Measuring Welfare by Matching Households Through Time

David Baqaee   Ariel Burstein   Yasutaka Koike-Mori

UCLA

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Introduction

- Money-metric utility function backbone of welfare economics:
  - Converts t-income into equivalent income in t_0,
  - essential tool to measure growth & inflation.
- Need to weigh price changes by “compensated” spending shares.
- Compensated shares unobserved, two options:
  - Assume homotheticity: compensated = uncompensated.
  - Assume strong functional form and estimate.

What We Do

- Develop non-parametric method to recover money-metric utility using prices & repeated cross-section of household expenditures.
  - W/o imposing homotheticity or parametric assumptions about preferences, and w/o estimating a demand system.
- Extension with missing prices under separability assumption.
- Find inflation understated for $<$ 60th percentile of income in UK.
- Extend to account for uncertainty and dynamics.

Selected Literature

- Non-parametric methods to estimate money-metric:

  Blundell et al (2003): calculate bounds using revealed-choice arguments, without requiring continuously observed data.

  Jaravel & Lashkari (2021): different method, can give large errors.

  Our approach also extends to allow for unobserved prices.

- Welfare measurement under missing prices using Engel Curves:

  Costa (2001), Hamilton (2001), Atkin-Faber-Fally-Gonzalez (2020)

  Our approach, which generalizes Feenstra (2004), has distinct intuition, assumptions, and data requirements.

- Welfare measurement with parametric demand system

  Boppart (2014), Comin et al. (2021), Fan et al. (2022), Matsuyama (2000), Fajgelbaum et al. (2011), Straub (2019), Auer et. al. (2022)

  Our approach can be applied for ex-post measurement, not for counterfactuals.

Agenda

Theory

Illustration with Artificial Data

UK Application

Unobserved prices

Conclusion

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Setup

- Preferences $\succeq$ defined over $\boldsymbol{c}$ in $\mathbb{R}^N$ represented by $\mathscr{U}(\boldsymbol{c})$.
- Indirect utility function

$$v(\boldsymbol{p}, I) = \max_{\boldsymbol{c}} \{\mathscr{U}(\boldsymbol{c}) : \boldsymbol{p} \cdot \boldsymbol{c} \leq I\},$$

- Expenditure function

$$e(\boldsymbol{p}, U) = \min_{c} \{\boldsymbol{p} \cdot \boldsymbol{c} : \mathscr{U}(\boldsymbol{c}) \geq U\}.$$

Definition (Money-Metric Utility)

Given base prices $\bar{\boldsymbol{p}}$, MM function maps $(\boldsymbol{p}, I)$ to $e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I))$.

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- $e(\bar{\boldsymbol{p}}, \cdot)$ is an indirect utility function with interpretable units, measures growth under base prices $\bar{\boldsymbol{p}}$.
- $e(\cdot, v(\boldsymbol{p}, I))$ is cost-of-living index to achieve $v(\boldsymbol{p}, I)$ under diff. $\bar{\boldsymbol{p}}$.

Observed data

- Suppose that for each $t \in [t_0, T]$ we observe
  - Prices $\boldsymbol{p}_t \in \mathbb{R}^N$, absolutely continuous over time.
  - Budget shares $\boldsymbol{B}(I, t) \in \mathbb{R}^N$ for expenditures $I \in [\underline{I}_t, \bar{I}_t]$.
  - Repeated cross-section of HHs with same preferences over time.
  - If preferences vary by observed characteristic, split sample.

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  - Repeated cross-section of HHs with same preferences over time.
  - If preferences vary by observed characteristic, split sample.
- Objective is money-metric utility, base prices $\boldsymbol{p}_{t_0}$, w.l.o.g.

$$u(I, t) \equiv e(\boldsymbol{p}_{t_0}, v(\boldsymbol{p}_t, I))$$

for $t \in [t_0, T]$ and $I \in [\underline{I}_t, \bar{I}_t]$.

Recasting money metric as a fixed point problem

1. Given compensated demand, obtain MM by chaining. derivation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i b_i(u(I,t),s) \frac{d \log p_{is}}{ds} ds$$

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   Equivalently, $b_i(\cdot,s) = B_i(u^{-1}(\cdot,s),s)$.

