Measuring Welfare with Income Effects using Cross-Sectional Data

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Abstract
We show how to recover the money-metric utility function, which converts income at one point in time into equivalent income at another point in time, using repeated cross-sectional household level data. Our procedure allows unrestricted preferences, but requires that households’ preferences be the same in both the cross-section and the time-series. Our idea is to trace out Hicksian (or compensated) demand curves through time by matching households on the same indifference curve at different points in time. Given Hicksian demand curves, we can construct ideal price indices and money-metric utility for every matched income level.
1 Introduction

In this paper, we show how using repeated cross-sectional household level data, and the assumption of common and unchanging preferences, one can recover the money-metric utility function that converts income at one point in time into equivalent income at other points in time (i.e. the income that makes households indifferent between their budget sets at both points in time). This problem was recently posed and investigated in an important paper by Jaravel and Lashkari (2022). Although we answer the same question under more or less the same assumptions, our approach and algorithm are different.

Our procedure, which builds on Baqaee and Burstein (2021), has an intuitive interpretation using Hicksian, or compensated, demand curves. To recover a money-metric utility function, one must integrate Hicksian demand curves with respect to changes in prices. More prosaically, this means that to construct a welfare-relevant inflation index, we must weigh changes in prices at each point in time using Hicksian demand for each good. In practice, one does not observe the Hicksian demand curves. In lieu of Hicksian demand, standard price indices (i.e. chain-weighted or Divisia) weigh changes in prices by observed — that is, Marshallian — demand curves instead. When preferences are homothetic, this practice is innocuous since for homothetic preferences income affects demand for all goods in the same way. However, when preferences are nonhomothetic, Marshallian and Hicksian budget shares are different and usual practice fails to recover a well-defined welfare measure.

To resolve this problem, we propose the following. Suppose we observe repeated cross-sections of households with identical preferences facing common prices. To construct the money-metric utility value in terms of some base time \( t_0 \) for a household with income \( I \) at time \( t \), we must know the compensated demand of this household when prices were different. This can be done at each point in time \( s \neq t \) by finding another household with a different income level \( I' \neq I \) at time \( s \) who is on the same indifference curve as the household with income \( I \) at \( t \). If we can find such a household at every point in time \( t_0 \leq s < t \), then we can trace the compensated Hicksian demand curve through time, and by using these demand curves, rather than observed demand, we can calculate the money-metric utility function.\(^1\) The key insight is that this process is recursive and can be solved by iteratively.

\(^1\)Hicksian demand can also be calculated given knowledge of elasticities of substitution. The procedure in this paper does not require nor allow one to estimate elasticities of substitution non-parametrically however. Intuitively, we only recover Hicksian demand curves evaluated at observed prices, whereas the elasticities of substitution allow one to compute counterfactual Hicksian demand even for prices that are not observed.
Like Jaravel and Lashkari (2022), our procedure is non-parametric in the sense that it works for any well-behaved preference relation. Similar to Jaravel and Lashkari (2022), the most important assumption we make is that all households in the sample have the same preferences and that these preferences are not changing through time. Our approach is complementary to theirs and relies on a different insight. One advantage of our approach is that we do not need to assume that the support of cross-sectional distribution of (ex-ante unknown) utility is constant and unchanging through time. Instead our procedure endogenously determines the region of the income distribution where a money-metric utility function can be constructed given the available information.

The outline of the paper is as follows. In Section 2 we define the cost-of-living index and money-metric utility and introduce a preliminary result taken from Baqaee and Burstein (2021). In section 3, we show how this result can be used to recover cost-of-living and money-metric utility with the aid of cross-sectional data. We also discuss some extensions and limitations of our approach, including how our results can be used when some of our baseline assumptions are relaxed. For example, with enough data, we discuss how to handle idiosyncratic taste shocks that are uncorrelated with income. Similarly, we can discuss how to apply our method if there is heterogeneity in preferences that is a function of observable characteristics. We also discuss how one can handle changes in quality that are not reflected in prices. In Section 4, we apply our results to artificial data generated using popular functional forms for non-homothetic preferences. We illustrate that our procedure quickly converges to the truth as the number of households increases and the time period shrinks. Our numerical examples are calibrated to match real-world data in terms of the frequency of observation, the number of households in the sample, and the rate at which prices and incomes are changing over time. In the next draft, we will apply our method to construct a money-metric utility function using household expenditure survey data from the United Kingdom from 1974 to 2017.

