Supplemental Appendix to "Transitional Dynamics in Aggregate Models of Innovative Investment"

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Outline summary In Appendix A, we present the model with time period of length Δ. In Appendix B, we provide additional details on the equilibrium. In Appendices C and D, we provide additional details of the BGP and transition dynamics, including the proofs of Proposition 1 and Corollary 1. In Appendix E, we present the log-linearized system of equations that we use to solve for a first-order approximation to the transition dynamics to a new BGP. In Appendix F, we describe how we calibrate the model. In Appendix G, we describe the experiments and present the tables with the quantitative results.

A Model setup

We present the model when the time period in calendar time is of length Δ. We have two reasons to consider a flexible period length. First, the stability properties of the balanced growth path of the model solved in discrete time depend upon the length of a time period. Second, the impact on the incentives of incumbent firms to invest in improving their own products coming from variation in the extent of business stealing due to changes in entry in the transition to a BGP also depends on the length of a time period. In our computations, we set the time period to one month.

Final consumption good

$$\Delta Y_t = \Delta C_t + K_{t+1} - \exp(-\Delta \delta_k) K_t, \tag{1}$$

where $(1 - \exp(-\Delta d_k))/\Delta$ represents the depreciation rate per unit time of physical capital. The preferences of the representative agent are given by

$$\Delta \sum_{t=0}^{\infty} \frac{\beta^{\Delta t}}{1-\gamma} L_t \left(C_t / L_t\right)^{1-\gamma} \tag{2}$$

with $\beta < 1$ and $\gamma > 0$. Population grows exogenously at rates g_{Lt} with $L_{t+1} = \exp(\Delta g_{Lt}) L_t$.

Aggregate output is produced according to the CES aggregator

$$Y_t = \left[\sum_z y_t(z)^{(\rho-1)/\rho} M_t(z) \right]^{\rho/(\rho-1)}, \tag{3}$$

where $\rho > 1$, $M_t(z)$ is the measure of intermediate goods with frontier technology indexed by z at time t. To simplify our notation, we assume that the support of z is a grid with countable elements z_n for integers n with $\log z_{n+1} - \log z_n$ equally spaced. The production of each intermediate input with productivity index z is given by

$$y_t(z) = z k_t(z)^{\alpha} l_t(z)^{1-\alpha}. \tag{4}$$

With common markups and competitive factor markets, aggregate output can be written in equilibrium as

$$Y_t = Z_t \left(K_t\right)^{\alpha} \left(L_{pt}\right)^{1-\alpha}, \tag{5}$$

where total labor hours employed in production satisfy $l_{pt} L_t = \sum_z l_t(z) M_t(z)$, the constraint on physical capital requires $K_t = \sum_z k_t(z) M_t(z)$, and Z_t is the measure of aggregate productivity [1] given by

$$Z_t = \left[\sum_z z^{\rho-1} M_t(z) \right]^{1/(\rho-1)}. \tag{6}$$

[1]This model-based measure of aggregate productivity, Z_t, does not correspond to measured total factor productivity (TFP), which is given by $TFP_t = GDP_t / \left(K_t^{\tilde{\alpha}} L_t^{1-\tilde{\alpha}} \right)$, where the definition of GDP depends on the measurement standard for expenditures on innovative investment being used (e.g., the definition of output of the final consumption good Y_t in equation (1) corresponds to the Bureau of Economic Analysis' pre-2013 measurement of GDP, which did not include expenditures on innovative investment), and $1 - \tilde{\alpha}$ denotes the share of labor compensation in measured GDP. Our analytic comparative statics can be used to construct alternative measures of TFP and GDP.

Lastly, $M_t = \sum_z M_t(z)$ is the total measure of products available, and $Z_t^{\rho-1}/M_t$ is the average productivity index of existing intermediate goods.

Research good Output of the research good, which is the input used for innovative investment by firms, is given by

$$Y_{rt} = A_{rt} Z_t^{\phi-1} l_{rt} L_t. \tag{7}$$

Here, A_{rt} represents the stock of freely available scientific progress, which grows at an exogenous rate $g_{A_t} = \bar{g}_A$. The term $Z_t^{\phi-1}$ with $\phi \leq 1$ reflects *intertemporal knowledge spillovers* in the production of the research good.

Innovative investment by entering firms Entering firms purchase units of the research good to invest in obtaining the frontier technology to produce an intermediate good. The resource requirement at t to enter and obtain a new product at $t+1$ is $\Delta M_t^{-\psi}$ units of the research good. When $\psi = 0$, the investment of the research good required to invest in a new product is independent of the number of existing products. Let $x_{et} M_t$ denote the measure of entering firms at t, where x_{et} denotes the measure of entering firms relative to the measure of existing products at t. Total expenditure of the research good by entering firms is $\Delta x_{et} M_t^{1-\psi}$. Note that when $\psi = 1$, the number of resources required to create one new product falls with the number of existing products. When $\psi = 0$, the investment of the research good required to invest in a new product is independent of the number of existing products. The total expenditure by entering firms is $\Delta x_{et} M_t^{1-\psi}$.

Each of these $x_{et} M_t$ entering firms acquires with probability $1 - \exp(-\Delta\lambda)$ a frontier technology to produce at the start of period $t+1$ an intermediate good with some productivity index z'. Under these assumptions, the effective entry cost, $\frac{\Delta M_t^{-\psi}}{1-\exp(-\Delta\lambda)}$, is approximately unchanged with Δ for small values of Δ.

With probability δ_e, this productivity index z' drawn by the entrant at $t+1$ is associated with an intermediate good that was already being produced by an incumbent firm at t, but with a lower productivity index. Since identical intermediate goods are perfect substitutes in the production of the final consumption good, competition in the product market between the entering firm and the previous incumbent producer of this intermediate good implies that the previous incumbent producer ceases production of the good. In this case, the innovative investment by the entering firm does not result in a net increase in the total measure of products available M_t. Instead, it only results in a positive increment to the average productivity index across existing products. As is common in the literature,we say that this intermediate good that is new to the entering firm represents *business stealing*.

With the complementary probability $1 - \delta_e$, this technology allows this entering firm to produce an intermediate good that is new to society as a whole. In this case, the innovative investment by the entering firm results in a net increase in the total measure of products available M_{t+1}. As is common in the literature, we say that this intermediate good that is new to this entering firm represents a contribution to productivity through *expanding varieties*.

Stolen products in entering firms at $t+1$ have a productivity index z' drawn from a distribution such that the expected value of the random variable z' raised to the power of $(\rho - 1)$ is equal to $\mathbb{E} z'^{\rho-1} = \eta_{es} Z_t^{\rho-1}/M_t$, with $\eta_{es} > 1$. New products in entering firms at $t+1$ have a productivity index z' drawn from a distribution such that $\mathbb{E} z'^{\rho-1} = \eta_{en} Z_t^{\rho-1}/M_t$, with $\eta_{en} > 0$. These assumptions imply that the average value of $z'^{\rho-1}$ across all products produced by entering firms at $t+1$ is given by $\mathbb{E}z'^{\rho-1} = \eta_e Z_t^{\rho-1}/M_t$, where $\eta_e \equiv \delta_e \eta_{es} + (1-\delta_e)\eta_{en}$.

Investments in new products by incumbent firms Incumbent firms have the opportunity to invest in acquiring new products to their firm and to improve their existing products. We assume that if an incumbent firm at t has the frontier technology to produce an intermediate good with index z, it also has the capacity to invest $\Delta x_{mt}(z)$ units of the research good to acquire an additional product (new to the firm) at $t+1$ with probability

$$1 - \exp\left(-\Delta h\left(\frac{x_{mt}(z)}{s_t(z)M_t^{1-\psi}}\right)\right),$$

where $s_t(z)$ denotes the size of the firm. Here, $h(\cdot)$ is a strictly increasing and concave function with $h(0) = 0$ and $h(x) < 1$ for all x.

As is the case with entry, acquisition of new products by incumbent firms may arise from business stealing from other incumbent firms or from expanding varieties. Consider the productivity index z' for a newly acquired product that an incumbent firm obtains at $t+1$ arising from innovative investment associated with a product with index z at t. With probability δ_m, the product acquired by the incumbent firm at $t+1$ is stolen from another incumbent firm and has productivity index z' at $t+1$ drawn at random from a distribution such that $\mathbb{E}z'^{\rho-1} = \eta_{ms} z^{\rho-1}$, with $\eta_{ms} > 1$. With complementary probability $1-\delta_m$, the newly acquired product is new to society. We assume that the productivity index z' in this case is drawn from a distribution such that $\mathbb{E}z'^{\rho-1} = \eta_{mn} z^{\rho-1}$, with $\eta_{mn} > 0$. The average value of $(z')^{\rho-1}$ of the new products acquired by an incumbent firm investing based on a current product with frontier productivity z is $\mathbb{E}z'^{\rho-1} = \eta_m z^{\rho-1}$ where we define $\eta_m = \delta_m \eta_{ms} + (1-\delta_m)\eta_{mn}$

Let $\Delta x_{mt} M_t^{1-\psi}$ denote the total expenditure by incumbent firms on acquiring new products. Then aggregation gives

$$\Delta x_{mt} M_t^{1-\psi} \equiv \Delta \sum_z x_{mt}(z)\, M_t(z).$$

Investment in continuing products by incumbent firms For each product that they produce at t, incumbent firms can lose its production capacity at $t+1$ either due to exogenous exit (with probability $(1-\exp(-\Delta\delta_0))$) or due to business stealing. For each product that they produce at t, incumbents have research capacity that allows them to invest to improve the index z of that product if they retain it at $t+1$. Specifically, if an incumbent firm with a product with productivity z at t spends $\Delta x_{ct}(z)$ of the research good on improving that product, it draws a new productivity index z', conditional on not losing that product to exogenous exit or business stealing, from a distribution such that

$$\mathbb{E}z'^{\rho-1} = \exp\left(\Delta\zeta\left(\frac{x_{ct}(z)}{s_t(z)M_t^{1-\psi}}\right)\right) z^{\rho-1}.$$

We assume that $\zeta(\cdot)$ is a strictly increasing and concave function, with $\zeta(x) > 0$ for all $x \geq 0$. In addition, we assume that $\eta_{es} > \exp(\Delta\zeta(x))$ and $\eta_{ms} > \exp(\Delta\zeta(x))$ for all x in equilibrium. These inequalities correspond to the requirement that a product that is stolen from incumbent firms is, in expectation, produced with a higher z' at $t+1$ in its new firm than it would have had as a continuing product in the firm that previously produced it. Equivalently, stolen products have larger average size than continuing products in incumbent firms. Note that these assumptions are necessarily satisfied if Δ is small enough.

We define the aggregate quantity of this type of innovative investment by incumbent firms as $\Delta x_{ct} M_t^{1-\psi}$. Again, aggregation gives

$$\Delta x_{ct} M_t^{1-\psi} \equiv \Delta \sum_z x_{ct}(z) M_t(z).$$

With these definitions, we can write the resource constraint for the research good as

$$\Delta (x_{ct} + x_{mt} + x_{et}) M_t^{1-\psi} = \Delta Y_{rt} \tag{8}$$

Dynamics of M and Z Under our assumptions, we show below that incumbent firms at every date t in equilibrium choose investment according to

$$x_{mt}(z) = s_t(z) x_{mt} M_t^{1-\psi} \tag{9}$$

and

$$x_{ct}(z) = s_t(z) x_{mt} M_t^{1-\psi}. \tag{10}$$

With these choices of innovative investment by incumbent firms proportional to product size $s_t(z)$, we have that the probability that each incumbent obtains a new product is constant across products at $1 - \exp(-\Delta h(x_{mt}))$ and the expected rate of growth of $z^{\rho-1}$ for incumbent products that do not exit is constant across products and given by $\exp(\Delta \zeta(x_{ct}))$.

