Welfare and Output with Income Effects and Taste Shocks

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[1] The conclusions and analysis are our own, calculated in part on data from Nielsen Consumer LLC and provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

Motivation

- How does welfare respond to changes in choice sets?

  "How much endowment at t_0 to make consumer indifferent between choice set in t_0 and t_1?"

- If preferences stable and homothetic: can express welfare as chained (or Divisia) index.

$$\Delta \text{Welfare} = \log \frac{spending_{t_1}}{spending_{t_0}} - \int_{t_0}^{t_1} \sum_i \frac{p_{i,t} c_{i,t}}{spend_t} \, d \log p_{i,t}$$

- Foundation for aggregation procedures to calculate aggregate quantities and prices.

- What if demand is unstable and/or non-homothetic.

What We Do

- Consider preferences that are unstable and/or non-homothetic.
- Characterize welfare change and chained objects in PE and GE in terms of suff. stats.
  - Chaining treats all expenditure-switching equally (due to prices, income, taste shocks), but welfare does not.
  - PE/GE distinction matters when pref. unstable or non-homoth.
- Quantitative applications for long-run growth and fluctuations.
  1. Welfare-relevant Baumol's cost disease.
  2. Firm-level shocks and industry-level outcomes.
  3. PE vs. GE during Covid-19 recession
     Path-dependence/Index-drift of RGDP and TFP.

Selected Literature

- Biases of real consumption/GDP:
  Fisher & Shell (1968), Hausman (1981), Feenstra (1994), Basu et al. (2012), Aghion et al. (2019), Syverson (2017), Jones & Klenow (2016).

- Index numbers with taste shocks or non-homotheticity:
  Caves, Christensen, Diewert (1982), Deaton & Muellbauer (1980), Feenstra & Reinsdorf (2007), Redding & Weinstein (2020).

- Growth accounting and disaggregated macro:
  Solow (1951), Domar (1961), Hulten (1978), Long & Plosser (1983), Gabaix (2011), Acemoglu et al. (2012), Baqaee & Farhi (2019).

- Structural Transformation
  Baumol (1961), Kongsamut, Rebelo, Xie (2001), Buera & Kaboski (2009), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2020), Alder et al. (2019).

Agenda

Microeconomic Problem

Macroeconomic Problem

Applications

Extensions

Conclusion

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Macroeconomic Problem

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Conclusion

Set up

- Preference relations $\{\succeq_x\}$ over vector of consumption goods c.
  (x is e.g. age, fads, advertising, state of nature, no choices/preferences over x)
- Represent preferences by $u(c; x)$ with indirect utility $v(p, \mathrm{I}; x)$.
- Consider change from $(p_{t_0}, \mathrm{I}_{t_0})$ to $(p_{t_1}, \mathrm{I}_{t_1})$.
- Consider how welfare changes.
- Consider how (chained) real consumption changes.
- Compare the difference.

Micro Welfare and Real Consumption

Definition (Real consumption)

Change in real consumption is

$$\Delta \log Y = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i \frac{p_{i,t} c_{i,t}}{\mathrm{I}_t} d \log p_{i,t}.$$

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Definition (Welfare)

Income needed under p_{t_0} to make $\succeq_{x_{t_1}}$ indifferent to $(p_{t_1}, \mathrm{I}_{t_1})$.

$$v(p_{t_0}, e^{\mathbf{EV^m}} I_{t_0}; x_{t_1}) = v(p_{t_1}, I_{t_1}; x_{t_1})$$

Change in real consumption versus welfare

Define $b(p, u, x)$ to be budget share given p, u, and x.

Change in real consumption versus welfare

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Lemma

$$\Delta \log Y = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p_t, u_t, x_t) d \log p_{i,t}.$$

$$EV^m = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p_t, u_{t_1}, x_{t_1}) d \log p_{i,t}.$$

where $b(p, u, x)$ denotes budget share at (p, u, x)

If preferences stable and homothetic, $b_i(p_t, u_t, x_t) = b_i(p_t, u_{t_1}, x_{t_1})$.