2 Money-Metrics and the Cost-of-Living

Consider a preference relation $\succeq$ defined over consumption bundles $c$ in $\mathbb{R}^N$. Suppose that we represent these preferences using a utility function $U(c)$ that maps consumption bundles to utility values. Given this utility function, we can define the indirect utility function

$$v(p, I) = \max_c\{U(c) : p \cdot c \leq I\},$$
mapping a vector of prices $p$ and income $I$ to utility values. We can also define the expenditure function

$$e(p, u) = \min_c \{ p \cdot c : U(c) \leq u \}.$$ 

The expenditure and indirect utility functions are useful because they can be used to construct money-metrics and cost-of-living indices.

**Definition 1 (Money-Metric).** For some reference vector of prices $\bar{p}$, the money-metric function is

$$m(\bar{p}, p, I) = e(\bar{p}, v(p, I)).$$

The money-metric $m(\bar{p}, \cdot)$ is a specific cardinalization of the indirect utility function in the sense that a budget set $(p, I) \succeq (p', I')$ if, and only if, $m(\bar{p}, p, I) \geq m(\bar{p}, p', I')$. That $m(\bar{p}, p, I)$ expresses the value of the budget set $(p, I)$ in terms of $\bar{p}$ prices. We use this cardinalization of utility throughout the rest of the paper.

**Definition 2 (Cost-of-living).** For some reference budget constraint $(\bar{p}, \bar{I})$, the cost-of-living function is

$$r(p, \bar{p}, \bar{I}) = e(p, v(\bar{p}, \bar{I})).$$

Note that $r(p, \bar{p}, \bar{I})$ converts the value of budget constraint $(\bar{p}, \bar{I})$ into equivalent income under prices $p$.

In other words, the key object is the function $e(p', v(p, I))$ which maps $(p', p, I)$ into a scalar. The “money-metric” is the cross-section of this function that holds $p'$ constant and the cost-of-living index is the cross-section that holds $(p, I)$ constant. The money-metric is useful for converting different budget sets into a common price system for comparison. On the other hand, the cost-of-living index is useful for converting a common utility level, attained by $v(p, I)$, into equivalent income under different price systems. As its name suggests, the cost-of-living index is necessary for providing a consumer with a cost-of-living adjustment.

Next, denote the Hicksian budget share for good $i$ to be $b_i(p, u)$ where $p$ is a vector of prices and $u$ is a utility level. Similarly, denote observed (or Marshallian) budget share for good $i$ at time $t$ for households with income $I$ to be $B_i(I, t)$, where $t$ indexes the vector of prices.

The following proposition, which is a corollary of Lemma 1 from Baqee and Burstein (2021), provides a characterization of both the cost-of-living index and the money-metric using Hicksian budget shares.

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2In index number theory, the cost-of-living index is also called the Konüs (1939) index.
Proposition 1 (Money-Metric and Cost-of-Living). The money-metric of a budget set \((p, I)\) in terms of \(\bar{p}\) prices can be expressed as

\[
\log m(\bar{p}, p, I) = \log I - \int_{\bar{p}}^{p} \sum_{i \in \mathbb{N}} b_i(\xi, v(p, I)) d \log \xi_i. \tag{1}
\]

The cost-of-living for a budget set \((\bar{p}, \bar{I})\) in terms of \(p\) prices can be expressed as

\[
\log r(p, \bar{p}, \bar{I}) = \log \bar{I} + \int_{\bar{p}}^{p} \sum_{i \in \mathbb{N}} b_i(\xi, v(\bar{p}, \bar{I})) d \log \xi_i. \tag{2}
\]

Intuitively, both the money-metric and the cost-of-living index can be expressed as integrals of Hicksian budget shares with respect to changes in prices. However, Hicksian demand curves are not directly observable, so operationalizing this result requires having a way to identify Hicksian budget shares. This is what we focus on in the next section.

3 Main Results

In this section, we discuss how Proposition 1 can be deployed to recover money-metric utility functions and cost-of-living indices if one has access to repeated cross-sectional data of consumers with common and stable preferences who all face common prices at each point in time but have different incomes.

3.1 Theoretical Result

Suppose we observe a smooth path of prices \(p_t\) at each point in time \(t \in [t_0, T]\) and, for consumers with income level \(I \in [I_t, \bar{I}_t]\) at time \(t\) we observe the vectors of expenditure shares \(B(I, t)\) across all goods.