With this pattern of investment by incumbents, the evolution of the total measure of intermediate products M_t is given by $\log M_{t+1} - \log M_t = H(x_{mt}, x_{et}; \Delta)$ where

$$H(x_{mt}, x_{et}; \Delta) \equiv \log\left(\exp(-\Delta \delta_{ct}) + 1 - \exp(-\Delta h(x_{mt})) + (1 - \exp(-\Delta \lambda)) x_{et}\right) \tag{11}$$

and $\exp(-\Delta \delta_{ct})$ is the probability that a product remains in the same incumbent firm at $t+1$, where $\delta_{ct} = \delta_c(x_{mt}, x_{et})$ is defined by the equation

$$\exp(-\Delta \delta_c(x_{mt}, x_{et})) = \exp(-\Delta \delta_0) - \delta_m (1 - \exp(-\Delta h(x_{mt}))) - \delta_e (1 - \exp(-\Delta \lambda)) x_{et}. \tag{12}$$

In the continuous time limit, we have $\delta_c(x_{mt}, x_{et}) = \delta_0 + \delta_m h(x_{mt}) + \delta_e \lambda x_{et}$ and[2]

$$\frac{\dot{M}_t}{M_t} = -\delta_0 + (1 - \delta_m) h(x_{mt}) + (1 - \delta_e) \lambda x_{et}. \tag{13}$$

Likewise, the evolution of aggregate productivity is given by $\log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et}; \Delta)$ where

$$G(x_{ct}, x_{mt}, x_{et}; \Delta) \equiv \tag{14}$$

$$\frac{1}{\rho - 1} \log\left(\exp(-\Delta \delta_{ct}) \exp(\Delta \zeta(x_{ct})) + \eta_m (1 - \exp(-\Delta h(x_{mt})) + \eta_e (1 - \exp(-\Delta \lambda)) x_{et}\right).$$

[2]For any variable X_t, we define $\dot{X}_t = \lim_{\Delta \to 0} \frac{X_{t+1} - X_t}{\Delta}$, which we refer to as the continuous time limit.

In the continuous time limit,

$$\frac{\dot{Z}_t}{Z_t} = \frac{1}{\rho - 1}\left(\zeta_c(x_{ct}) - \delta_c(x_{mt}, x_{et}) + \eta_m h(x_{mt}) + \eta_e \lambda x_{et}\right). \tag{15}$$

In our derivations below, we make use of the share of production labor employed in new firms (that is, firms that just entered) as $t+1$, which is given by

$$S_{et+1}(\Delta) = \eta_e \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \frac{M_{et+1}}{M_t} = \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \left(1 - \exp\left(-\Delta\lambda\right)\right) \eta_e x_{et}. \tag{16}$$

In the continuous time limit,

$$\lim_{\Delta \to 0} \frac{S_{et+1}(\Delta)}{\Delta} = \eta_e \lambda x_{et}.$$

B Equilibrium

Here we describe additional equilibrium details omitted from the text. These equilibrium conditions take into account the length of the time period, Δ.

Final consumption good producers Final consumption good producers purchase intermediate goods to produce using the technology in equation (3). These firms are competitive and choose output and inputs to maximize profits taking the price of the final consumption good (which we normalize to one) and the prices of intermediate goods, $p_t(z)$, as given. They receive a production subsidy τ_y per unit sold.

Profit maximization implies standard CES input demands for each intermediate good with demand elasticities determined by ρ. Because the technology in equation (3) is constant returns to scale, these firms have no profits in equilibrium.

Physical capital stock holding firm The physical capital stock is managed by competitive firms that rent out physical capital to intermediate goods producing firms and invest in physical capital. Each firm takes the price of the final consumption good, the rental rate for physical capital R_{kt}, and intertemporal prices for the final consumption good $\{Q_t\}$ as given, and is subject to a corporate profits tax τ_{corp} with expensing for investment in physical capital of λ_k. These firms seek to maximize the discounted present value of after-tax dividends, with dividends given by

$$\sum_{t=0}^{\infty} Q_t \Delta D_{kt},$$

with dividends per unit of time of

$$D_{kt} = (1 - \tau_{corp}) R_{kt} K_t - \left(1 - \tau_{corp}\lambda_k\right) \frac{1}{\Delta} \left(K_{t+1} - \exp(-\Delta d_k) K_t\right) \tag{17}$$

Intermediate goods producers Intermediate goods firms' dividends at time t per unit of time associated with a product with frontier technology z are given by

$$D_t(s_t(z)) = (1-\tau_{corp})\frac{\mu-1}{\mu}(1+\tau_y)Y_t s_t(z) - (1-\tau_{corp}\lambda_I)P_{rt}\left[(1-\tau_c)x_{ct}(z) + (1-\tau_m)x_{mt}(z)\right], \tag{18}$$

which can be written as $D_t\left(s_t\left(z\right)\right) = D_t s_t\left(z\right)$, where

$$D_t = (1-\tau_{corp})\frac{\mu-1}{\mu}\left(1+\tau_y\right)Y_t - \left(1-\tau_{corp}\lambda_I\right)P_{rt}M_t^{1-\psi}\left[\left(1-\tau_c\right)x_{ct} + \left(1-\tau_m\right)x_{mt}\right] \tag{19}$$

Given this strategy for investment, each incumbent product remains in the same firm at $t+1$ with probability $\exp(-\Delta\delta_{ct})$ and has expected size conditional on survival in the same firm equal to $\exp(\Delta\zeta(x_{ct}))s_t(z)Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$. In addition, this firm anticipates acquiring a new product with expected size of $\eta_m s_t(z)Z_t^{\rho-1}/Z_{t+1}^{\rho-1}$ with probability $1-\exp(-\Delta h(x_{mt}))$. Thus, under this investment strategy, the expected discounted present value of dividends associated with a product of size $s_t(z)$ at t inclusive of the dividend at t is directly proportional to the size of the product; that is it can be written as $V_t s_t(z)$ where the factors of proportionality $\{V_t\}$ satisfy the recursion

$$V_t = \Delta D_t + \exp(-\Delta R_t)V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\left[\exp\left(-\Delta\delta_{ct}\right)\exp\left(\Delta\zeta\left(x_{ct}\right)\right) + \eta_m\left(1-\exp\left(-\Delta h\left(x_{mt}\right)\right)\right)\right] \tag{20}$$

and where the interest rate R_t is defined by $\exp(-\Delta R_t) \equiv Q_{t+1}/Q_t$.

Now consider the first-order conditions that govern the investment choices of an incumbent firm managing a product with frontier technology z at t conditional on following our assumed investment strategy in equations (9) and (10) from period $t+1$ on. The current dividend that the firm earns is given as a function of its investment choices in equation (18). The expected value of the products at $t+1$ that the firm expects to gain from these investments is given by

$$V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\left[\exp\left(-\Delta\delta_{ct}\right)\exp\left(\Delta\zeta\left(\frac{x_{ct}\left(z\right)}{s_t(z)M_t^{1-\psi}}\right)\right) + \eta_m\left(1-\exp\left(-\Delta h\left(\frac{x_{mt}\left(z\right)}{s_t\left(z\right)M_t^{1-\psi}}\right)\right)\right)\right]s_t(z).$$

Taking the first-order conditions trading off the impact of investment on dividends at t and expected value at $t+1$ confirms that the optimal choice of innovative investment at t is of the form (9) and (10) where x_{mt} and x_{ct} satisfy

$$(1-\tau_{corp}\lambda_I)(1-\tau_m)P_{rt}M_t^{1-\psi} = \exp\left(-\Delta R_t\right)V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\eta_m\exp\left(-\Delta h\left(x_{mt}\right)\right)h'\left(x_{mt}\right) \tag{21}$$

and

$$\left(1-\tau_{corp}\lambda_I\right)\left(1-\tau_c\right)P_{rt}M_t^{1-\psi} = \exp\left(-\Delta R_t\right)V_{t+1}\frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}}\exp\left(-\Delta\delta_{ct}\right)\exp\left(\Delta\zeta\left(x_{ct}\right)\right)\zeta'\left(x_{ct}\right). \tag{22}$$

Entering firms We now consider the incentives of entering firms. As described above, an investment of $\Delta M_t^{-\psi}$ units of the research good at t yields a new product with probability of $1 - \exp(-\Delta\lambda)$ and expected size $\eta_e Z_t^{\rho-1}/M_t Z_{t+1}^{\rho-1}$. Thus, in equilibrium, we must have

$$\left(1 - \tau_{corp}\lambda_E\right)\left(1 - \tau_e\right) P_{rt} M_t^{1-\psi} \geq \frac{1 - \exp\left(-\Delta\lambda\right)}{\Delta} \exp\left(-\Delta R_t\right) V_{t+1} \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \eta_e, \tag{23}$$

where this expression is an equality if there is positive investment in entry in period t.

In any period t with positive entry, we can combine equations (21) and (23) to obtain a static equation determining x_{mt} given by

$$\frac{\left(1 - \tau_{corp}\lambda_I\right)\left(1 - \tau_m\right)}{\left(1 - \tau_{corp}\lambda_E\right)\left(1 - \tau_e\right)} \frac{1 - \exp\left(-\Delta\lambda\right)}{\Delta} \frac{\eta_e}{\eta_m} = \exp\left(-\Delta h\left(x_{mt}\right)\right) h'\left(x_{mt}\right). \tag{24}$$

This condition implies that x_{mt} is constant in any periods in which entry is positive. Likewise, in any period t with positive entry, we can combine equations (22) and (23) to obtain a static equation relating x_{et}, x_{mt}, and x_{ct} given by

$$\frac{\left(1 - \tau_{corp}\lambda_I\right)\left(1 - \tau_c\right)}{\left(1 - \tau_{corp}\lambda_E\right)\left(1 - \tau_e\right)} \frac{1 - \exp\left(-\Delta\lambda\right)}{\Delta} \eta_e = \exp\left(-\Delta\delta_c\left(x_{mt}, x_{et}\right)\right) \exp\left(\Delta\zeta\left(x_{ct}\right)\right) \zeta'\left(x_{ct}\right). \tag{25}$$

Since x_{mt} is constant in all periods t in which entry is positive, equation (25) defines an implicit function $x_c\left(x_{et}\right)$ that determines x_{ct} as a function of x_{et} in every period in which entry is positive. Note that the derivative on this function is given by

$$\frac{dx_c}{dx_e} = \frac{\Delta\zeta'\left(x_c\right) \frac{\partial}{\partial x_e}\delta_c\left(x_m, x_e\right)}{\Delta\zeta'\left(x_c\right) + \frac{\zeta''\left(x_c\right)}{\zeta'\left(x_c\right)}}. \tag{26}$$

As $\Delta \to 0$, $\frac{\partial}{\partial x_e}\delta_c(x_m, x_e)$ approaches a constant (see equation (12)). Then, $dx_c/dx_e \to 0$, even if there is business stealing. This result implies that as $\Delta \to 0$, x_{ct} is constant in all periods t with positive entry.

Household and government expenditures The intertemporal budget constraint for the household is given by

$$\sum_{t=0}^{\infty} Q_t \left[C_t + T_t + (1 - \tau_{corp}\lambda_E)(1 - \tau_e) P_{rt} M_t^{-\psi} x_{et}\right] \leq$$
$$\sum_{t=0}^{\infty} Q_t \left[W_t L_t + D_{kt} + D_t\right].$$

Government fiscal expenditures net of corporate tax receipts is

$$\begin{aligned} \Delta E_t = \Delta P_{rt} M_t^{1-\psi} \left[\tau_c x_{ct} + \tau_m x_{mt} + \tau_e x_{et}\right] + \Delta\tau_{corp}\lambda_E \left(1 - \tau_e\right) P_{rt} M_t^{1-\psi} x_{et} \\ -\tau_{corp}\left[\Delta R_{kt} K_t - \lambda_k \left(K_{t+1} - \exp(-\Delta d_k) K_t\right)\right] \\ -\tau_{corp}\Delta\left[\frac{\mu - 1}{\mu}(1 + \tau_y) Y_t - \lambda_I P_{rt} M_t^{1-\psi}\left[(1 - \tau_c) x_{ct} + (1 - \tau_m) x_{mt}\right]\right]. \end{aligned}$$

The government budget constraint is $T_t = E_t$ for all t.

Definition of equilibrium The economy starts with initial conditions for $K_0, Z_0, and M_0$ as given. The paths for $\{L_t\}$ and $\{A_{rt}\}$ are given exogenously as well.

An allocation in this model is a sequence of variables $\{K_{t+1}, l_{pt}, l_{rt}, C_t, Y_t, Y_{rt}, Z_{t+1}, M_{t+1}, \delta_{ct}\}$ and innovative investments $\{x_{ct}, x_{mt}, x_{et}\}$. An allocation is feasible if it satisfies equations 1-14. The vector of prices and values in this economy are $\{Q_t, W_t, R_{kt}, P_{rt}, V_t, D_{kt}, D_t\}$ together with a markup of price over marginal cost μ.

The policies in this economy are the corporate tax rate τ_{corp}, subsidy rates for innovative investment τ_c, τ_m, and τ_e, a subsidy rate for output of the final consumption good τ_y, and expensing allowances for physical investment λ_K, innovative investment by incumbents λ_I, and innovative investment in entry λ_E.

An equilibrium given policies is a feasible allocation together with a vector of prices and values and a markup such that firms producing the final consumption good, intermediate goods, and the research good, as well as the holding company for physical capital maximize profits, households maximize utility given their budget constraint, and the government budget constraint is satisfied each period.

