Transitional Dynamics in Aggregate Models of Innovative Investment

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Abstract

What quantitative lessons can we learn from models of endogenous technical change through innovative investments by firms for the impact of changes in the economic environment on the dynamics of aggregate productivity in the short, medium, and long run? We present a unifying model that nests a number of canonical models in the literature and characterize both their positive implications for the transitional dynamics of aggregate productivity and their welfare implications in terms of two sufficient statistics. We review the current state of measurement of these two sufficient statistics and discuss the range of positive and normative quantitative implications of our model for a wide array of counterfactual experiments, including the link between a decline in the entry rate of new firms and a slowdown in the growth of aggregate productivity. We conclude with a summary of the lessons learned from our analysis to help direct future research aimed at building models of endogenous productivity growth useful for quantitative analysis.

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1 Introduction

The Economics Nobel Prize Committee recently honored the contributions of Paul Romer and others in developing models of endogenous technical change through innovative investments by firms.[1] What quantitative lessons have we learned from these models regarding how changes in policies or in the economic environment, such as the extent of monopoly profits, the costs of firm entry, or the rate of population growth, affect aggregate productivity growth through their impact on firms' incentives to engage in these investments in innovation? To date, the quantitative implications of these models for counterfactual analysis of the transitional dynamics of aggregate productivity have not received as much attention as their implications for long-run growth.[2] Moreover, there is a great deal of uncertainty regarding the quantitative implications of these models for the analysis of positive questions such as: to what extent are the driving factors responsible for the decline in firm entry rates observed in the US economy over the past several decades also responsible for the disappointing growth in aggregate productivity observed over this same time period?

In this paper, we look to summarize the current state of knowledge regarding the quantitative implications of models of endogenous technical change through innovative investment by firms for the transitional dynamics of aggregate productivity. To do so, we build on our work in Atkeson and Burstein (2018) (henceforth AB2018) to present a model that nests many of the canonical models of the interaction of firms' innovative investments and aggregate productivity. We use this model to study, quantitatively, the dynamics of aggregate productivity in response to a variety of counterfactual changes in policies and the economic environment.[3] We characterize, up to a first-order approximation, the implications of our model for the equilibrium relationship between the transitional dynamics of aggregate productivity and innovative investment measured by the

[1] See the surveys of this literature by Jones (2005), Acemoglu (2009), Aghion et al. (2014), Aghion et al. (2015), Akcigit (2017), and the Nobel Committee (2018).

[2] The insights from these models of endogenous technical change have also not been widely applied by policy institutions in analyses of the impact of policy changes on macroeconomic dynamics over the short- and medium-term horizons. For example, the Congressional Budget Office relies on variants of the Solow neoclassical growth model in their 10- and 25-year forecasts of the budget and economic outlook. The Bureau of Economic Analysis has begun to incorporate measures of firms' investments in intangible capital into the national income and product accounts starting in 2013 (see e.g. Corrado et al. 2009).

[3] For examples of analyses of the impact of tax policies on aggregate productivity through their impact on firms' investments in innovation, see McGrattan and Prescott (2005), Peretto (2007), Acemoglu (2009), Jaimovich and Rebelo (2017), and Ferraro et al. (2017). Karahan et al. (2016) study the impact of changes in the growth rate of the labor force on firm entry, and Comin and Gertler (2006), De Loecker and Eeckhout (2017) and Edmond et al. (2018) study the macroeconomic implications of changes in markups. Important contributions in the public finance literature on the impact of tax policy on business entry (entrepreneurship) include Gentry and Hubbard (2005), Cullen and Gordon (2006) and Giroud and Rauh (2015).

innovation intensity of the economy, the allocation of labor to research, or the entry rate of new firms.[4] We use this approximate solution of our model to highlight two sufficient statistics that play a key role in shaping our model's quantitative positive and normative implications. We illustrate the range of quantitative answers implied by our model to our counterfactual experiments given plausible measures of these statistics. We see our results as a useful benchmark for future research aimed at developing models of endogenous technical change that are useful for quantitative policy analysis.

Our model, which extends the model of firm dynamics in Garcia-Macia et al. (2016), features innovative investments by firms that may be directed either at acquiring products new to those firms or at improving the productivity with which these firms produce one of their current products. Firms acquire new products either by inventing products that are new to society as a whole (as in models of growth through *expanding varieties*) or by inventing a better technology for producing a product currently produced by some other incumbent firm (as in neo-Schumpeterian models of innovation through *business stealing*).[5] We nest a wider class of models of the spillovers from innovative investment in the literature than that in AB2018. Specifically, in addition to the semi-endogenous growth framework of Jones (2002), we nest models that use what are referred to as *second generation endogenous growth* technologies for research.[6]

To develop our sufficient statistics, we solve analytically for a first-order approximation to the model-implied path of aggregate productivity following an increase in the innovation intensity of the economy (and the aggregate allocation of labor to innovative investment) induced by a policy change that has a uniform impact on the incentives of incumbent and entering firms to invest in innovation. The first of our sufficient statistics, which we term the *impact elasticity*, is the elasticity of the growth of aggregate productivity between this period and the next with respect to a given change in the log of the labor input allocated to research. The second of our sufficient statistics, which we refer to as the extent of *intertemporal knowledge spillovers in research*, determines the persistence of this impulse to aggregate productivity; that is, the rate at which the level of aggregate productivity returns to its baseline growth path.[7] We show how these sufficient statis-

[4] See Fernald and Jones (2014) for a similar approach to modeling the transition dynamics of aggregate productivity and the allocation of labor to research.

[5] As noted in Nobel Committee (2018), the existence of business stealing has the important implication that equilibrium investment in innovation may be too high from a social perspective since business stealing amounts to a negative spillover to other firms.

[6] See, for example, Peretto (1998), Young (1998), Dinopoulos and Thompson (1998), Howitt (1999), and Ha and Howitt (2007).

[7] In the limit, with full intertemporal knowledge spillovers, the half-life of the level of aggregate productivity in our model is infinite. In this case, if we abstract from the dynamics of physical capital, the growth rate of aggregate productivity jumps immediately to its new long-run level following a change in policies.

tics shape the model's quantitative implications for the transitional dynamics of aggregate productivity following not only uniform changes in innovation subsidies, but also changes in markups and changes in corporate taxes with equal expensing of innovative investment by entering and incumbent firms. We also show that these sufficient statistics shape our model's normative implications for the welfare consequences of these changes in the economic environment.

How can one measure these two sufficient statistics? The impact elasticity of the model is measured using data on firm dynamics — in particular, data on the contribution of firm entry to aggregate productivity growth relative to the share of innovative investment carried out by entering firms on the economy's balanced growth path. Our second sufficient statistic, the extent of intertemporal knowledge spillovers, is much harder to measure because it requires one to estimate whether the half-life of a shock to aggregate productivity is a decade, a century, or a millennium.[8] We show that uncertainty regarding the extent of intertemporal knowledge spillovers is not very important for positive quantitative analysis of the medium-term dynamics (e.g., 20 years) of aggregate productivity in response to the changes in the economic environment that we consider. In contrast, this uncertainty in measurement of intertemporal knowledge spillovers is much more important for the normative implications of our model since these normative implications are driven by the model's predictions for the long-term response of aggregate productivity to policy changes or other changes in the economic environment.

We next consider the transitional dynamics of aggregate productivity in our model in response to policy changes or other changes in the economic environment that do not have a uniform impact on the incentives of incumbent and entering firms to invest in innovation. Such policies lead, in the long term, to changes in the share of innovative investment carried out by entering versus incumbent firms. In this case, a number of additional forces come into play in shaping the model's transitional dynamics, because one must use the full model to compute its counterfactual implications for our sufficient statistics and the level and growth rate of aggregate productivity on the new balanced growth path to which the economy converges. We illustrate the additional mechanisms at work in our model with an analysis of the impact of a change in corporate profit taxes when the expensing of innovative investment for tax purposes is not the same for incum-

[8]The specific form of intertemporal knowledge spillovers in research was central to the debate in the literature 20 years ago about scale effects in endogenous growth models. See the surveys in Jones (2005), Ha and Howitt (2007), and Akcigit (2017). While interest in this debate in the literature has diminished, we find that the issues raised there are central to understanding the normative implications of counterfactual policy experiments in our model. See Bloom et al. (2017) for a recent empirical contribution to the measurement of intertemporal knowledge spillovers.

bent and entering firms.

We finally use our model to interpret quantitatively the joint dynamics of firm entry and aggregate productivity. Recent research has focused attention on the observation that the rate of entry of new firms in the US economy has declined substantially in recent decades (see, e.g., Decker et al. 2014 and Alon et al., 2017). Viewed through the lens of our model, this decline in firm entry reflects reduced investment in innovation by entering firms relative to investments by incumbent firms. We use our analytical results and sufficient statistics to examine the quantitative implications of our model for the relationship between firm entry rates and aggregate productivity, given different possible driving factors behind these phenomena. We consider three candidate explanations for the decline in the entry rate of new firms in the model: a uniform increase in the cost of innovation for incumbent and entering firms, an increase in the cost of innovation for entering firms relative to incumbent firms, and a decline in the population growth rate. In each case, we choose the magnitude of the change in costs or demographics to induce the same decline in the entry rate of firms over a 20-year period. We then consider the quantitative implications of different specifications of our model for the associated cumulative change in aggregate productivity. In almost all of the cases that we consider, a large decline in entry over a 20-year period is associated with a modest cumulative decline in the level of aggregate productivity. Thus, our model implies that these factors likely account for only a modest portion of the decline in aggregate productivity growth over the past two decades.

The remainder of the paper is organized as follows. In Section 2, we present our model and discuss the models in the literature that are nested in our framework. In Section 3, we characterize the growth rates and levels of variables on the balanced growth path of the model. In Section 4, we characterize our model's implications for the equilibrium relationships between the transitional dynamics of aggregate productivity and those of the innovation intensity of the economy, the allocation of labor to research, or the rate of entry by new firms. We use these formulas for the model's transitional dynamics to highlight which features of the data on firm dynamics and the aggregate economy are critical for identifying the parameters of the model that are key to its dynamic implications. In Section 5, we describe the counterfactual experiments that we conduct with the model and the results we obtain regarding the short- and long-term responses of aggregate productivity and welfare induced by these various experiments. In Section 6, we examine our model's implications for the cumulative change in aggregate productivity associated with a gradual decline in the rate of entry of new firms over a 20-year period. We conclude with a summary of the lessons learned from our analysis to help direct future research aimed at building models of endogenous productivity growth useful for quantitative analysis. In a supplemental online appendix, we present a full description of our model, proofs of our propositions, the calibration of the model parameters, and a wide range of supporting technical material.

2 The Model

The model is specified so that it is possible to aggregate production and various types of innovative investment across heterogeneous firms. It is this aggregation that makes analysis of our model's transitional dynamics tractable. We present the key elements of the model here and point out the assumptions we make to allow for aggregation. Further details regarding our model and the definition of equilibrium are provided in the online appendix. In this section, we also discuss some of the models in the literature that are nested in our model.