For any cardinalization of the indirect utility function and its associated Hicksian demand curves, the following identity holds

\[
b_i(p_t, v(p_t, I)) = B_i(I, t),
\]

Since \(m(p_{t_0}, p_t, I)\) is the cardinalization of indirect utility that we are working with, we can write

\[
b_i(p_t, m(p_{t_0}, p_t, I)) = B_i(I, t).
\]

Using this identity, Proposition 1 can be rewritten as the following recursive integral.
equation.

**Proposition 2** (Money-metric as Solution to Integral Equation). For \( t \in [t_0, T] \), the money-metric is \( m(p_{t_0}, p_t, I) = \log u(I, t) \), where \( \log u(I, t) \) solves the following integral equation

\[
\log u(I, t) = \log I - \int_{t_0}^{t} \sum_i B_i(u^{-1}(u(I, t), s), s) \frac{d \log p_{is}}{ds} ds
\]

with boundary condition \( u(I, t_0) = I \). Here, \( u^{-1}(\cdot, s) \) is the inverse of \( u \) with respect to its first argument (income) given its second argument equal to \( s \).

In words, \( u(I, t) \) converts the value of the budget constraint defined by prices \( p_t \) and income \( I \) into income under \( p_{t_0} \). That is, \( u(I, t) = e(p_{t_0}, v(p_t, I)) \). This is the money-metric for \( (p_t, I) \) in terms of \( p_{t_0} \). The solution to the integral equation above is \( m(p_{t_0}, p_t, I) \) where \( p_{t_0} \) are prices at the initial condition. By varying the initial condition and evaluating \( m(p_{t_0}, p_t, I) \) for different \( p_{t_0} \) for fixed \( (p_t, I) \) we recover the cost-of-living index \( r(p_{t_0}, p_t, I) \).

Proposition 2 follows immediately from Proposition 1 if we recognize that in the integral equation above, \( B_i(u^{-1}(\cdot, s), s) : \mathbb{R}_+ \rightarrow [0, 1] \) maps utility values into budget shares at time \( s \). That is, it is the Hicksian budget share.

If we can solve the integral equation in Proposition 2, then we can recover the money-metric and cost-of-living functions without direct knowledge of the elasticities of substitution or income elasticities. This is because we can compute the Hicksian budget shares \( b(u_t, p_s) \) of a household with utility \( u_t \) at time \( t \) under prices \( p_s \) at time \( s \) by using the budget shares \( B(u^{-1}(u_t, s), s) \) of a household on the same indifference curve at time \( s \). Following Proposition 1, we use Marshallian demand for this matched household, \( B(u^{-1}(u_t, s), s) \), to weigh prices changes at time \( s \) for a household with utility \( u_t \).\(^3\)

### 3.2 Numerical Procedure

Equation (3) is a system of nonlinear equations, albeit an infinite-dimensional one, and there are many established numerical procedures for solving such equations. Here, we show a very simple iterative procedure that converges to a true solution as we approach the continuous-time limit.

For some interval of time \([t_0, T]\), suppose we have data on a grid of points \([t_0, \ldots, t_M]\) where \( t_n < t_{n+1} \), with \( t_M = T \). Then, use the following iterative procedure for each

\(^3\)Proposition 2 can be used to measure changes in microeconomic welfare if we have repeated cross-sectional data and common stable preferences. However, we cannot use this procedure to answer counterfactual or macroeconomic welfare questions like those studied by Baqee and Burstein (2021).
$n \in \{1, \ldots, M\}$ starting with $n = 1$:

$$
\log u(I, t_n) \approx \log I - \sum_{s=0}^{n-1} b(u(I, t_{n-1}), t_s) \cdot \Delta \log p_{t_s},
$$

(4)

$$
b(v, t_s) = B(u^{-1}(v, t_s), t_s).
$$

(5)

with the boundary condition $u(I, t_0) = I$ and $b(u, t_0) = B(I, t_0)$. The summation in (4) above approximates the integral in (3) using a Riemann sum and becomes exact in the continuous-time limit because the Riemann sum becomes an integral and $u(I, t_{n-1}) \to u(I, t_n)$.

For those values of $u$ that can be inverted, this procedure recovers welfare as the time interval shrinks to zero. This procedure endogenously delineates those values of $(I, t)$ for which $u(I, t)$ can be computed, and it does not require an assumption of full overlapping support over time on either the set of observed incomes or unobserved utilities.

