research technology with \( \phi = 1 \) and \( \psi = 1 \).

The second specification of the technology for research has \( \phi = -1.67 \) and \( \psi = 1 \). This specification is similar to that in Jones (2002), Kortum (1997), or Segerstrom (1998). In this specification of the technology for research, “ideas become harder to find” in the sense that increases in aggregate productivity \( Z_t \) relative to the pace of scientific progress \( A_{rt} \) lead to a reduction in the productivity of labor allocated to research in producing real innovative investment per product. Bloom et al. (2017) offer evidence for this specification of the technology for research. We refer to this specification as Jones/Kortum/Segerstrom or J/K/S semi-endogenous growth specification of the research technology.
The third specification of the technology for research has $\phi = 0.96$ and $\psi = 0$. This specification is similar to that in Peretto (1998), Segerstrom (1998), Young (1998), Dinopoulos and Thompson (1998), Howitt (1999), and Ha and Howitt (2007). In this specification, increases in the mass of products $M_t$ relative to the pace of scientific progress $A_{rt}$ lead to a reduction in the productivity of labor allocated to research in producing real innovative investment per product. We refer to this as the Second Generation Endogenous Growth specification of the technology for research.

3 Balanced Growth Path

We now describe how to solve for a balanced growth path (BGP) of this economy given policies and model parameters. On a BGP, policies rates are constant, and the exogenous sequences for $A_{rt}$ and $L_t$ grow at constant rates $\bar{g}_A$ and $\bar{g}_L$. Output $Y_t$, physical capital $K_{t+1}$, and consumption $C_t$ grow at a common rate $\bar{g}_Y$. Aggregate productivity $Z_{t+1}$ and the measure of products $M_t$ grow at rates $\bar{g}_Z$ and $\bar{g}_M$. Innovative investment rates per product remain constant over time at $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$. This last assumption implies from equation (8) that the term $Y_{rt}M_t^{\psi-1}$ remains constant over time as well.

We first discuss how to solve for the BGP growth rates. We then discuss how to solve for the levels of variables on the BGP.

BGP growth rates

To solve for BGP growth rates, it is useful to consider the variable $J_t \equiv Z_t^{1-\phi}M_t^{1-\psi}$ together with the physical capital stock $K_t$ as the endogenous state variables of the economy. Since the term $Y_{rt}M_t^{\psi-1}$ is constant over time on a BGP, equations (7) and (8) imply that the growth rate of $J$ on a BGP depends only on the sum the growth of scientific progress and population and not on policies:

$$\bar{g}_J = (1-\phi)\bar{g}_Z + (1-\psi)\bar{g}_M = \bar{g}_A + \bar{g}_L$$

Note that if $\psi = 1$, then the growth rate of productivity along the BGP, $\bar{g}_Z$, is independent of policies. More generally, the division of the growth of $J$ into components due to growth in aggregate productivity $Z$ and growth in the number of products $M$ depends on the parameters $\phi$ and $\psi$ and on policies and the corresponding mix of innovative investment on a BGP as follows.

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11 In the literature, it is typical to parameterize the research technology in this case with $\phi = 1$ and $\psi = 0$. Again, the implications of the model are not substantially altered with $\phi = 0.96$. 

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The value of $\bar{x}_m$ on a BGP with positive entry is determined from equation (17) while the implicit function $\bar{x}_c(x_e)$ is determined from equation (18). The BGP level of $\bar{x}_e$ is then determined from the equation

$$\bar{g}_J = (1 - \phi) G (\bar{x}_c(\bar{x}_e), \bar{x}_m, \bar{x}_e) + (1 - \psi) H (\bar{x}_m, \bar{x}_e) \quad (21)$$

In the appendix we present conditions under which the right hand side of this expression is strictly increasing in $\bar{x}_e$ so at most one positive solution to this equation exists.

Once one solves for innovative investments $\bar{x}_c, \bar{x}_m,$ and $\bar{x}_e,$ the growth rates of aggregate productivity and the measure of products are given by

$$\bar{g}_Z = G(\bar{x}_c(\bar{x}_e), \bar{x}_m, \bar{x}_e) \quad (22)$$

and

$$\bar{g}_M = H(\bar{x}_m, \bar{x}_e). \quad (23)$$

We then have the standard result for the growth model that $\bar{g}_Y = \bar{g}_Z / (1 - \alpha) + \bar{g}_L.$

**BGP levels**

We now describe how to solve for the equilibrium levels of variables on a BGP given levels of $A_{rt}$ and $L_{rt}.$ The level of $J_t \equiv Z_t^{1-\phi} M_t^{1-\psi}$ is determined on a BGP, but the levels of $Z_t$ and $M_t$ individually on the BGP are not pinned down by BGP equations alone. As discussed below, these are determined by the initial conditions of the economy and the transition path to the new BGP. We use the following equations to solve for the level of variables on a BGP.