For welfare, substitution effect $\neq$ income effect & taste shocks.

derivation

Key Intuition for Consumption vs. Welfare

$$\Delta \log Y = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p_t, u_t, x_t) d \log p_{i,t}.$$

$$EV^m = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p_t, u_{t_1}, x_{t_1}) d \log p_{i,t}.$$

- Consider change in welfare comparing 1950 to 2014.
- Spend more on healthcare in 2014 due to aging and income.
- Chained index uses 1950 demand to weight prices in 1950.
- Welfare-relevant uses 2014 demand to weight prices in 1950.
- Welfare uses x_{t_1} since households are older and u_{t_1} since we are giving them income in 1950 to make them indifferent to 2014.

Implementation

- Consider e.g non-homothetic CES with taste shocks where

$$e(p_t, u_t, x_t) = \left(\sum_i \omega_i x_{it} p_{it}^{1-\theta} u_t^{\xi_i}\right)^{\frac{1}{1-\theta}} u_t = \tilde{P}_t u_t.$$

$(\Delta u, \Delta \tilde{P})$ not good (quantity, price) measure if non-homo./unstable.

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$(\Delta u, \Delta \tilde{P})$ not good (quantity, price) measure if non-homo./unstable.

- To measure welfare, integrate $b(p, u_{t_1}, x_{t_1}) d \log p$:

$$EV^m = \Delta \log I - \log \left(\sum_i b_{it_1} \left(\frac{p_{it_1}}{p_{it_0}} \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

- If we know elasticity of substitution, we don't need to know income elasticities or taste shocks (or even separate them).

Second-order Approx.

Real consumption is

$$\Delta \log Y = \Delta \log I_t - \int_{t_0}^{t_1} \sum_{i \in N} b_{it} \frac{d \log p_{it}}{dt} dt.$$

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Consider Taylor expansion in t_1 around t_0:

$$\Delta \log Y \approx \underbrace{\Delta \log I - \mathbb{E}_b(\Delta \log p)}_{\text{first-order}}$$

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Consider Taylor expansion in t_1 around t_0:

$$\Delta \log Y \approx \underbrace{\Delta \log I - \mathbb{E}_b(\Delta \log p)}_{\text{first-order}} - \underbrace{\frac{1}{2} Cov_b(\Delta \log b, \Delta \log p)}_{\text{second-order}},$$

If spending rises for goods that got more expensive, then $\Delta \log Y$ falls.

Consumption vs. Welfare: Second-order Approx.

Consider Taylor expansion in t_1 around t_0:

$$\Delta \log Y \approx \underbrace{\Delta \log I - \mathbb{E}_b[\Delta \log p]}_{\text{First-order}}$$

,

$$EV^m \approx \Delta \log I - \mathbb{E}_b[\Delta \log p]$$

.

To a first-order, welfare and real consumption are the same.

Consumption vs. Welfare: Second-order Approx.

Consider Taylor expansion in t_1 around t_0:

$$\Delta \log Y \approx \underbrace{\Delta \log I - \mathbb{E}_b\left[\Delta \log p\right]}_{\text{First-order}} - \underbrace{\frac{1}{2}(1-\theta_0)\, Var_b\left(\Delta \log p\right)}_{\substack{\text{expenditure-switching} \\ \text{due to substitution effect}}}$$

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$$EV^m \approx \Delta \log I - \mathbb{E}_b[\Delta \log p] - \frac{1}{2}(1-\theta_0)\, Var_b(\Delta \log p)$$
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No bias if covariances are zero. quality change

Agenda

Set up

- Perfectly competitive neoclassical economy, representative agent
- F primary factors, N goods

$$y_i = A_i G_i \left(\{m_{ij}\}_{j \in N}, \{l_{ij}\}_{j \in F} \right)$$

- Macro indirect utility of representative agent is

$$V(A, L; x) = \max\{u(c; x) : c \text{ is feasible}\}.$$

- Consider changes in technologies from (A_{t_0}, L_{t_0}) to (A_{t_1}, L_{t_1}) and preferences from x_{t_0} to x_{t_1}.
- These imply some (p_{t_0}, I_{t_0}) and (p_{t_1}, I_{t_1}).