There are three types of goods: a final good used for consumption and investment in physical capital, a second final good that we term the research good that is the input into innovative investment by firms, and differentiated intermediate goods produced by innovating firms.

The Final Consumption Good The representative agent has standard preferences over final consumption $\sum_{t=0}^{\infty} \frac{\beta^t}{1-\gamma} L_t \left(C_t/L_t\right)^{1-\gamma}$, with $\beta < 1$ and $\gamma > 0$. Population L_t grows exogenously at rate $\exp(g_{Lt}) = L_{t+1}/L_t$.

Output of the final consumption good, Y_t, is produced as a constant elasticity of substitution (CES) aggregate of the output of a continuum of differentiated intermediate goods, with the elasticity of substitution between intermediate goods equal to $\rho > 1$. At each date t, the technology with which any particular intermediate good can be produced is summarized by its productivity index z. Production of an intermediate good with productivity index z is carried out with physical capital, k, and labor, l, according to

$$y_t\left(z\right) = z k_t\left(z\right)^{\alpha} l_t\left(z\right)^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$. For each intermediate good that can be produced at time t, we refer to the technology with the highest value of z as the *frontier technology* for this good. Intermediate goods are produced by firms that own the exclusive rights to use the frontier technology for producing one or more intermediate goods.

The productive capacity of the economy at time t is determined by its population L_t, its current stock of physical capital K_t, and the measure of intermediate goods with frontier technology indexed by z, which we denote by $M_t(z)$. Total labor hours employed in the production of intermediate goods is denoted by $l_{pt}L_t$, with $l_{pt} \in [0,1]$ representing the fraction of the population engaged in current production. The constraints on production labor and physical capital require that $l_{pt}L_t = \sum_z l_t(z)M_t(z)$ and $K_t = \sum_z k_t(z)M_t(z)$.

We make the following standard assumptions to allow for simple aggregation of current production. In each period, physical capital and labor are freely mobile across intermediate goods producing firms, and the markup $\mu \geq 1$ of price over marginal cost charged by intermediate goods firms is constant across intermediate goods and over time.[9] With these assumptions, aggregate output of the final good can be written as $Y_t = Z_t\,(K_t)^{\alpha}\,\left(l_{pt}L_t\right)^{1-\alpha}$, where Z_t is given by

$$Z_t = \left[\sum_z z^{\rho-1} M_t(z)\right]^{1/(\rho-1)}. \tag{2}$$

Throughout this paper, we refer to Z_t as *aggregate productivity* at time t. We refer to $M_t = \sum_z M_t(z)$ as the *total measure of products available,* and to the ratio $Z_t^{\rho-1}/M_t$ as the *average productivity* index of existing intermediate goods (specifically, the average of $z^{\rho-1}$ across intermediate goods). We refer to $s_t(z) = z^{\rho-1}/Z_t^{\rho-1}$ as the *size* of a product with frontier technology z. This is because the share of revenue and physical capital and production employment at t associated with this product in equilibrium is given by $s_t(z)$.

The Research Good The second final good in this economy is the input used for innovative investment by firms. Output of the research good is produced using research labor hours $l_{rt}L_t$, according to

$$Y_{rt} = A_{rt} Z_t^{\phi-1} l_{rt} L_t. \tag{3}$$

Here, $l_{rt} = 1 - l_{pt}$ denotes the fraction of the labor force employed in research.In this case, A_{rt} represents the stock of freely available scientific progress, which grows at an exogenous rate $g_{At} = \bar{g}_A$.[10] The term $Z_t^{\phi-1}$ with $\phi \leq 1$ reflects *intertemporal knowledge*

[9] We assume that it is technologically possible for latent competitors to copy the frontier technology for each intermediate good but that intellectual property protection forces them to use a version of the technology with productivity that is fraction $1/\bar{\mu}$ of the productivity of the frontier technology for that good. With Bertrand competition and limit pricing, the gross markup μ charged by the incumbent producer of each product is the minimum of the monopoly markup, $\rho/(\rho-1)$, and the technology gap between the frontier and the latent technology, $\bar{\mu}$. When we consider a change in the equilibrium markup, we assume that it is due to a change in policy regarding protection of intellectual property determining $\bar{\mu}$.

[10] Some papers in the theoretical literature on economic growth with innovating firms assume that aggregate productivity growth is driven entirely by firms' expenditures on R&D (Griliches 1979, p. 93). As noted in Corrado et al. (2011) and Akcigit (2017) this view ignores the productivity-enhancing effects of invest-

spillovers in the production of the research good, as in the model of Jones (2002). Using the language of Bloom et al. (2017), $A_{rt}Z_t^{\phi-1}$ denotes the productivity with which research labor $l_{rt}L_t$ translates into a real flow of "ideas" Y_{rt} available to be applied to innovative investment. If $\phi < 1$, then increases in the level of aggregate productivity Z_t reduce research productivity in the sense that "ideas become harder to find." Because the impact of advances in Z_t on research productivity is external to any particular firm, we call it a "spillover." The parameter ϕ indexes the extent of this spillover. While this is a negative spillover, our model also features the classic positive spillovers through which new ideas build on old ideas as described below.

Innovation Innovative investment is undertaken by intermediate goods producing firms. We refer to those firms producing intermediate goods at t that also produced at $t-1$ as *incumbent* firms. We refer to those firms at t that are new (and hence did not produce intermediate goods at $t-1$) as *entering* firms. There are three types of innovative investments: (i) new firms invest to enter, each acquiring the frontier technology to produce a single product that is new for that firm, (ii) incumbent firms invest to acquire the frontier technologies for products that are new to those firms, and (iii) incumbent firms invest to improve the frontier technologies of their existing products.

We now describe the assumptions that generate a tractable relationship between these investments and the dynamics of the total measure of products M_t and aggregate productivity Z_t, summarized in equations (6) and (7) below.

Innovation by Entering Firms Each entering firm at t must invest $M_t^{-\psi}$ units of the research good to acquire a product at $t+1$, with $\psi \leq 1$. Let $x_{et}M_t^{1-\psi}$ denote the total use of the research good by entering firms. Then, $x_{et}M_t$ is the measure of products acquired by entering firms at $t+1$, where x_{et} denotes the ratio of products acquired by entering firms at $t+1$ relative to the total measure of products at t.

As we discuss in our review of nested models below, two values of ψ are typically considered in the literature. The first value is $\psi = 1$. In this case, the resources required to create one new product through entry fall with the number of existing products. The second value is $\psi = 0$. In this case, the resources required to create one new product

ments by actors other than business firms. We capture all of these other productivity-enhancing effects with A_r. Akcigit et al. (2013) consider a growth model that distinguishes between basic and applied research and introduce a public research sector. As we discuss below, the only role served by the exogenous growth of scientific progress A_{rt} in our analysis is that, by adjusting the parameter $\bar{g}_A$, we can target a given baseline growth rate of output in the balanced growth path for a given growth rate of population, $\bar{g}_L$, as we vary the parameters ϕ and ψ.

through entry are independent of the number of existing products.

The frontier technology newly acquired by an entering firm may apply to an intermediate good that was previously produced by an incumbent firm or may apply to an intermediate good that is new to society. Specifically, with probability δ_e, the intermediate good acquired by the entrant at $t+1$ was already being produced by an incumbent firm at t, but with a lower productivity index than the new frontier technology for that good acquired by the entrant. Since identical intermediate goods are perfect substitutes in the production of the final consumption good, competition in the product market between the entering firm and the previous incumbent producer of this intermediate good implies that the previous incumbent producer ceases production of the good. In this case, the innovative investment by the entering firm does not result in a net increase in the total measure of products available M_{t+1}. Instead, it only results in a positive increment to the average productivity index across existing products $Z_{t+1}^{\rho-1}/M_{t+1}$. As is common in the literature, we say that this intermediate good that is new to the entering firm represents *business stealing* from an incumbent firm.[11]

With the complementary probability $1-\delta_e$, this newly acquired frontier technology allows this entering firm to produce an intermediate good that is new to society as a whole. In this case, the innovative investment by the entering firm results in a net increase in the total measure of products available M_{t+1}. As is common in the literature, we say that this intermediate good that is new to this entering firm represents a contribution to productivity through *expanding varieties*.

Let the average value of $z'^{\rho-1}$ across all products obtained by entering firms at $t+1$ be given by $\mathbb{E}z'^{\rho-1} = \eta_e Z_t^{\rho-1}/M_t$.[12] Note that the term $Z_t^{\rho-1}/M_t$ is the average value of $z^{\rho-1}$ across existing intermediate goods at t. Hence, in the model, there is a positive spillover from the average value of $z^{\rho-1}$ across existing products at t to the expected value of productivity $z'^{\rho-1}$ for entering firms at $t+1$. These assumptions imply that the fraction of products at $t+1$ produced by entering firms is given by $x_{et}M_t/M_{t+1}$ and the share of production employment in entering firms is given by $x_{et}\eta_e Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$. The term η_e is an exogenous parameter that determines the average size of products produced by entering

[11] We assume that the probability that a new product acquired by either an entering or an incumbent firm is stolen from another incumbent firm is an exogenous parameter. A richer model might aim to make this probability endogenous.

[12] The productivity index z' for stolen products in entering firms is drawn as an increment over the previous frontier technology of an existing intermediate good in a manner similar to that in Klette and Kortum (2004) and other standard quality ladder models. The productivity index z' for products that are new to society in entering firms is drawn by imperfect copying of the frontier technology for existing intermediate goods in a manner similar to that in Luttmer (2007). The transitional dynamics of the measure of products and aggregate productivity implied by our model do not depend on the specifics of these assumptions regarding draws of z' for entering firms beyond our assumption regarding the average value of $z'^{\rho-1}$ above.

firms relative to the average size of all products at $t+1$.

Innovation by Incumbent Firms to Acquire Products An incumbent firm that owns the frontier technology z for producing a particular intermediate good possesses the capacity to acquire the frontier technology on additional goods new to that firm through innovative investment. The investment technology is specified so that to attain any given probability of acquiring a new product at $t+1$, a firm must invest $x_{mt}(z)$ in proportion to the term $s_t(z)M_t^{1-\psi}$. Hence, if $\psi = 1$, then the investment to attain a given probability of gaining a new product is proportional to the size of the product $s_t(z)$. If $\psi = 0$, then the required investment is proportional to the ratio of $z^{\rho-1}$ to its average value across existing products, $Z_t^{\rho-1}/M_t$. We show in the online appendix that, in equilibrium, incumbent firms choose to invest in proportion to $s_t(z)M_t^{1-\psi}$. Thus, if $x_{mt}M_t^{1-\psi}$ is aggregate investment by incumbent firms in obtaining new goods, then the probability that incumbent firms gain a new product for each product that they currently produce is given by $1-\exp\left(-h\left(x_{mt}\right)\right)$, where $h(\cdot)$ is a strictly increasing and concave function with $h(0)=0$ and $h(x)<1$ for all $x \geq 0$.[13] As is the case with entering firms, new products acquired by incumbent firms may be stolen from other incumbent firms (with probability δ_m) or are new to society (with probability $1-\delta_m$).