The consumer’s intertemporal Euler equation gives the equilibrium BGP interest rate $\bar{R}.$ From equation (14), dividends from intermediate goods firms relative to output inclusive of production subsidies, $d_t \equiv D_t / ((1 + \tau_y) Y_t),$ are

$$\bar{d} = (1 - \tau_{corp}) \frac{\mu - 1}{\mu} - (1 - \tau_{corp} \lambda t) \bar{p}_r [(1 - \tau_c) \bar{x}_c + (1 - \tau_m) \bar{x}_m] \quad (24)$$

where

$$\bar{p}_r \equiv \frac{P_{rt} M_t^{1-\phi}}{(1 + \tau_y) Y_t}. \quad (25)$$
From equation (15), the value of a product relative to output, \( v_t \equiv V_t/Y_t \), is
\[
\bar{v} = \frac{\bar{d}}{1 - \exp \left(- (\bar{R} - \bar{g}_Y) (1 - \bar{S}_e) \right)},
\]
(26)
where \( S_e \) represents the share of labor employed in new firms within the period on a BGP, given by \( S_e = \eta_e \bar{x}_e \exp \left(- (\rho - 1) \bar{g}_Z \right) \). The BGP value of \( \bar{p}_r \) can be found as the solution to the BGP version of the free entry condition
\[
(1 - \tau_{corp} \lambda_E) \left(1 - \tau_e \right) \bar{p}_r \bar{x}_e = \exp \left(- (\bar{R} - \bar{g}_Y) \right) \bar{v} \bar{S}_e
\]
(27)
in combination with equations (24) and (26). The BGP value of the research intensity of the economy is \( \bar{i}_r = \bar{p}_r (\bar{x}_c + \bar{x}_m + \bar{x}_e) \), and the allocation of labor \( \bar{L}_r / (1 - \bar{I}_r) \) is obtained from equation (13).

The ratio of physical capital to output is given by the standard Euler equation from the profit maximization problem of the physical capital holding company
\[
\exp(\bar{R}) = \frac{(1 - \tau_{corp} \lambda_E)}{(1 - \tau_{corp} \lambda_K)} \frac{\alpha \left(1 + \tau_y \right) \bar{Y}_{t+1}}{\bar{K}_{t+1}} + \exp \left(- \delta _{K} \right)
\]
(28)
Note that we cannot solve for the levels of \( \bar{K}_{t+1} \) and \( \bar{Y}_{t+1} \) until we solve for the level of aggregate productivity \( Z_{t+1} \) as described below. Finally, from equations (7) and (8) together with the definition of \( J_t \), we get that the level of \( J_t \) on a BGP is given by
\[
\bar{x}_c + \bar{x}_m + \bar{x}_e = A_{r_t} \bar{I}_t L_t / J_t.
\]
(29)
When \( \psi = 1 \), this equation is sufficient to pin down the BGP level of productivity \( Z_t \). More generally, with \( \psi < 1 \), there is a continuum of pairs of \( Z_t \) and \( M_t \) each consistent with the same value of \( J_t \) that are candidate values of aggregate productivity and the measure of products on a BGP. The particular values of \( Z_t \) and \( M_t \) that arise on a particular BGP depend upon the initial conditions of the economy \( Z_0 \) and \( M_0 \) and the transition path that the economy takes to converge to BGP. Specifically, an equilibrium sequence of innovative investments \( \{x_{ct}, x_{mt}, x_{et}\} \) implies, through the functions \( H \) and \( G \) defined above in equations (10) and (11), a sequence of growth rates of \( M_t \) and \( Z_t \). This sequence of growth rates can then be used to trace out the paths for the levels of \( M_{t+1} \) and \( Z_{t+1} \) from their initial conditions to their levels on the BGP. We use a first order approximation of these dynamics to solve for the levels of \( Z_t \) and \( M_t \) on a BGP as follows.
BGP level of aggregate productivity

To construct our first order approximation to the dynamics of \( \{J_{t+1}, Z_{t+1}, M_{t+1}\} \) we need to compute the elasticities of the growth rates of these variables with respect to changes in investment in entry around the BGP. In computing these elasticities, we make use of the result that \( x_{mt} = \bar{x}_m \) in every period of an equilibrium with positive entry. Thus, the results we develop below are conditional on the assumption that the transition path of the equilibrium to BGP has positive entry in every period.

Define the elasticity of the growth of aggregate productivity with respect to entry from equation (22) as

\[
\Theta_G = \left[ \frac{\partial}{\partial \bar{x}_e} G(\bar{x}_c, \bar{x}_m, \bar{x}_e) \cdot \frac{d}{d \bar{x}_e} x_c(\bar{x}_e, \bar{x}_m, \bar{x}_e) + \frac{\partial}{\partial \bar{x}_c} G(\bar{x}_c, \bar{x}_m, \bar{x}_e) \right] \bar{x}_e
\]  

(30)

where \( \frac{d}{d \bar{x}_e} x_c(\bar{x}_e) \) is computed from equation (18). Similarly, define the elasticity of the growth in the number of products with respect to entry from equation (23) by

\[
\Theta_H = \frac{\partial}{\partial \bar{x}_e} H(\bar{x}_m, \bar{x}_e) \bar{x}_e.
\]  

(31)

From the definition of \( J_t \) we can define the elasticity \( \Theta_J \) of the growth rate of \( J \) with respect to changes in entry as

\[
\Theta_J = (1 - \phi) \Theta_G + (1 - \psi) \Theta_H
\]  

(32)

We then have the following proposition that allows us to pin down the BGP levels of aggregate productivity and the measure of products, \( \bar{Z}_0 \) and \( \bar{M}_0 \), from initial conditions \( Z_0 \) and \( M_0 \) as follows.