C Balanced growth path

We now describe how to solve for a balanced growth path (BGP) of this economy given policies and model parameters. On a BGP, taxes are constant, and the exogenous sequences for A_{rt} and L_t grow at constant rates $\Delta \bar{g}_A$ and $\Delta \bar{g}_L$. Output Y_t, physical capital K_{t+1}, and consumption C_t grow at a common rate $\Delta \bar{g}_y$. Aggregate productivity Z_{t+1} and the measure of products M_t grow at rates $\Delta \bar{g}_Z$ and $\Delta \bar{g}_M$. Innovative investment rates per product remain constant over time at $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$. This last assumption implies from equation (8) that the product $Y_{rt} M_t^{\psi-1}$ remains constant over time as well.

C.1 BGP growth rates

We consider the variable $J_t \equiv Z_t^{1-\phi} M_t^{1-\psi}$ together with the physical capital stock K_t as the endogenous state variables of the economy. By construction, we have

$$\bar{g}_J = (1-\phi)\bar{g}_Z + (1-\psi)\bar{g}_M. \tag{27}$$

Since the product $Y_{rt} M_t^{\psi-1}$ is constant over time on a BGP, equations (7) and (8) imply that the growth rate of J on a BGP depends only on the sum of the growth of scientific progress and population and not on policies:

$$\bar{g}_J = \bar{g}_A + \bar{g}_L. \tag{28}$$

The division of the growth of J into components due to growth in aggregate productivity Z and growth in the number of products M depends on the parameters ϕ and ψ and on policies as follows. The value of $\bar{x}_m$ on a BGP with positive entry is determined from equation (24), while the implicit function $\bar{x}_c(x_e)$ is determined from equation (25). The BGP level of $\bar{x}_e$ is then determined from the equation

$$\Delta \bar{g}_J = (1-\phi)\, G\left(\bar{x}_c\left(\bar{x}_e\right), \bar{x}_m, \bar{x}_e\right) + (1-\psi)\, H\left(\bar{x}_m, \bar{x}_e\right). \tag{29}$$

Recall that as $\Delta \to 0$, $\bar{x}_c(x_e)$ approaches a constant. This implies that for small enough Δ, as long as $\eta_e > \delta_e \zeta(\Delta \bar{x}_c)$ and $\delta_e \leq 1$, then the right-hand side of this expression is strictly increasing in $\bar{x}_e$, so at most one positive solution to this equation exists.

Once one solves for innovative investments $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$, the growth rates of aggregate productivity and the measure of products are given by

$$\Delta \bar{g}_Z = G(\bar{x}_c(\bar{x}_e), \bar{x}_m, \bar{x}_e), \tag{30}$$

$$\Delta \bar{g}_M = H(\bar{x}_m, \bar{x}_e), \tag{31}$$

and

$$\bar{g}_Y = \bar{g}_Z / (1-\alpha) + \bar{g}_L. \tag{32}$$

Impact of a change in population growth on productivity growth Here we derive the expression for the impact of changes in the growth rate of population g_L and scientific progress g_{Ar} on the BGP growth rate of aggregate productivity,

$$\frac{d\bar{g}_Z}{d\bar{g}_L} = \frac{\Theta_G}{\Theta_J} = \frac{1}{(1-\phi) + (1-\psi)\,\frac{\Theta_H}{\Theta_G}}, \tag{33}$$

where Θ_G, Θ_H and Θ_J are defined in the next Subsection (and in the main paper). To derive this expression, observe that a change in the growth rate of labor has no impact on equations (24) and (25) that determine $\bar{x}_m$ or on the implicit function $x_c(x_e)$. Thus, we can differentiate equations (28) and (29) to get:

$$d\bar{g}_L = d\bar{g}_J = \Theta_J d \log \bar{x}_e \tag{34}$$

From the definitions of Θ_G and Θ_H, we have $d\bar{g}_Z = \Theta_G d \log \bar{x}_e$ and $d\bar{g}_M = \Theta_H d \log \bar{x}_e$. Using $d\bar{g}_J = (1-\phi)\, d\bar{g}_Z + (1-\psi)\, d\bar{g}_M$ and $d\bar{g}_M = \Theta_H / \Theta_G d\bar{g}_Z$, we obtain (34).

C.2 BGP levels

We now describe how to solve for the equilibrium levels of variables on a BGP given levels of A_{rt} and L_{rt}. The level of $J_t \equiv Z_t^{1-\phi} M_t^{1-\psi}$ is determined on a BGP, but the levels of Z_t and M_t individually on the BGP are not pinned down by BGP equations alone. As discussed below, these are determined by the initial conditions of the economy and the transition path to the new BGP. We use the following equations to solve for the level of variables on a BGP.

The consumer's intertemporal Euler equation gives

$$\exp(\Delta \bar{R}) = \beta^{-\Delta} \exp\left(\gamma \Delta \left(\bar{g}_Y - \bar{g}_L\right)\right). \tag{35}$$

From equation (19), dividends from intermediate goods firms relative to output inclusive of production subsidies are given by $\bar{d} \equiv D_t / \left(\left(1+\tau_y\right) Y_t\right)$ where

$$\bar{d} = (1-\tau_{corp}) \frac{\mu - 1}{\mu} - (1 - \tau_{corp} \lambda_I) \bar{p}_r \left[(1-\tau_c)\bar{x}_c + (1-\tau_m)\bar{x}_m\right] \tag{36}$$

and

$$\bar{p}_r \equiv \frac{P_{rt} M_t^{1-\psi}}{\left(1+\tau_y\right) Y_t}. \tag{37}$$

From equation (20), the value of a product relative to output is given by $\bar{v} \equiv V_t / Y_t$ where

$$\bar{v} = \frac{\Delta \bar{d}}{1 - \exp\left(-\Delta\left(\bar{R} - \bar{g}_Y\right)\right)\left(1 - \bar{S}_e\left(\Delta\right)\right)}, \tag{38}$$

where $\bar{S}_e(\Delta)$ is given as a function of innovative investments and the growth rate of productivity from equation (80) below. The BGP value of $\bar{p}_r$ is then found as the solution to the BGP version of the free entry condition

$$\left(1 - \tau_{corp}\lambda_E\right)\left(1 - \tau_e\right)\bar{p}\bar{x}_e = \exp\left(-\Delta\left(R - \bar{g}_Y\right)\right)\bar{v}\frac{\bar{S}_e\left(\Delta\right)}{\Delta}. \tag{39}$$

The innovation intensity of the economy on a BGP is given by $P_{rt}Y_{rt} / \left(\left(1 + \tau_y\right)Y_t\right) = \bar{p}_r\left(\bar{x}_c + \bar{x}_m + \bar{x}_e\right)$, and the allocation of labor to research is given by

$$\frac{\bar{l}_r}{\bar{l}_p} = \frac{\mu}{1 - \alpha}\frac{1}{1 + \tau_y}\bar{p}_r\left(\bar{x}_c + \bar{x}_m + \bar{x}_e\right). \tag{40}$$

The ratio of physical capital to output is given by the standard Euler equation from the profit maximization problem of the physical capital holding company:

$$\exp(\Delta\bar{R}) = \Delta\frac{\left(1 - \tau_{corp}\right)}{\left(1 - \tau_{corp}\lambda_K\right)}\frac{\alpha\left(1 + \tau_y\right)}{\mu}\frac{Y_{t+1}}{K_{t+1}} + \exp\left(-\Delta\delta_K\right). \tag{41}$$

Finally, from equations (7) and (8), together with the definition of J_t, we get that the level of J_t on a BGP is given by

$$\bar{x}_c + \bar{x}_m + \bar{x}_e = A_{rt}\bar{l}_r L_t / \bar{J}_t. \tag{42}$$

The equilibrium sequence of innovative investments $\{x_{ct}, x_{mt}, x_{et}\}$ implies, through the functions H and G defined above in equations (11) and (14), a sequence of growth rates of the log of M_{t+1} and Z_{t+1}. This sequence of growth rates can then be used to trace out the paths for the levels of M_{t+1} and Z_{t+1} from their initial conditions to their levels on the BGP (should the equilibrium converge to a BGP). We use a first-order approximation of these dynamics to solve for the levels of Z_t and M_t on a BGP as follows.

Consider a first-order approximation to the dynamics of $\{J_{t+1}, Z_{t+1}, M_{t+1}\}$ from initial condition $J_0 = Z_0^{1-\phi}M_0^{1-\psi}$, Z_0, and M_0 to its BGP path $\{\bar{J}_t\}$ taking as given a path for investment in entry $\{x_{et}\}$ that converges to its BGP level $\bar{x}_e$. We use the fact that the initial and BGP values of J_t are known to solve, up to a first-order approximation, for the BGP values of Z_t and M_t.

The elasticity of the growth of aggregate productivity with respect to entry is defined as

$$\Theta_G = \frac{1}{\Delta}\left[\frac{\partial}{\partial x_c}G\left(\bar{x}_c\left(\bar{x}_e\right), \bar{x}_m, \bar{x}_e\right)\frac{d}{dx_e}x_c\left(\bar{x}_e\right) + \frac{\partial}{\partial x_e}G\left(\bar{x}_c\left(\bar{x}_e\right), \bar{x}_m, \bar{x}_e\right)\right]\bar{x}_e. \tag{43}$$

Note that, given the expression for $\exp\left(-\Delta\delta_{ct}\left(x_m, x_c\right)\right)$ in equation (12), we have

$$\frac{1}{\Delta}\frac{\partial}{\partial x_e}G\left(\bar{x}_c\left(\bar{x}_e\right), \bar{x}_m, \bar{x}_e\right)\bar{x}_e = \frac{1}{\rho - 1}\left(1 - \frac{\delta_e}{\eta_e}\exp\left(\Delta\zeta\left(\bar{x}_c\right)\right)\right)\frac{\bar{S}_e(\Delta)}{\Delta} \tag{44}$$

and

$$\frac{1}{\Delta}\frac{\partial}{\partial x_c}G\left(\bar{x}_c\left(\bar{x}_e\right),\bar{x}_m,\bar{x}_e\right)=\frac{1}{\rho-1}S_c\left(\Delta\right)\zeta'\left(\bar{x}_c\right). \tag{45}$$

The derivative $\frac{d}{dx_e}x_c(x_e)$ is given in equation (26). Similarly, define the elasticity of the growth in the number of products with respect to entry by

$$\Theta_H=\frac{1}{\Delta}\frac{\partial}{\partial x_e}H\left(\bar{x}_m,\bar{x}_e\right)\bar{x}_e=\left(1-\delta_e\right)\frac{F_e\left(\Delta\right)}{\Delta}. \tag{46}$$

Note that as $\Delta\to 0$,

$$\Theta_G\to\frac{1}{\rho-1}\left(1-\frac{\delta_e}{\eta_e}\right)\eta_e\lambda\bar{x}_e, \tag{47}$$

$$\Theta_H\to(1-\delta_e)\lambda\bar{x}_e. \tag{48}$$

By the definition of J, we have

$$\frac{\log J_{t+1}-\log J_t}{\Delta}\equiv\left(1-\phi\right)\frac{1}{\Delta}G\left(x_{ct},x_{mt},x_{et}\right)+\left(1-\psi\right)\frac{1}{\Delta}H\left(x_{mt},x_{et}\right). \tag{49}$$

Thus, we can define the elasticity Θ_J of the growth rate of J with respect to changes in entry from

$$\Theta_J=\left(1-\phi\right)\Theta_G+\left(1-\psi\right)\Theta_H. \tag{50}$$

The following expressions allow us to pin down, up to a first-order approximation, the BGP levels of aggregate productivity and the measure of products, $\bar{Z}_0$ and $\bar{M}_0$, from initial conditions Z_0 and M_0 as follows:

$$\log\bar{Z}_0=\log Z_0+\frac{\Theta_G}{\Theta_J}\left(\log\bar{J}_0-\log J_0\right) \tag{51}$$

$$\log\bar{M}_0=\log M_0+\frac{\Theta_H}{\Theta_J}\left(\log\bar{J}_0-\log J_0\right) \tag{52}$$

where Θ_G, Θ_H and Θ_J are evaluated at the BGP.