Let the average value of $z'^{\rho-1}$ across all products obtained by entering firms at $t+1$ be given by $\mathbb{E}z'^{\rho-1} = \eta_m Z_t^{\rho-1}/M_t$. Note that these assumptions imply that the fraction of products at $t+1$ that are newly acquired by incumbent firms is given by $(1-\exp\left(-h(x_{mt})\right)$ $\times M_t/M_{t+1}$, and the share of production employment at $t+1$ in products newly acquired by incumbent firms is given by $(1-\exp\left(-h(x_{mt})\right)\eta_m Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$. The term η_m is an exogenous parameter that determines the average size of products newly acquired by incumbent firms relative to the average size of all products.

Innovations by Incumbent Firms to Improve Continuing Products Incumbent firms lose existing products due to exogenous exit and business stealing by entering and other incumbent firms.[14] We refer to the products that they retain from period t to $t+1$ as *continuing products*.

[13] In the online appendix, we present the equations of our model with the length of a time period in calendar time as a parameter. We use the exponential function to denote rates per unit of time so that we can derive the standard continuous time expressions for the equations of the model in the limit as the length of a time period in our model shrinks to zero. As discussed below, several important economic implications of the model are derived as the length of a time period in the model measured in units of calendar time shrinks to zero.

[14] We do not consider here the endogenous exit of products due to fixed operating costs, which is featured in other papers in the literature but makes the model less tractable analytically.

Incumbent firms can invest to improve the frontier technology for their continuing products. This investment technology is specified so that to attain any given percentage growth in this frontier technology, the firm must invest at a rate $x_{ct}(z)$ in proportion to $s_t(z)M_t^{1-\psi}$. The interpretation of the parameter ψ is the same as above. We show in the appendix that, in equilibrium, incumbent firms invest in proportion to $s_t(z)M_t^{1-\psi}$. Thus, if $x_{ct}M_t^{1-\psi}$ is aggregate investment by incumbent firms in improving their products, the expected growth rate of the frontier technology for continuing products with productivity index z satisfies $\mathbb{E}z'^{\rho-1}/z^{\rho-1} = \exp\left(\zeta\left(x_{ct}\right)\right)$. We assume that $\zeta(\cdot)$ is a strictly increasing and concave function, with $\zeta(x) > 0$ for all $x \geq 0$. In terms of observables, these assumptions imply that, in expectation, surviving products in incumbent firms grow in size at a common rate of $\exp\left(\zeta\left(x_{ct}\right)\right) Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$.

With these definitions, we can write the resource constraint for the research good as

$$\left(x_{ct} + x_{mt} + x_{et}\right) M_t^{1-\psi} = Y_{rt}. \tag{4}$$

Dynamics of M and Z We now characterize the dynamics of the measure of intermediate goods M_t and aggregate productivity Z_t as functions of innovation rates by entering and incumbent firms.

The evolution of the total measure of products M_t is determined by the rates at which incumbent firms lose the frontier technologies for products that they produced at t and the rate at which incumbent and entering firms gain the frontier technologies for products new to these firms. Consider first the rate at which an incumbent firm lose products. For each product that they produce at t, incumbent firms can lose its frontier technology at $t+1$, due to either exogenous exit (with probability $\left(1-\exp\left(-\delta_0\right)\right)$) or business stealing. Since x_{et} is the ratio of the measure of products gained by entering firms relative to the measure of existing products produced by incumbent firms M_t, incumbent firms lose a fraction $\delta_e x_{et}$ of their existing products to business stealing by entrants. Likewise, since $\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right)$ is the ratio of the measure of new products obtained by incumbent firms to the measure of existing products in incumbent firms, incumbent firms lose a fraction $\delta_m\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right)$ of their existing products to business stealing by other incumbent firms. Let $\exp(-\delta_{ct})$ denote the probability that a product remains in the same incumbent firm at $t+1$, where $\delta_{ct} = \delta_c(x_{mt}, x_{et})$ is defined by the equation

$$\exp(-\delta_c(x_{mt}, x_{et})) = \exp\left(-\delta_0\right) - \delta_m\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right) - \delta_e x_{et}. \tag{5}$$

Adding back in the measures of new products obtained by entering and incumbent firms as described above, we get that the corresponding evolution of the total measure of intermediate products M_t is given by $\log M_{t+1} - \log M_t = H(x_{mt}, x_{et})$ where

$$H(x_{mt}, x_{et}) \equiv \log\left(\exp\left(-\delta_c\left(x_{mt}, x_{et}\right)\right) + 1 - \exp\left(-h\left(x_{mt}\right)\right) + x_{et}\right). \tag{6}$$

The evolution of aggregate productivity can be expressed as a weighted sum of the average values of $z'^{\rho-1}$ at $t+1$ across the three types of products: new products obtained by entering firms, new products obtained by incumbent firms, and continuing products in incumbent firms, with the weights given by the fractions of products in each category. This weighted sum is given by

$$\frac{Z_{t+1}^{\rho-1}}{M_{t+1}} = \left[\exp\left(-\delta_{ct}\right)\frac{M_t}{M_{t+1}}\right]\exp\left(\zeta\left(x_{ct}\right)\right)\frac{Z_t^{\rho-1}}{M_t} +$$

$$\left[\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right)\frac{M_t}{M_{t+1}}\right]\eta_m\frac{Z_t^{\rho-1}}{M_t} + \left[x_{et}\frac{M_t}{M_{t+1}}\right]\eta_e\frac{Z_t^{\rho-1}}{M_t},$$

where the three terms on the right hand side of this expression correspond to a weight in square brackets times an average value of $z'^{\rho-1}$ for continuing products in incumbent firms, new products in incumbent firms, and new products in entering firms respectively. This equation then implies that the evolution of aggregate productivity is given by $\log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et})$ where

$$G(x_{ct}, x_{mt}, x_{et}) \equiv \tag{7}$$

$$\frac{1}{\rho-1}\log\left(\exp\left(-\delta_{ct}\right)\exp\left(\zeta\left(x_{ct}\right)\right) + \eta_m\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right) + \eta_e x_{et}\right).$$

Note that equations (6) and (7) imply that the dynamics of the total measure of products M_t and aggregate productivity Z_t depend only on three aggregate indicators of innovative investment: x_{ct}, x_{mt}, and x_{et} . It is not necessary to record further attributes of the measure $M_t(z)$ describing the distribution of frontier productivities across intermediate goods. It is this result that makes the transitional dynamics of the model tractable.

Mechanically, equations (6) and (7) are derived from assumptions that ensure that in each period of the transition, incumbent firms lose and gain products at common rates regardless of the frontier technologies for those products, and that the expectation of $z'^{\rho-1}$ for continuing products in incumbent firms, products newly acquired by incumbent firms, and products newly acquired by entering firms are each common proportions of the average value of $z^{\rho-1}$ for products last period. These assumptions imply that data on the dynamics of products within firms generated by the model satisfy certain regularities. These include a strong version of Gibrat's law at the product level for incumbent firms — in expectation, incumbent firms lose products at a common rate regardless of the current size of the product, and continuing products in incumbent firms grow at a rate independent of current size. A related property holds for new products gained by incumbent firms — in expectation, the number of products in an incumbent firm grows at a common rate independent of the current number of products in that firm, and the contribution of newly acquired products to the growth rate of incumbent firms is common across firms each period. Finally, on a balanced growth path, the average size of entering firms is constant. These implications of our model are similar to those of the model of Garcia-Macia et al. (2016). Our model also implies that the research intensity of firms, measured as the ratio of sales to expenditure on innovative investment, is independent of size and that the value of firms is directly proportional to size.[15]

In addition, we must also assume that all aggregate spillovers affecting the productivity of research labor in producing innovations are specified in terms of these same two aggregates M_t and Z_t.

The Allocation of Innovative Investment in Equilibrium In the appendix, we fully describe the market structure, policies, and equilibrium in our model economy. Here we focus in particular on deriving equilibrium relationships between policies and the levels of the three types of innovative investment by firms in this economy that we use in characterizing the transitional dynamics of our model economy. These equilibrium relationships are presented in equations (8), (12), and (13) below.

Intermediate goods producing firms hire production labor and rent physical capital to produce intermediate goods according to the technology in equation (1). They set prices $p_t(z)$ at a markup of $\mu > 1$ over their marginal cost of production. Standard arguments give that, in equilibrium, these firms expend a fraction $(1-\alpha)/\mu$ of revenue $p_t(z)y_t(z)$ on wages for production labor and a fraction α/μ of revenue on rental payments for physical capital. A fraction $(\mu-1)/\mu$ of revenue is left over as pre-tax variable profits. These same arguments imply that aggregate output is split into wage payments to production labor, rental payments for physical capital, and aggregate variable profits in the same manner.

The research good is produced by competitive firms (or in-house by intermediate good producing firms) using the technology in equation (3). These firms take the productiv-

[15] Several recent papers in the literature aim to account for deviations of firm- and product-level data from some of these extreme regularities, but these papers typically focus on the balanced growth paths of their models. See, for example, Lentz and Mortensen (2008), Luttmer (2011), Akcigit and Kerr (2018), and Acemoglu et al. (2018). It is an open question whether these more complex models show, quantitatively, significantly different transitional dynamics from those that we derive below.

ity of research labor as determined by A_{rt} and $Z_t^{\phi-1}$ as given. They hire research labor at wage W_t and sell the research good at price P_{rt}. In equilibrium, $P_{rt} = W_t / A_{rt} Z_t^{\phi-1}$. Because the technology in equation (3) is constant returns to scale, the wage bill for research exhausts revenues, so these research good producing firms earn no profits. That is, $P_{rt}Y_{rt} = W_t l_{rt} L_t$ in all periods t.

These results regarding factor shares imply a simple relationship between the innovation intensity of the economy as measured by the ratio of spending on the research good to output , $i_{rt} \equiv \frac{P_{rt}Y_{rt}}{Y_t}$, and the allocation of labor between research and current production given by

$$\frac{l_{rt}}{l_{pt}} = \frac{\mu}{1-\alpha} i_{rt}. \tag{8}$$

For each intermediate good that they manage, intermediate good firms choose innovative investment to maximize the expected discounted present value of dividends. In the appendix, we show that the dividend associated with a product with frontier technology z is given by $D_t(z) = D_t s_t(z)$, where

$$D_t = (1-\tau_{corp})\frac{\mu-1}{\mu}\left(1+\tau_y\right) Y_t - \left(1-\tau_{corp}\lambda_I\right) P_{rt} M_t^{1-\psi}\left[(1-\tau_c)\, x_{ct} + (1-\tau_m)\, x_{mt}\right], \tag{9}$$

where τ_c and τ_m are rates at which incumbent firms' innovative investments are subsidized, τ_{corp} is the corporate profits tax rate, λ_I is the rate at which incumbent firms can expense their innovative investment for tax purposes, and τ_y is a production subsidy.[16]

We compute the expected discounted value of the dividends associated with a product as follows. Each existing product remains in the same incumbent firm at $t+1$ with probability $\exp(-\delta_{ct})$ and has expected size conditional on continuing in the same firm equal to $\exp(\zeta(x_{ct}))s_t(z)Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$. In addition, this firm anticipates acquiring a new product with expected size of $\eta_m s_t(z) Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$ with probability $(1-\exp(-h(x_{mt})))$. Thus, the expected discounted present value of dividends associated with a product of size $s_t(z)$ at t inclusive of the dividend at t is directly proportional to the size of the product; that is, it can be written as $V_t s_t(z)$, where the factor of proportionality V_t satisfies the recursion

$$V_t = D_t + \exp(-R_t)V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\left[\exp\left(-\delta_{ct}\right)\exp\left(\zeta\left(x_{ct}\right)\right) + \eta_m\left(1-\exp\left(-h\left(x_{mt}\right)\right)\right)\right], \tag{10}$$

with R_t denoting the interest rate.