Micro Welfare Poor Measure of Technology Change

- Suppose households age from t_0 to t_1 but technology unchanged.

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  A society of old people indifferent between tech in t_0 and t_1.
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- But technology has not changed:
  A society of old people indifferent between tech in t_0 and t_1.
- Macro welfare takes into account that prices respond to demand.
- Similar for cross-country comparisons.

Macro Welfare

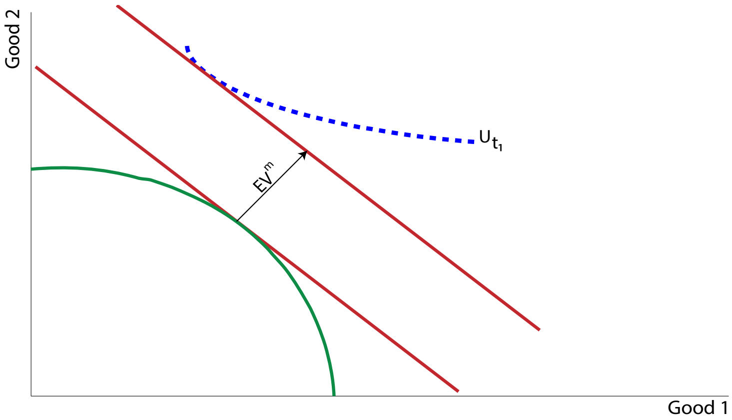
To quantify welfare effect of changes in technologies, we ask:

Factors needed in t_0 to make $\succeq_{x_{t_1}}$ indifferent to t_1 technologies.

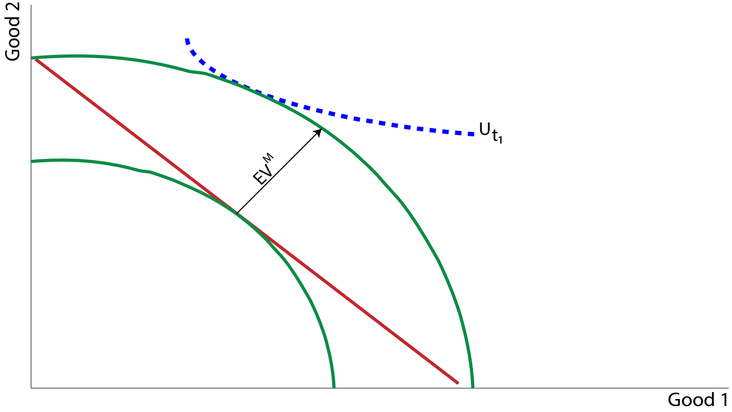
$$V(A_{t_0}, e^{\mathbf{EV^M}} L_{t_0}; x_{t_1}) = V(A_{t_1}, L_{t_1}; x_{t_1}).$$

- When preferences are stable + homothetic, or PPF is linear:
  - macro welfare is the same as micro welfare.
  - macro welfare is the same as “consumption equivalents”.

Graphical Representation – Micro EV^m



Graphical Representation – Macro EV^M



Real GDP and Welfare

- Let $\lambda_i(A, u, x)$ be sales shares, $\frac{p_i y_i}{GDP}$, with demand $b(p, u, x)$.

Proposition

Change in real GDP and welfare in response to $(\Delta x, \Delta A)$ *is*

$$\Delta \log Y = \int_{t_0}^{t_1} \sum_i \lambda_i(A_t, u_t, x_t) d \log A_{i,t},$$

$$EV^M = \int_{t_0}^{t_1} \sum_i \lambda_i(A_t, u_{t_1}, x_{t_1}) d \log A_{i,t}.$$

Notation

- Let
$$\Omega_{ij} = \frac{p_j m_{ij}}{p_i y_i},$$
be input-output matrix.

