A Fixed Point Approach to Measuring Growth

David Baqaee   Ariel Burstein   Yasutaka Koike-Mori

UCLA

March 2023

Introduction

- Money-metric utility function backbone of welfare economics:
  - Converts income under different prices into equivalent income under some base prices,
  - cardinalization of utility with interpretable units,
  - essential input to measure growth & cost-of-living.
- To construct MM deflators, we need to weight price changes every period using "compensated" budget shares.
- Compensated budget shares unobserved, two options:
  - Assume homotheticity so that compensated = uncompensated (justifies national accounts),
  - Assume strong functional form and estimate.

What We Do

- Develop non-parametric method to recover money-metric utility using prices & repeated cross-section of household expenditures.
- Key assumption: preferences stable in time series & x-section.
- Recover MM utility as solution to a fixed-point problem:
  - If you know Hicksian demand, then you know *MM* via chaining.
  - If you know *MM*, then you know Hicksian demand by matching "compensated" households in the x-section over time.
- Extension with missing prices under separability assumption.
  - generalizes Feenstra (1994) to non-homoth., non-CES.
- App: inflation understated for $<$ 60th percentile of income in UK.

Selected Literature

- Non-parametric methods to estimate money-metric:

  Blundell et al (2003): calculate bounds using revealed-choice arguments.

  Jaravel & Lashkari (2021): different method and requires fixed support.

  Our approach calculates point estimate, does not require fixed support, and allows for unobserved prices.

- Welfare measurement under missing prices using Engel Curves:

  Costa (2001), Hamilton (2001), Atkin et al. (2020)

  Our approach has distinct intuition, assumptions, and data requirements.

- Welfare measurement with parametric demand system

  Boppart (2014), Comin et al. (2021), Fan et al. (2022), Matsuyama (2000), Fajgelbaum et al. (2011), Straub (2019), Auer et. al. (2022)

  Our approach can be applied for ex-post measurement, not for counterfactuals.

Agenda

Theory

Illustration with Artificial Data

UK Application

Unobserved prices

Conclusion

Agenda

Theory

Illustration with Artificial Data

UK Application

Unobserved prices

Conclusion

Setup

- Preferences $\succeq$ defined over $\boldsymbol{c}$ in $\mathbb{R}^N$ represented by $U(\boldsymbol{c})$.
- Indirect utility function

$$v(\boldsymbol{p}, I) = \max_{\boldsymbol{c}}\{U(\boldsymbol{c}) : \boldsymbol{p} \cdot \boldsymbol{c} \leq I\},$$

- Expenditure function

$$e(\boldsymbol{p}, u) = \min_{c}\{\boldsymbol{p} \cdot \boldsymbol{c} : U(\boldsymbol{c}) \geq u\}.$$

Definition (Money-Metric Utility)

Given base prices $\bar{\boldsymbol{p}}$, MM function maps $(\boldsymbol{p}, I)$ to $e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I))$.

Setup

- Preferences $\succeq$ defined over $\boldsymbol{c}$ in $\mathbb{R}^N$ represented by $U(\boldsymbol{c})$.
- Indirect utility function

$$v(\boldsymbol{p}, I) = \max_{\boldsymbol{c}} \{U(\boldsymbol{c}) : \boldsymbol{p} \cdot \boldsymbol{c} \leq I\},$$

- Expenditure function

$$e(\boldsymbol{p}, u) = \min_{c} \{\boldsymbol{p} \cdot \boldsymbol{c} : U(\boldsymbol{c}) \geq u\}.$$

Definition (Money-Metric Utility)

Given base prices $\bar{\boldsymbol{p}}$, MM function maps $(\boldsymbol{p}, I)$ to $e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I))$.

- MM is an indirect utility function with monetary cardinalization.
  - $e(\bar{\boldsymbol{p}}, \cdot)$ ranks budget sets, measures growth under base prices $\bar{\boldsymbol{p}}$.
  - $e(\cdot, v(\boldsymbol{p}, I))$ is cost of living index to achieve $v(\boldsymbol{p}, I)$ under diff. $\bar{\boldsymbol{p}}$.