[16]We introduce this production subsidy to allow us to undo the distortion in incentives for physical capital accumulation arising from markups and the corporate profits tax in some of our counterfactual experiments.

In equilibrium, entering firms must earn non-positive profits, so

$$\left(1-\tau_{corp}\lambda_E\right)\left(1-\tau_e\right)P_{rt}M_t^{1-\psi}\geq\exp\left(-R_t\right)V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\eta_e, \tag{11}$$

where this expression is an equality if there is positive investment in entry in period t.

Combining (11) (assuming positive entry in period t) and the first-order condition for the optimal choice of x_{mt} (to maximize the right-hand side of equation 10) implies a static equation determining x_{mt} given by

$$\frac{\left(1-\tau_{corp}\lambda_I\right)\left(1-\tau_m\right)}{\left(1-\tau_{corp}\lambda_E\right)\left(1-\tau_e\right)}\frac{\eta_e}{\eta_m}=\exp\left(-h\left(x_{mt}\right)\right)h'\left(x_{mt}\right). \tag{12}$$

This condition implies that x_{mt} is constant in any period in which entry is positive.[17] Likewise, in any period t with positive entry, the first-order condition for the optimal choice of x_{ct} to maximize the right hand side of (10) can be combined with (11) to obtain a static equation relating x_{et}, x_{mt}, and x_{ct} given by

$$\frac{\left(1-\tau_{corp}\lambda_I\right)\left(1-\tau_c\right)}{\left(1-\tau_{corp}\lambda_E\right)\left(1-\tau_e\right)}\eta_e=\exp\left(-\delta_c\left(x_{mt},x_{et}\right)\right)\exp\left(\zeta\left(x_{ct}\right)\right)\zeta'\left(x_{ct}\right). \tag{13}$$

Since x_{mt} is constant in all periods t in which entry is positive, equation (13) defines an implicit function $x_c\left(x_{et}\right)$ that determines x_{ct} as a function of x_{et} in every period in which entry is positive. In the appendix, we show that the derivative dx_c/dx_e approaches zero in the limit as the length of a time period in the model approaches zero. In this case, in the continuous time limit, equation (13) implies that x_{ct} is also constant in any period in which entry is positive.

Nested Models

Our model has two important features shared by several commonly used models in the literature.

[17] Equation (12) follows from the assumption that the entry margin is linear in the sense that entry is replicable: a doubling of investment in entry x_{et} in period t doubles the measure of new products obtained by entering firms at $t+1$. This replicability implies, through equation (11), that the ratio of the value of a product of a given size next period to the cost of another unit of the research good is pinned down by policies (if entry is positive). Because incumbent firms have strictly convex costs of investing to acquire a new product, this implies that this type of innovative investment by incumbent firms is held fixed at the level that equates the marginal cost of acquiring another product for incumbent firms to the marginal benefit, which is fixed over time from the zero-profits entry condition. Equation (13) below follows from the same argument.

The first important feature of our model is that it permits sufficient aggregation in equilibrium so that the evolution of aggregate productivity and the total measure of products is a simple function of a few types of aggregate innovative expenditure as in equations (6) and (7). A number of commonly used expanding varieties and neo-Schumpeterian models in the literature share this aggregation property. Each of these models focuses on a subset of the types of innovative investment considered in our model. In nesting these models, we are able to study the extent to which consideration of different types of innovative investment affect the transitional dynamics of the model. We now discuss these models.

Expanding varieties models assume that there is no business stealing and hence all new products acquired by incumbent and entering firms are new products for society, expanding the measure of products M_t. We nest such models by setting $\delta_e = \delta_m = 0$. Luttmer (2007) is an example of an expanding varieties model in which there is only innovative investment in entry. Atkeson and Burstein (2010) is an example of an expanding varieties model in which there is innovative investment in entry and by incumbent firms in continuing products. Luttmer (2011) is an example of an expanding varieties model in which there is innovative investment in entry and in the acquisition of new products by incumbent firms.

Neo-Schumpeterian models based on the quality ladder framework typically assume $\delta_e = \delta_m = 1$ and $\delta_0 = 0$. With these assumptions, the measure of products M_t is constant over time. Grossman and Helpman (1991) and Aghion and Howitt (1992) are examples of neo-Schumpeterian models in which there is only innovative investment in entry. Klette and Kortum (2004) is an example of a neo-Schumpeterian model in which there is innovative investment in entry and by incumbent firms in acquiring new products (new to the firm, not to society). Acemoglu and Cao (2015) is an example of a neo-Schumpeterian model in which there is innovative investment in entry and by incumbent firms in improving their own products.

The second important feature of our model is that it allows for a simple and flexible reduced-form specification of the technology for producing real innovative investment (what we call the *technology for research*) that nests three specifications of this technology that have played an important role in the literature. Specifically, equations (3) and (4) can be combined into a single constraint on real innovative investment per product

$$x_{ct} + x_{mt} + x_{et} = A_{rt} Z_t^{\phi-1} M_t^{\psi-1} l_{rt} L_t. \tag{14}$$

We focus on three specifications of this technology for research that have played an important role in the literature (see Jones (2005) and Ha and Howitt (2007) for a more extensive discussion).

The first specification of the technology for research that we consider has $\phi = 0.96$ and $\psi = 1$. We refer to this first specification as the *first generation endogenous growth* specification of the technology for research. As we show below and as discussed in Jones (2005), with ϕ close to 1 and $\psi = 1$, the economic implications of our model with this technology for research for the first century of a transition to a new balanced growth path resemble the transitional dynamics of a fully endogenous growth model with a research technology with $\phi = 1$ and $\psi = 1$.

The second specification of the technology for research has $\phi = -1.67$ and $\psi = 1$. This specification is similar to that in Jones (2002), Kortum (1997), and Segerstrom (1998). In this specification of the technology for research, "ideas become harder to find" in the sense that increases in aggregate productivity Z_t relative to the pace of scientific progress A_{rt} lead to a reduction in the productivity of labor allocated to research in producing real innovative investment per product. We refer to this specification as the Jones/Kortum/Segerstrom *(J/K/S) semi-endogenous growth* specification of the research technology.

The third specification of the technology for research has $\phi = 0.96$ and $\psi = 0$. This specification is similar to that in Peretto (1998), Young (1998), Dinopoulos and Thompson (1998), Howitt (1999), and Ha and Howitt (2007).[18] In this specification, increases in the mass of products M_t relative to the pace of scientific progress A_{rt} lead to a reduction in the productivity of labor allocated to research in producing real innovative investment per product. We refer to this as the *second generation endogenous growth* specification of the technology for research.

3 Balanced Growth Paths

We now describe how to solve for a balanced growth path (BGP) of this economy given policies and model parameters. We use this solution procedure in our counterfactual policy experiments below. On a BGP, policies are constant, the exogenous sequences for A_{rt} and L_t grow at constant rates $\bar{g}_A$ and $\bar{g}_L$, and the fraction of the population engaged in current production is constant over time at $\bar{l}_p$. Output, physical capital, and consumption grow at a common rate $\bar{g}_Y$. Aggregate productivity and the measure of products grow at rates $\bar{g}_Z$ and $\bar{g}_M$. Innovative investments of each type remain constant over time at $\bar{x}_c$, $\bar{x}_m$,

[18] In the literature, it is typical to parameterize the research technology with $\phi = 1$ and $\psi = 0$. Again, the implications of the model for the transition in the first century are not substantially altered with $\phi = 0.96$.

and $\bar{x}_e$.

BGP growth rates To solve for BGP growth rates, it is useful to consider the variable $J_t \equiv Z_t^{1-\phi} M_t^{1-\psi}$ together with the physical capital stock K_t as the endogenous state variables of the economy. Since innovative investment rates are constant over time on a BGP, equation (14) implies that the growth rate of J on a BGP depends only on the sum of the growth of scientific progress and population and not on policies:

$$\bar{g}_J = (1-\phi)\bar{g}_Z + (1-\psi)\bar{g}_M = \bar{g}_A + \bar{g}_L. \tag{15}$$

As discussed above in our description of the models nested in our framework, we restrict parameters to $\phi < 1$ and $\psi \leq 1$. Note that if $\psi = 1$, then the growth rate of productivity along the BGP, $\bar{g}_Z$, is also independent of policies. More generally, the division of the growth of J into components arising from growth in aggregate productivity and growth in the number of products depends on the parameters ϕ and ψ and on policies due to the impact of these parameters on the mix of innovative investment on a BGP, which is determined as follows.

The value of $\bar{x}_m$ on a BGP with positive entry is determined from equation (12), while the implicit function $\bar{x}_c(x_e)$ is determined from equation (13). Note that policies enter into these two equations. The BGP level of $\bar{x}_e$ is then determined as the solution to the equation $g_J(\bar{x}_e) = \bar{g}_J$, where $g_J(.)$ is defined as

$$g_J(x_e) \equiv (1-\phi)\, G\left(\bar{x}_c(x_e), \bar{x}_m, x_e\right) + (1-\psi)\, H\left(\bar{x}_m, x_e\right). \tag{16}$$

In the appendix we present conditions under which the function $g_J(x_e)$ is strictly increasing in $\bar{x}_e$ so at most one positive solution to this equation exists.

Once one solves for innovative investments on the BGP, $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$, the growth rates of aggregate productivity and the measure of products are given by the functions G and H evaluated at these levels of innovative investments: $\bar{g}_Z = G(\bar{x}_c(\bar{x}_e), \bar{x}_m, \bar{x}_e)$ and $\bar{g}_M = H(\bar{x}_m, \bar{x}_e)$. The growth rate of aggregate output is $\bar{g}_Y = \bar{g}_Z/(1-\alpha) + \bar{g}_L$.[19]

BGP levels We describe in the appendix how to use the equations for the value function (10) and the zero profits at entry condition (11) on a BGP to solve for the value of $\bar{p}_r$, which is defined as the BGP value of the ratio $p_{rt} \equiv P_{rt} M_t^{1-\psi}/Y_t$. The BGP value of the research intensity of the economy is then given by $\bar{i}_r = \bar{p}_r\left(\bar{x}_c + \bar{x}_m + \bar{x}_e\right)$, and the BGP allocation

[19] If $\phi < 1$, the growth rate of aggregate productivity on the BGP is independent of the levels of scientific knowledge and population, A_{rt} and L_t.

of labor $\bar{l}_r / \left(1 - \bar{l}_r\right)$ is obtained from equation (8). The BGP level of J_t, given levels of A_{rt} and L_{rt}, can be calculated from equation (14).