These expressions are derived as follows. To a first-order approximation, we have

$$\left(\log J_{t+1}-\log\bar{J}_{t+1}\right)-\left(\log J_t-\log\bar{J}_t\right)=\Delta\Theta_J\left(\log x_{et}-\log\bar{x}_e\right), \tag{53}$$

$$\left(\log Z_{t+1}-\log\bar{Z}_{t+1}\right)-\left(\log Z_t-\log\bar{Z}_t\right)=\Delta\Theta_G\left(\log x_{et}-\log\bar{x}_e\right) \tag{54}$$

and

$$\left(\log M_{t+1}-\log\bar{M}_{t+1}\right)-\left(\log M_t-\log\bar{M}_t\right)=\Delta\Theta_H\left(\log x_{et}-\log\bar{x}_e\right). \tag{55}$$

Substituting equation (53) into (54) and (52), we obtain

$$\left(\log Z_{t+1}-\log\bar{Z}_{t+1}\right)-\frac{\Theta_G}{\Theta_J}\left(\log J_{t+1}-\log\bar{J}_{t+1}\right)=\left(\log Z_t-\log\bar{Z}_t\right)-\frac{\Theta_G}{\Theta_J}\left(\log J_t-\log\bar{J}_t\right), \tag{56}$$

$$\left(\log M_{t+1}-\log\bar{M}_{t+1}\right)-\tfrac{\Theta_H}{\Theta_J}\left(\log J_{t+1}-\log\bar{J}_{t+1}\right)=\left(\log M_t-\log\bar{M}_t\right)-\tfrac{\Theta_H}{\Theta_J}\left(\log J_t-\log\bar{J}_t\right). \tag{57}$$

By direct recursion, we have

$$(\log Z_{t+1} - \log \bar{Z}_{t+1}) - \frac{\Theta_G}{\Theta_J}(\log J_{t+1} - \log \bar{J}_{t+1}) = (\log Z_0 - \log \bar{Z}_0) - \frac{\Theta_G}{\Theta_J}(\log J_0 - \log \bar{J}_0) \quad (58)$$

and

$$(\log M_{t+1} - \log \bar{M}_{t+1}) - \tfrac{\Theta_H}{\Theta_J}(\log J_{t+1} - \log \bar{J}_{t+1}) = (\log M_0 - \log \bar{M}_0) - \tfrac{\Theta_H}{\Theta_J}(\log J_0 - \log \bar{J}_0). \quad (59)$$

To be consistent with convergence to the BGP path, we must have that $(\log Z_{t+1} - \log \bar{Z}_{t+1})$ and $(\log M_{t+1} - \log \bar{M}_{t+1})$ converge to zero as $(\log J_{t+1} - \log \bar{J}_{t+1})$ converges to zero. Setting the left-hand side in equations (58) and (59) to zero, the unique values of the initial conditions $(\log Z_0 - \log \bar{Z}_0)$ and $(\log M_0 - \log \bar{M}_0)$ consistent with this terminal condition are given by equations (51) and (52).

With these solutions for the BGP levels of aggregate productivity and the measure of products, it is straightforward to solve for the BGP level of the physical capital stock given the BGP allocation of labor to current production obtained from equation (40) and the BGP physical-capital-to-output ratio obtained from equation (41).

D Characterizing dynamics of J, Z, and M

We now consider the dynamics of aggregate productivity implied by our model when the path of research labor $\{l_{rt}L_t\}$ is taken as exogenous such that the fraction of labor allocated to research converges to its BGP value $\bar{l}_r$ and population L_t converges to its BGP path $\{\bar{L}_{rt}\}$. We assume that the ratios $\frac{(1-\tau_{corp}\lambda_I)(1-\tau_m)}{(1-\tau_{corp}\lambda_E)(1-\tau_e)}$ and $\frac{(1-\tau_{corp}\lambda_I)(1-\tau_c)}{(1-\tau_{corp}\lambda_E)(1-\tau_e)}$ remain constant.

Equations (7), (8), (11), (14), (24), (25), and (49) can be used to solve for the path $\mathcal{Z}_t = \{Y_{rt}, x_{et}, x_{mt}, x_{ct}, J_{t+1}, Z_{t+1}, M_{t+1}\}$ given initial conditions for J_0, Z_0, M_0 and the path of research labor $\{l_{rt}L_t\}$. As discussed above, equation (24) implies that with positive entry, $x_{mt} = \bar{x}_m$. Thus, equations (7), (8), and (25) imply that, to a first-order approximation

$$(\log x_{et} - \log \bar{x}_e) = A\left[\,(\log l_{rt} - \log \bar{l}_r) + (\log L_t - \log \bar{L}_t) - (\log J_t - \log \bar{J}_t)\,\right], \quad (60)$$

where

$$A \equiv \frac{\bar{x}_c + \bar{x}_m + \bar{x}_e}{\bar{x}_e}\left[\frac{1}{\frac{d}{dx_e}x_c(\bar{x}_e) + 1}\right] \quad (61)$$

and $\frac{d}{dx_e}x_c(\bar{x}_e)$ is given by equation (26) evaluated at the BGP values of investment. Since in the limit, as Δ approaches zero x_c is independent of x_e, then the continuous time limit of A is

$$A = \frac{\bar{x}_c + \bar{x}_m + \bar{x}_e}{\bar{x}_e}. \quad (62)$$

Define

$$\Theta \equiv A\Theta_J. \quad (63)$$

From equation (53), we have

$$\log J_{t+1} - \log \bar{J}_{t+1} = \Delta\Theta\left[(\log l_{rt} - \log \bar{l}_r) + (\log L_t - \log \bar{L}_t)\right] + (1 - \Delta\Theta)(\log J_t - \log \bar{J}_t). \quad (64)$$

The initial condition of this AR1 process for J, $\log J_0 - \log \bar{J}_0$, is given (since the BGP level $\bar{J}_t$ and growth rate $\bar{g}_J$ are both pinned down).

The analog of equation (24) in Proposition 1 of the paper, allowing for a length of time period Δ, is

$$\log J_{t+1} - \log \bar{J}_{t+1} = \sum_{j=0}^{t} \Delta\Theta \left(1 - \Delta\Theta\right)^{j} \left[\left(\log l_{rt-j} - \log \bar{l}_r\right) + \left(\log L_{t-j} - \log \bar{L}_t\right)\right] + \\ + \left(1 - \Delta\Theta\right)^{t+1} \left(\log J_0 - \log \bar{J}_0\right). \tag{65}$$

Once we solve for the dynamics of J, the dynamics of aggregate productivity and the measure of products are given by

$$\log Z_t - \log \bar{Z}_t = \frac{\Theta_G}{\Theta_J} \left(\log J_t - \log \bar{J}_t\right) \tag{66}$$

$$\log M_t - \log \bar{M}_t = \frac{\Theta_H}{\Theta_J} \left(\log J_t - \log \bar{J}_t\right). \tag{67}$$

Equations (66) and (67) are implied by equations (58) and (59) together with the initial condition from Proposition 1, $(\log Z_0 - \log \bar{Z}_0) = \frac{\Theta_G}{\Theta_J} (\log J_0 - \log \bar{J}_0)$ and $(\log M_0 - \log \bar{M}_0) = \frac{\Theta_H}{\Theta_J} (\log J_0 - \log \bar{J}_0)$. Note that the dynamics of aggregate productivity to the BGP, as given by $\log Z_t - \log \bar{Z}_t$, do not depend, to a first-order approximation, on the initial decompositon of J_0 into Z_0 and M_0.

The analog of equation (27) in Corollary 1 of the paper, allowing for a length of time period Δ, is

$$\log Z_{t+1} - \log Z_0 - t\bar{g}_Z = \Delta A \Theta_G \sum_{j=0}^{t} \left(1 - \Delta\Theta\right)^{j} \left(\log l_{rt-j} - \log l_{r0}\right) \tag{68}$$

Proof of Corollary 1: Under the assumptions that the growth rates of scientific knowledge and population are unchanged, we have that the terms $\log L_{t-j} - \log \bar{L}_t = 0$ in equation (65). Likewise, we have that the path of $\{A_{rt}\}$ is unchanged. Note that since we assume that the economy starts on an initial BGP, the terms l_{r0}, J_0, and Z_0 correspond to the levels of these variables at $t = 0$ on that initial BGP. The terms $\bar{l}_r$, $\bar{J}_0$, and $\bar{Z}_0$ correspond to the values of these variables on the new BGP. The term $\bar{g}_Z$ corresponds to the growth rate of aggregate productivity on the new BGP. Under the assumption that the allocation of innovative investment $\bar{x}_c$,$\bar{x}_m$, and $\bar{x}_e$ is unchanged, we have that the growth rate of productivity on the new BGP is equal to its growth rate on the initial BGP. In addition, under the assumption that the allocation of investment is unchanged, we have $\bar{x}_c + \bar{x}_m + \bar{x}_e = \bar{Y}_r = Y_{r0}$, and hence from equation (42), we have that $\log J_0 - \log \bar{J}_0 = \log l_{r0} = \log \bar{l}_r$. Using these equations and equations (63) and (66), we can then rewrite equation (65) as

$$\log Z_{t+1} - \log \bar{Z}_{t+1} =$$

$$\frac{\Theta_G}{\Theta_J} \left[\sum_{j=0}^{t} \Delta\Theta \left(1 - \Delta\Theta\right)^{j} \left[\left(\log l_{rt-j} - \log l_{r0}\right) + \left(\log l_{r0} - \log \bar{l}_r\right)\right] + \left(1 - \Delta\Theta\right)^{t+1} \left(\log l_{r0} - \log \bar{l}_r\right)\right] =$$

$$\Delta A \Theta_G \sum_{j=0}^{t} \left(1 - \Delta\Theta\right)^{j} \left(\log l_{rt-j} - \log l_{r0}\right) + \log Z_0 - \log \bar{Z}_0$$

Using the fact that $\log \bar{Z}_{t+1} - \log \bar{Z}_0 = (t+1)\bar{g}_Z$, and that the growth rate of aggregate productivity on the initial and the new BGP is the same, this gives us equation (68). This proves the result.

When solving for the allocation of labor to production and research on the optimal BGP, assume that the production subsidy τ_y is set as required to undo the distortions of corporate taxes and markups on the accumulation of physical capital in equation (41). That is, we assume that τ_y is set so that, on the BGP

$$\tau_y = \mu \left(\frac{1 - \tau_{corp}\lambda_K}{1 - \tau_{corp}} \right) - 1. \tag{69}$$

The log of the consumption-equivalent variation in welfare that arises from a perturbation of the BGP allocation is given, to a first-order approximation, by

$$\log \xi \approx (1 - \exp(\Delta(\bar{g}_Y - \bar{R}))) \sum_{t=0}^{\infty} \exp(t\Delta(\bar{g}_Y - \bar{R})) \frac{\bar{Y}}{\bar{C}} \left[(\log Z'_t - \log \bar{Z}_t) - (1-\alpha) \frac{\bar{l}_r}{\bar{l}_p} (\log l'_{rt} - \log \bar{l}_r) \right]$$

The allocation of labor to production and research on the optimal BGP is such that $\log \xi = 0$, which implies

$$\frac{\bar{l}^*_r}{\bar{l}^*_p} = \left(\frac{1}{1-\alpha} \right) A\Theta_G \frac{\Delta \exp(\Delta(\bar{g}_Y - \bar{R}))}{1 - \exp(\Delta(\bar{g}_Y - \bar{R}))(1 - \Delta\Theta)}. \tag{70}$$

In the continuous time limit we have

$$\frac{\bar{l}^*_r}{\bar{l}^*_p} = \left(\frac{1}{1-\alpha} \right) A\Theta_G \frac{1}{\bar{R} - \bar{g}_Y + \Theta},$$

where the continuous time expressions for A and Θ_G are given above.

E System of linearized equations

To solve for the transitional dynamics of the model economy from its initial conditions to a new BGP, we log-linearize the equations of the model around the new BGP. We treat the variables J_t and K_t as the endogenous state variables of the model. When we consider demographic changes, we consider exogenously specified paths for the growth rate of populations denoted by $g_{Lt} = \log L_{t+1} - \log L_t$. We assume that the sequence $\{g_{Lt}\}$ converges to a new BGP growth rate $\bar{g}'_L$.

The log-linearized versions of the model equations are as follows. The log-linearized versions of equation (1) are given by

$$0 = \tilde{y}_t - \frac{\bar{C}}{\bar{Y}} \tilde{c}_t + \exp(-\Delta\delta_K) \frac{\bar{K}}{\Delta\bar{Y}} \tilde{k}_t - \exp(\Delta\bar{g}_Y) \frac{\bar{K}}{\Delta\bar{Y}} \tilde{k}_{t+1},$$

where a tilde over a variable indicates the difference between the log of a variable and the log of its level on the BGP. Equation (5) becomes

$$0 = -\tilde{y}_t + \tilde{z}_t + \alpha\tilde{k}_t + (1-\alpha)\left(\tilde{l}_{pt} + \tilde{L}_t\right).$$

The resource constraint on labor is

$$0 = \bar{l}_r \tilde{l_{rt}} + \bar{l}_p \tilde{l}_{pt}.$$

As discussed above, equations (7), (8), (11), (14), (24), (25), and (49) give us equation (64) here written as

$$\tilde{j}_{t+1} = \Delta\Theta \left[\tilde{l}_{rt} + \tilde{L}_t\right] + (1 - \Delta\Theta)\tilde{j}_t,$$

together with

$$\tilde{z}_t = \frac{\Theta_G}{\Theta J}\tilde{j}_t$$

$$\tilde{m}_t = \frac{\Theta_H}{\Theta J}\tilde{j}_t.$$

From equation (60) we have

$$A\tilde{x}_{et} = \tilde{l}_{rt} + \tilde{L}_t - \tilde{J}_t,$$

where A is defined in equation (61) and

$$\tilde{x}_{ct} = \frac{dx_c}{dx_e}\frac{\bar{x}_c}{\bar{x}_c}\tilde{x}_{et},$$

where $\frac{d}{dx_e}x_c(\bar{x}_e)$ is given by equation (26) evaluated at the BGP values of investment.