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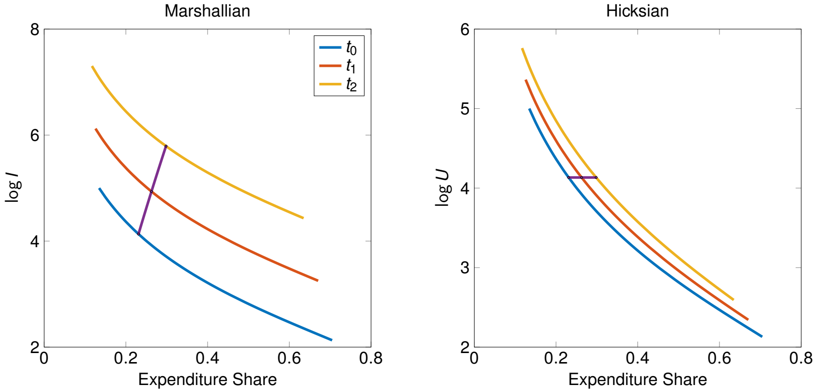
- 1.+ 2. $\implies$ $u(I,t)$ is a fixed point of integral equation:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i B_i(u^{-1}(u(I,t),s),s) \frac{d \log p_{is}}{ds} ds.$$

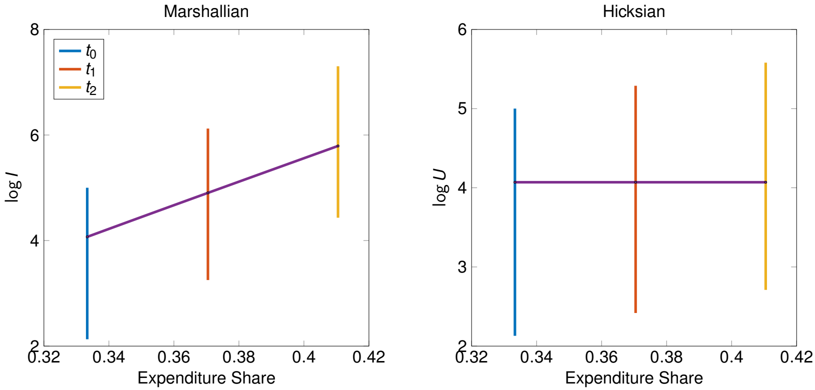
RHS operator is a contraction mapping. Solution to integral equation is unique.

Graphical illustration

Plot budget share on some good against income or utility over time.



Homothetic Case



- Homothetic preferences, budget shares indep. of u

$$\log u(I, t) = \log I - \int_{t_0}^{t} \sum_i B_i(s) \frac{d \log p_{is}}{ds} ds$$

- Justifies deflators using aggregate budget shares.

Two methods to solve integral equation

- Solve fixed point via:
  1. Iterative procedure.
  2. Recursive procedure.
- Given discrete data, we interpolate budget shares $\boldsymbol{B}(I, t)$:
  - Between income bins I.
  - Between time periods t.
- As time periods and household incomes get closer together:
  - Interpolation error $\rightarrow 0$.
  - Solution of fixed point converges to true money-metric.

Iterative Implementation

- Need numerical solution for following integral equation:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i B_i(u^{-1}(u(I,t),s),s) \frac{d \log p_{is}}{ds} ds.$$

- Suppose we have data on grid of points $\{t_0, \ldots, t_M\}$.

$$\log u(I,t_n) \approx \log I - \sum_{m=0}^{n-1} \boldsymbol{B}(I_m^*, t_m) \cdot \Delta \log \boldsymbol{p}_{t_m}$$

where I_m^* satisfies $u(I_m^*, t_m) = u(I, t_{n-1})$, and $u(I, t_0) = I$.

- Calculate $u(I, t_n)$ for I's for which we can find I_m^* for $m \leq M - 1$.
- Converges to exact $u(I, t)$ as we approach continuous time limit.

Recursive Implementation

- Iterative procedure assumes $t_{n-1} \approx t_n$, but we can do a bit better.
- Call the outcome from iterative procedure $u^0(I, t)$.
- Start with $u^0(I, t)$ as guess and repeat until converges to $u^\infty(I, t)$.