- Let
$$\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \ldots$$
be Leontief inverse.

- Denote
$$\lambda^{ev}(A) \equiv \lambda(A, u_{t_1}, x_{t_1}).$$

Implementation

- Consider non-homothetic CES consumer + CES producers.

- Observed sales:

$$d\lambda_{it} = \underbrace{\sum_{j \in \{0\}+N+F} \lambda_{jt}(\theta_j - 1) Cov_{\Omega_{(j,:),t}} \left(-d \log p_t, \Psi_{(:,i),t} \right)}_{\text{substitution effect}}$$

$$+ \underbrace{Cov_{\Omega_{(0,:),t}} \left(d \log x_t, \Psi_{(:,i),t} \right)}_{\text{taste shocks}} + \underbrace{Cov_{\Omega_{(0,:),t}} (\varepsilon_t, \Psi_{(:,i),t}) \left(\sum_{kt \in N} \lambda_k d \log A_{kt} \right)}_{\text{income effect}}.$$

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- Welfare-relevant sales (starting at t_1 and going back to t_0)

$$d\lambda_{it}^{ev} = \sum_{j\in\{0\}+N+F} \lambda_{jt}^{ev}(\theta_j - 1) Cov_{\Omega_{(j,:),t}^{ev}}\left(-d\log p_t^{ev}, \Psi_{(:,i),t}^{ev}\right).$$

- Real GDP uses λ_i, welfare uses λ_i^{ev}.

prices

Simple Examples

- One sector economy with no intermediates and one factor:

$$EV^M - \Delta \log Y \approx \frac{1}{2} \underbrace{Cov_b(\Delta \log x, \Delta \log A)}_{\text{gap due to taste shocks}} + \frac{1}{2} \underbrace{Cov_b(\varepsilon, \Delta \log A)\mathbb{E}_b[\Delta \log A]}_{\text{gap due to income effects}}.$$

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- With roundabout (larger changes in sales shares due to taste):

$$EV^M - \Delta \log Y \approx \frac{1}{2} \frac{1}{(1-\Omega_{ii})} \left[Cov_b(\Delta \log x, \Delta \log A) + Cov_b\left(\varepsilon, \Delta \log A\right) \mathbb{E}_b[\Delta \log A] \right],$$

where Ω_{ii} is intermediate input share.

Simple Example with Nonlinear PPF

- One sector with decreasing returns to scale:

$$EV^M - \Delta \log Y \approx \frac{1}{2} \frac{Cov_{\Omega^{(0)}}(\Delta \log x, \Delta \log A)}{1 + (\theta_0 - 1)(1 - \gamma)}.$$

Sales shares respond more to taste shocks if complements and DRS.

Simple Example with Nonlinear PPF

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  Sales shares respond more to taste shocks if complements and DRS.

- With only taste shocks, Micro and Macro welfare are different.

$$EV^m \approx -\frac{1}{2} \frac{(1-\gamma)}{(1 + (\theta_0 - 1)(1 - \gamma))} Var_{\Omega^{(0)}}(\Delta \log x) \neq 0 = EV^M.$$

Agenda

Implications for Long-Run Growth

As economies grow, low-productivity-growth sectors expand.
(Baumol's cost disease)

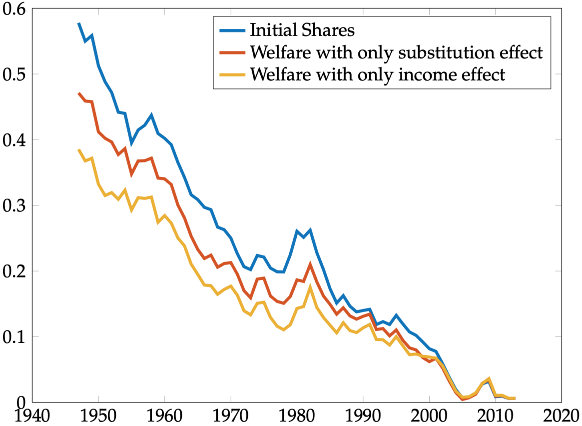
Corollary

If structural transformation is due only to:

- *substitution effects, then welfare-TFP uses chained sales shares.*
- *income effects or taste shocks, then welfare-TFP uses terminal sales shares.*

Growth in welfare TFP between *t* and 2014 in the US

61 private-sector industries, gross-output TFP as in Jorgenson et al. (2005) and Carvalho and Gabaix (2013)



- Settling dispute about income v. substitution effects has important welfare implications.