**Proposition 1.** Given initial values for \( Z_0, M_0, \) and \( J_0 = Z^{1 - \phi}_0 M^{1 - \psi}_0 \) and the BGP level of \( J_0 \) from equation (29), the BGP values of \( Z_0 \) and \( M_0 \) are, to a first order approximation given by

\[
\log \bar{Z}_0 = \log Z_0 + \frac{\Theta_G}{\Theta_J} (\log J_0 - \log J_0)
\]  

(33)

\[
\log \bar{M}_0 = \log M_0 + \frac{\Theta_H}{\Theta_J} (\log J_0 - \log J_0)
\]  

(34)

where \( \Theta_G, \Theta_H \) and \( \Theta_J \) are evaluated at the BGP.

With these solutions for the BGP levels of aggregate productivity and the measure of products, it is straightforward to solve for the BGP levels of the physical capital stock and output given the BGP allocation of labor to current production obtained from equation...
evaluated at the BGP levels of $\bar{p}_r$ and innovative investment, the BGP physical capital to output ratio obtained from equation (28), and the aggregate production function (5).

4 Dynamics of Aggregate Productivity

We now consider the dynamics of aggregate productivity implied by our model when the path of research labor $\{l_t, L_t\}$ is taken as exogenous. Our aim is to study which parameters of our model determine the implications of the model for the short and long term responses of aggregate productivity to a policy or demographically induced change in research labor as we discuss in the next section.

Consider the paths of quantities $Z_t = \{Y_{rt}, x_{et}, x_{mt}, x_{ct}, J_{t+1}, Z_{t+1}, M_{t+1}\}$ given initial conditions for $J_0, Z_0, M_0$ and a path of research labor $\{l_t, L_t\}$. Here we consider paths for research labor $\{l_t, L_t\}$ such that the fraction of labor allocated to research converges to its BGP value $\bar{l}_r$ and population, $L_t$ converges to its BGP path $\{\bar{L}_rt\}$ and we construct a first order approximation to the paths for the variables in $Z_t$ relative the the BGP values of these variables that the economy is converging to. Here we assume that, along the transition to the BGP, the ratios $(1 - \tau_{corp})(1 - \tau_{m})$ and $(1 - \tau_{corp})(1 - \tau_{e})$ remain constant.

This first order approximation is obtained from the following equations. As discussed above, equation (17) implies that with positive entry, $x_{mt} = \bar{x}_m$. Thus, equations (7), (8), and (18) imply that, to a first order approximation

$$A \times (\log x_{et} - \log \bar{x}_e) = \left( \log l_{rt} - \log \bar{l}_r \right) + (\log L_t - \log \bar{L}_t) - (\log J_t - \log \bar{J}_t)$$

(35)

where

$$A \equiv \bar{x}_e \left[ d \frac{d}{d \bar{x}_e} x_c (\bar{x}_e) + \frac{1}{\bar{x}_c + \bar{x}_m + \bar{x}_e} \right],$$

(36)

and $\frac{d}{d \bar{x}_e} x_c (\bar{x}_e)$ is computed from equation (18) evaluated at the BGP values of investment. Define

$$\Theta \equiv \frac{\theta_l}{A}.$$  

(37)

Then, from the first order approximation around equation (21), we have

$$\log J_{t+1} - \log \bar{J}_{t+1} = \Theta \left[ (\log l_{rt} - \log \bar{l}_r) + (\log L_t - \log \bar{L}_t) \right] + (1 - \Theta) (\log J_t - \log \bar{J}_t).$$  

(38)

The initial condition of this AR1 process, $\log J_0 - \log \bar{J}_0$, is given.

We now develop the following analog of Proposition 6 from AB2018 regarding the
dynamics of the variable \( J_t \).

**Proposition 2.** From the AR1 representation (38), we have that to a first order approximation, the dynamics of \( \log J_t \) in the transition to a BGP with positive entry are given by

\[
\log J_{t+1} - \log J_{t} = \sum_{j=0}^{t} \Theta (1 - \Theta)^j \left[ (\log l_{rt-j} - \log \bar{l}_r) + (\log L_{t-j} - \log \bar{L}_t) \right] + \Theta (1 - \Theta)^{t+1} (\log J_0 - \log J_0)
\]

(39)

The proof is by direct calculation.

Once we solve for the dynamics of \( J_t \), the dynamics of aggregate productivity \( Z \) and the measure of products are given by the time \( t \) version of equations (33) and (34),

\[
\log Z_t - \log \bar{Z}_t = \frac{\Theta_G}{\Theta_J} (\log J_t - \log \bar{J}_t)
\]

(40)

\[
\log M_t - \log \bar{M}_t = \frac{\Theta_H}{\Theta_J} (\log J_t - \log \bar{J}_t)
\]

(41)

For many of our policy experiments, we make use of the following corollary to Proposition 2.