To give more intuition, it helps to explicitly spell out the first few steps of this iterative procedure. We start with the boundary condition $u(I, t_0) = I$ since $t_0$-equivalent income in $t_0$ is just initial income. At time $t_1$, we use

$$
u(I, t_1) \approx \log I - b(u(I, t_0), t_0) \cdot \Delta \log p_{t_1} = \log I - B(I, t_0) \cdot \Delta \log p_{t_1}
$$

where the last equation uses the boundary condition, which implies $b(u(I, t_0), t_0) = B(I, t_0)$ and becomes exact in the continuous time limit as the gap between $t_0$ and $t_1$ shrinks to zero.

Given $u(I, t_1)$, we construct Hicksian budget shares at $t_1$:

$$
b(u, t_1) = B(u^{-1}(u, t_1), t_1)
$$

for all $u$’s for which $B(u^{-1}(u, t_1), t_1)$ is observed. That is, to each budget share $B(I, t_1)$ in $t_1$, we assign a utility value based on $u(I, t_1)$. Hence, we now have Hicksian budget shares $b(u, t_0)$ and $b(u, t_1)$ for all values of $u \in [I_0, \bar{I}_0]$. For $u(I, t_1)$ values outside of $[I_0, \bar{I}_0]$, we cannot compute Hicksian budget shares in $t_0$ since there are no households in $t_0$ who are on the same indifference curve as $u(I, t_1)$.

\footnote{In our computations, we use the trapezoid rule rather than the left Riemann sum in equation (4) to approximate the integral in (3) since it is a better numerical approximation.}

\footnote{Invertibility at $(u, s)$ means that we observe an income level $I$ such that $u(I, s) = u$. When applying the algorithm to the data in the next section, the value of the budget share in period $s$ corresponding to $u(I, t_{n-1})$ is obtained by linear interpolation of the grid of $u$ at the time $s$.}
Next, we construct

$$u(I, t_2) \approx \log I - b(u(I, t_1), t_1) \cdot \Delta \log p_{t_2} - b(u(I, t_1), t_0) \cdot \Delta \log p_{t_1},$$

and given $u(I, t_2)$, we construct Hicksian budget shares in $t_2$:

$$b(u, t_2) = B(u^{-1}(u, t_2), t_2).$$

That is, for each budget share $B(I, t_2)$ in $t_2$, we assign a utility value based on $u(I, t_2)$. We continue this iterative process until $t_M$. Note that we can only calculate $u(I, t)$ for those $I$’s for which $B(u^{-1}(u(I, t), s), s)$ is observed for all $s < t$.

Figure 1: Non-homothetic preferences. Expenditure share for some good against log nominal income and log money-metric utility at three points in time.

To see this procedure graphically, consider the left panel of Figure 1 showing the expenditure share on some good against nominal income for three different points in time. The fact that the lines are downward sloping means that higher incomes are associated with lower expenditures on the good. In this example, incomes grow over time, so the range of nominal income levels shifts up over time. Whereas in the data we observe budget shares as a function of income over time, to construct the money metric we require budget shares as a function of utility (Hicksian budget shares). The right panel of Figure 1 displays the Hicksian budget shares for the same good. The purple line in the right panel of Figure 1 shows for each period the Hicksian expenditure share for the good evaluated at some fixed utility level $\bar{u}$. The change in expenditures, holding utility constant, are pure
substitution effects over time due to changes in relative prices. As implied by Proposition 1, multiplying the Hicksian budget shares by log price changes and summing over time gives the money-metric utility for the household with utility $\bar{u}$ at time $t_2$.

But, we cannot directly observe the figure on the right. How do we infer Hicksian budget shares? The purple line in the left panel of Figure 1 plots, for each period $s$, the income that gives the utility of $\bar{u}$, that is $u^{-1}(\bar{u}, s)$, and the associated budget share, $B_1(u^{-1}(\bar{u}, s), s)$. In other words, we can infer Hicksian budget shares for $\bar{u}$ by using the observed budget share along the purple line in the left panel. Then we can construct the mapping between income and utility at each point (the purple line) by iteratively applying the summation in (4).

Figure 2: Homothetic preferences. Expenditure share for some good against log nominal income and log money-metric utility at three points in time.

To understand why Proposition 2 is unnecessary when preferences are homothetic, Figure 2 plots the same information as Figure 1 but for homothetic preferences. Since there are no income effects, budget shares at a point in time do not vary with household income or utility. That is, observed and Hicksian budget shares coincide. Therefore, we can construct the money metric using a price index based on observed budget shares by good.
3.3 Extensions and Limitations

In practice, data is imperfect and noisy. Specifically, recorded expenditure shares can change through time for reasons other than changes in observed prices and income. Under some additional assumptions, our procedure can be modified to account for some of these issues.