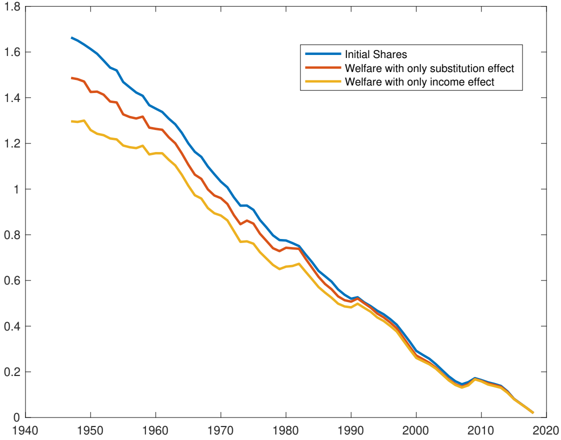
Quantitative Experiment

- θ_0 elasticity of sub. between primary/manufacturing/services.
- θ_1 elasticity of sub. within primary/manufacturing/services.
- θ_2 elasticity of sub. between materials.
- θ_4 elasticity of sub. between VA-materials.

Table: US from 1948 to 2015.

$(\theta_0, \theta_1, \theta_2, \theta_3)$	(1,1,1,1)	(0.5,1,1,1)	(1,0.5,1,1)	(1,1,0.5,1)	(1,1,1,0.5)
Welfare TFP	46%	46%	54%	48%	55%
Measured TFP	60%	60%	60%	60%	60%
Constant-share TFP	78%	78%	78%	78%	78%

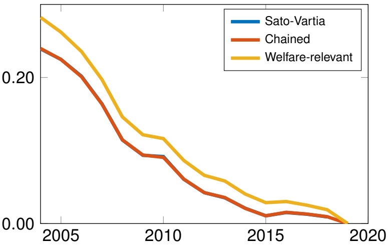
US welfare between *t* and 2019 using consumption data



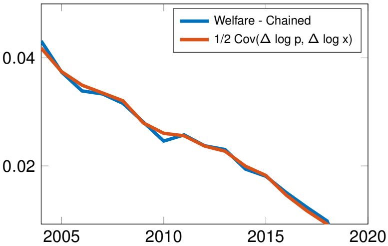
Implications for Short-Run Growth

- Biases can also matter at more disaggregated level.
- The more we disaggregate, the more shares vary.
  (increasing scope for bias).
- Within-industry firm-level shocks can cause industry-level price/quantity to be mismeasured.
- Use Nielsen UPC-level data 2004-2019 and CES with taste shocks and elasticity of substitution 4.5 to quantify.

UPC-level inflation rate for continuing varieties



(a) Inflation rates



(b) Bias

- Welfare-relevant inflation is higher than chained.
  Supply and demand shocks are negatively correlated.
- Gap widens as we extend horizon due to persistence.
- Covariance formula works well.

Implications for Business Cycles

- Consider application to Covid-19 recession: mixture of supply and demand shifters.
- 66 sector model with input-output linkages and complementarities in production, as in Baqaee-Farhi (2021).
- Labor market segmented by sector.
- Supply shocks: labor supplied to match reductions in employment from Q1 to Q2 of 2020.
- Demand shocks: taste shifters to match changes in PCE expenditures from Q1 to Q2 of 2020.