**Corollary 1.** Suppose the economy starts at \( t = 0 \) on some initial BGP, and there is a change in the economic environment that leaves the growth rates \( \{g_{At}\} \) and \( \{g_{Lt}\} \) unchanged, and the allocation of innovative investment \( \bar{x}_c, \bar{x}_m, \bar{x}_e \) is unchanged across BGPs. Then, to a first order approximation, the dynamics of aggregate productivity relative to its initial BGP path are given by

\[
\log Z_{t+1} - \log Z_0 - t \bar{g}_Z = \frac{\Theta_G}{A} \sum_{j=0}^{t} (1 - \Theta)^j (\log l_{rt-j} - \log l_{r0})
\]

(42)

We see here from equation (42) that, under the conditions of Corollary 1, the dynamics of aggregate productivity relative to its trend on the initial BGP can be summarized by two sufficient statistics. We refer to the first of these statistics, \( \Theta_G / A \), as the impact elasticity of a change in research labor at \( t \) on the level of aggregate productivity at \( t + 1 \) relative to its initial level. We refer to the second of these statistics, \( 1 - \Theta \) as the persistence of the response of aggregate productivity to an exogenous reallocation of labor to research.

Observe that the impact elasticity of an increase in labor devoted to research on aggregate productivity relative to its new BGP level is independent of the specification of the research good technology indexed by \( \phi \) and \( \psi \). Instead, it is determined by the parameters
that shape our model’s implications for firm dynamics and the allocation of innovative investment across incumbent and entering firms. Specifically, the elasticity $\Theta_G$ is bounded above by the contribution of entry to aggregate productivity growth, i.e.

$$\Theta_G \leq \frac{1}{\rho - 1} G(\bar{x}_c, \bar{x}_m, \bar{x}_e)^{\rho - 1} - G(\bar{x}_c, \bar{x}_m, 0)^{\rho - 1} = \frac{1}{\rho - 1} \left(1 - \frac{\delta_e}{\eta_e}\right) \bar{S}_e, \quad (43)$$

where $\bar{S}_e$ denoted the share of production employment in entering firms on the BGP. The parameter $A$ is bounded above by the share of innovative investment by entering firms in total innovative investment. As we show in the appendix, both $\Theta_G$ and $A$ converge to these upper bounds as the length a time period in calendar time shrinks to zero. Likewise, the elasticity $\Theta_H$ is given by the contribution of entry to the growth in the measure of products, that is

$$\Theta_H = \frac{H(\bar{x}_m, \bar{x}_e) - H(\bar{x}_m, 0)}{\exp(\bar{g}_M)} = (1 - \delta_e) \bar{F}_e,$$

where $\bar{F}_e$ denotes the fraction of products produced by newly entered firms.

In contrast, the persistence of this impact, which is determined by the parameter $1 - \Theta$ in equation (37), is highly sensitive to the the parameters $\phi$ and $\psi$ of the research good technology. This is because the elasticity $\Theta_J$, defined in equation (32), depends on these parameters. We discuss the quantitative implications of these observations when we consider the transition dynamics for aggregate productivity implied by various specifications of our model next.

**Quantitative implications of analytic results**

We now conduct experiments to explore the quantitative implications of different specifications of our model for the dynamics of aggregate productivity at short, medium, and long term horizons.

We calibrate all specifications of our model as described in greater detail in the appendix. We set the elasticity of substitution between intermediate goods in production to $\rho = 4$. As discussed above, the elasticities $\Theta_G$ and $\Theta_H$ on the initial BGP can be identified by calibrating our model to match data on firm dynamics. We consider two specifications of these parameters. In the first, we assume that there is no business stealing by entrants, i.e. $\delta_e = 0$. This specification is of interest because it delivers the maximum values of $\Theta_G$ and $\Theta_H$ consistent with data on the shares of employment and products in entering firms. In our second specification, we set the business stealing parameter $\delta_e$ so that the contribution of entrants to aggregate productivity growth in equation (43) is equal to
that estimated in Akcigit and Kerr (2018). As described in our calibration appendix, to calibrate the parameter $A$ on the initial BGP we measure expenditures on innovation by incumbent firms using NIPA data and infer expenditure on innovation by entering firms from equation (16). We consider the three specifications of the technology for research described in subsection 2. This gives us a total of six model specifications to consider.

We consider a change in the allocation of labor to research at time $t = 0$ of 10% (specifically of magnitude $\log l_{r0} - \log \bar{l}_r = 0.10$) starting from an initial BGP calibrated as above, and assume that the allocation of labor to research remains at this elevated level permanently. We assume that the allocation of real innovative investment $\bar{x}_c, \bar{x}_m, \bar{x}_e$ is the same at $t = 0$ as it is on the BGP to which the economy is converging. These assumptions satisfy the conditions in Corollary 1, so we can use equation (42) to characterize the dynamics of aggregate productivity and data from the economy at $t = 0$ to measure the elasticities required to use this equation.

Given the impact elasticity $\Theta_G / A$ implied by data on firm dynamics and our measurement of the share of innovative investment undertaken by entering firms, the response on impact of aggregate productivity growth at an annual frequency relative to the initial BGP trend with respect to a reallocation of labor to research of size $\log l_{r0} - \log \bar{l}_r = 0.10$ is 0.00282 (that is, aggregate productivity would be 0.28 percent higher after the first year) if there is no business stealing and 0.00102 with business stealing. As discussed above, these implications of our model are independent of the specification of the technology for research.