From equation (16), we have

$$\tilde{s}_{et+1} = (\rho - 1)\left(\tilde{z}_t - \tilde{z}_{t+1}\right) + \tilde{x}_{et}.$$

The first-order condition for consumption is the standard equation

$$0 = \Delta\bar{R}\tilde{R}_t - \gamma(\tilde{c}_{t+1} - \tilde{L}_{t+1}) + \gamma(\tilde{c}_t - \tilde{L}_t).$$

The Euler equation for physical capital is the standard equation

$$\bar{R}\tilde{R}_t = \frac{\exp(\Delta\bar{R}) - \exp(-\Delta\delta_k)}{\exp(\Delta\bar{R})\Delta}\left[\tilde{y}_{t+1} - \tilde{k}_{t+1}\right].$$

Finally, we derive the Euler equation governing investment in entry by log-linearizing equations (20) and (23). We let $v_t = V_t/Y_t$, $d_t = D_t/Y_t$ and $p_t = P_{rt}/M_t^{\psi-1}Y_t$. The equations to be log-linearized include

$$p_t = \frac{1-\alpha}{\mu}\frac{1}{A_{rt}J_t l_{pt} L_t},$$

which is derived from the first-order condition of the profit maximization problem of the firm producing the research good and the formula for the fraction of output of the final consumption good paid as wages, as well as

$$d_t = (1 - \tau_{corp})\frac{\mu - 1}{\mu}(1 + \tau_y) - (1 - \tau_{corp}\lambda_I)p_t\left[(1 - \tau_c)x_{ct} + (1 - \tau_m)x_{mt}\right],$$

which is derived from equation (19), the equation

$$v_t = \Delta d_t + \exp(-\Delta R_t)v_{t+1}\frac{Y_{t+1}}{Y_t}\left[1 - S_{et+1}(\Delta)\right],$$

which is derived from equation (20), (14), and (16), and the zero-profits entry condition derided from (23)

$$(1-\tau_{corp}\lambda_E)(1-\tau_e)p_{rt}x_{et} = \exp(-\Delta R_t)v_{t+1}\frac{Y_{t+1}}{Y_t}\frac{S_{et+1}(\Delta)}{\Delta}.$$

We use the zero-profits entry condition as the second intertemporal Euler equation in the model. We use this condition to substitute into the equation for the value function to get

$$v_t = \Delta d_t + \Delta(1-\tau_{corp}\lambda_E)(1-\tau_e)p_{rt}x_{et}\frac{[1-S_{et+1}(\Delta)]}{S_{et+1}(\Delta)}.$$

The log-linearized versions of these equations are

$$\tilde{p}_t = -\tilde{J}_t - \tilde{l}_{pt} - \tilde{L}_t$$

$$\tilde{d}_t = -\frac{(1-\tau_{corp}\lambda_I)\bar{p}\left[(1-\tau_c)\bar{x}_c + (1-\tau_m)\bar{x}_m\right]}{\bar{d}}\tilde{p}_t - \frac{(1-\tau_{corp}\lambda_I)\bar{p}(1-\tau_c)\bar{x}_c}{\bar{d}}\tilde{x}_{ct}$$

$$\tilde{v}_t = \left[1-\exp(-\Delta(\bar{R}-\bar{g}_Y))\left(1-\bar{S}_e(\Delta)\right)\right]\tilde{d}_t + \exp\left(-\Delta(\bar{R}-\bar{g}_Y)\right)\left[1-\bar{S}_e(\Delta)\right]\left[\tilde{p}_{rt}+\tilde{x}_{et}\right] - \exp\left(-\Delta(\bar{R}-\bar{g}_Y)\right)\tilde{s}_{et+1}$$

and

$$\tilde{p}_{rt} + \tilde{x}_{et} = -\Delta\bar{R}\tilde{R}_t + \tilde{v}_{t+1} + \tilde{y}_{t+1} - \tilde{y}_t + \tilde{s}_{et+1}.$$

F Calibration

Our calibration strategy is similar, but not identical, to that in Atkeson and Burstein (2018) (henceforth AB2018). We impose the following restrictions on policies on the initial BGP. Incumbent firms can deduct all of their innovative investments ($\lambda_I = 1$), while entering firms cannot ($\lambda_E = 0$) since they are not incorporated at the time of their investments. We follow Barro and Furman (2018) in setting the corporate profit tax rates, $\tau_{corp} = 0.38$ (and, in the corporate profits tax experiment, we set the new tax rate to $\tau'_{corp} = 0.26$).[3] We set the extent of physical investment expensing λ_k as discussed below. We set $\tau_c = \tau_m = \tau_e = \tau_{rd} = 0.03$ as in AB2018. We allow for a production subsidy τ_y to remove the distortions in the allocation of physical capital induced by the markup and the corporate profits tax. The choice of τ_y only affects our welfare calculations.

We set the time period to 1 month, $\Delta = 1/12$, and $\lambda = 1$ (without loss of generality for our model's loglinearized dynamics, as long as $\lambda > 0$). We consider three combinations of the spillover parameters ϕ and ψ: $\{0.96, 1\}$, $\{-1.6, 1\}$, and $\{0.96, 0\}$. We set $\rho = 4$ and $\gamma = 1$ as in AB2018. We set $\bar{g}_L = 0.007$ and $\bar{g}_M = 0.01$ as in AB2018, and choose $\bar{g}_A$ so that $\bar{g}_Y = 0.025$ as in AB2018, where

$$\bar{g}_Y = \frac{\bar{g}_Z}{1-\alpha} + \bar{g}_L$$

and

$$(1-\phi)\bar{g}_Z + (1-\psi)\bar{g}_M = \bar{g}_A + \bar{g}_L,$$

. We set β to satisfy (by equation (35))

$$\exp(-\Delta\bar{R}) = \beta\exp\left(-\gamma\Delta\left(\bar{g}_Y - \bar{g}_L\right)\right)$$

[3] In contrast to AB2018, we do not pick the corporate profit tax rates to match the ratio of payments of taxes on income and wealth relative to the tax base of the corporate profits tax in the data.

with $\bar{R} = \log(1 + 0.04)$ as in AB2018.

As in AB2018, we use the annual growth of the measure of products $g^A_{Mt} = \log(M_{t+1}/M_t)$, the fraction of products that are continuing products in incumbent firms f^A_{ct+1}, the fraction of products that are new to incumbent firms measured as the sum of those that are new to society and stolen f^A_{mt+1}, and the fraction of products that are produced in entering firms measured as the sum of those that are new to society and stolen $f^A_{et+1} = 1 - f^A_{ct+1} - f^A_{mt+1}$, the aggregate size of continuing products in incumbent firms s^A_{ct+1}, the aggregate size of products that are new to incumbent firms measured as the sum of those that are new products and those that are stolen s^A_{mt+1}, and the aggregate size of products that are new to entering firms measured as the sum of those that are new products and those that are stolen $s^A_{et+1} = 1 - s^A_{ct+1} - s^A_{mt+1}$. Time averages of these variables are denoted with a bar. In the next Subsection F.1, we show how to convert these annual measures to per unit of time measures $\bar{f}_i(\Delta)$ and $\bar{s}_i(\Delta)$ for $i = e, m, c$. For simplicity, in what follows we omit the argument (Δ), so $\bar{f}_i = \bar{f}_i(\Delta)$ and $\bar{s}_i = \bar{s}_i(\Delta)$.

We calibrate δ_0, η_e, η_m and the initial BGP values of $\bar{\delta}_c$, $h(\bar{x}_m)$, $\zeta(\bar{x}_c)$, $\bar{x}_e$ to satisfy the following equations:

$$\begin{aligned}
\bar{f}_e &= \exp(-\Delta \bar{g}_M)(1 - \exp(-\Delta\lambda))\bar{x}_e \\
\bar{f}_m &= \exp(-\Delta \bar{g}_M)\eta_m (1 - \exp(-\Delta h(\bar{x}_m))) \\
1 - \bar{f}_e - \bar{f}_m &= \exp(-\Delta \bar{g}_M)\exp(-\Delta\bar{\delta}_c) \\
\bar{s}_e &= \exp(-(\rho - 1)\Delta \bar{g}_Z)(1 - \exp(-\Delta\lambda))\eta_e \bar{x}_e \\
\bar{s}_m &= \exp(-(\rho - 1)\Delta \bar{g}_Z)\eta_m (1 - \exp(-\Delta h(\bar{x}_m))) \\
\bar{s}_c &= \exp(-(\rho - 1)\Delta \bar{g}_Z)\exp(-\Delta\bar{\delta}_c)\exp(\Delta\zeta(\bar{x}_c)) \\
\exp(-\Delta\bar{\delta}_c) &= \exp(-\Delta\delta_0) - \delta_m(1 - \exp(-\Delta h(\bar{x}_m))) - \delta_e(1 - \exp(-\Delta\lambda))\bar{x}_e
\end{aligned}$$

In the specification without business stealing, we set $\delta_m = \delta_e = 0$. In the specification with business stealing, we set $\delta_m = \delta_e$ so that, as in AB2018,

$$\frac{(\bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0))}{\bar{g}_Z} = 0.257,$$

where

$$G(x_c, x_m, 0) = \frac{1}{\rho - 1}\log\Big(\exp(-\Delta\delta_c^0)\exp(\Delta\zeta(x_c)) + \eta_m(1 - \exp(-\Delta h(x_m))\Big)$$

and

$$\exp(-\Delta\delta_c^0) = \exp(-\Delta\delta_0) - \delta_m(1 - \exp(-\Delta h(\bar{x}_m))).$$

We calibrate $\zeta'(\bar{x}_c)$ and $h'(\bar{x}_m)$ using equations (24) and (25):

$$(1 - \tau_{corp})\,\frac{(1 - \exp(-\Delta\lambda))}{\Delta}\eta_e = \eta_m \exp(-\Delta h(\bar{x}_m))h'(\bar{x}_m)$$

$$(1 - \tau_{corp})\,\frac{1 - \exp(-\Delta\lambda)}{\Delta}\eta_e = \exp(-\Delta\bar{\delta}_c)\exp(\Delta\zeta(\bar{x}_c))\zeta'(\bar{x}_c).$$

We specify the functions $h(\cdot)$ and $\zeta(\cdot)$ as

$$h\left(x_m\right) = h_1 x_m^{h_2}$$

$$\zeta\left(x_c\right) = \zeta_0 + \zeta_1 x_c^{\zeta_2}.$$

In order to calibrate the parameters h_1, h_2, ζ_1, and ζ_2, we must know the values of $\bar{x}_c$ and $\bar{x}_m$. Our calibration procedure uses as an input a measure of $\left(\bar{x}_c + \bar{x}_m\right)/\bar{Y}_r$ and implies a value of $\bar{x}_e/\bar{Y}_r$, but does not pin down $\bar{x}_c$ and $\bar{x}_m$ separately. To determine the value of $\bar{x}_c$, we follow the same logic as in AB2018. The contribution of investment in acquiring products each period to firm value must be nonnegative. That is, on a BGP, we must have $\bar{v}$ at least as large as the value that the firm would obtain if it were to set investment into acquiring new products equal to zero in every period. Given the assumption that $h(0) = 0$, this alternative value $\tilde{v}$ of incumbent firms on a BGP is given by

$$\tilde{v} = \frac{\Delta}{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\bar{s}_c}\left(1 - \tau_{corp}\right)\left[\left(\frac{\mu - 1}{\mu}\right) - \left(1 - \tau_{rd}\right)\bar{p}_r \bar{x}_c\right],$$

and the value of the firm $\bar{v}$ is given by

$$\bar{v} = \frac{\Delta}{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\left(1 - \bar{s}_e\right)}\left(1 - \tau_{corp}\right)\left[\left(\frac{\mu - 1}{\mu}\right) - \left(1 - \tau_{rd}\right)\bar{p}_r\left(\bar{x}_c + \bar{x}_m\right)\right].$$

The requirement that $\tilde{v} \leq \bar{v}$ implies that the research expenditures of incumbents on improving continuing products relative to value added must lie between the bounds

$$\begin{aligned} &\bar{p}_r\left(\bar{x}_c + \bar{x}_m\right) \geq \bar{p}_r \bar{x}_c \geq \\ &\frac{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\bar{s}_c}{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\left(1 - \bar{s}_e\right)}\bar{p}_r\left(\bar{x}_c + \bar{x}_m\right) - \frac{\exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\bar{s}_m}{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\left(1 - \bar{s}_e\right)}\frac{1}{\left(1 - \tau_{rd}\right)}\left(\frac{\mu - 1}{\mu}\right). \end{aligned} \tag{71}$$

In our calibration, we set $\bar{p}_r \bar{x}_c$ in the middle point between the two bounds. Given values of $\bar{x}_m$, $h\left(\bar{x}_m\right)$, and $h'\left(\bar{x}_m\right)$, we determine the values of h_0 and h_1. Given values of $\bar{x}_c$, $\zeta\left(\bar{x}_c\right)$, and $\zeta'\left(\bar{x}_c\right)$ (which are assigned as described above, independently of ζ_2) and a value of $0 < \zeta_2 < 1$, we determine the values of ζ_0 and ζ_1. We set ζ_2 halfway between its two bounds, that is, $\zeta_2 = 0.5$.