Macro vs. Micro welfare

Welfare Feb to May, 2020 using structural model

Elasticities	High compl.	Medium compl.	Cobb-Douglas
Micro final preferences	-13.2%	-12.3%	-10.9%
Macro final preferences	-10.1%	-9.4%	-9.0%
Chained real consumption	-12.1%	-10.6%	-9.8%

- Micro welfare losses larger because prices rise for goods with increasing demand.
- Macro losses smaller than micro since part of price rises were due to changes in demand.
- Chaining under-measures micro losses, and over-measures macro losses

RGDP as Measure of Production

- What are implications for comparing RGDP to model predictions?
- Consider two different time-paths of shocks:
  - Supply shocks first, then demand shifters.
  - Demand shifters first, then supply shocks.

$\Delta RGDP$ Q1 to Q2, 2020 using structural model.

Elasticities	High compl.	Medium compl.	Cobb-Douglas
Supply then demand	-16.2%	-12.5%	-10.8%
Demand then supply	-10.1%	-9.4%	-9.0%

- RGDP falls by more if supply shocks first, because supply and demand shocks positively correlated.
- If recovery not mirror image of downturn, RGDP off by up to 6%.

Agenda

Extension to Heterogeneous Agents

- Homogeneous agent assumption simplifies exposition, but Baqaee & Burstein (2021) generalizes:

  "*What is the minimum change in endowments in t_0 so that it is possible to make every consumer indifferent between t_0 and t_1?*"

- With this definition, all macro and micro results readily generalize.

Extensions to Extensive Margin Adjustments

- Results continue to apply if adjustments are on extensive margin.
- For example, consider

$$\left(\int_0^{x^*} c(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}.$$

- Then we have

$$\Delta \log EV^m \approx \Delta \log Y + \frac{1}{2} b(x^*) \Delta x^* \left[\mathbb{E}_b \left[\Delta \log p\right] - \Delta \log p(x^*)\right].$$

- Very different to Feenstra (1994) adjustment.

Steady-state comparisons in dynamic model

- Intertemporal preferences

$$\mathscr{U}_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s), \qquad \sum_i \omega_{i0} x_{it} \left(\frac{c_{is}}{C_s^{\xi_i}} \right)^{\frac{\theta_0 - 1}{\theta_0}} = 1$$

- Goods: $y_{is} = A_{is} G_i \left(\{m_{ijs}\}_{j \in N}, H(l_{is}, k_{is}) \right)$
- Investment: $I_s = A_{Is} I \left(\{m_{Ijs}\}_{j \in N}, H(l_{Is}, k_{Is}) \right)$.
- Capital accumulation $K_{s+1} = (1 - \delta)(K_s + I_s)$

Proposition

Consider two dynamic economies, denoted t_0 and t_1, that are in steady-state. The change in macro welfare is given by

$$EV^M = \log \left(\frac{\sum_i p_{it_1} c_{it_1}}{\sum_i p_{it_0} c_{it_0}} \right) - \log \left(\sum_i b_{it_1} \left(\frac{p_{it_1}}{p_{it_0}} \right)^{1-\theta_0} \right)^{\frac{1}{1-\theta_0}}.$$

Other Extensions

- Distorted economies.
- Beyond CES.
- Endogenous arguments in utility.

Conclusion

- Toolbox for welfare computation. How to modify Divisia/Hulten for taste shocks and non homoth.
- Chaining doesn't correctly account for expenditure-switching caused by taste shocks or income effects.
- In these cases, distinct macro and micro notions of welfare.
- Characterize both and show gap with chained quantity significant when demand shifters covary with supply shifters.

Changes in Prices

$$d\log p_{it} = -\underbrace{\sum_{j\in\{0\}+N+F} \Psi_{ijt}\, d\log A_{jt}}_{\text{upstream TFP changes}} + \underbrace{\sum_{f\in F} \Psi_{ift}\, d\log \lambda_{ft}}_{\text{upstream factor price changes}} .$$

and

$$d\log p_{it}^{ev} = -\sum_{j\in\{0\}+N+F} \Psi_{ijt}^{ev}\, d\log A_{jt} + \sum_{f\in F} \Psi_{ift}^{ev}\, d\log \lambda_{ft}^{ev}.$$

back

With Quality Change

Change in real consumption

$$\Delta \log Y \approx \Delta \log I - \mathbb{E}_b [\Delta \log \tilde{p}] - \frac{1}{2}(1-\theta_0) Var_b(d \log \tilde{p})$$
$$+ \frac{1}{2}(1-\theta_0) Cov_b(d \log q, d \log \tilde{p}) - \frac{1}{2} Cov_b(d \log x, d \log \tilde{p}),$$