In contrast, the specification of the research technologies as indexed by $\phi$ and $\psi$ has a tremendous impact on the implications of the model for the persistence of any impulse to aggregate productivity. In Table 1, we report the level of aggregate productivity relative to its prior trend at horizons of 20 and 100 years that results from a hundred year long reallocation of labor to research of magnitude $\log l_{rt} - \log \bar{l}_r = 0.10$. As a theoretical matter, this cumulative impact over various horizons of a long-lasting reallocation of labor to research are the manifestation of the model’s implied persistence of an impulse to aggregate productivity from a one-time reallocation of labor. Hence, in Table 1 we also report the half-life of a one-time impulse to aggregate productivity.

We see that for the First Generation Endogenous Growth specification of the research technology, an impulse to aggregate productivity from a one-time reallocation of labor to research is essentially permanent. This implies that the cumulative response of aggregate productivity relative to its initial trend after 20 and 100 years is equal to 20 and 100 times the response of aggregate productivity growth on impact respectively. For the J/K/S and Second Generation Endogenous Growth specifications of the research technology, this is
no longer the case. In these cases, because one-time impulses to aggregate productivity decay over time, the cumulative impact of a long-lasting reallocation of labor to research on aggregate productivity does not grow linearly with the time horizon. Instead, the cumulative impact stops growing over time. This means that our model has dramatically different positive implications for the impact of a long-lasting reallocation of labor to research on the level of aggregate productivity relative to trend after 100 years depending on the specification of the technology for research. As a quantitative matter, these differences in the model’s implications are much smaller at a 20 year horizon.

**Impact of a change in BGP population growth on BGP productivity growth** We now use our analytical results to examine the impact on the BGP growth rate of aggregate productivity of a change in the BGP population growth rate. In our model, the growth rate of population $g_L$ impacts the growth rate of aggregate productivity because it impacts the growth rate of labor allocated to research. We can derive simple formulas using the elasticities $\Theta_G, \Theta_H$, and $\Theta_J$ for our model’s predictions for the impact of a change in the BGP growth rate of population on the BGP growth rate of aggregate productivity.\(^{12}\)

Observe that a change in the growth rate of labor has no impact on the equations (17) and (18) that determine $\bar{x}_m$ and the implicit function $x_c(x_e)$. Thus, we can differentiate equations (20) and (21) to get that $d\bar{g}_L = d\bar{g}_J = \Theta_J d \log \bar{x}_e$. From the definitions of $\Theta_G$ and $\Theta_H$, we have $d\bar{g}_Z = \Theta_G d \log \bar{x}_e$ and $d\bar{g}_M = \Theta_H d \log \bar{x}_e$. These observations imply that

\(^{12}\)In this case, we cannot use equation 42 in Corollary 1 to characterize the transition dynamics of aggregate productivity because $g_L$ changes. In the appendix we describe the transition dynamics of the economy in response to a permanent reduction in the growth rate of population that occurs gradually over time.
\[
\frac{d\bar{g}Z}{d\bar{g}L} = \frac{\Theta_G}{\Theta_J},
\]
which corresponds to the ratio between the impact elasticity and one minus the persistence parameter introduce in Corollary 1.

Using the calibration of the model described above, we can use equation (44) to compute a first order approximations to the decline in the BGP growth rate of aggregate productivity corresponding to a one percentage point drop in the BGP population growth rate for each of our three research technologies. For the First Generation Endogenous Growth and the J/K/S technologies, since they have \( \psi = 1 \), \( \Theta_G/\Theta_J = 1/(1 - \phi) \). With \( \phi = 0.96 \), the First Generation Endogenous Growth research technology implies a catastrophic decline in the BGP growth rate of aggregate productivity. This is simply a reflection of the well known finding that this research technology in the limiting case of \( \phi \rightarrow 1 \) cannot accomodate any growth in research labor on a BGP. With \( \phi = -1.67 \), the J/K/S research technology predicts a decline in productivity growth of \(-0.0037\). This result is consistent with the calculations in Jones (2002) and Fernald and Jones (2014) regarding the impact on productivity growth in the long run of a slowdown in growth of the quantity of labor devoted to research. With the Second Generation Endogenous Growth technology for research, the decline in BGP productivity growth is \(-0.0011\) with no business stealing and \(-0.0005\) with business stealing. Here, changes in the growth rate of population have a smaller impact on aggregate productivity growth on the BGP because the growth rate of the measure of products also adjusts as described in Ha and Howitt (2007).

5 Quantitative experiments

We now consider counterfactual experiments that alter the BGP levels and/or growth rates of variables on the BGP and in the model with fully endogenous responses of investment in physical capital, innovative investment, and the allocation of labor to research. We first consider a uniform subsidy to innovation. We next consider an exogenous change in the markup. We next consider a change in the corporate profits tax. Using a log-linear approximation to the model’s equilibrium transition dynamics, we examine the evolution of aggregate productivity in the first 20 and 100 years after the start of the experiment. We choose the magnitude of the experiment so that these results can be compared to those in Table 1. In the online supplement for this paper, we also report the values of the relevant elasticities, and the responses of aggregate output, entry, the share of labor compensation in output, and the valuation of firms as measured by Tobin’s Q.
Uniform innovation subsidy  We first consider the impact of a permanent increase in innovation subsidies $\tau_c, \tau_m,$ and $\tau_e$ that is uniform in the sense that it leaves the ratios $(1 - \tau_c)/(1 - \tau_e)$ and $(1 - \tau_m)/(1 - \tau_e)$ unchanged. If we hold all other parameters and policies fixed, we find that the new BGP has the same allocation of innovative investment $\bar{x}_c, \bar{x}_m,$ and $\bar{x}_e$ and hence the same growth rates of aggregate productivity and the measure of products. In addition, the employment shares and fractions of products in entering firms are also unchanged from the old to the new BGP. This is because equations (17), (18), (20), and (21) that determine the BGP growth rate of $J$ and the allocation of investment on a BGP are not altered by a uniform change in innovation subsidies.