We now describe how we set $\{\alpha, \delta_k, \mu, \lambda_k\}$. We set δ_k such that $\exp(-\delta_K) = 1 - 0.055$ as in AB2018. We choose μ so that Tobin's q is equal to 1.15, where Tobin's q is defined as

$$\bar{q} = \frac{\bar{v} + \bar{v}_k}{\frac{\bar{K}}{\bar{Y}\left(1 + \tau_y\right)}} \tag{72}$$

given the expressions for values and dividends above,

$$\bar{v} = \frac{\Delta \bar{d}}{1 - \exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\left(1 - \bar{s}_e\right)}$$

$$\bar{v}_k = \frac{\Delta \bar{d}_k}{1 - \exp\left(\Delta\left(g_Y - \bar{R}\right)\right)}$$

$$\bar{d} = \left(1 - \tau_{corp}\right)\left[\frac{\mu - 1}{\mu} - \left(1 - \tau_{rd}\right)\bar{p}_r\left(\bar{x}_c + \bar{x}_m\right)\right]$$

$$\bar{d}_k = \frac{D_{kt}}{(1+\tau_y)\,\bar{Y}_t} = (1-\tau_{corp})\frac{R_{kt}\bar{K}}{\bar{Y}_t\,(1+\tau_y)} - (1-\tau_{corp}\lambda_k)\frac{\bar{K}_{t+1} - \exp(-\Delta d_k)\,\bar{K}_{t+1}}{\Delta\bar{Y}_t\,(1+\tau_y)}$$
$$= (1-\tau_{corp})\frac{\alpha}{\mu} - (1-\tau_{corp}\lambda_k)\frac{\bar{K}}{\bar{Y}\,(1+\tau_y)}\left(\frac{\exp(\Delta\bar{g}_y) - \exp(-\Delta d_k)}{\Delta}\right)$$

and where we set $\frac{\bar{K}}{\bar{Y}(1+\tau_y)} = 2.12$ and $\bar{p}_r\,(\bar{x}_c + \bar{x}_m) = 0.061$ as in AB2018. We solve for α using $\frac{(1-\alpha)}{\mu} = \frac{\bar{W}_t\bar{L}_{pt}}{(1+\tau_y)\bar{Y}_t} = 0.654$ (as in AB2018) and for λ_K using

$$\frac{\exp(\Delta\bar{R}) - \exp(-\Delta\delta_K)}{\Delta} = \left(\frac{1-\tau_{corp}}{1-\tau_{corp}\lambda_K}\right)\frac{\alpha}{\mu}\frac{\bar{Y}\,(1+\tau_y)}{\bar{K}}.$$

In some exercises, we set τ_y in order to remove the distortion on the Euler equation for physical capital as given in equation (69).

We calculate unmeasured investment by entrants $\bar{p}_r\bar{x}_e$ as

$$\bar{p}_r\bar{x}_e = \frac{\bar{s}_e}{(1-\bar{\tau}_{rd})}\exp\left(\Delta\left(\bar{g}_Y - \bar{R}\right)\right)\bar{v}$$

, total innovative investment as

$$\bar{i}_r = \bar{p}_r\left(\bar{x}_c + \bar{x}_m + \bar{x}_e\right),$$

and the ratio of production to research labor as

$$\frac{\bar{l}_r}{\bar{l}_p} = \frac{\mu}{1-\alpha}\bar{i}_r.$$

F.1 Firm dynamics at an annual frequency on a BGP

Given a choice of Δ, we calibrate our model to statistics on firm dynamics at an annual frequency. Let $\Delta = 1/N$ for some integer N. Thus, if M_t is the measure of products at the beginning of the year, then M_{t+N} is the measure of products at the beginning of the next year. On a BGP, we assume that all per-period growth rates, entry rates, and exit rates are constant. We now describe the method we use to match parameters governing per-period rates to data at an annual frequency.

Let $\exp(-\bar{\delta}_c)M_t$ denote the measure of products produced at $t+N$ that were produced by the same incumbent firm at t. Let F_c denote the ratio of this measure to M_{t+N} or, equivalently, the fraction of products at $t+N$ that are incumbent products (from period t) in incumbent firms. This fraction satisfies

$$\exp(-\bar{\delta}_c)\exp(-\bar{g}_m) = F_e. \tag{73}$$

Then we can interpret $\exp(-\delta_c)$ as the probability at an annual frequency that any product at t continues in the same firm until $t+N$. The corresponding per period survival rate is $\exp(-\Delta\delta_c)$. We assume that this per period survival rate is constant across all incumbent products.

Let $F_m M_{t+N}$ denote the measure of products produced at $t+N$ that are produced in incumbent firms (firms that existed at t) but that are new to that firm. Then $(F_m + F_c)\,M_{t+N}$ is the measure of products at $t+N$ that are produced in incumbent firms (firms that existed at t). Let $(1-\exp(-\Delta h(x_m)))$ denote the probability each period that incumbent firms add new products per product that they currently produce. Let M^I_{t+j} denote the measure of products at $t+j$ produced in firms that existed at time t. These firms will add $(1-\exp(-\Delta h(x_m))M^I_{t+j}$ products in period $t+j+1$ and lose $(1-\exp(-\Delta\bar{\delta}_c))M^I_{t+j}$ products so that

$$M^I_{t+j+1} = \left[\exp(-\Delta\bar{\delta}_c) + (1-\exp(-\Delta h(x_m))\right] M^I_{t+j}.$$

Note that $M^I_{t+j} = M_t$ for $j=0$ and $M^I_{t+N} = (F_m + F_c)\, M_{t+N}$ or $j=N$. Thus, the total measure of products produced at $t+N$ in firms that existed at t is given by

$$(F_c + F_m) = \left[\exp(-\Delta\bar{\delta}_c) + (1-\exp(-\Delta h(x_m))\right]^N \exp(-\bar{g}_m).$$

Hence, the total measure of products produced at $t+N$ in firms that existed at t that are new to those firms is

$$F_m = \left[\exp(-\Delta\bar{\delta}_c) + (1-\exp(-\Delta h(x_m))\right]^N \exp(-\bar{g}_m) - \exp(-\bar{\delta}_c)\exp(-\bar{g}_m). \tag{74}$$

We have that the fraction of products at $t+N$ that are produced in firms that did not exist at t is then given by $F_e = 1 - F_c - F_m$.

Note that these equations imply that if we define $F_c(\Delta)$ to be the fraction of products at $t+1$ that were produced in the same firm at t and $t+1$, we have

$$F_c(\Delta) = \exp(-\Delta\bar{\delta}_c)\exp(-\Delta\bar{g}_m) = F_c^\Delta \tag{75}$$

and if we define $F_c(\Delta) + F_m(\Delta)$ to be the fraction of products at $t+1$ that are produced in a firm that operated at t and $t+1$, we have

$$F_c(\Delta) + F_m(\Delta) = \left[\exp(-\Delta\bar{\delta}_c) + (1-\exp(-\Delta h(x_m))\right]\exp(-\Delta\bar{g}_m) = (F_c + F_m)^\Delta.$$

Thus, we have

$$F_m(\Delta) = (F_c + F_m)^\Delta - F_c^\Delta = (1-\exp(-\Delta h(x_m))\exp(-\Delta\bar{g}_m). \tag{76}$$

Finally, we have that the fraction of products at $t+1$ produced in firms that did not exist at t is given by

$$F_e(\Delta) = 1 - F_c(\Delta) - F_m(\Delta) = (1-\exp(-\Delta\lambda))\bar{x}_e\exp(-\Delta\bar{g}_m), \tag{77}$$

where this last equation follows from equations (11) and (12).

We proceed in a parallel fashion to convert annual employment shares S_c, S_m, and S_e to per-period shares $S_c(\Delta)$, $S_m(\Delta)$, $S_e(\Delta)$.

Let S_c denote the fraction of employment at $t+N$ in firms producing products that they also produced at t. As shown above, there are $\exp(-\bar{\delta}_c)M_t$ of these products. These products had an average value of $z^{\rho-1}$ of $Z_t^{\rho-1}/M_t$. The average value of $z^{\rho-1}$ for these products grew at rate $\exp(\Delta\zeta(\bar{x}_c))$ for N periods, so these products end up with an average value of $z^{\rho-1}$ equal to $\exp(\zeta(\bar{x}_c))Z_t^{\rho-1}/M_t$. Thus, we have

$$S_c(\Delta) = \exp(-\Delta\bar{\delta}_c)\exp(\Delta\zeta(\bar{x}_c))\exp(-\Delta(\rho-1)\bar{g}_z) = S_c^\Delta. \tag{78}$$

Let $S_c + S_m$ denote the fraction of employment at $t + N$ in firms that were also active at t. By definition, S_m is the share of employment in these firms at $t + N$ producing products that they did not produce at t. Parallel arguments to those above give

$$S_c(\Delta) + S_m(\Delta) = \left[\exp(-\Delta\bar{\delta}_c)\exp(\Delta\zeta(\bar{x}_c)) + \eta_m(1 - \exp(-\Delta h(x_m))\right]\exp(-\Delta(\rho - 1)\bar{g}_z) = (S_c + S_m)^{\Delta}.$$

Thus, we have

$$S_m(\Delta) = (S_c + S_m)^{\Delta} - S_c^{\Delta} = \eta_m(1 - \exp(-\Delta h(x_m))\exp(-\Delta(\rho - 1)\bar{g}_z). \tag{79}$$

Finally, we have

$$S_e(\Delta) = 1 - S_c(\Delta) - S_m(\Delta) = \eta_e(1 - \exp(-\Delta\lambda))\bar{x}_e\exp(-\Delta(\rho - 1)\bar{g}_z), \tag{80}$$

where this last equation follows from equation (14).

G Quantitative results

Here we report the responses of aggregate productivity and output, welfare, the allocation of labor in research, the share of production and incumbents' research labor compensation in output (i.e., $(1 - \alpha)/\mu + p_r(x_c + x_m)$), the share in employment of entrants (i.e., S_e), the valuation of firms as measured by Tobin's q defined in equation (72), and the elasticities (Θ_G, Θ_H, Θ_J, and $A\Theta_G$) for the experiments that we consider in Sections 5 and 6 of the paper.

G.1 Uniform increase in innovation subsidies

We consider a uniform increase in innovation subsidies to implement an increase in the innovation intensity of the economy from 0.090 on the initial BGP to 0.102 on the new BGP. The corresponding allocation of labor to research rises from $\bar{l}_r/\bar{l}_p = 0.139$ on the initial BGP to a value of 0.155 on the new BGP. We show results from this experiment in Tables 1 - 8. The responses of aggregate productivity at 20 and 100 years along the equilibrium transition path shown in Table 2 are similar to those shown in our experiment in Table 1 in the main paper. The responses of aggregate output shown in Table 3 are smaller than the corresponding responses of aggregate productivity because the amount of labor allocated to current production is permanently reduced. As shown in Table 4, this innovation policy experiment has very little impact on the valuation of firms as measured by Tobin's q, except for the case of the first generation endogenous growth research technology. In this case, the valuation of firms drops considerably. This is because the share of employment in entering firms rises for a very long time along the transition in the case with the first generation endogenous growth research technology, as shown in Table 6. This persistent increase in entry leads to a decline in the ratio of product value V_t to dividends D_t along the transition. As shown in Table 5, the share of labor compensation (including both production and research labor by incumbents) in output rises slightly due to the increase in the incumbents' innovation intensity of the economy.