For example, if there is classical measurement error or idiosyncratic taste shocks at the individual consumer level, uncorrelated with any observable, then we can eliminate this noise by averaging over multiple households with the same (or similar) income level. If the noise is caused by idiosyncratic taste shocks, then our money-metric utility function will apply to preferences in the absence of the taste shocks. At the opposite extreme, suppose that there are persistent differences in preferences that are functions of observable characteristics, for example households with children have different preferences to those without. This is related to the assumption considered in Section 2.3 of Jaravel and Lashkari, 2022). In this case, we can handle this by splitting the sample in two and applying our method to each sample separately.6

If there are unobservable demand shifters that affect the entire distribution of households, then we cannot deal with that by averaging or conditioning on observable characteristics. This happens if there are aggregate taste shocks that affect the entire distribution of households, or if there are changes in quality over time that are not reflected in prices. If there are changes in quality, then our method can be applied to the quality-adjusted version of prices (following standard quality-adjustment practice) without issue. However, if there are unobservable shocks to preferences that are not idiosyncratic, and cannot be eliminated by averaging, then our methodology cannot be used. An example is if household preferences over time are systematically different to preferences in the past in ways we cannot model.

4 Illustrative Example Using Artificial Data

In this section, we illustrate and evaluate our algorithm using artificial data from fully parameterized preferences. We consider generalized non-homothetic CES preferences

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6Similarly, if we observe two groups of households that face different prices at a point in time (e.g. households living in different locations), then we can apply our method to each sample separately.
from Fally (2022). The expenditure function is

\[ e(p, u) = \left( \sum_i \omega_i (u^e p_i)^{1-\sigma(u)} \right)^{\frac{1}{1-\sigma(u)}}. \]

By Shephard’s lemma, Hicksian budget shares \( b(p, u) \) are

\[ b_i(p, u) = \frac{\omega_i (u^e p_i)^{1-\sigma(u)}}{\sum_j \omega_j (u^e p_j)^{1-\sigma(u)}}, \]

and Marshallian budget shares are \( B(I, t) = b(p_t, u) \) where \( u \) solves \( I = e(p_t, u) \). Income elasticities can vary across goods and the elasticity of substitution \( \sigma \) can vary across indifference curves (but is constant along any indifference curve, as under standard CES). As shown by Baqae and Burstein (2021), the money-metric function for \( t_0 \) reference prices is

\[ m(p_{t_0}, p, I) = \left( \sum_i \omega_i (u^e p_{t_0})^{1-\sigma(u)} \right)^{\frac{1}{1-\sigma(u)}} \]

where \( u \) is the solution to \( I = e(p, u) \). To evaluate the accuracy of our algorithm, we compare \( m(p_{t_0}, p_T, I) \) with estimates applying our numerical procedure to artificial data generated by these preferences.

We generate repeated cross-sectional data on income and expenditure shares over 3 goods for 100 households that face a common price vector for \( T = 40 \) periods. The distribution of income in the first period is lognormal (parameterized to match the distribution of household expenditures in the 1974 UK household survey, described in the next section). All incomes grow by a factor of 10 in the period sample at a constant annual growth rate. Good 1 has the lowest income elasticity (and hence we refer to it as a necessity) and highest inflation rate. Figure 3 displays the paths of household income and price data in our illustrative example.

We consider three parameterizations. The first one is the homothetic case with \( \varepsilon_1 = \varepsilon \) and \( \sigma = 0.25 \). The second case allows for income effects but the elasticity of substitution is independent of \( u \). We follow Comin et al. (2021), and set \( \varepsilon_1 = 0.2, \varepsilon_2 = 1, \varepsilon_3 = 1.65 \), and \( \sigma = 0.25 \). The third case further assumes that the elasticity of substitution is a log-linear decreasing function of \( u \), consistent with estimates in Auer et al. (2021).
\( \sigma(u) = 10 - 2 \log u \), with the intercept value ensuring that elasticities of substitution remain higher than unity. The share parameter \( \omega \) is calibrated separately in each case so that the budget shares of each good for the median household in the first period are all the same (equal to one third for each good).