In contrast, on the new BGP, the economy has a higher innovation intensity of the economy measured as the ratio of expenditures on innovative investment relative to output as a result of the subsidy. This innovation intensity of the economy on the new BGP is found by solving equations (24), (26), and (27) for the new BGP level of $\bar{p}_r$. From equation (13), we find that the new BGP has a higher fraction of labor allocated to research $\bar{l}_r$. Since the conditions in Corollary 1 are satisfied, we can calculate the dynamics of aggregate productivity using equation 42.

Given these analytical results, we see that a permanent and uniform innovation subsidy has much the same impact on aggregate variables as what we found in our experiments in section 4 above in which we took as given a perturbation of the allocation of labor to research. We compute the transition path of the economy from the initial to the new BGP corresponding to the specific experiment of raising innovation subsidies in a uniform manner to implement an increase in the allocation of labor to research in the long run of the same magnitude as we considered in the results in Table 1 above. The responses of aggregate productivity at 20 and 100 years along the equilibrium transition path are similar to those shown in our experiment above in Table 1. That is, consideration of the endogenous timing of the reallocation of labor to research induced by the subsidy does not substantially alter the quantitative implications of the model for aggregate productivity. The responses of aggregate output are smaller than the corresponding responses of aggregate productivity because the amount of labor allocated to current production is permanently reduced.

Welfare  We now consider the normative implications of our model given this counterfactual policy experiment. To do so, we consider an economy on a BGP in which policies include a production subsidy as required to undo the distortions of corporate taxes and markups on the accumulation of physical capital in the equation (28). That is, we assume
that \( \tau_y \) is set so that, on the BGP\(^{13}\)

\[
\tau_y = \mu \left( \frac{1 - \tau_{corp} \lambda_k}{1 - \tau_{corp}} \right) - 1. \tag{45}
\]

In this case, as shown in AB2018, the log of the consumption equivalent variation in welfare that arises from a perturbation of the BGP allocation is given, to a first order approximation, by

\[
\log \xi \approx \left(1 - \exp((g_Y - R))\right) \sum_{t=0}^{\infty} \exp\left( t (g_Y - R) \right) \frac{\bar{Y}}{C} \left[ (\log Z'_t - \log Z_t) - (1 - \alpha) \frac{\bar{I}_r}{\bar{I}_p} (\log l'_rt - \log \bar{I}_r) \right] \tag{46}
\]

Note that the direct effect of any reallocation of labor to research on welfare is negative as it results in a reduction in current production labor and hence of current output of size \(-(1 - \alpha)\bar{I}_r/\bar{I}_p\). This reallocation of labor to research results in a welfare gain to the extent that the discounted present value of the impulse to aggregate productivity generated by this reallocation outweighs the direct cost in terms of lost output. If consumers are patient in the sense that the interest rate is not that much larger than the growth rate of output, then the normative implications of our model for our policy experiment are highly sensitive to the specification of the technology for research (as are model’s implications for the long term response of aggregate productivity). In Table 2, we present the consumption equivalent welfare gain from the uniform innovation subsidies considered above. We see that the uniform increase in innovation subsidies considered here leads to a significant increase in welfare in the specification of the model with the First Generation Endogenous Growth research technology, a more moderate increase in welfare with the JKS research technology, and a decline in welfare with the Second Generation Endogenous Growth research technology because the subsidy raises the allocation of labor to research above its constrained optimal level discussed below.

We can also use equation (46) to compute the normative implications of our model for the allocation of labor to research on the BGP of our economy once the optimal uniform innovation subsidy has been put in place. If the economy is on a BGP corresponding to the constrained socially optimal uniform innovation subsidy (i.e. the planner can make uniform changes to innovation subsidies), then a one-time policy-induced perturbation to the fraction of labor allocated to research at \(t\) and the induced dynamics of aggregate productivity as described in equation (42) should result in no change in welfare, i.e. \(\log \xi = 0\).

\(^{13}\)The value of \(\tau_y\) does not matter for the positive results we discussed above.
Thus, the allocation of labor to production and research on the BGP corresponding to the constrained optimal uniform innovation subsidy is given by

$$\bar{l}_r = \left( \frac{1}{1 - \alpha} \right) \frac{\Theta_G}{A} \frac{\exp \left( (\bar{g}_Y - \bar{R}) \right)}{1 - \exp \left( (\bar{g}_Y - \bar{R}) \right)} \left( 1 - \Theta \right).$$

We present results for this allocation of labor in Table 2. As anticipated by our discussion above, this normative implications of our model are highly sensitive to the specification of the technology for research because the persistence parameter $\Theta$ is highly sensitive to the specification of this technology. For the First Generation Endogenous Growth research technology, the BGP allocation of labor under the optimal uniform innovation subsidy has half of the labor force or more engaged in research. For the J/K/S technology, at least one quarter of the labor force should be engaged in research. For the Second Generation Endogenous Growth technology, the portion of the labor force that should be engaged in research is roughly equal to or even less than that current allocation of labor to research.