When $\psi = 1$, this equation is sufficient to pin down the BGP level of productivity $\bar{Z}_t$. More generally, with $\psi < 1$, there is a continuum of pairs of $\bar{Z}_t$ and $\bar{M}_t$, each consistent with the same BGP value of $\bar{J}_t$. The particular values of $\bar{Z}_t$ and $\bar{M}_t$ that arise on a BGP depend upon the initial conditions of the economy Z_0 and M_0 and the transition path that the economy takes to converge to the BGP. Specifically, an equilibrium sequence of innovative investments $\{x_{ct}, x_{mt}, x_{et}\}$ implies, through the functions G and H defined above, a sequence of growth rates of Z_t and M_t. This sequence of growth rates can then be used to trace out the paths for the levels of Z_{t+1} and M_{t+1} from their initial conditions to their levels on the BGP, as described in the next section.

4 Transitional Dynamics of Aggregate Productivity

We now consider the equilibrium relationship implied by our model for the dynamics of aggregate productivity and the transition path of research labor $\{l_{rt}L_t\}$ (or equivalently, from equation (8), a transition path for the innovation intensity of the economy). This exercise is analogous to a growth accounting exercise relating the equilibrium dynamics of labor productivity to a given transition path for investment in physical capital in a standard growth model. Our aim is to study which parameters of our model determine its implications for the short- and long-term responses of aggregate productivity to changes in the allocation of labor to research induced by a change in policies or other features of the economic environment. We then use these results to develop formulas for the equilibrium relationship between the dynamics of aggregate productivity and the transition path of entry rates as measured by the share of employment and output in newly entered firms.

Consider the paths of quantities $\mathcal{Z}_t = \{Y_{rt}, x_{et}, x_{mt}, x_{ct}, J_{t+1}, Z_{t+1}, M_{t+1}\}$ given initial conditions for J_0, Z_0, M_0, and a path of research labor $\{l_{rt}L_t\}$. We consider paths for research labor $\{l_{rt}L_t\}$ such that the fraction of labor allocated to research converges to a BGP value $\bar{l}_r$ and population converges to a BGP path $\{\bar{L}_t\}$. We construct a first-order approximation to the paths for the variables in $\mathcal{Z}_t$ relative to the BGP values of these variables to which the economy is converging.

We assume that, along the transition to the BGP, the ratios of policies $\frac{\left(1-\tau_{corp}\lambda_I\right)(1-\tau_m)}{\left(1-\tau_{corp}\lambda_E\right)(1-\tau_e)}$ and $\frac{\left(1-\tau_{corp}\lambda_I\right)(1-\tau_c)}{\left(1-\tau_{corp}\lambda_E\right)(1-\tau_e)}$ remain constant. Under this assumption, $x_{mt} = \bar{x}_m$ along the transition path (determined by equation 12), and the implicit function $\bar{x}_c(x_e)$ (defined in equation 13) does not vary over time in every period with positive entry. The results we develop below are conditional on the assumption that the transition path to the BGP has positive entry in every period.

When we construct our first order approximation to the transitional dynamics of our model economy, we make use of the following elasticities of the growth rates of Z, M, and J with respect to changes in investment in entry around the BGP to which the economy is converging. Define the elasticity of the growth of aggregate productivity with respect to changes in investment in entry evaluated on the BGP to which the economy is converging as

$$\Theta_G = \left[\frac{\partial}{\partial x_c} G\left(\bar{x}_c\left(x_e\right), \bar{x}_m, x_e\right) \frac{d}{dx_e} x_c\left(x_e\right) + \frac{\partial}{\partial x_e} G\left(\bar{x}_c\left(x_e\right), \bar{x}_m, x_e\right)\right] \bar{x}_e, \tag{17}$$

where $\frac{d}{dx_e} x_c\left(x_e\right)$ is computed from equation (13) and all derivatives are evaluated at $x_e = \bar{x}_e$. Similarly, define the elasticity of the growth in the number of products with respect to changes in investment in entry by

$$\Theta_H = \frac{\partial}{\partial x_e} H\left(\bar{x}_m, \bar{x}_e\right) \bar{x}_e. \tag{18}$$

From the definition of J_t, we can define the elasticity Θ_J of the growth rate of J with respect to changes in investment in entry as

$$\Theta_J = (1-\phi)\,\Theta_G + (1-\psi)\,\Theta_H. \tag{19}$$

Our first-order approximation to the dynamics of aggregate productivity is obtained as follows. Equations (14) and (13) together with $x_{mt} = \bar{x}_m$ imply that the equilibrium change in investment in entry is, to a first-order approximation,

$$(\log x_{et} - \log \bar{x}_e) = A\left[\left(\log l_{rt} - \log \bar{l}_r\right) + \left(\log L_t - \log \bar{L}_t\right) - \left(\log J_t - \log \bar{J}_t\right)\right], \tag{20}$$

where

$$A \equiv \left(\frac{\bar{x}_c + \bar{x}_m + \bar{x}_e}{\bar{x}_e}\right)\left(\frac{1}{\frac{d}{dx_e} x_c\left(\bar{x}_e\right) + 1}\right). \tag{21}$$

Define

$$\Theta \equiv A\Theta_J. \tag{22}$$

Then, combining equation (20) and a first-order approximation to equation (16) around

$\bar{x}_e$, we have

$$\log J_{t+1} - \log \bar{J}_{t+1} = \Theta \left[\left(\log l_{rt} - \log \bar{l}_r \right) + \left(\log L_t - \log \bar{L}_t \right) \right] + (1-\Theta) \left(\log J_t - \log \bar{J}_t \right) . \tag{23}$$

The initial condition of this auto-regressive process for J, $\log J_0 - \log \bar{J}_0$, is given (since the BGP level $\bar{J}_t$ and growth rate $\bar{g}_J$ are both pinned down).

Using equation (23), we develop the following analog of Proposition 6 from AB2018 regarding the dynamics of the variable J_t as the standard moving average of past perturbations to the allocation of labor to research, $l_{rt}L_t$, and its initial condition in period $t=0$.

Proposition 1. *From the AR1 representation* (23)*, we have that to a first-order approximation, the dynamics of* $\log J_t$ *in the transition to a BGP with positive entry are given by*

$$\begin{aligned} \log J_{t+1} - \log \bar{J}_{t+1} = \sum_{j=0}^{t} \Theta \left(1-\Theta\right)^{j} \left[\left(\log l_{rt-j} - \log \bar{l}_r \right) + \left(\log L_{t-j} - \log \bar{L}_t \right) \right] + \\ + \left(1-\Theta\right)^{t+1} \left(\log J_0 - \log \bar{J}_0 \right) . \end{aligned} \tag{24}$$

In the appendix we show that, once we solve for the dynamics of J_t, the dynamics of aggregate productivity and the measure of products are given by

$$\log Z_t - \log \bar{Z}_t = \frac{\Theta_G}{\Theta_J} \left(\log J_t - \log \bar{J}_t \right) \tag{25}$$

$$\log M_t - \log \bar{M}_t = \frac{\Theta_H}{\Theta_J} \left(\log J_t - \log \bar{J}_t \right) . \tag{26}$$

We also use equations (25) and (26) to solve for the BGP levels $\bar{Z}_t$ and $\bar{M}_t$ to which the economy converges, given initial conditions Z_0 and M_0.

For many of our policy experiments, we make use of the following corollary to Proposition 1.

Corollary 1. *Suppose that the economy starts at* $t=0$ *on some initial BGP; that there is a change in the economic environment that leaves the growth rates* $\{g_{At}\}$ *and* $\{g_{Lt}\}$ *unchanged; and that the allocation of innovative investment* $\bar{x}_c, \bar{x}_m, \bar{x}_e$ *is unchanged across BGPs. Then, to a first order approximation, the dynamics of aggregate productivity relative to its initial BGP path are given by*

$$\log Z_{t+1} - \log Z_0 - t\bar{g}_Z = A\Theta_G \sum_{j=0}^{t} \left(1-\Theta\right)^{j} \left(\log l_{rt-j} - \log l_{r0} \right) . \tag{27}$$

Two sufficient statistics We see from equation (27) that, under the conditions of Corollary 1, the dynamics of aggregate productivity relative to its trend on the initial BGP can be summarized by two statistics. We refer to the first of these statistics, $A\Theta_G$, as the *impact elasticity* of aggregate productivity at $t+1$ with respect to a change in research labor at t. Note that Θ_G is the corresponding impact elasticity of aggregate productivity at $t+1$ with respect to a change in investment by entrants at t. We refer to the second of these statistics, $1-\Theta$, as the extent of *intertemporal knowledge spillovers* in research. This second statistic captures the persistence of the response of aggregate productivity to a one-time policy-induced reallocation of labor to research.

Observe that the impact elasticity of an increase in labor devoted to research on aggregate productivity relative to its new BGP level is independent of the specification of the research technology indexed by ϕ and ψ. Instead, it is determined by the parameters that shape our model's implications for firm dynamics and the allocation of innovative investment across incumbent and entering firms. Specifically, the elasticity Θ_G is bounded above by

$$\Theta_G \leq \frac{1}{\rho-1}\left(1-\frac{\delta_e}{\eta_e}\right)\bar{S}_e, \tag{28}$$

where $\bar{S}_e$ denotes the share of production employment in entering firms on the BGP.[20] Note that this bound on Θ_G can be interpreted as a measure of the contribution of firm entry to aggregate productivity growth on the BGP. The parameter A is bounded below by the inverse of the share of innovative investment by entering firms in total innovative investment. As we show in the appendix, both Θ_G and A converge to these bounds as our model approaches a continuous time model, shrinking the length of a time period in units of calendar time to zero. Likewise, the elasticity Θ_H is given by the contribution of entry to the growth in the measure of products, that is,

$$\Theta_H = \frac{H(\bar{x}_m,\bar{x}_e)-H(\bar{x}_m,0)}{\exp(\bar{g}_M)} = (1-\delta_e)\bar{F}_e, \tag{29}$$

where $\bar{F}_e$ denotes the fraction of products produced by newly entered firms. If products in entering firms are small relative to the average product ($\eta_e < 1$), as is the case in our calibration, then $\bar{S}_e < \bar{F}_e$, which implies $\Theta_G < \Theta_H$.

In contrast, the persistence of this impact, which is determined by the parameter $1-\Theta$ in equation (22), is sensitive to the parameters ϕ and ψ of the research technology, governing the extent of intertemporal knowledge spillovers. This is because the elasticity Θ_J, defined in equation (19), depends on these parameters. In our calibration, Θ_G is sub-

[20] The inequality in (28) uses the fact that $\frac{\partial}{\partial x_c} G\left(\bar{x}_c\left(x_e\right), \bar{x}_m, x_e\right) \frac{d}{dx_e} x_c\left(x_e\right) \leq 0$.

stantially smaller than Θ_H. As a result, the persistence parameter, $1-\Theta$, is substantially smaller when $\phi \approx 1$ and $\psi = 0$ (as with the second generation endogenous growth research technology) compared to when $\psi = 1$ and $\phi \ll 1$ (as with the J/K/S research technology). We discuss the quantitative implications of these observations when we consider the transitional dynamics for aggregate productivity implied by various specifications of our model.