As shown in Table 1, this uniform increase in innovation subsidies leads to a significant increase in welfare in the specification of the model with the first generation endogenous growth research technology, a more moderate increase in welfare with the J/K/S research technology, and a decline in welfare with the second generation endogenous growth research technology.

Table 1: Welfare gains, uniform subsidy experiment

(ϕ, ψ)	Long-run $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	1.230	1.029	0.999
With business stealing	1.076	1.019	0.992

Table 2: Aggregate productivity, uniform subsidy experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.056	0.036	0.011	0.254	0.037	0.011
With business stealing	0.021	0.021	0.005	0.099	0.037	0.005

Note: The values in each column are in log deviations relative to the initial trend.

Table 3: Aggregate output, uniform subsidy experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.050	0.031	0.000	0.313	0.035	0.000
With business stealing	0.010	0.007	-0.008	0.114	0.034	-0.007

The values in each column are in log deviations relative to the initial trend.

Table 4: Tobin's q, uniform subsidy experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	1.014	1.160	1.151	1.034	1.149	1.149
With business stealing	1.059	1.139	1.150	1.065	1.151	1.149

Note: The values in each column are in levels. The initial BGP level is equal to 1.15.

Table 5: Labor share, uniform subsidy experiment

	20 years			100 years		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.717	0.723	0.724	0.718	0.724	0.724
With business stealing	0.717	0.721	0.724	0.717	0.724	0.724

Note: The values in each column are in levels. The initial BGP level is 0.716.

Table 6: Employment share of entrants, uniform subsidy experiment

	20 years			100 years		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.036	0.027	0.027	0.035	0.027	0.027
With business stealing	0.036	0.032	0.027	0.035	0.027	0.027

Note: The calculation of the employment share of entrants corresponds to the annual share at the beginning of the 20th year after the policy shock. The values in each column are in levels. The initial BGP level is 0.027.

Table 7: Allocation of labor L_r/L_p, uniform subsidy experiment

	20 years			100 years		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.154	0.155	0.155	0.154	0.155	0.155
With business stealing	0.154	0.158	0.155	0.154	0.155	0.155

Note: The values in each column are in levels. The initial BGP level is 0.139

Table 8: Elasticities, uniform subsidy experiment

	Θ_G			Θ_H		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.009	0.009	0.009	0.08	0.08	0.08
With business stealing	0.003	0.003	0.003	0.064	0.064	0.064
	Θ_J			Impact elasticity		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.000	0.024	0.081	0.028	0.028	0.028
With business stealing	0.000	0.009	0.064	0.011	0.011	0.011

G.2 Increase in markup μ

We calibrate the increase in the markup μ so that the change in the allocation of labor to research from the initial to the new BGP is the same as that we considered with the uniform innovation subsidies. In this experiment, we set to zero the production subsidy, τ_y. We report the results in Tables 9-16.

A permanent increase in the markup μ has qualitatively the same effects as a uniform increase in innovation subsidies. Again, if we hold all other parameters and policies fixed, we find that the new BGP has the same allocation of innovative investment $\bar{x}_c$, $\bar{x}_m$, and $\bar{x}_e$ and hence the same growth rates of aggregate productivity and the measure of products. As was the case with a uniform change in innovation subsidies, however, on the new BGP, the economy has a higher innovation intensity of the economy measured as the ratio of expenditures on innovative investment relative to output as a result of the increase in markups. In particular, this change moves the innovation intensity of the economy from 0.090 on the initial BGP to 0.100 on the new BGP.

By comparing Table 7 and Table 15, we can see that the change in the allocation of labor to research at the 20-year and 100-year horizons is also very similar across experiments. From Table 2 and Table 10, these similar perturbations to the allocation of labor to research produce similar responses of the level of aggregate productivity relative to trend at horizons of 20 and 100 years. However, because markups are higher in this case, as indicated in Tables 3 and 11, the response of aggregate output at the 20- and 100-year horizons is smaller. The increase in markups discourages the accumulation of physical capital. In Table 12, we see that Tobin's q is higher in this experiment than what we found with uniform innovation subsidies. The increase in markups has a very similar impact on the share of labor compensation in output as a uniform increase in innovation subsidies. The impact of this experiment on firm dynamics as measured by the share of employment in entrants is also very similar to that found with a uniform change in innovation subsidies. The impact of this change in markups on welfare is only very slightly smaller than that found with uniform innovation subsidies.

Table 9: Welfare gains, markup experiment

	Long-run		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	1.230	1.028	0.998
With business stealing	1.075	1.018	0.991

Table 10: Aggregate productivity, markup experiment

	20 years			100 years		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.057	0.036	0.011	0.254	0.037	0.011
With business stealing	0.021	0.021	0.005	0.100	0.037	0.005

Note: The values in each column are in log deviations relative to the initial trend.

Table 11: Aggregate output, markup experiment

(ϕ, ψ)	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	0.046	0.026	-0.005	0.308	0.03	-0.005
With business stealing	0.005	0.003	-0.013	0.109	0.029	-0.013

Note: The values in each column are in log deviations relative to the initial trend.

Table 12: Tobin's q, markup experiment

(ϕ, ψ)	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	1.070	1.218	1.209	1.091	1.209	1.209
With business stealing	1.113	1.196	1.209	1.119	1.21	1.209

Note: The values in each column are in levels. The initial BGP level is is 1.15.

Table 13: Labor share, markup experiment

(ϕ, ψ)	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	0.719	0.726	0.726	0.72	0.726	0.726
With business stealing	0.719	0.723	0.726	0.72	0.726	0.726

Note: The values in each column are in levels. The initial BGP level is 0.716.

Table 14: Employment share of entrants, markup experiment

(ϕ, ψ)	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	0.036	0.027	0.027	0.035	0.027	0.027
With business stealing	0.036	0.032	0.027	0.035	0.027	0.027

Note: The calculation of the employment share of entrants corresponds to the annual share at the beginning of the 20th year after the policy shock. The values in each column are in levels. The initial BGP level is 0.027.

Table 15: Allocation of labor L_r/L_p, markup experiment

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	0.154	0.155	0.155	0.154	0.155	0.155
With business stealing	0.154	0.158	0.155	0.154	0.155	0.155

Note: The values in each column are in levels. The initial BGP level is 0.139

Table 16: Elasticities, markup experiment

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	Θ_G $(0.96, 1)$	Θ_G $(-1.67, 1)$	Θ_G $(0.96, 0)$	Θ_H $(0.96, 1)$	Θ_H $(-1.67, 1)$	Θ_H $(0.96, 0)$
No business stealing	0.009	0.009	0.009	0.08	0.08	0.08
With business stealing	0.003	0.003	0.003	0.064	0.064	0.064

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	Θ_J $(0.96, 1)$	Θ_J $(-1.67, 1)$	Θ_J $(0.96, 0)$	Impact elasticity $(0.96, 1)$	Impact elasticity $(-1.67, 1)$	Impact elasticity $(0.96, 0)$
No business stealing	0	0.024	0.081	0.028	0.028	0.028
With business stealing	0	0.009	0.064	0.011	0.011	0.011

G.3 Reduction in corporate profits tax rate

We follow Barro and Furman (2018) (henceforth BF2018) in changing the corporate profit tax rate from $\tau_{corp} = 0.38$ to $\tau'_{corp} = 0.26$. We choose the new value $\lambda'_k = 1.05$ to match the change in the user cost of capital across BGPs implied by the BF2018 calibration (so that the increase in the log of the output-capital ratio across BGPs is equal to 0.08). We set to zero the production subsidy, τ_y. In addition to the values for the three specifications for the research technology, we show results in a specification of the model in which innovative investments by entering and incumbent firms are fixed exogenously at their initial BGP levels, so that aggregate productivity grows exogenously in BF2018. We also consider a specification of our model in which innovation by incumbent firms is fixed at the initial BGP levels, and only innovation by entrants responds to the policy change. We report the results in Tables 17-25.

Consider first the results from our version of a standard model with exogenous productivity. By construction, the response of aggregate productivity reported in the first row of Table 17 is zero at all horizons. In the first row of Table 18, we find that output per worker rises by 2.4% in a horizon of 20 years relative to the path for output per worker that would have occurred without the policy change and 2.5% in a horizon of 100 years. So the response of aggregate output is relatively modest and the convergence to the new BGP is fast. The welfare impact of the change in corporate profits taxes in this version of our model is reported in the first row of Table 20. Here, the consumption equivalent change in welfare is 0.4%.

In the specification of the model where firms change their innovation policies after the reduction in the corporate profits' tax, the allocation of labor to research rises from $\bar{l}_r/\bar{l}_p = 0.139$ on the initial BGP to a value of approximately 0.144 across all specifications. In addition, innovation investments by incumbents fall in the new BGP. In the first two specifications of the research technology, whether or not reallocation of investment toward entry is desirable depends on the extent of business stealing and the extent to which the initial corporate profits tax policies had favored investment by incumbent firms over investment by entering firms on the initial BGP. The change in aggregate productivity over a 20- and 100-year horizon can be either positive (without business stealing) or negative (with business stealing). In the 20-year horizon, the change in aggregate productivity relative to trend is between $+3.5\%$ and 5.3% without business stealing and roughly -2% with business stealing. The change in output per worker over a 20-year horizon is between $+6.3\%$ and 7.7% without business stealing and roughly -0.5% with business stealing. The implications for productivity and output over a 100-year horizon and for long-term welfare are very large in absolute terms (positive without business stealing and negative without business stealing), especially under the first research technology.

For the third specification of the research technology (second generation endogenous growth), the reallocation towards innovation by entrants implies a reduction in the BGP growth rate of productivity, with very negative implications for aggregate productivity, output and welfare (with or without business stealing).

Another way of understanding the role of reallocation of innovation from incumbents to entrants is to compare the implications of the baseline model with the alternative specification in which innovative investments by incumbents are exogenously fixed at their initial BGP level (see Tables 26-25). We can see in Table 26 that aggregate productivity rises in all cases, in contrast to the baseline model in which investments by incumbents respond to the policy change. The impact elasticity on aggregate productivity in this case is identical to that in our uniform subsidy experiment. The change in aggregate productivity is slightly smaller than under the uniform innovation subsidy experiment reported in Table 2 due to a smaller extent of labor reallocation from production to research.

As shown in Table 19, this experiment implies a large increase in the valuation of firms (as measured by Tobin's q) in all cases except for the first generation endogenous growth research technology. The increase in valuation occurs because the market share of incumbents is relatively stable so the value of the firm scales almost one to one with the after-tax value of profits, which rises with the reduction in the corporate profits tax. In contrast, in the first generation endogenous growth research technology, the valuation of firms drops considerably because the share of employment in entering firms rises for a very long time.

As shown in Table 21, despite the overall increase in the research intensity induced by the change in corporate profit taxation, the share of labor compensation (including both production and incumbents' research labor) in output drops slightly. This is because the research share is reallocated to entrants, and the wage bill of entering firms is not measured in aggregate output.

Table 17: Aggregate productivity, corporate profits tax experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0	0	0	0	0	0
No business stealing	0.053	0.035	-0.042	0.265	0.038	-0.315
With business stealing	-0.023	-0.02	-0.056	-0.103	-0.03	-0.33

Note: The values in each column are in log deviations relative to the initial trend. In the second generation endogenous growth models, the growth rate of TFP gZ falls from 0.0136 to 0.0102 and that of products M increases from 0.0100 to 0.0101.

Table 18: Aggregate output, corporate profits tax experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.024	0.024	0.024	0.025	0.025	0.025
No business stealing	0.077	0.063	-0.022	0.36	0.071	-0.382
With business stealing	-0.007	-0.004	-0.04	-0.112	-0.02	-0.401

Note: The values in each column are in log deviations relative to the initial trend. In the second generation endogenous growth models, the growth rate $g_{Y/L}$ falls from 0.0181 to 0.0136.

Table 19: Tobin's q, corporate profits tax experiment

	20 years			100 years		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	1.281	1.281	1.281	1.273	1.273	1.273
No business stealing	1.086	1.256	1.299	1.106	1.235	1.278
With business stealing	1.248	1.156	1.297	1.235	1.15	1.278

Note: The values in each column are in levels. The initial BGP level is 1.15.

Table 20: Welfare gains, corporate profits tax experiment

	Long-run		
(ϕ, ψ)	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	1.004	1.004	1.004
No business stealing	1.257	1.043	0.764
With business stealing	0.913	0.97	0.751

Table 21: Labor share, corporate profits tax experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.716	0.716	0.716	0.716	0.716	0.716
No business stealing	0.698	0.702	0.709	0.698	0.702	0.709
With business stealing	0.698	0.696	0.709	0.698	0.695	0.709

Note: The values in each column are in levels. The initial BGP level is 0.716.