Change in welfare

$$EV^m \approx \Delta \log I - \mathbb{E}_b [\Delta \log \tilde{p} - \Delta \log q] - \frac{1}{2}(1-\theta_0) Var_b(\Delta \log \tilde{p})$$
$$- \frac{1}{2}(1-\theta_0) Var_b(\Delta \log q) + (1-\theta_0) Cov_b(\Delta \log \tilde{p}, \Delta \log q) - Cov_b(\Delta \log x, \Delta \log p),$$

Hence,

$$EV^m - \Delta \log Y \approx \underbrace{\mathbb{E}_b [\Delta \log q]}_{\text{average quality}} + \frac{1}{2} \underbrace{(\theta_0 - 1) Var_b(\Delta \log q)}_{\text{dispersion in quality}} + \frac{1}{2} \underbrace{(1-\theta_0) Cov_b(\Delta \log \tilde{p}, \Delta \log q)}_{\text{covariance of price and quality}}$$
$$- \frac{1}{2} \underbrace{Cov_b(\Delta \log x, \Delta \log \tilde{p})}_{\text{covariance of taste and price}} + \underbrace{Cov_b(\Delta \log x, \Delta \log q)}_{\text{covariance of taste and quality}}.$$

back

Micro Welfare

Lemma

$$EV^m = \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p_t, u_{t_1}, x_{t_1}) d \log p_{i,t}.$$

$$\begin{aligned} EV^m &= \log \frac{e(p_{t_0}, v(p_{t_1}, \mathrm{I}_{t_1}; x_{t_1}); x_{t_1})}{e(p_{t_0}, v(p_{t_0}, \mathrm{I}_{t_0}; x_{t_1}); x_{t_1})} = \log \frac{\mathrm{I}_{t_1}}{\mathrm{I}_{t_0}} \times \frac{e(p_{t_0}, u_{t_1}; x_{t_1})}{e(p_{t_1}, u_{t_1}; x_{t_1})} \\ &= \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i \frac{\partial \log e(p, v(p_{t_1}, \mathrm{I}_{t_1}; x_{t_1}); x_{t_1})}{\partial \log p_i} d \log p_i \\ &= \Delta \log \mathrm{I} - \int_{t_0}^{t_1} \sum_i b_i(p, u_{t_1}, x_{t_1}) \frac{d \log p_i}{dt} dt, \end{aligned}$$

back

Second-order Approximation

- Real GDP:

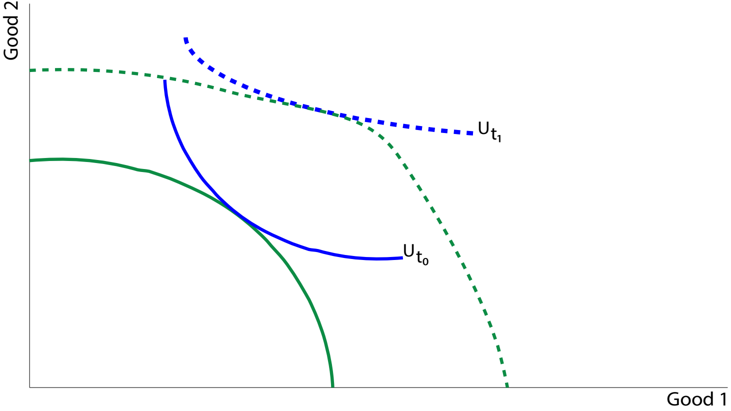
$$\Delta \log Y \approx \sum_i \lambda_i \Delta \log A_i + \frac{1}{2} \underbrace{\sum_i \Delta \lambda_i \Delta \log A_i}_{\text{expenditure switching}} .$$