Taste shocks

- If idiosyncratic (mean-zero) taste shocks at the HH level
  - average multiple HH's with similar income.
  - our approach recovers preferences in absence of taste shocks
- If unobservable changes to preferences that are not idiosyncratic, then we may have a problem.

Unobserved Taste shocks

- Suppose we observe

$$\tilde{\boldsymbol{B}}(I, t|\kappa) = \boldsymbol{B}(I, t) + \kappa \boldsymbol{\varepsilon}(I, t),$$

where $\varepsilon(I, t)$ is the error and κ controls size of error.

- $\tilde{u}(I, t|\kappa)$ is the solution to wrong (i.e. with error) integral equation.

Proposition

Suppose $Cov(\boldsymbol{\varepsilon}(I, s), d \log \boldsymbol{p}/ds) = 0$. *Then, around* $\kappa \approx 0$*,*

$$\tilde{u}(I, t|\kappa) \approx u(I, t).$$

- If error uncorrelated with price shocks, no bias to first order.

Taste shocks uncorrelated with Engel curve slopes

Proposition

Suppose $Cov(\partial \boldsymbol{B}(I,s)/\partial I, d\log \boldsymbol{p}/ds) = 0$. *Then, to a first-order approximation around* $\kappa \approx 0$*,*

$$\tilde{u}(I,t|\kappa) - u(I,t) \approx -\kappa \int_{t_0}^{t} Cov(\boldsymbol{\varepsilon}(u(I,t),s), d\log \boldsymbol{p}/ds)ds.$$

- If error uncorrelated with Engel curves, bias straightforward function of error.
- If budget shares are overstated for goods where prices rising faster, then estimated money metric downward biased.
- Need covariance with Engel curve slopes zero to avoid systematic errors in matching as we solve integral equation forward.

Agenda

Illustration with Artificial Data

Generalized non-homothetic CES preference from Comin et al (2021)

$$e(\boldsymbol{p}, U) = \left(\sum_i \omega_i \left(U^{\varepsilon_i} p_i \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Compensated budget shares:

$$b_i(\boldsymbol{p}, U) = \frac{\omega_i \left(U^{\varepsilon_i} p_i \right)^{1-\gamma}}{\sum_j \omega_j \left(U^{\varepsilon_j} p_j \right)^{1-\gamma}},$$

Money-metric:

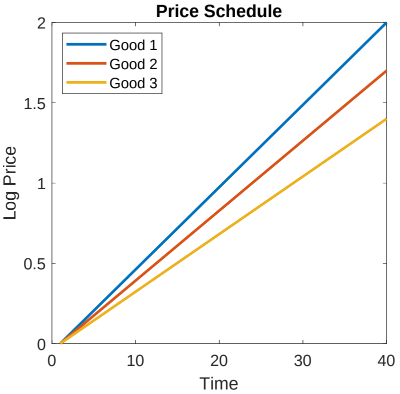
$$u(I, t) = \left(\sum_i \omega_i \left(V^{\varepsilon_i} p_{i,t_0} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}, \text{ where } I = e(\boldsymbol{p}_t, V),$$

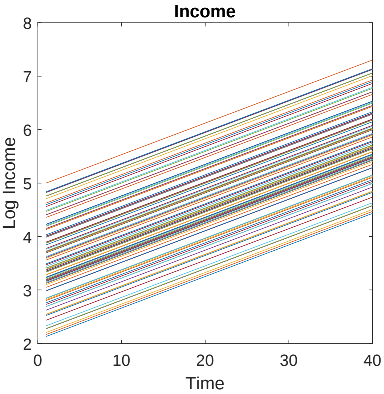
In terms of observables:

$$u(I, t) = I \times \left(\sum_i B(I, t) \left(\frac{p_{i,t_0}}{p_{i,t}} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Exogenous path of price and income

- Three goods
- Initialized by from 1974 UK income dist. with constant growth.





Convergence

- $\varepsilon = 0.2$, $\varepsilon = 1$, $\varepsilon = 1.65$ (Comin et al., 2021)
- $error = \max(|logU^{TRUE} - logU|)$.