Impact of a change in entry rates on aggregate productivity growth We now use the results above to characterize the model-implied relationship between observed entry rates, measured as the share of employment in entering firms, and the dynamics of aggregate productivity observed along a transition of the economy to a new BGP. Specifically, let the growth rates and investment levels on the new BGP to which the economy is converging be given by $\bar{g}'_z, \bar{g}'_m$, $\bar{x}'_c$, $\bar{x}'_m$, and $\bar{x}'_e$. Let Θ'_G, Θ'_H, and Θ'_J be the corresponding elasticities defined in equations (17), (18), and (19). Let S_{et+1} denote the share of production employment in entering firms in period $t+1$ and $\bar{S}'_e$ the corresponding share on the new BGP to which the economy is converging. Since this employment share of entrants is given by $S_{et+1} = x_{et}\eta_e Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$, we have

$$\log S_{et+1} - \log \bar{S'}_e = \left(\log x_{et} - \log \bar{x'}_e\right) + (\rho-1)\left(\log Z_t - \log \bar{Z}'_t\right) - (\rho-1)\left(\log Z_{t+1} - \log \bar{Z}'_{t+1}\right).$$

Then, from this equation and equations (20), (22), (23), and (25), we have that

$$\left(\log Z_{t+1} - \log \bar{Z}'_{t+1}\right) - \left(\log Z_t - \log \bar{Z'}_t\right) = \frac{\Theta'_G}{\left(1-(\rho-1)\Theta'_G\right)}\left(\log S_{et+1} - \log \bar{S}'_e\right). \quad (30)$$

This formula implies that the transition path of the share of employment in entrering firms is related in equilibrium to cumulative changes in aggregate productivity relative to its initial trend *initial* trend by

$$\log Z_{t+1} - \log Z_0 - t\bar{g}_Z = \frac{\Theta'_G}{\left(1-(\rho-1)\Theta'_G\right)}\sum_{j=0}^{t}\left(\log S_{ej+1} - \log \bar{S}'_e\right) + t\left(\bar{g}'_Z - \bar{g}_Z\right). \quad (31)$$

To apply this formula, one must specify a particular counterfactual change across BGPs in policies or the economic environment that gives rise to a change in the BGP to which the economy then converges, with corresponding values of $\bar{g}'_Z$, $\bar{S}'_e$, and Θ'_G. We illustrate the use of this formula with specific counterfactual experiments in Section 6.

Impact of a change in population growth on aggregate productivity growth Here, we consider a decline in the growth rate of population as a potential driving force behind the decline in the rate of investment in entry. We make reference to the following result on the impact of changes in the growth rate of population on the BGP growth rate of aggregate productivity (proved in the appendix):

$$\frac{d\bar{g}_Z}{d\bar{g}_L} = \frac{\Theta_G}{\Theta_J} = \frac{1}{(1-\phi) + (1-\psi)\,\frac{\Theta_H}{\Theta_G}}. \tag{32}$$

We use equation (32) to compute a first-order approximation to the decline in the BGP growth rate of aggregate productivity corresponding to a 1 percentage point drop in the BGP population growth rate. For the first generation endogenous growth and the J/K/S technologies, since they have $\psi = 1$, $\Theta_G/\Theta_J = 1/(1-\phi)$. With $\phi = 0.96$, the first generation endogenous growth research technology implies a catastrophic decline in the BGP growth rate of aggregate productivity. This is simply a reflection of the well -known finding that this research technology in the limiting case of $\phi \to 1$ cannot accommodate any growth in research labor on a BGP. With $\phi = -1.67$, the J/K/S research technology predicts a decline in productivity growth of -0.0037. With the second generation endogenous growth technology for research, the decline in BGP productivity growth depends on the specific values of Θ_G and Θ_H. From equation (32), and since $\frac{\Theta_H}{\Theta_G}$ does not depend on the research technology, it follows that $\frac{d\bar{g}_Z}{d\bar{g}_L}$ is increasing in ψ for given ϕ. With the second generation endogenous growth technology, changes in the growth rate of population have a smaller impact on aggregate productivity growth on the BGP because the growth rate of the measure of products adjusts, as described in Ha and Howitt (2007).[21]

5 Counterfactual Experiments

We now consider counterfactual experiments that alter the BGP and compute the model-implied transition path for aggregate variables in our economy. We first consider a uniform subsidy to innovation. We next consider a change in the equilibrium markup due to changes in policies governing patents and intellectual property. In these first two counterfactual experiments, the conditions for Corollary 1 are satisfied. We next consider a change in the corporate profits tax. In this experiment, the policy change results in a change in the allocation of innovative investment on the new BGP relative to the allocation of this investment on the initial BGP to which the model is calibrated. Hence, in this

[21] In the appendix, we report the transition path of the economy to a gradual decline in the growth rate of population under the J/K/S and second generation endogenous growth research technologies.

experiment, Corollary 1 does not apply. Instead, the equilibrium transitional dynamics of the model are characterized by Proposition 1 applied on the new BGP.

We calibrate all specifications of our model as described in greater detail in the appendix. We set the elasticity of substitution between intermediate goods in production to $\rho = 4$. As discussed above, the elasticities Θ_G and Θ_H on the initial BGP can be identified by calibrating our model to match data on firm dynamics. We consider two specifications of these parameters. In the first, we assume that there is no business stealing (i.e., $\delta_e = \delta_m = 0$). This specification is of interest because it delivers the maximum values of Θ_G and Θ_H consistent with data on the shares of employment and products in entering firms. In our second specification, we set the business stealing parameters $\delta_e = \delta_m$ so that the contribution of entrants to aggregate productivity growth in equation (28) is equal to that estimated in Akcigit and Kerr (2018). As described in our calibration section in the online appendix, to calibrate the parameter A on the initial BGP, we measure expenditures on innovation by incumbent firms using National Income and Product Accounts data and infer expenditures on innovation by entering firms from equation (11). We consider the three specifications of the technology for research described in Section 2. This gives us a total of six model specifications to consider.

Uniform innovation subsidy We first consider the impact of a permanent increase in innovation subsidies $\tau_c, \tau_m,$and τ_e which is uniform in the sense that it leaves the ratios $(1-\tau_c)/(1-\tau_e)$ and $(1-\tau_m)/(1-\tau_e)$ unchanged. Note that equations (12), (13), (15), and (16) which determine the BGP growth rate of J and the allocation of investment on a BGP, are not altered by a uniform change in innovation subsidies. Thus, if we hold all other parameters and policies fixed, we find that the new BGP has the same allocation of innovative investment $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$ and hence the same growth rates of aggregate productivity and the measure of products. Thus, this experiment satisfies the conditions in Corollary 1. In addition, the employment shares and fractions of products in entering firms are also unchanged from the initial to the new BGP.

In contrast, on the new BGP, the economy has a higher innovation intensity, measured as the ratio of expenditures on innovative investment relative to output, as a result of the subsidy. This innovation intensity of the economy on the new BGP is found by solving for the new BGP level of $\bar{p}_r$. From equation (8), we find that the new BGP also has a higher fraction of labor allocated to research $\bar{l}_r$. We pick the increase in innovation subsidies so that the allocation of labor to research increase by 10% (i.e., $\log \bar{l}_r' - \log \bar{l}_r = 0.10$).

To calculate the dynamics of aggregate productivity using equation (27), we assume for simplicity that the allocation of labor to research increases at $t = 0$ by 10% and stays at

Research Technology	Productivity at 20 years	Productivity at 100 years	Half-Life in years
First Generation EG			
no business stealing	0.056	0.260	7561
with business stealing	0.022	0.098	20794
Jones/Kortum/Segerstorm			
no business stealing	0.029	0.037	110
with business stealing	0.017	0.035	287
Second Generation EG			
no business stealing	0.011	0.011	33
with business stealing	0.005	0.005	41

Table 1: Permanent 10% increase in research labor: 20- and 100-year response and half-life of aggregate productivity

this elevated level permanently.[22] In Table 1, we report the level of aggregate productivity relative to its prior trend at horizons of 20 and 100 years using equation (27). We also report the half-life of a one-time impulse to aggregate productivity as determined by $1 - \Theta$ to illustrate the implied extent of intertemporal knowledge spillovers.

To interpret these results, note that the (annualized) impact elasticity $A\Theta_G$ implied by our data on firm dynamics and by our measurement of the share of innovative investment undertaken by entering firms, is 0.0282 without business stealing and 0.0102 with business stealing. Therefore, the response on impact of aggregate productivity growth at an annual frequency relative to the initial BGP trend with respect to a reallocation of labor to research of size $\log l_{r0} - \log \bar{l}_r = 0.10$ is 0.00282 (that is, aggregate productivity would be 0.28% percent higher after the first year) if there is no business stealing and 0.00102 with business stealing. As discussed above, these implications of our model are independent of the specification of the technology for research.

In contrast, the specification of the research technologies as indexed by ϕ and ψ has a tremendous impact on the implications of the model for the persistence of any impulse to aggregate productivity (which is determined by $1 - \Theta$). We see that for the first generation endogenous growth specification of the research technology, an impulse to the growth rate of aggregate productivity from a permanent reallocation of labor to research

[22]In the online appendix, we report the transition path of the economy following a uniform increase in innovation subsidies, taking into account the transitional dynamics of the allocation of labor to research, rather than assuming a once-and-for-all 10% increase. The responses of aggregate productivity at 20 and 100 years along the equilibrium transition path are similar to those shown in our experiment in Table 1. That is, consideration of the endogenous timing of the reallocation of labor to research induced by the subsidy does not substantially alter the quantitative implications of the model for aggregate productivity. The responses of aggregate output are smaller than the corresponding responses of aggregate productivity because the amount of labor allocated to current production is permanently reduced.

is essentially permanent. This implies that the cumulative response of aggregate productivity relative to its initial trend after 20 and 100 years is equal to nearly 20 and 100 times the response of aggregate productivity growth on impact, respectively. For the J/K/S and second generation endogenous growth specifications of the research technology, this is no longer the case. In these cases, because impulses to the growth rate of aggregate productivity decay over time, the cumulative impact on aggregate productivity of a long-lasting reallocation of labor to research does not grow linearly with the time horizon. As discussed above, in our calibration, Θ_G is substantially smaller than Θ_H. As a result, the persistence parameter $1 - \Theta$ is substantially smaller with the second generation endogenous growth research technology compared to the J/K/S research technology. The persistence parameter $1 - \Theta$ captures the impact of intertemporal knowledge spillovers in research on the transitional dynamics of aggregate productivity. Because of these differences in intertemporal knowledge spillovers across specifications of the research technology, our model has dramatically different positive implications for the impact of a long-lasting reallocation of labor to research on the level of aggregate productivity relative to trend after 100 years, depending on the specification of the technology for research.As a quantitative matter, these differences in the model's positive implications for aggregate productivity are much smaller at a 20-year horizon.