Observed data

- Suppose that for each $t \in [t_0, T]$ we observe
  - Prices $\boldsymbol{p}_t \in \mathbb{R}^N$, assumed smooth over time.
  - Expenditure shares $\boldsymbol{B}(I, t) \in \mathbb{R}^N$ for consumers with preferences $\succeq$ and income levels $I \in [\underline{I}_t, \bar{I}_t]$.
- Repeated cross-section, not a panel.
- In what follows use base prices $\boldsymbol{p}_{t_0}$, w.l.o.g.
- Objective: money-metric $u(I, t) \equiv e(\boldsymbol{p}_{t_0}, v(\boldsymbol{p}_t, I))$ for $t \in [t_0, T]$.

Recasting money metric as a fixed point problem

- Let $b_i(u, t)$ denote Hicksian budget share of i with prices $\boldsymbol{p}_t$.

1. Given Hicksian demand, obtain MM by chaining: derivation

$$\log u(I, t) = \log I - \int_{t_0}^{t} \sum_i b_i(u(I, t), s) \frac{d \log p_{is}}{ds} ds$$

Recasting money metric as a fixed point problem

- Let $b_i(u,t)$ denote Hicksian budget share of i with prices $\boldsymbol{p}_t$.

1. Given Hicksian demand, obtain MM by chaining: derivation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i b_i(u(I,t),s) \frac{d \log p_{is}}{ds} ds$$

2. Given MM, obtain Hicksian demand using observed demand:

$$b_i(u(I,t),s) = B_i(I^*(I,t,s),s),$$

where "matched" household $I^*(I,t,s)$ solves $u(I^*,s) = u(I,t)$.

Recasting money metric as a fixed point problem

- Let $b_i(u, t)$ denote Hicksian budget share of i with prices $\boldsymbol{p}_t$.

1. Given Hicksian demand, obtain MM by chaining: derivation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i b_i(u(I,t), s) \frac{d \log p_{is}}{ds} ds$$

2. Given MM, obtain Hicksian demand using observed demand:

$$b_i(u(I,t), s) = B_i(I^*(I,t,s), s),$$

where "matched" household $I^*(I,t,s)$ solves $u(I^*, s) = u(I,t)$.

Equivalently,

$$b_i(\cdot, s) = B_i(u^{-1}(\cdot, s), s).$$

Recasting money metric as a fixed point problem

- Let $b_i(u,t)$ denote Hicksian budget share of i with prices $\boldsymbol{p}_t$.

1. Given Hicksian demand, obtain MM by chaining: derivation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i b_i(u(I,t),s) \frac{d \log p_{is}}{ds} ds$$

2. Given MM, obtain Hicksian demand using observed demand:

$$b_i(u(I,t),s) = B_i(I^*(I,t,s),s),$$

where "matched" household $I^*(I,t,s)$ solves $u(I^*,s) = u(I,t)$.
Equivalently,

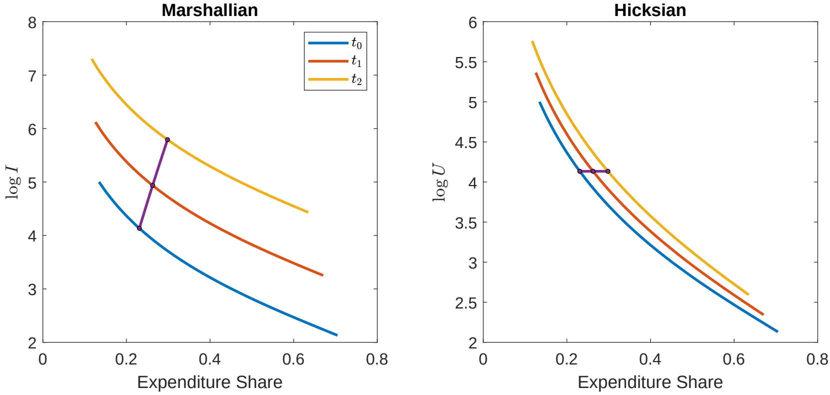
$$b_i(\cdot,s) = B_i(u^{-1}(\cdot,s),s).$$

- 1.+ 2. $\Longrightarrow$ $u(I,t)$ is a fixed point of integral equation:

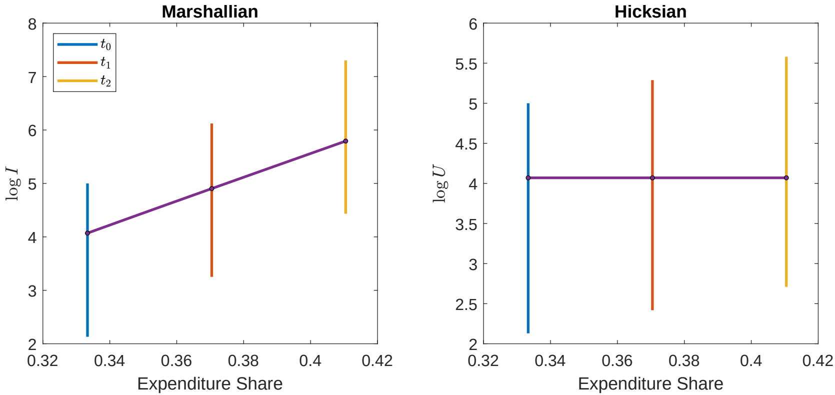
$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i B_i(u^{-1}(u(I,t),s),s) \frac{d \log p_{is}}{ds} ds.$$

Graphical illustration

- Plot share on some good against income or utility over time.
- Fixed point recovers the purple line.
- Each t: find income s.t. observed share $=$ Hicksian share.



Homothetic Case



- Homothetic preferences, budget shares indep. of u

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i B_i(s) \frac{d \log p_{is}}{ds} ds$$

- Justifies deflators using aggregate budget shares.

Iterative Implementation

- Need numerical solution for following integral equation:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_i B_i(u^{-1}(u(I,t),s),s)\frac{d\log p_{is}}{ds}ds.$$

- Suppose we have data on a grid of points $\{t_0,\ldots,t_M\}$ where $t_n < t_{n+1}$, with $t_M = T$.

$$\log u(I,t_n) \approx \log I - \sum_{s=0}^{n-1} \boldsymbol{B}(I_s^*,t_s)\cdot\Delta\log \boldsymbol{p}_{t_s}$$

  where I_s^* satisfies $u(I_s^*,t_s) = u(I,t_{n-1})$ and boundary $u(I,t_0) = I$.

- To illustrate intuition, let's look at step-by-step construction.

Period 1

- Start with the boundary condition $u(I, t_0) = I$.

$$\log u(I, t_1) \approx \log I - \boldsymbol{b}(u(I, t_0), t_0) \cdot \Delta \log \boldsymbol{p}_{t_1} = \log I - \boldsymbol{B}(I, t_0) \cdot \Delta \log \boldsymbol{p}_{t_1}.$$

- Can only compute $u(I, t_1)$ for values of $I \in [\underline{I}_0, \bar{I}_0]$.

- With $u(I, t_1)$ in hand, construct Hicksian budget shares for t_1:

$$\boldsymbol{b}(u(I, t_1), t_1) = \boldsymbol{B}(I, t_1).$$

Period 2

- Calculate

$$\log u(I,t_2) \approx \log I - \boldsymbol{b}(u(I,t_1),t_1)\cdot\Delta\log \boldsymbol{p}_{t_2} - \boldsymbol{b}(u(I,t_1),t_0)\cdot\Delta\log \boldsymbol{p}_{t_1}$$

- Construct Hicksian budget shares for period t_2:

$$\boldsymbol{b}(u(I,t_2),t_2) = \boldsymbol{B}(I,t_2).$$

- Continue this iterative process until t_M.
- Can only calculate $u(I,t)$ for those I's for which $\boldsymbol{B}(u^{-1}(u(I,t),s),s)$ is observed for all $s \leq t$.

Recursive Implementation

- Iterative procedure assumes $t_{n-1} \approx t_n$, but we can do a bit better.
- Call the outcome from iterative procedure $u^0(I, t)$.
- Start with $u^0(I, t)$ as guess and repeat until converges to $u^\infty(I, t)$.

Taste shocks

- If idiosyncratic taste shocks at the HH level
  - average multiple HH's with similar income.
  - our approach recovers preferences in absence of taste shocks
- If taste shock is a function of observable characteristics (e.g. HHs with and without children)
  - split sample and apply method to each sample separately
- If unobservable changes to preferences that are not idiosyncratic, then our method cannot be used
  - likely to be less prevalent with sectoral data

Agenda

Illustration with Artificial Data

Generalized non-homothetic CES preference from Fally (2022)

$$e(\boldsymbol{p}, u) = \left(\sum_i \omega_i \left(u^{\varepsilon_i} p_i \right)^{1-\gamma(u)} \right)^{\frac{1}{1-\gamma(u)}}.$$