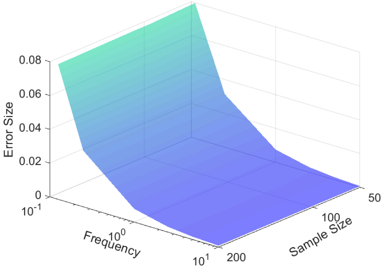


Figure: Iterative

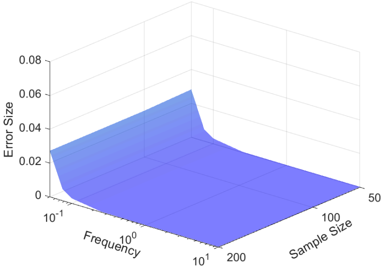


Figure: Recursive

- Error declines rapidly as frequency and sample size increase.

Agenda

UK Application

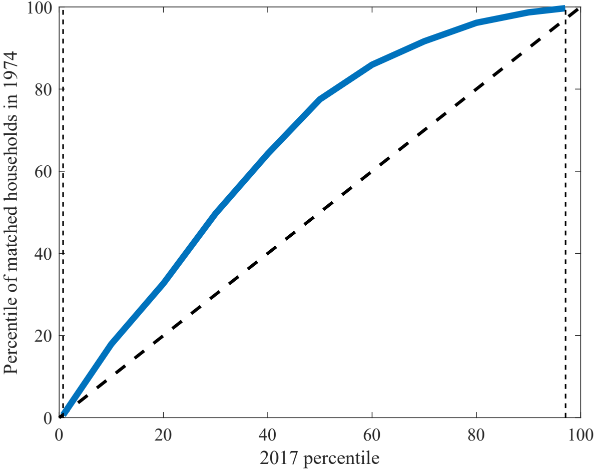
- Household Consumption budget data from 1974 to 2017
- 17 categories of goods with corresponding price index
- Engel curve: by fitting the following curve for each t and i.

$$B_{iht} = \alpha_{it} + \beta_{it} \log I_{ht} + \kappa_{it} \left(\log I_{ht} \right)^2 + \varepsilon_{iht},$$

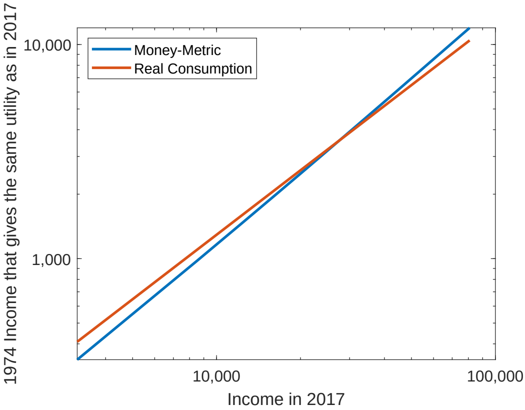
  where i is the good, h is the household, and t is the time period.

  - robustness: estimate in sub-samples by HH type

Matched Households in 2017 vs 1974



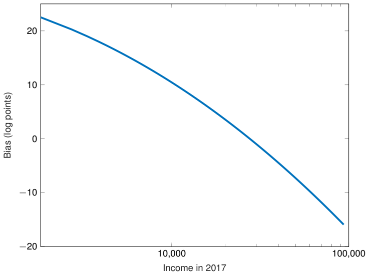
Recovered Money-Metric and Real Consumption



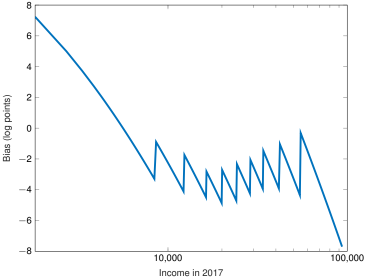
- Converts income in 1974 into equivalent in 2017 and vice versa.

by household type

Bias of Chained Price Index, 1974-2017



(a) Aggregate chain-weighted inflation and true cost-of-living inflation.



(b) Decile-specific chain-weighted inflation and true cost-of-living.

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Incorporating Unobserved Price Changes

- Partition set of goods into X and Y.
- Prices and spending on X observed, those in Y are not.
- Suppose preferences are *indirectly separable*:

$$e(\boldsymbol{p}, U) = e(e^X(\boldsymbol{p}^X, U), e^Y(\boldsymbol{p}^Y, U), U),$$

- Compensated elasticity of substitution btw X and Y: $\sigma(\boldsymbol{p}, U)$
  - Elasticity different to 1, almost everywhere.