- Welfare:

$$EV^M \approx \lambda' \Delta \log A + \frac{1}{2} \sum_i \Delta \lambda_i \Delta \log A_i$$
$$+ \frac{1}{2} \sum_i \left[\Delta \log x' \frac{\partial \lambda_i}{\partial \log x} + \Delta \log A' \frac{\partial u}{\partial \log A} \frac{\partial \lambda_i}{\partial \log u} \right] \Delta \log A_i.$$

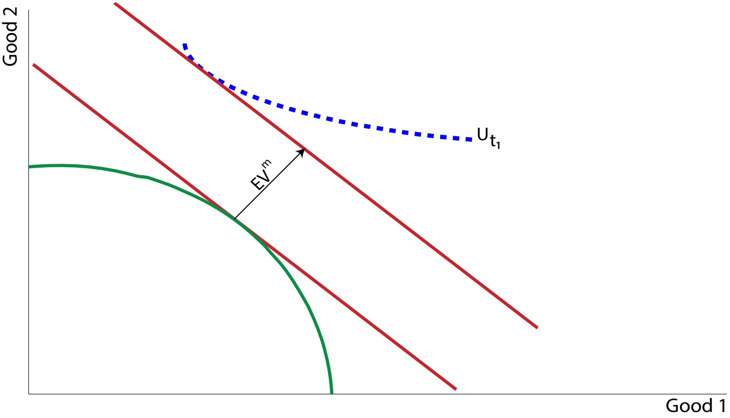
- Changes due to income effects and taste shocks are undercounted.

Graphical Representation

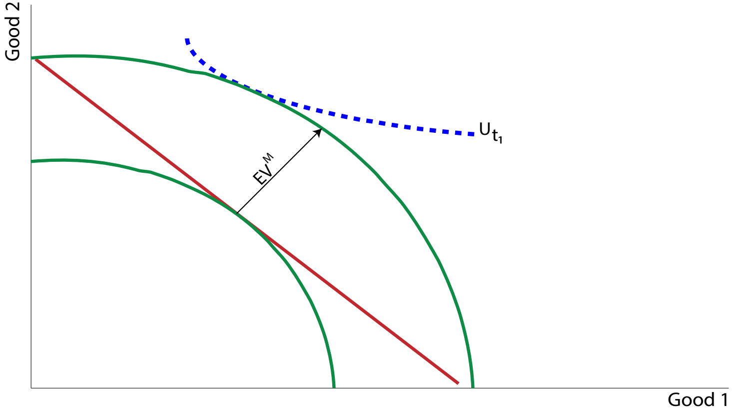


- Objective is to compare value of initial and final technology.

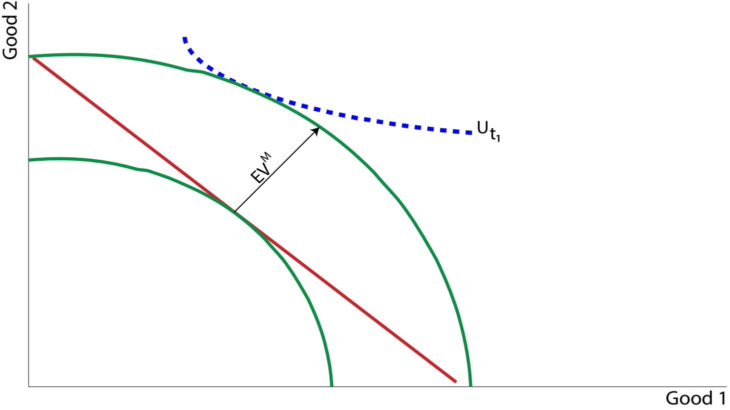
Graphical Representation – Micro EV^m



Graphical Representation – Macro EV^M



Graphical Representation – Macro EV^M



- $EV^M = EV^m$ if PPF is linear or preferences homothetic + stable.