Increase in the markup $\mu$  We now conduct a counterfactual experiment in which the markup $\mu$ increases permanently. As discussed above, this could arise from a change in the patent system. A permanent increase in the markup $\mu$ has qualitatively the same effects as a uniform increase in innovation subsidies. Again, if we hold all other parameters and policies fixed, we find that the new BGP has the same allocation of innovative investment $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$ and hence the same growth rates of aggregate productivity and the measure of products. In addition, firm dynamics as measured by employment shares and fractions of products in entering firms are also unchanged from the old to the new BGP. Again, this is because the equations (17),(18), (20), and (21) that determine the BGP

<table>
<thead>
<tr>
<th>Research Technology</th>
<th>Cons. Equivalent</th>
<th>Optimal $l^<em>_r / l^</em>_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Generation EG</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>1.230</td>
<td>2.407</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>1.076</td>
<td>0.969</td>
</tr>
<tr>
<td><strong>Jones/Kortum/Segerstrom</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>1.029</td>
<td>0.418</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>1.019</td>
<td>0.332</td>
</tr>
<tr>
<td><strong>Second Generation EG</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>0.999</td>
<td>0.141</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>0.992</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Table 2: Welfare implications of uniform increase in innovation subsidies
growth rate of $J$ and the allocation of investment on a BGP are not altered by a uniform change in the markup.

As was the case with a uniform increase in innovation subsidies, however, on the new BGP the economy has a higher innovation intensity of the economy measured as the ratio of expenditures on innovative investment relative to output as a result of the increase in markups. That is, again one solves equations (24), (26), and (27) for the new BGP level of $\bar{p}_r$. And again, the new BGP has a higher fraction of labor allocated to research $\bar{L}_r$.

We report the results from this experiment in the appendix. We calibrate the increase in markups so that the change in the allocation of labor to research from the initial to the new BGP is the same as that we considered with the uniform innovation subsidies. These similar perturbations to the allocation of labor to research produce similar responses of the level of aggregate productivity relative to its original trend at horizons of 20 and 100 years. However, because markups are higher in this case, the response of aggregate output at the 20 and 100 year horizons is smaller that is the case with uniform innovation subsidies since the increase in markups discourages the accumulation of physical capital.

**A reduction in corporate profits tax rate** The impact of a permanent reduction in corporate profits tax rate $\tau_{corp}$ depends upon the details of expensing of innovative investment for tax purposes for incumbent firms and entering firms, as indexed by $\lambda_I$ and $\lambda_E$.

Consider first the case with equal expensing by both entrants and incumbents, i.e. $\lambda_I = \lambda_E$. Here, as with the case with uniform innovation subsidies and the change in the markup, the allocation of investment $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$ and BGP growth rates are not altered by a change in the corporate tax rate $\tau_{corp}$. In this case, the impact of a reduction in the corporate tax rate on the innovation intensity of the economy depends on the extent of expensing. If there is full expensing (i.e. $\lambda_I = \lambda_E = 1$) then a change in the corporate profits tax rate leaves net-of-tax variables profits unchanged relative to net-of-subsidy innovation costs, so $\bar{p}_r$ and the innovation intensity of the economy do not change on the new BGP. Thus all variables on the new BGP are equal to what they were on the old BGP except that the economy accumulates more physical capital if investment on physical capital is partially expensed (i.e. $\lambda_k < 1$). If there is only partial expensing of innovative investments (i.e. $\lambda_I = \lambda_E < 1$), then a reduction in the corporate profits tax rate raises net-of-tax variables profits relative to net-of-subsidy innovation costs, which results in an increase in $\bar{p}_r$, in the innovation intensity of the economy, and in the share of labor in research on the new BGP. As is the case with a uniform innovation subsidy and the change in markup, the magnitude of the effect of the corporate profits tax change on aggregate productivity is determined by the induced change in the the allocation of labor.
Consider next the case in which the expensing of investment by entrants is less than that for incumbents (i.e. $\lambda_I > \lambda_E$). In this case, a change in the corporate profits tax rate has an impact on the allocation of investment on the new BGP. This is because equations (17) and (18) are altered by the change in $\tau_{\text{corp}}$. Specifically, under these assumptions, if the corporate profits tax rate is reduced, the term $(1 - \tau_{\text{corp}}\lambda_I)/(1 - \tau_{\text{corp}}\lambda_E)$ rises. From these equations, we see that a reduction in the tax rate leads to a fall in innovative investments by incumbents $\tilde{x}_c$ and $\tilde{x}_m$ from the initial BGP to the new BGP, since the functions $h$ and $\zeta$ are strictly concave. Since the BGP growth rate of $J$ is not altered by the tax change, this implies an increase in investment by entrants $\tilde{x}_e$ from the initial BGP to the new BGP. This reallocation of investment implies that we cannot directly apply Corollary 1. Instead, we must compute the dynamics of $J$ given in Proposition 2 and here the change in research output across BGP (i.e. $\tilde{Y}_r$ relative to $Y_{r0}$) becomes relevant for the dynamics of aggregate productivity. In particular, in equation (29) the BGP level of $J$ (i.e. $\tilde{J}_0$) is impacted. Hence, from equation (40), for a given path of research labor, the change in aggregate productivity relative to its initial trend, $\log Z_{t+1} - \log Z_0 - t \bar{g}_Z$, can be higher or lower than what we found in the previous experiments in which $\tilde{Y}_r = Y_{r0}$. Note that in the case in which the research technology has $\psi = 1$, the BGP growth rate of aggregate productivity $\bar{g}_Z$ is unchanged by a change in corporate profits taxes. In contrast, when $\psi = 0$, the BGP growth rate of aggregate productivity changes.\(^{15}\)