Money-Metric with Missing Prices

Proposition (Money-Metric with Missing Prices)

Under assumptions, $u(I,t)$ solves the following integral equation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} b_{Xi}(\boldsymbol{p}_s, u(I,t)) \frac{d \log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d \log b_X(\boldsymbol{p}_s, u(I,t))/ds}{\sigma(\boldsymbol{p}_s, u(I,t)) - 1} ds$$

where

$$b_{Xi}(\boldsymbol{p}_s, u(I,t)) = B_{Xi}(u^{-1}(u(I,t),s),s),$$

$$b_X(\boldsymbol{p}_s, u(I,t)) = B_X(u^{-1}(u(I,t),s),s).$$

- Infer missing prices using observed prices, *compensated* changes in expenditures, and elasticity.
- Compensation requires solving a fixed-point problem.

Homothetic case

- Homothetic case easy because no fixed point:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} B_{Xi}(\boldsymbol{p}_s) \frac{d \log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d \log B_X / ds}{\sigma(\boldsymbol{p}_s) - 1}.$$

- If also σ constant, then becomes Feenstra (1994) adjustment:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} b_{Xi}(\boldsymbol{p}_s) \frac{d \log p_{is}}{ds} ds - \frac{\log b_X(\boldsymbol{p}_t) - \log b_X(\boldsymbol{p}_{t_0})}{\sigma - 1}.$$

Non-homothetic case

- Relative to Feenstra, elasticity & change in share compensated:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} B_{Xi}(I_s^*, s) \frac{d \log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d \log B_X(I_s^*, s)/ds}{\sigma(\boldsymbol{p}_s, u(I,t)) - 1} ds,$$

where I_s^* is implictly defined by $u(I_s^*, s) = u(I, t)$.

Identification of $\sigma(B_x, u)$

- Estimate

$$\Delta \log B_X(h,t) = \varepsilon_X \sum_{i \in X} B_{Xi}(h,t) \Delta \log p_{it} + \text{controls} + \text{error},$$

where h is quantile of expenditure distribution and t is time.

- ε_X is uncompensated elasticity of B_X wrt price of X bundle.

- Paper shows that

$$\sigma(\boldsymbol{p}_s, u(I,t)) = 1 - \frac{\varepsilon_X(I_s^*, s) + B_X(I_s^*, s) \sum_{i \in X} (\eta_i(I_s^*, s) - 1) B_{Xi}(I_s^*, s)}{1 - B_X(I_s^*, s)},$$

where I_s^* is defined by $u(I_s^*, s) = u(I, t)$.

Estimation of ε_X in UK data

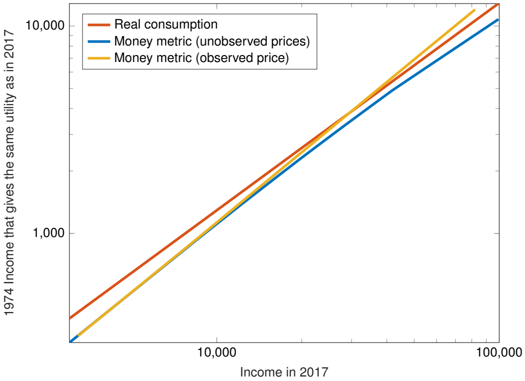
- Y is leisure services and X is other goods.

Table: Elasticity of budget share of X with respect to price index of X

	(1) OLS	(2) IV	(3) OLS	(4) IV
$\sum_{i \in X} B_{Xi}(h,t)\Delta \log p_{it}$	0.144**	0.073***	0.146**	0.061***
	(0.069)	(0.019)	(0.069)	(0.021)
$\sum_{i \in X} B_{Xi}(h,t)\Delta \log p_{it} \times 1(h \geq \text{median})$			0.005	0.025
			(0.007)	(0.039)
F-stat		403,945		177,760
Quantile FE	Y	Y	Y	Y
Year FE	Y	N	Y	N
Obs	41,000	41,000	41,000	41,000

Notes: Columns (2) and (4) use the log difference in world oil prices as an instrument. All lags are two-year differences (results are similar for annual and triennial differences). The sample years are 1974-2017. Standard errors are clustered at the household quantile level (we have 1000 quantiles). Two and three stars indicate statistical significance at the 5% and 1% level.