Hicksian budget shares:

$$b_i(\boldsymbol{p}, u) = \frac{\omega_i \left(u^{\varepsilon_i} p_i \right)^{1-\gamma(u)}}{\sum_j \omega_j \left(u^{\varepsilon_j} p_j \right)^{1-\gamma(u)}},$$

Money-metric:

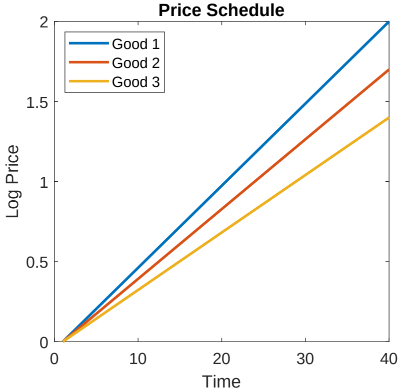
$$u(I, t) = \left(\sum_i \omega_i \left(v^{\varepsilon_i} p_{i,t_0} \right)^{1-\gamma(v)} \right)^{\frac{1}{1-\gamma(v)}}, \text{where } I = e(\boldsymbol{p}_t, v),$$

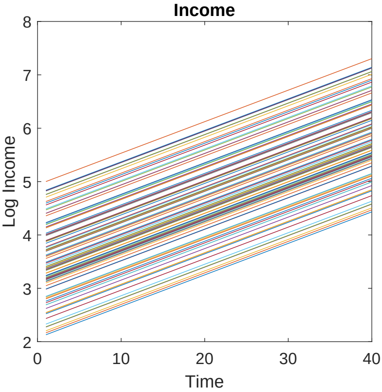
In terms of observables:

$$u(I, t) = I \times \left(\sum_i B(I, t) \left(\frac{p_{i,t_0}}{p_{i,t}} \right)^{1-\Gamma(I,t)} \right)^{\frac{1}{1-\Gamma(I,t)}}.$$

Exogenous path of price and income

- Three goods
- Initialized by from 1974 UK income dist. with constant growth.





Convergence

- $\varepsilon = 0.2$, $\varepsilon = 1$, $\varepsilon = 1.65$ (Comin et al., 2021)
- $\gamma(u) = 10 - 2\log u$ (Auer et al., 2021)
- $error = \max(|logU^{TRUE} - logU|)$.

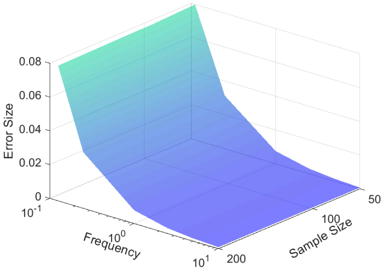


Figure: Iterative

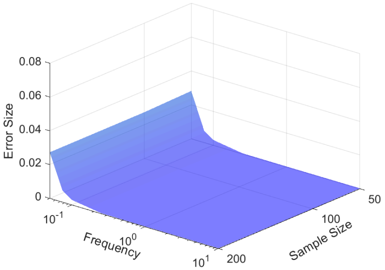


Figure: Recursive

- Error declines rapidly as frequency and sample size increase.

Agenda

UK Application

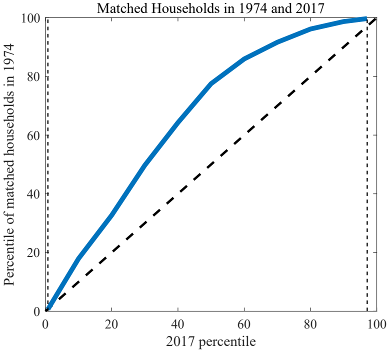
- Household Consumption budget data from 1974 to 2017
- 17 categories of goods with corresponding price index
- Engel curve: by fitting the following curve for each t and i.

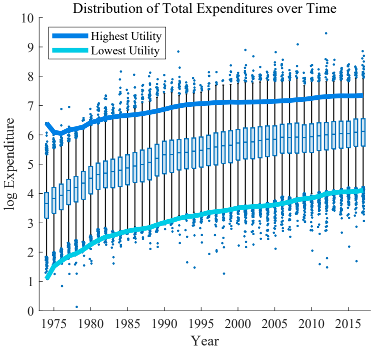
$$B_{iht} = \alpha_{it} + \beta_{it} \log I_{ht} + \kappa_{it} \left(\log I_{ht}\right)^2 + \varepsilon_{iht},$$

  where i is the good, h is the household, and t is the time period.