We now illustrate some of these effects with a specific experiment, shown in Table 3. We set the corporate profit tax rates $\tau_{\text{corp}}$ as in Barro and Furman (2018), and assume that incumbent firms can deduct all of their innovative investments ($\lambda_I = 1$), while entering firms cannot ($\lambda_E = 0$) since they are not incorporated at the time of their investments. Further details are given in the Appendix. As we now discuss, the implications of the model for the impact of the tax cut on aggregate productivity and welfare are highly sensitive to the amount of business stealing and the specification of the technology for research.

Consider first the results under the first two research technologies ($\psi = 1$), in which case the growth rate of aggregate productivity is unchanged between BGPs. We see in the top four rows of Table 3 that aggregate productivity relative to its initial BGP trend rises when there is no business stealing and falls when there is business stealing. This


\(^{15}\)Peretto (2007) discusses this impact of changes in corporate profits tax rates on the BGP growth rate of aggregate productivity in a model with a Second Generation Endogenous Growth research technology.
occurs despite the fact that the change in tax policy induces a similar reallocation of labor to research across these model specifications. Without business stealing, the reallocation of innovative investment towards entry induced by the change in corporate tax policies reduces the research output $Y_r$ required to attain the unchanged BGP growth rate because the initial tax policy favors investment by incumbent firms (since these firms can deduct their innovative investments for tax purposes while entering firms cannot). This effect amplifies the positive effects of the tax change on the level aggregate productivity. With business stealing (and in spite of the bias of initial policies in favor of investment by incumbent firms), the increase in investment in entry induced by the change in corporate tax policies reduces the research output $Y_r$ required to attain the BGP growth rate. This effect now dampens the positive effects of the tax change on aggregate productivity or even make them negative. In the third specification of the research technology ($\psi = 0$), reported in the bottom two rows of Table 3, the BGP growth rate of aggregate productivity falls on the new BGP. As a result, the reduction in aggregate productivity (and welfare) is large at medium and long term horizons.

<table>
<thead>
<tr>
<th>Research Technology</th>
<th>Productivity at 20 years</th>
<th>Productivity at 100 years</th>
<th>Cons. Equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FIRST GENERATION EG</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>0.053</td>
<td>0.265</td>
<td>1.193</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>-0.023</td>
<td>-0.103</td>
<td>0.913</td>
</tr>
<tr>
<td><strong>JONES/KORTUM/SEGERSTROM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>0.035</td>
<td>0.038</td>
<td>1.027</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>-0.02</td>
<td>-0.03</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>SECOND GENERATION EG</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no Business Stealing</td>
<td>-0.042</td>
<td>-0.315</td>
<td>0.732</td>
</tr>
<tr>
<td>with Business Stealing</td>
<td>-0.056</td>
<td>-0.33</td>
<td>0.751</td>
</tr>
</tbody>
</table>

Table 3: Corporate profits tax experiment: 20 and 100 year response of aggregate productivity, and equivalent variation in consumption

6 Conclusion

In this paper, we have shown how to characterize the transition dynamics for aggregate productivity implied by a variety of models of firms’ investments in innovation in terms of two sufficient statistics: the impact elasticity of aggregate productivity with respect to a change in the allocation of labor to research and the persistence of the response of aggregate productivity to that impulse. We have shown how the normative implications
of these models for uniform innovation subsidies are also determined by these two sufficient statistics. We have shown how to discipline the first of these sufficient statistics using data on firm dynamics and inference regarding the share of innovative investment undertaken by entering firms. We have discussed the challenge of disciplining the second of these sufficient statistics due to uncertainty regarding the nature and magnitude of intertemporal knowledge spillovers in research. Finally, we have considered policy experiments for which the transition dynamics depend on additional model details beyond our two sufficient statistics.

In order to obtain sufficient aggregation to allow for analytically tractable transition dynamics, our model abstracts from some of the richness in recently developed models of innovative investment with heterogeneous firms, including those in Peters (2016), Akcigit and Kerr (2018), Lentz and Mortensen (2008), Lentz and Mortensen (2016), Acemoglu et al. (forthcoming) and Luttmer (2011). And yet, even in our model with strong assumptions to allow us to aggregate across heterogeneous firms, there is a lot of uncertainty about the positive and normative implications of policy changes that lead to a long run reallocation of innovative investment between incumbent and entering firms, as we saw with our final corporate profits tax experiment. One important challenge for future research in this area is to find reliable metrics for evaluating the positive implications of these richer models for the transition dynamics of aggregate productivity and the normative implications of these models for which types of firms should be doing relatively more innovative investment and which types of firms should be doing less.

References


16McGrattan (2017) pursues an approach to modeling intangible investment as a non-rivalrous input that is not nested in the class of models that we consider here.