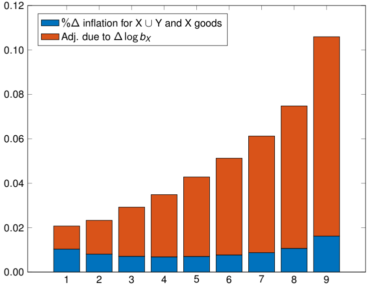
UK Money Metric with Unobserved Prices



- The rich are less well-off than baseline, but poor are unchanged.

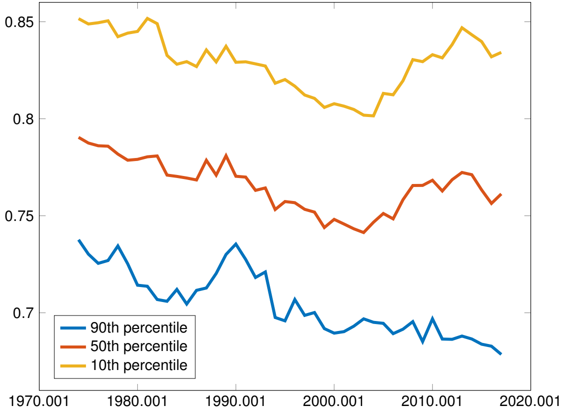
Decomposition

Figure: Log difference in estimated money metrics under observed and unobserved prices by decile of the expenditures distribution.



- Differences can be split into two terms:
  - Difference in average inflation between X goods and $X + Y$ goods,
  - Changes in the compensated share of expenditures on X goods.
- Most of the action due to the change in the compensated share.

Compensated share of *X* goods by 2017 percentile



- Compensated share of services constant for poor, rising for rich.
- X and Y complements
- Rich not as well off as service price data suggests.

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Conclusion

- Develop non-parametric method to recover static money-metric.
- Inflation understated for $<$ 60th percentile of spending in UK.
- Extension with missing prices under separability assumption.

Money-metric and compensated budget shares

- By definition of the expenditure function

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I + \log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) - \log e(\boldsymbol{p}, v(\boldsymbol{p}, I)).$$

- Gradient theorem

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I + \int_{\boldsymbol{p}}^{\bar{\boldsymbol{p}}} \sum_{i \in N} \frac{\partial \log e(\xi, v(\boldsymbol{p}, I))}{\partial \log \xi_i} d \log \xi_i.$$

- Using Shephard's lemma, $\frac{\partial \log e(\xi, v(\boldsymbol{p}, I))}{\partial \log \xi_i} = b_i(\xi, v(\boldsymbol{p}, I))$,

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I - \int_{\bar{\boldsymbol{p}}}^{\boldsymbol{p}} \sum_{i \in N} b_i(\xi, v(\boldsymbol{p}, I)) d \log \xi_i.$$

- Use compensated b/c MM compensates to be indifferent with $(\boldsymbol{p}, I)$.

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Money-Metric by Household Type

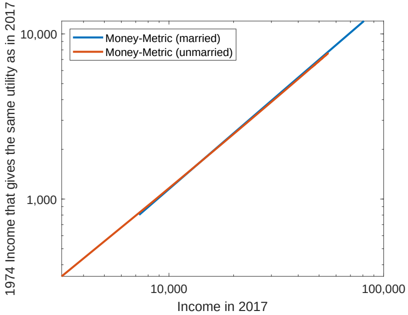

Figure: Married and unmarried

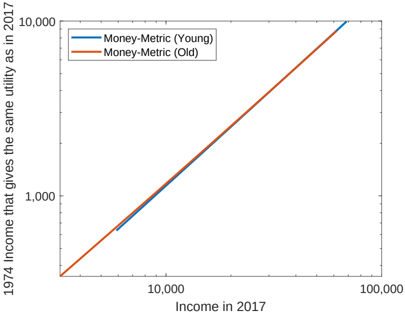

Figure: Above and below median age

Figure: Money-metric $e(\boldsymbol{p}_{1974}, v(\boldsymbol{p}_{2017}, I_{2017}))$ by household characteristic (annualized pounds, log scale) for the UK data

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