  - robustness: estimate in sub-samples by HH type

Matched Household Over Time





Recovered Money-Metric and Real Consumption

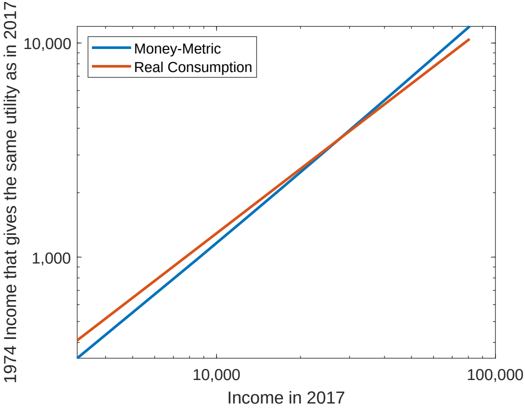


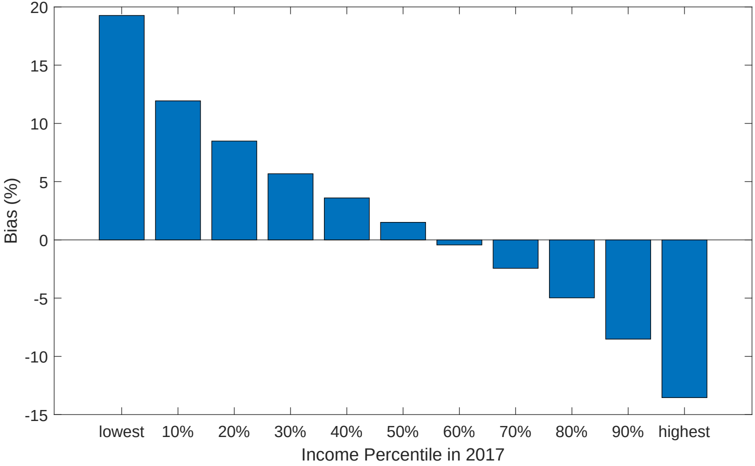
Figure: $e(\boldsymbol{p}_{1974}, v(\boldsymbol{p}_{2017}, I_{2017}))$ and real consumption using aggregate chain-weighted inflation between 1974 to 2017 (annualized pounds).

- Converts income in 1974 into equivalent in 2017 and vice versa.

by household type

Bias of Aggregate Chained Price Index

- log difference between real consumption and money-metric in 1974 for different percentiles of the income distribution in 2017.



Agenda

Incorporating Unobserved Price Changes

- Partition set of goods into X and Y.
- Prices and spending on X observed, those in Y are not.

Assumption (Indirect Separability)

Suppose preferences are indirectly separable:

$$e(\boldsymbol{p}, u) = e(e^X(\boldsymbol{p}^X, u), e^Y(\boldsymbol{p}^Y, u), u),$$

where $\boldsymbol{p}^X$ and $\boldsymbol{p}^Y$ are vectors of prices in X and Y, and e^X and e^Y are non-decreasing and homogeneous of degree one in prices.

Incorporating Unobserved Price Changes

- Partition set of goods into X and Y.
- Prices and spending on X observed, those in Y are not.

Assumption (Indirect Separability)

Suppose preferences are indirectly separable:

$$e(\boldsymbol{p}, u) = e(e^X(\boldsymbol{p}^X, u), e^Y(\boldsymbol{p}^Y, u), u),$$

where $\boldsymbol{p}^X$ and $\boldsymbol{p}^Y$ are vectors of prices in X and Y, and e^X and e^Y are non-decreasing and homogeneous of degree one in prices.

Assumption (Monotone Budget Share)

Suppose that $b_X(\boldsymbol{p}, u)$ is strictly monotone in the price of some $i \in X$.