Welfare We now consider the normative implications of our model given a uniform change in innovation subsidies. The direct effect on welfare of a reallocation of labor to research is negative as it results in a reduction in current production labor and hence of current output. This reallocation of labor to research results in a welfare gain to the extent that the discounted present value of the impulse to aggregate productivity generated by this reallocation outweighs the direct cost in terms of lost output. As we have seen from Corollary 1, the shape of this impulse is determined by our two key sufficient statistics; the impact elasticity, $A\Theta_G$, and the persistence parameter, $1 - \Theta$.

We can use this logic to compute the allocation of labor to research on the BGP of our economy once the optimal uniform innovation subsidy has been put in place. If the economy is on a BGP corresponding to the constrained socially optimal uniform innovation subsidy (i.e., the planner can make uniform changes to innovation subsidies but cannot change the subsidy to entrants relative to incumbents), then a one-time policy-induced perturbation to the fraction of labor allocated to research at t and the induced dynamics of aggregate productivity as described in equation (27) should result in no change in welfare; that is, the loss of current output arising from a reallocation of labor to research should balance the gains from the resulting dynamic response of aggregate productivity.

In the appendix we show that the allocation of labor to production and research on the BGP corresponding to the optimal uniform innovation subsidy is given by

$$\frac{\bar{l}_r^*}{\bar{l}_p^*} = \left(\frac{1}{1-\alpha}\right) A\Theta_G \frac{\exp\left(\left(\bar{g}_Y - \bar{R}\right)\right)}{1 - \exp\left(\left(\bar{g}_Y - \bar{R}\right)\right)(1-\Theta)}. \tag{33}$$

Note, again, the important role of our two key sufficient statistics, together with the elasticity of aggregate output with respect to production labor $1-\alpha$ and the gap between the interest rate and the growth rate in shaping the normative implications of our model.

In Table 2, we present the consumption-equivalent welfare gain from the uniform innovation subsidies considered above. As anticipated by our previous discussion, these normative implications of our model are highly sensitive to the specification of the technology for research because the model's implications for the extent of intertemporal knowledge spillovers $1-\Theta$ are highly sensitive to the specification of this technology.

We see that the uniform increase in innovation subsidies considered here leads to a significant increase in welfare in the specification of the model with the first generation endogenous growth research technology, a more moderate increase in welfare with the J/K/S research technology, and a decline in welfare with the second generation endogenous growth research technology. This ranking of the welfare gain across models is simply a consequence of the ranking of the persistence parameter, $1-\Theta$, implied by these models since the impact elasticity, $A\Theta_G$, is invariant to the research technology. These different specifications of the research technology also lead to very different implications for the constrained optimal allocation of labor to research on a BGP. For the first generation endogenous growth research technology, the BGP allocation of labor under the optimal uniform innovation subsidy has half of the labor force or more engaged in research. For the J/K/S technology, at least one-quarter of the labor force should be engaged in research. For the second generation endogenous growth technology, the portion of the labor force that should be engaged in research is roughly equal to or even less than the current allocation of labor to research to which the model is calibrated.

Increase in the markup μ We now conduct a counterfactual experiment in which the markup μ increases permanently due to a change in policy regarding the protection of intellectual property. A permanent increase in the markup μ has qualitatively the same effects as a uniform increase in innovation subsidies. This is because equations (12), (13), (15), and (16), which determine the BGP growth rate of J and the allocation of investment on a BGP, are not altered by a uniform change in the markup. Thus, the new BGP has the same allocation of innovative investment $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$, the same growth rates of

	Cons. Equiv.	Optimal
Research Technology	Welfare	$\bar{l}_r^* / \bar{l}_p^*$
First Generation EG		
no business stealing	1.230	2.407
with business stealing	1.076	0.969
Jones/Kortum/Segerstrom		
no business stealing	1.029	0.418
with business stealing	1.019	0.332
Second Generation EG		
no business stealing	0.999	0.141
with business stealing	0.992	0.067

Table 2: Welfare implications of uniform increase in innovation subsidies

aggregate productivity and the same measure of products as the initial BGP. In addition, firm dynamics as measured by employment shares and fractions of products in entering firms are also unchanged from the initial to the new BGP. So, Corollary 1 applies to this counterfactual experiment.

As is the case with a uniform increase in innovation subsidies, on the new BGP the economy has a higher level of $\bar{p}_r$ as a result of the increase in markups, which implies a higher innovation intensity of the economy, measured as the ratio of expenditures on innovative investment relative to output and a higher fraction of labor allocated to research.

We report the results from this experiment in the online appendix. We calibrate the increase in markups so that the change in the allocation of labor to research from the initial to the new BGP is the same as the change we considered with the uniform innovation subsidies. These similar perturbations to the allocation of labor to research produce similar responses of the level of aggregate productivity relative to its original trend at horizons of 20 and 100 years. However, because markups are higher in this case, the response of aggregate output at the 20- and 100-year horizons is smaller that is the case with uniform innovation subsidies since the increase in markups discourages the accumulation of physical capital.

A reduction in corporate profits tax rate The impact of a permanent reduction in corporate profits tax rate τ_{corp} depends upon the details of expensing of innovative investment for tax purposes for incumbent firms and entering firms, as indexed by λ_I and λ_E. This is true for two reasons. First, the extent of expensing alters the impact of a given change in the corporate profits tax rate on the incentives to engage in innovative investment. Second, differences in the ability of incumbent and entering firms to expense expenditures on innovative investment mean that a given change in the tax rate alters the mix of incentives for these different types of firms to engage in innovative investment.

Consider first the case with equal expensing by both entrants and incumbents (i.e., $\lambda_I = \lambda_E$). Here, as in the case with uniform innovation subsidies and the change in the markup, the allocation of investment on the new BGP $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$ and BGP growth rates are not altered by a change in the corporate tax rate τ_{corp}. In this case, the impact of a reduction in the corporate tax rate on the innovation intensity of the economy depends on the extent of expensing. If there is full expensing (i.e., $\lambda_I = \lambda_E = 1$), then a change in the corporate profits tax rate leaves net-of-tax variables profits unchanged relative to net-of-subsidy innovation costs, so $\bar{p}_r$, the innovation intensity of the economy, and the share of labor in research do not change on the new BGP. Thus, all variables on the new BGP are equal to what they were in the initial BGP except that the economy accumulates more physical capital if investment on physical capital is partially expensed. If there is only partial expensing of innovative investments (i.e., $\lambda_I = \lambda_E < 1$), then a reduction in the corporate profits tax rate raises net-of-tax variable profits relative to net-of-subsidy innovation costs, which results in an increase in $\bar{p}_r$, in the innovation intensity of the economy, and in the share of labor in research on the new BGP. As is the case with a uniform innovation subsidy and the change in markup, the magnitude of the effect of the corporate profits tax change on aggregate productivity is determined by the induced change in the allocation of labor to research.

Consider next the case in which the expensing of investment by entrants is less than that for incumbents (i.e. $\lambda_I > \lambda_E$). In this case, a change in the corporate profits tax rate has an impact on the allocation of investment on the new BGP. This is because equations (12) and (13) are altered by the change in τ_{corp}. Specifically, under these assumptions, if the corporate profits tax rate is reduced, the term $(1 - \tau_{corp}\lambda_I)/(1 - \tau_{corp}\lambda_E)$ rises. From these equations, we see that a reduction in the tax rate leads to a fall in innovative investments by incumbents $\bar{x}_c$ and $\bar{x}_m$ from the initial BGP to the new BGP, since the functions h and ζ are strictly concave. Since the BGP growth rate of J is not altered by the tax change, this implies an increase in investment by entrants $\bar{x}_e$ from the initial BGP to the new BGP.

This reallocation of investment implies that we cannot directly apply Corollary 1. Instead, we must compute the dynamics of J given in Proposition 1. In particular, the BGP level of J (i.e., $\bar{J}_0$) is affected. Hence, from equation (25), for a given path of research labor, the change in aggregate productivity relative to its initial trend, $\log Z_{t+1} - \log Z_0 - t\bar{g}_Z$, can be higher or lower than what we found in the previous experiments in which $\bar{Y}_r = Y_{r0}$. Note that in the case in which the research technology has $\psi = 1$, the BGP growth rate of aggregate productivity $\bar{g}_Z$ is unchanged by a change in corporate profits taxes. In

Research Technology	Productivity at 20 years	Productivity at 100 years	Cons. Equiv. Welfare
First Generation EG			
no business stealing	0.053	0.265	1.193
with business stealing	-0.023	-0.103	0.913
Jones/Kortum/Segerstrom			
no business stealing	0.035	0.038	1.027
with business stealing	-0.020	-0.030	0.970
Second Generation EG			
no business stealing	-0.042	-0.315	0.732
with business stealing	-0.056	-0.330	0.751

Table 3: Corporate profits tax experiment: 20- and 100-year response of aggregate productivity and equivalent variation in consumption

contrast, when $\psi = 0$, as is the case with the second generation endogenous growth technology for research, the BGP growth rate of aggregate productivity changes.

We now illustrate some of these effects with a specific experiment, shown in Table 3. We set the corporate profit tax rates τ_{corp} as in Barro and Furman (2018) and assume that incumbent firms can deduct all of their innovative investments ($\lambda_I = 1$), while entering firms cannot ($\lambda_E = 0$) since they are not incorporated at the time of their investments. Further details are given in the appendix. As we now discuss, the implications of the model for the impact of the tax cut on aggregate productivity and welfare are highly sensitive to the amount of business stealing and the specification of the technology for research.

Consider first the results under the first two research technologies (with $\psi = 1$), in which case the growth rate of aggregate productivity is unchanged between BGPs. We see in the top four rows of Table 3 that aggregate productivity relative to its initial BGP trend rises when there is no business stealing and falls when there is business stealing. This occurs even though the change in tax policy induces a similar reallocation of labor to research across these model specifications.[23] In the third specification of the research technology (with $\psi = 0$), reported in the bottom two rows of Table 3, the BGP growth rate of aggregate productivity falls on the new BGP. As a result, the reduction in aggregate productivity (and welfare) can be very large at long-term horizons.[24]

To this point, we have used Proposition 1 and its Corollary 1 to study quantitatively

[23] This finding is related to the results in Acemoglu and Cao (2015).

[24] Note that our welfare results imply that the initial allocation of innovative investment is far from optimal, especially under the second generation endogenous growth technology for research. Peretto (2007) also discusses this impact of changes in corporate profits tax rates on the BGP growth rate of aggregate productivity in a model with a second generation endogenous growth research technology.

the relationship in equilibrium between the transition path of the allocation of labor to research (or equivalently, from equation (8), the research intensity of the economy) and the transition path of aggregate productivity. In the next section, we use equation (31) to study quantitatively the relationship in equilibrium between the transition path for entry rates, as measured by the share of production labor employed in entering firms, and the transition path for aggregate productivity.

6 Declining Entry Rates and Aggregate Productivity

The entry rate of new firms in the United States, measured as the share of employment in new firms, has fallen dramatically over the past several decades. What are the implications of this decline in the firm entry rate for the evolution of aggregate productivity over these past decades? To use our model to answer this question, one must take a stand on the nature of the change in policies or the economic environment that has driven this decline in the firm entry rate. In this section, we consider three alternative counterfactual changes in policies or the economic environment that might account for the observed decline in entry. For each counterfactual, we use the first-order approximation to the dynamics of aggregate productivity developed in equation (31) to illustrate how different assumptions regarding the technology for research and the driving force behind the decline in entry lead to different conclusions regarding the associated evolution of aggregate productivity.