Table 22: Employment share of entrants, corporate profits tax experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.027	0.027	0.027	0.027	0.027	0.027
No business stealing	0.047	0.039	0.028	0.046	0.038	0.028
With business stealing	0.047	0.051	0.028	0.047	0.054	0.028

Note: The calculation of the employment share of entrants corresponds to the annual share at the beginning of the 20th year after the policy shock. The initial BGP level is 0.027.

Table 23: Allocation of labor L_r/L_p, corporate profits tax experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.139	0.139	0.139	0.139	0.139	0.139
No business stealing	0.144	0.144	0.142	0.144	0.144	0.143
With business stealing	0.144	0.143	0.143	0.144	0.145	0.143

Note: The values in each column are in levels. The initial BGP level is 0.139.

Table 24: Elasticities, corporate profits tax experiment

(ϕ, ψ)	Θ_G $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	Θ_H $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.013	0.013	0.01	0.115	0.115	0.085
With business stealing	0.007	0.007	0.004	0.129	0.129	0.067

(ϕ, ψ)	Θ_J $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	Impact elasticity $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.001	0.035	0.085	0.026	0.026	0.023
With business stealing	0.000	0.019	0.067	0.012	0.012	0.009

Table 25: Welfare gains, corporate profits tax experiment with fixed innovation decisions by incumbents

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	Long-run $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	1.004	1.004	1.004
No business stealing	1.193	1.027	1.004
With business stealing	1.068	1.020	0.999

Table 26: Aggregate productivity, corporate profits tax experiment with fixed innovation decisions by incumbents

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0	0	0	0	0	0
No business stealing	0.042	0.028	0.009	0.214	0.030	0.009
With business stealing	0.016	0.016	0.004	0.084	0.030	0.004

Note: The values in each column are in log deviations relative to the initial trend. In the second generation endogenous growth models, the growth rates g_Z and g_M are unchanged between BGPs.

Table 27: Aggregate output, corporate profits tax experiment with fixed innovation decisions by incumbents

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.024	0.024	0.024	0.025	0.025	0.025
No business stealing	0.059	0.047	0.024	0.289	0.054	0.026
With business stealing	0.016	0.016	0.004	0.084	0.030	0.004

Note: The values in each column are in log deviations relative to the initial trend. In the second generation endogenous growth models, the growth rate g_Y is unchanged between BGPs.

Table 28: Allocation of labor L_r/L_p, corporate profits tax experiment with fixed innovation decisions by incumbents

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
Exogenous Growth	0.139	0.139	0.139	0.139	0.139	0.139
No business stealing	0.152	0.152	0.152	0.152	0.152	0.152
With business stealing	0.152	0.154	0.152	0.152	0.152	0.152

Note: The values in each column are in levels. The initial BGP level is 0.139.

G.4 Population growth decline

We consider a gradual and permanent reduction in the population growth rate g_{Lt}. In particular, the growth rate of the population falls by half a percentage point (broadly consistent with the projections by the Congressional Budget Office summarized in Shackleton et al. (2018)) from the old BGP rate of 0.7% to 0.2% with an AR1 coefficient of 0.93 on an annual basis. This implies that the half-life of the transition of the population is roughly 10 years. We report the results in Tables 29-36.

Changes in the BGP growth rates, reported in Table 29, are calculated using the two sufficient statistics discussed in the main paper. Given the large response of the growth rate in the first generation endogenous growth research technology, we do not present results in this case.

The reallocation of labor away from research implies that aggregate productivity falls relative to the initial BGP trend. As shown in Table 30, the magnitude of the reduction in the first 20 years is quite modest. Output per capita falls by less than productivity (or rises slightly) due to the reallocation of labor toward production. As shown in Tables 32 and 33, the decline in the population growth rates has a very small impact on the valuation of firms as measured by Tobin's q. The share of labor compensation in output over a 20-year horizon falls slightly due to the increase in the innovation intensity of the economy.

Table 29: BGP growth rates, population growth experiment

	Z		M		Y/L	
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(-1.67, 1)$	$(0.96, 0)$	$(-1.67, 1)$	$(0.96, 0)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	0.012	0.013	-0.007	0.005	0.016	0.017
With business stealing	0.012	0.013	-0.024	0.005	0.016	0.018

Note: The values in the table can be computed analytically and are highly sensitive to the specification of the research technologies (see Section 4 in the main paper). The BGP rates g_Z, g_M, and $g_{Y/L}$ are 0.0136, 0.01, and 0.0181 initially.

Table 30: Aggregate Productivity, population growth experiment

	20 years			100 years		
$(\boldsymbol{\phi}, \boldsymbol{\psi})$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	$(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	–	-0.025	-0.013	–	-0.174	-0.058
With business stealing	–	-0.009	-0.006	–	-0.130	-0.028

Note: The values in the table are log deviations relative to the initial trend, corresponding to $g_L = 0.0067$.

Table 31: Output per capita

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	–	-0.001	0.012	–	-0.196	-0.047
With business stealing	–	0.013	0.020	–	-0.138	-0.009

Note: The values in the table are in levels. The BGP value, corresponding to $g_L = 0.0067$, is 0.7162.

Table 32: Tobin's q, population growth experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	–	1.148	1.127	–	1.140	1.125
With business stealing	–	1.179	1.131	–	1.206	1.130

Note: The values in each column are in levels. The BGP value, corresponding to a growth rate in population of 0.0067., is 1.15.

Table 33: Labor share, population growth experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	–	0.717	0.714	–	0.717	0.714
With business stealing	–	0.720	0.714	–	0.725	0.714

Note: The values in each column are in levels. The BGP value, corresponding to a growth rate in population of 0.0067, is 0.7162.

Table 34: Employment share of entrants, population growth experiment

(ϕ, ψ)	20 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$	100 years $(0.96, 1)$	$(-1.67, 1)$	$(0.96, 0)$
No business stealing	–	0.022	0.025	–	0.021	0.025
With business stealing	–	0.024	0.025	–	0.013	0.025

Note: The calculation of the employment share of entrants corresponds to the annual share at the beginning of the 20th year after the policy shock. The values in each column are in levels. The BGP value, corresponding to a growth rate in population of 0.0067, is 0.027.

Table 35: Allocation of labor L_r/L_p, population growth experiment

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	20 years $(0.96, 1)$	20 years $(-1.67, 1)$	20 years $(0.96, 0)$	100 years $(0.96, 1)$	100 years $(-1.67, 1)$	100 years $(0.96, 0)$
No business stealing	–	0.130	0.130	–	0.130	0.130
With business stealing	–	0.135	0.130	–	0.131	0.130

Note: The initial BGP value is 0.0139.

Table 36: Elasticities, population growth experiment

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	Θ_G $(0.96, 1)$	Θ_G $(-1.67, 1)$	Θ_G $(0.96, 0)$	Θ_H $(0.96, 1)$	Θ_H $(-1.67, 1)$	Θ_H $(0.96, 0)$
No business stealing	–	0.007	0.009	–	0.064	0.076
With business stealing	–	0.002	0.003	–	0.030	0.059

$(\boldsymbol{\phi}, \boldsymbol{\psi})$	Θ_J $(0.96, 1)$	Θ_J $(-1.67, 1)$	Θ_J $(0.96, 0)$	Impact elasticity $(0.96, 1)$	Impact elasticity $(-1.67, 1)$	Impact elasticity $(0.96, 0)$
No business stealing	–	0.019	0.076	–	0.026	0.028
With business stealing	–	0.004	0.059	–	0.009	0.011

H Declining Entry Rates and Aggregate Productivity

The model predicts a specific relationship between the dynamics of aggregate productivity and business formation, depending on the nature of the shock driving innovation decisions. For this reason, the model can be used to study the implications of the recent decline in firm entry on aggregate productivity. In this section, we consider three alternative counterfactual changes in policies or the economic environment that might account for the observed decline in entry. In our first counterfactual exercise, we assume that the decline in entry is driven by a decrease in innovation subsidies that is uniform in the sense that $(1-\tau_c)/(1-\tau_e)$ and $(1-\tau_m)/(1-\tau_e)$ are unchanged across the initial and new BGPs. In our second counterfactual exercise, we assume that the decline in entry is driven by a decrease in innovation subsidies for entry, with subsidies for innovative investment by incumbents left unchanged. This counterfactual exercise is similar to a change in the corporate income tax, where entrants can not expense at the same rate as incumbents their R&D investments. In our third counterfactual exercise, we assume that the decline in entry is driven by a decline in the BGP growth rate of population, $\bar{g}_L'$, chosen in each specification so that the entry rate on the new BGP is $\bar{S}_e' = 1.64\%$. In Tables 37 and 38, we report the inputs Θ_G', $\log \bar{S}_e' - \log S_0$, and $\bar{g}_Z' - \bar{g}_Z$ needed to implement the formula in equation (31) in the main paper for the model-implied cumulative change in aggregate productivity relative to initial trend over the first 20 years of transition, $\log Z_{20} - \log Z_0 - 20\bar{g}_Z$.

Experiment	Research Tech.	Θ_G'	$\log \bar{S}_e'/S_{e0}$	$\bar{g}'_Z - \bar{g}_Z$	$\log Z_{20}/Z_0 - 20\bar{g}_Z$
Uniform Innovation Tax	First Gen. EG	0.009	0	0	-0.0466
	J/K/S	0.009	0	0	-0.0466
	Second Gen. EG	0.009	0	0	-0.0466
Entry Tax	First Gen. EG	0.0055	-0.5	0	0.0280
	J/K/S	0.0055	-0.5	0	0.0280
	Second Gen. EG	0.0055	-0.5	0.0257	0.5428
Decline in Pop.Growth	First Gen. EG	-	-	-	-
	J/K/S	0.0055	-0.5	-0.0036	-0.0436
	Second Gen. EG	0.0055	-0.5	-0.0036	-0.0436

Table 37: Reduction in firm entry and aggregate productivity: without business stealing

We draw three lessons from the results of this experiment. First, if the decline in the employment share of new firms is the result of a decline in innovative investment by entrants with no change in innovative investment by incumbents, as in our first experiment with a uniform tax on innovative investment, then the predicted decline in aggregate productivity over a 20 year horizon is relatively small (less than 2% with business stealing and less than 5% without business stealing).

Second, if the decline in the employment share of new firms is the result of a reallocation of innovative investment, decreasing investment by entrants and raising innovative investment by incumbents, as in our second experiment with a tax on innovative investment by entrants, then the model predicts an increase in aggregate productivity over a 20 year horizon. This predicted increase is small if the long run growth rate is unchanged as with the J/K/S research technology. This predicted increase can be extremely large if the long run growth rate changes as with the Second Generation Endogenous Growth technology for research. The intuition for this result is as follows. At the new levels of innovative investment for incumbents and entrants induced by the

Experiment	Research Tech.	Θ_G'	$\log \bar{S}_e'/S_{e0}$	$\bar{g}'_Z - \bar{g}_Z$	$\log Z_{20}/Z_0 - 20\bar{g}_Z$
Uniform Innovation Tax	First Gen. EG	0.003	0	0	-0.0176
	J/K/S	0.003	0	0	-0.0176
	Second Gen. EG	0.003	0	0	-0.0176
Entry Tax	First Gen. EG	0.0021	-0.5	0	0.0106
	J/K/S	0.0021	-0.5	0	0.0106
	Second Gen. EG	0.0021	-0.5	0.0256	0.5316
Decline in Pop.Growth	First Gen. EG	-	-	-	-
	J/K/S	0.0021	-0.5	-0.0014	-0.0168Z
	Second Gen. EG	0.0021	-0.5	-0.0014	-0.0168

Table 38: Reduction in firm entry and aggregate productivity: with business stealing

entry tax, the economy can sustain the same or even higher BGP growth rate of aggregate productivity with a much smaller share of employment in entering firms. In our second experiment, the employment share of entrants in the 20 years of the transition is higher than the new BGP share of employment. Thus, since our model predicts that innovative investment by incumbents rises immediately to its new BGP level with the imposition of the entry tax, it also predicts that productivity should grow faster than its BGP growth rate as long as observed entry in the transition is above the new BGP level of entry.

Third, if the decline in the employment share of new firms is the result of a decline in the population growth rate, then the model's prediction for the response of aggregate productivity over 20 years is the sum of two separate effects, one negative and one positive. For the two technologies for research that we consider, a reduction in the population growth rate reduces the BGP growth rate of productivity. If the economy adjusts to its new BGP growth rate immediately (by having the entry rate drop immediately to the new BGP employment share in entering firms), then the decline in aggregate productivity at 20 years should be 20 times the reduction in the BGP growth rate of productivity. But, to the extent that the employment share of entrants is above its BGP level during the first 20 years of the transition, then, as in our entry tax experiment, the model predicts an offsetting positive impact of entry on productivity during the transition to the new BGP.

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