Incorporating Unobserved Price Changes

Definition (Relative Budget Shares)

Denote *relative* Marshallian and Hicksian shares on $i \in X$ by

$$B_{Xi}(I,t) = \frac{B_i(I,t)}{B_X(I,t)}, \qquad \text{and} \qquad b_{Xi}(\boldsymbol{p},u) = \frac{b_i(\boldsymbol{p},u)}{b_X(\boldsymbol{p},u)}.$$

Let $\sigma(\boldsymbol{p},u)$ be compensated elasticity of substitution between X and Y

Money-Metric with Missing Prices

Proposition (Money-Metric with Missing Prices)

Under assumptions, $u(I,t)$ solves the following integral equation

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i\in X} b_{Xi}(\boldsymbol{p}_s, u(I,t)) \frac{d\log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d\log b_X(\boldsymbol{p}_s, u(I,t))/ds}{\sigma(b_X(\boldsymbol{p}_s, u(I,t)), u(I,t)) - 1} ds$$

where

$$b_{Xi}(\boldsymbol{p}_s, u(I,t)) = B_{Xi}(u^{-1}(u(I,t),s),s),$$

$$b_X(\boldsymbol{p}_s, u(I,t)) = B_X(u^{-1}(u(I,t),s),s).$$

Intuition & Examples

- Infer missing prices using observed prices, *compensated* changes in expenditures, and elasticity.
- Compensation requires solving a fixed-point problem.
- Homothetic case is easy:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} B_{Xi}(\boldsymbol{p}_s) \frac{d \log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d \log B_X / ds}{\sigma(B_X(s)) - 1}.$$

- If σ is constant, then this becomes Feenstra (1994) adjustment:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} b_{Xi}(\boldsymbol{p}_s) \frac{d \log p_{is}}{ds} ds - \frac{\log b_X(\boldsymbol{p}_t) - \log b_X(\boldsymbol{p}_{t_0})}{\sigma - 1}.$$

Intuition & Examples

- Relative to Feenstra, elasticity & change in share compensated:

$$\log u(I,t) = \log I - \int_{t_0}^{t} \sum_{i \in X} B_{Xi}(I_s^*, s) \frac{d \log p_{is}}{ds} ds - \int_{t_0}^{t} \frac{d \log B_X(I_s^*, s)/ds}{\sigma(B_X(I_s^*, s), u(I,t)) - 1} ds,$$

  where I_s^* is implictly defined by $u(I_s^*, s) = u(I,t)$.

- The necessary compensation satisfies

$$\underbrace{\frac{d \log B_X(I_s^*, s)}{ds}}_{\substack{\Delta \text{ budget share due to prices} \\ \text{holding fixed utility}}} = \underbrace{\frac{\partial \log B_X(I_s^*, s)}{\partial s}}_{\substack{\Delta \text{ budget share due to prices} \\ \text{holding fixed income}}} + \underbrace{\frac{\partial \log B_X(I_s^*, s)}{\partial I} \frac{dI_s^*}{ds}}_{\substack{\Delta \text{ budget share} \\ \text{due to compensating income}}}.$$

- When preferences homothetic, second term disappears.

Example of $\sigma(B_x, u)$

Example (Indirect Addilog)

Suppose expenditure function is implicitly defined as

$$f(u) = \frac{\omega_X}{\sigma_X - 1}\left(\frac{e(\boldsymbol{p}, u)}{e^X(\boldsymbol{p}^X, u)}\right)^{\sigma_X - 1} + \frac{\omega_Y}{\sigma_Y - 1}\left(\frac{e(\boldsymbol{p}, u)}{e^Y(\boldsymbol{p}^Y, u)}\right)^{\sigma_Y - 1},$$

where $f(\cdot)$ is an increasing function. Then

$$\sigma(b_X(\boldsymbol{p}, u), u) = b_X(\boldsymbol{p}, u)\sigma_Y + (1 - b_X(\boldsymbol{p}, u))\sigma_X,$$

which varies both as a function of utility and as a function of prices.

Identification of $\sigma(B_X, u)$

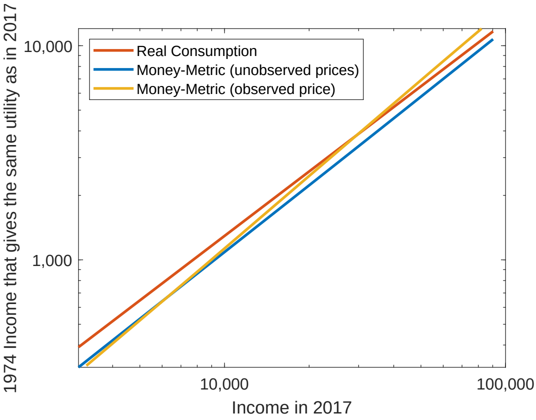
- Paper shows we can identify σ globally given:
  - income elasticity for each $i \in X$ at time t,
  - uncompensated price elasticity of X w.r.t. p_k for some $k \in X$ at t.
- Estimating income elasticities easy curve fitting in each t.
- Estimating price elasticity more challenging, but we only require a single elasticity per income group and period.
- Let $\eta_i(I,t) - 1 = \partial \log B_i(I,t) / \partial \log I$ for each $i \in X$, and $1 - \varepsilon_{Xk}(I,t) = \partial \log B_X(I,t) / \partial \log p_k$ for any $k \in X$.