Note that we can rewrite the term $\sum_{j=0}^{t}\left(\log S_{ej+1} - \log \bar{S}_e'\right)$ in equation (31) as the sum of $\sum_{j=0}^{t}\left(\log S_{ej+1} - \log S_{e0}\right)$ and $(t+1)\left(\log S_{e0} - \log \bar{S}_e'\right)$. In each counterfactual exercise, we assume that the economy starts on the BGP corresponding to our baseline calibration with a share of employment in entering firms of $S_{e0} = 2.7\%$, and for illustrative purposes, we assume that from period $t = 0$ to $t = 19$, this share falls so that $\sum_{t=0}^{19} \log S_{et+1} - \log S_0 = -5$. A fall in entry of this magnitude corresponds to a decline in the share of employment in entrants on an annual basis from 2.7% to 1.64% occurring in equal log steps over 20 years (so $-5 = \frac{20}{2}\left(\log 0.0164 - \log 0.027\right)$).

In our first counterfactual exercise, we assume that the decline in entry is driven by a decrease in innovation subsidies that is uniform in the sense that $(1-\tau_c)/(1-\tau_e)$ and $(1-\tau_m)/(1-\tau_e)$ are unchanged across the initial and new BGPs. As we showed above, such a decrease in innovation subsidies lowers the long-run innovation intensity of the economy and the long-run level of aggregate productivity, but it leaves BGP levels of investment $\bar{x}_c'$, $\bar{x}_m'$, and $\bar{x}_e'$ unchanged. This implies that the entry share $\bar{S}_e'$ and growth rate of aggregate productivity $\bar{g}_Z'$ are also unchanged. That is, in this counterfactual exercise, the decline in the entry share and the growth rate of aggregate productivity is a temporary, transitional phenomenon.

In our second counterfactual exercise, we assume that the decline in entry is driven by a decrease in innovation subsidies for entry, with subsidies for innovative investment by incumbents left unchanged. Such a change in policies has effects similar to our corporate profits tax experiment above in that it leads to a reallocation of innovative investment from the initial to the new BGP, with investment in entry falling permanently relative to investment by incumbents. In this experiment, we choose the decline in entry subsidies so that the entry share on the new BGP, $\bar{S}'_e$, falls from 2.7% to 1.64% ($\log S_{e0} - \log \bar{S}'_e = 0.5$). In this experiment, as in the corporate profits tax experiment above, the growth rate of the aggregate productivity on the new BGP, $\bar{g}'_Z$, is equal to that on the initial BGP if one uses the first generation endogenous growth research technology or the J/K/S research technology and differs from that on the initial BGP if one uses the second generation endogenous growth research technology.

In our third counterfactual exercise, we assume that the decline in entry is driven by a decline in the BGP growth rate of population, $\bar{g}'_L$, chosen in each specification so that the entry rate on the new BGP is $\bar{S}'_e = 1.64\%$. Here, the decline in the population growth rate results in both a decline in the new BGP growth rate of aggregate productivity (as described in equation (32)) and a reduction in investment in entry with little or no change in innovative investment by incumbent firms.[25]

We report results from these three experiments in Table 4 in the specifications of the model with business stealing. We report results for the model without business stealing in the appendix. For each experiment and each specification of the research technology, we report the inputs Θ'_G, $\log \bar{S}'_e - \log S_0$, and $\bar{g}'_Z - \bar{g}_Z$ needed to implement the formula in equation (31) for the model-implied cumulative change in aggregate productivity relative to initial trend over the first 20 years of transition, $\log Z_{20} - \log Z_0 - 20\bar{g}_Z$, associated with the decline in the entry rate specified above.

We draw three lessons from the results in Table 4. First, if the decline in the employment share of new firms is the result of a decline in innovative investment by entrants with no change in innovative investment by incumbents, as in our first experiment with a uniform tax on innovative investment, then the predicted decline in aggregate productivity over a 20 year horizon is relatively small (less than 2% with business stealing and less than 5% without business stealing, as reported in the appendix).

[25] For this experiment, we do not report results for the first generation endogenous growth research technology since this specification implies that changes in population growth are associated with implausibly large changes in aggregate productivity growth (see equation 32).

Experiment	Θ'_G	$\log \bar{S}'_e / S_{e0}$	$\bar{g}'_Z - \bar{g}_Z$	Productivity at 20 years
Uniform Innovation Tax				
First Gen. EG	0.003	0	0	-0.018
Jones/Kortum/Segerstorm	0.003	0	0	-0.018
Second Gen. EG	0.003	0	0	-0.018
Entry Tax				
First Gen. EG	0.002	-0.5	0	0.011
Jones/Kortum/Segerstorm	0.002	-0.5	0	0.011
Second Gen. EG	0.002	-0.5	0.026	0.532
Decline in Pop.Growth				
First Gen. EG	-	-	-	-
Jones/Kortum/Segerstorm	0.002	-0.5	-0.001	-0.017
Second Gen. EG	0.002	-0.5	-0.001	-0.017

Table 4: Reduction in firm entry and aggregate productivity: with business stealing

Second, if the decline in the employment share of new firms is the result of a reallocation of innovative investment, decreasing investment by entrants and raising innovative investment by incumbents, as in our second experiment with a tax on innovative investment by entrants, then the model predicts an increase in aggregate productivity over a 20 year horizon. This predicted increase is small if the long run growth rate is unchanged as with the J/K/S research technology. This predicted increase can be extremely large if the long run growth rate changes as with the Second Generation Endogenous Growth technology for research. The intuition for this result is as follows. At the new levels of innovative investment for incumbents and entrants induced by the entry tax, the economy can sustain the same or even higher BGP growth rate of aggregate productivity with a much smaller share of employment in entering firms. In our second experiment, the employment share of entrants in the 20 years of the transition is higher than the new BGP share of employment. Thus, since our model predicts that innovative investment by incumbents rises immediately to its new BGP level with the imposition of the entry tax, it also predicts that productivity should grow faster than its BGP growth rate as long as observed entry in the transition is above the new BGP level of entry.

Third, if the decline in the employment share of new firms is the result of a decline in the population growth rate, then the model's prediction for the response of aggregate productivity over 20 years is the sum of two separate effects, one negative and one positive. For the two technologies for research that we consider, a reduction in the population growth rate reduces the BGP growth rate of productivity.[26] If the economy adjusts to its

[26]Note that our finding that a decline in entry rates driven by a change in the growth rate of population has the same impact on aggregate productivity growth under the J/K/S and second generation endoge-

new BGP growth rate immediately (by having the entry rate drop immediately to the new BGP employment share in entering firms), then the decline in aggregate productivity at 20 years should be 20 times the reduction in the BGP growth rate of productivity. But, to the extent that the employment share of entrants is above its BGP level during the first 20 years of the transition, then, as in our entry tax experiment, the model predicts an offsetting positive impact of entry on productivity during the transition to the new BGP.

7 Conclusion

In this paper, we have shown how to characterize the transitional dynamics for aggregate productivity associated with policy-induced transition paths for the allocation of labor to research and entry rates of new firms implied by a variety of models of firms' investments in innovation. We have shown that when the change in policy leading to the transition has a uniform impact on incentives for innovative investment by both entering and incumbent firms, these transitional dynamics can be characterized in terms of two sufficient statistics: the impact elasticity of aggregate productivity with respect to a change in the allocation of labor to research or to a change in entry, and the persistence of the response of aggregate productivity to that impulse, as determined by the extent of intertemporal knowledge spillovers in research. We have shown how the normative implications of these models for uniform innovation subsidies are also determined by these two sufficient statistics. We have shown how to discipline the first of these sufficient statistics, the impact elasticity, using data on firm dynamics and model-based inference of the share of innovative investment undertaken by entering firms. We have discussed the challenge of disciplining the second of these sufficient statistics due to uncertainty regarding the nature and magnitude of intertemporal knowledge spillovers in research. Finally, we have considered policy experiments for which the transitional dynamics of aggregate productivity depend on additional model details beyond our two sufficient statistics because these policy changes lead to a reallocation of innovative investment between incumbent and entering firms.

What have we learned from our model about the transitional dynamics of aggregate productivity in response to changes in policies and the associated welfare implications of those changes in policies? We offer three conclusions from our analysis.

nous growth specifications is driven by our assumption that the change in the population growth rate is endogenously chosen to match a given decline in the entry rate on the new BGP. From equation (32), we see that the impact of a given change in the population growth rate on BGP growth rates of aggregate productivity is sensitive to the specification of the technology for research.

First, for policy changes or other changes in the economic environment that have a uniform impact on the incentives of incumbent and entering firms to invest in innovation, the short- and medium-term (up to 20-year) impacts on aggregate productivity of such policy changes are likely to be modest. We see this in the first column of Table 1, which reports the change in aggregate productivity over a 20-year horizon from a substantial and permanent reallocation of labor to research (or, equivalently, an increase in the innovation intensity of the economy). Likewise, as reported in the uniform innovation tax experiment in Table 4, large declines in firm entry rates do not have a substantial impact on aggregate productivity over a 20 year period if they have been induced by policies that have a uniform impact on innovation incentives.

Second, the quantitative implications of current models for the long-run and welfare impacts of policies that have a uniform impact on the incentives to invest are highly uncertain — as indicated in Table 1 for the response of aggregate productivity at a horizon of 100 years and for welfare in Table 2. This is because there is considerable uncertainty about the extent of intertemporal knowledge spillovers in research. Thus, it is difficult to come to firm conclusions about the normative implications of our model for innovation policies absent further progress in measuring intertemporal knowledge spillovers in research.

Third, there is also a great deal of uncertainty regarding the impact of changes in policies or the economic environment that induce, in the long run, increased investment in entry relative to investment by incumbents. As we see in Table 3, with either of our first two specifications of the technology for research, a cut in corporate tax rates that reallocates innovative investment toward entry due to differences in expensing of that investment may raise or lower aggregate productivity depending on the extent of business stealing. With the second generation endogenous growth specification of the technology for research, the negative spillovers from product creation through entry on research productivity are so large that they imply that such a cut in the corporate profits tax rate would, by stimulating entry, have a devastating negative impact on aggregate productivity and welfare. Likewise, in Table 4, we see that with this technology for research, an entry tax that permanently reduces the entry rate has a large beneficial impact on aggregate productivity.

In order to obtain sufficient aggregation to allow for analytically tractable transitional dynamics, our model abstracts from some of the richness in recently developed models of innovative investment with heterogeneous firms, including those in Lentz and Mortensen (2008), Luttmer (2011), Lentz and Mortensen (2016), Peters (2016), Acemoglu et al. (2018), and Akcigit and Kerr (2018). These models feature additional forces regarding the efficiency of the allocation of innovative investment across firms beyond those that we consider in our model. One important challenge for future research in this area is to find reliable metrics for evaluating the positive implications of these richer models for the transitional dynamics of aggregate productivity and the normative implications of these models for which types of firms should be doing relatively more innovative investment and which types of firms should be doing less.

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