We define real income to be income deflated by a chain-weighted price index based on observed aggregate budget shares (as is standard practice in national income accounting). Define the non-homotheticity bias to be the log difference between real income and the money-metric. Figure 4 plots the gap between real income and the money-metric in our examples under fixed and variable elasticities of substitution. We do not plot the homothetic case since the bias is zero. In both cases, since inflation is higher for necessities, and poor households are more reliant on necessities, the bias is larger for poorer households. We do not report the bias for households above the 70th percentile because for those values of income, there do not exist similarly well-off households in the past whose demand can be used (i.e. the invertibility condition fails for those households).

To assess the accuracy of our procedure, we use the infinity norm – that is, the maximum absolute value of the log difference between the true money-metric function and our estimate at time \( T \). Under both parameterizations, constant and variable \( \sigma \), the error is very small: 0.0020 and 0.0025. This is equivalent to roughly 1/5 of 1% of income. Figure 5 shows how this error varies as we vary the number of households and the frequency of observations using the constant \( \sigma \) non-homothetic specification as an example. As expected, the error converges to zero as we approach the continuous-time limit. The error
Figure 4: Non-homotheticity bias: Log difference between real income and the money-metric also falls as the number of households in the sample increases.

Figure 5: This figure displays the maximum error as a function of the frequency of observation, holding the path of price and income changes in Figure 3 constant. Our baseline calibration is annual frequency corresponding to a value of 1 observation per year on the x-axis. If we observe the data once every decade, then the frequency is 1/10, and if we observe the data every quarter, the frequency is 4. The blue line is the case where only 20 households can be observed, while the red line corresponds to a case where 500 households can be observed.
5 Empirical Results

TO BE ADDED

6 Conclusion

In this paper, we provide a simple and intuitive procedure for constructing money-metric representations of utility using repeated cross-sectional data. Our insight is that one can trace out Hicksian, or compensated, demand curves through time by matching households whose incomes are equivalent in utility terms. Although our approach is non-parametric, it relies on the assumption that preferences are the same in both the cross-section and the time-series dimensions and that all consumers face common prices. Relaxing these assumptions is an interesting avenue for future work.

References

Online Appendix

A Using artificial data from Almost Ideal Demand System
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In this appendix, we redo our analysis of Section 4 using another popular form of non-homothetic preferences: Almost Ideal Demand System (AIDS). The expenditure function is

\[ e(p, u) = c(p) u^d(p) \]

where \( c(p) \) and \( d(p) \) are given by:

\[
c(p) = \exp \left( a_0 + \sum_{i=1}^{l} a_i \log p_i + \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \gamma_{ij} \log p_i \log p_j \right)
\]

\[
d(p) = \exp \left( \sum_{i=1}^{l} \beta_i \log p_i \right)
\]

where \( \sum \alpha_i = 1, \sum \beta_i = \sum \gamma_{ij} = 0 \) and \( \gamma_{ij} = \gamma_{ji} \) for all \( i \) and \( j \).

By Shephard’s lemma, Hicksian budget shares \( b(p, u) \) are

\[
b_i(p, u) = \alpha_i + \sum_{j=1}^{l} \gamma_{ij} \log p_j + \beta_i d(p) \log u.
\]

The money-metric function for \( t_0 \) reference prices is\(^8\)

\[
m(p_0, p, I) = c(p_0) \left( \frac{I}{c(p)} \right)^{\frac{d(p_0)}{d(p)}}.
\]

In assigning parameter values, we assume that the expenditure share is decreasing in utility for good 1 and increasing for good 3, as in the non-homothetic CES example in Section 4. Specifically, we consider the following parameter values, that also ensure that the expenditure share on all goods is positive in all periods in the artificial dataset.

\[
\begin{bmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 & \beta_3 & \gamma_{11} & \gamma_{22} & \gamma_{33} & \gamma_{12} & \gamma_{13} & \gamma_{23}
\end{bmatrix}
\begin{bmatrix}
2 & 1/3 & 1/3 & 1/3 & -0.15 & -0.05 & 0.2 & -1/4 & -1/4 & -1/4 & 1/8 & 1/8 & 1/8
\end{bmatrix}
\]

Table 1: Parameters for AIDS

Figure 6 presents the non-homotheticity bias. The approximation error of the money-metric in period \( T \), \( \max_i \left| \log u(I, T) - \log u(I, T)^{TRUE} \right| \), is \( 5.13 \times 10^{-4} \).

\(^8\)To obtain the expression for the money metric, we use \( m(p_0, p, I) = e(p_0, u) \), where \( I = c(p) u^d(p) \).
Figure 6: Log difference between the inflation implied by the cost-of-living function and aggregate chain-weighted inflation