$$\sigma(b_X(\boldsymbol{p}_s, u(I,t)), u(I,t)) = 1 - \frac{1 - \varepsilon_{Xk}(I_s^*, s) + B_k(I_s^*, s) \sum_{i \in X} (\eta_i(I_s^*, s) - 1) B_{Xi}(I_s^*, s)}{(1 - B_X(I_s^*, s)) B_{Xk}(I_s^*, s)},$$

where I_s^* is implicitly defined by $u(I_s^*, s) = u(I,t)$. Nh-CES

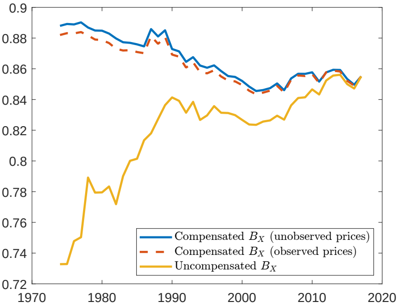
UK Application with Unobserved Price

- Assume constant $\sigma = 0.25$ from Comin et al. (2021)
- Suppose "leisure goods & services" prices mismeasured.

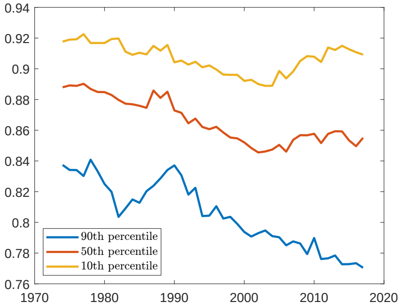


- The rich are less well-off than baseline, but poor are unchanged.

Share of X goods



(a) HH with median 2017 income.



(b) Different 2017 incomes.

- Compensated share of services constant for poor, rising for rich.
- Hence, rich not as well off as service price data suggests.

Agenda

Conclusion

- Develop non-parametric method to recover money-metric.
- Extension with missing prices under separability assumption.
- Application: inflation understated for $<$ 60th percentile of income in UK.

Money-metric and Hicksian budget shares

- By definition of the expenditure function

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I + \log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) - \log e(\boldsymbol{p}, v(\boldsymbol{p}, I)).$$

- Fundamental theorem of calculus

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I + \int_{\boldsymbol{p}}^{\bar{\boldsymbol{p}}} \sum_{i \in N} \frac{\partial \log e(\xi, v(\boldsymbol{p}, I))}{\partial \log \xi_i} d \log \xi_i.$$

- Using Shephard's lemma, $\frac{\partial \log e(\xi, v(\boldsymbol{p}, I))}{\partial \log \xi_i} = b_i(\xi, v(\boldsymbol{p}, I))$,

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I - \int_{\bar{\boldsymbol{p}}}^{\boldsymbol{p}} \sum_{i \in N} b_i(\xi, v(\boldsymbol{p}, I)) d \log \xi_i.$$

- Use Hicksian b/c MM compensates to be indifferent with $(\boldsymbol{p}, I)$.

back

Cost-of-Living

Definition (Cost-of-Living)

Holding fixed some reference budget set $(\bar{\boldsymbol{p}}, \bar{I})$, the *cost-of-living* index maps $\boldsymbol{p}$ to

$$e(\boldsymbol{p}, v(\bar{\boldsymbol{p}}, \bar{I})).$$

back

General Version

Proposition (Money-Metric and Cost-of-Living)

The money-metric of a budget set $(\boldsymbol{p}, I)$ in terms of $\bar{\boldsymbol{p}}$ prices can be expressed as

$$\log e(\bar{\boldsymbol{p}}, v(\boldsymbol{p}, I)) = \log I - \int_{\bar{\boldsymbol{p}}}^{\boldsymbol{p}} \sum_{i \in N} b_i(\xi, v(\boldsymbol{p}, I)) d \log \xi_i.$$

The cost-of-living for a budget set $(\bar{\boldsymbol{p}}, \bar{I})$ in terms of $\boldsymbol{p}$ prices can be expressed as

$$\log e(\boldsymbol{p}, v(\bar{\boldsymbol{p}}, \bar{I})) = \log \bar{I} + \int_{\bar{\boldsymbol{p}}}^{\boldsymbol{p}} \sum_{i \in N} b_i(\xi, v(\bar{\boldsymbol{p}}, \bar{I})) d \log \xi_i.$$

Money-Metric by Household Type

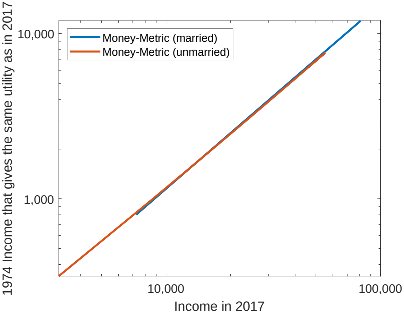


Figure: Married and unmarried

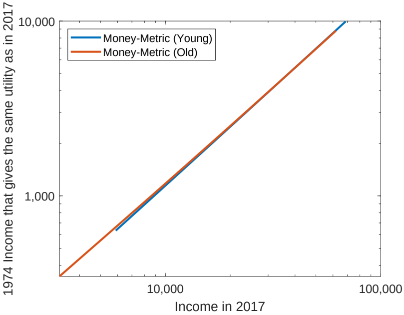


Figure: Above and below median age

Figure: Money-metric $e(\boldsymbol{p}_{1974}, v(\boldsymbol{p}_{2017}, I_{2017}))$ by household characteristic (annualized pounds, log scale) for the UK data

back

Blundell (2003): Artificial Data Results

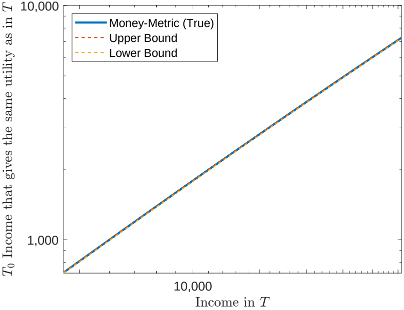


Figure: Constant Elasticity

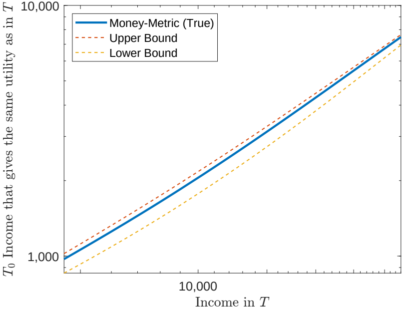


Figure: Variable Elasticity

Comparison with Jaravel & Lashkari (2022): Quintiles

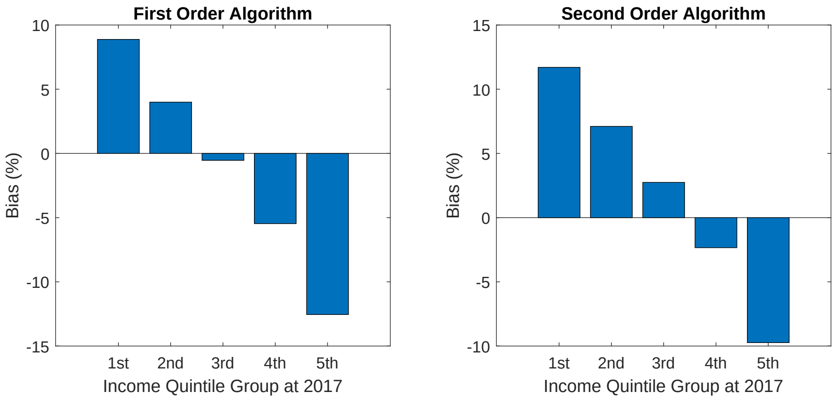


Figure: Results of the first/second order algorithm of Jaravel and Lashkari (2022) with K=1 to the aggregated (by quintile) UK household data: Log difference between real consumption and the money-metric

Comparison with Jaravel & Lashkari (2022): Disaggregated

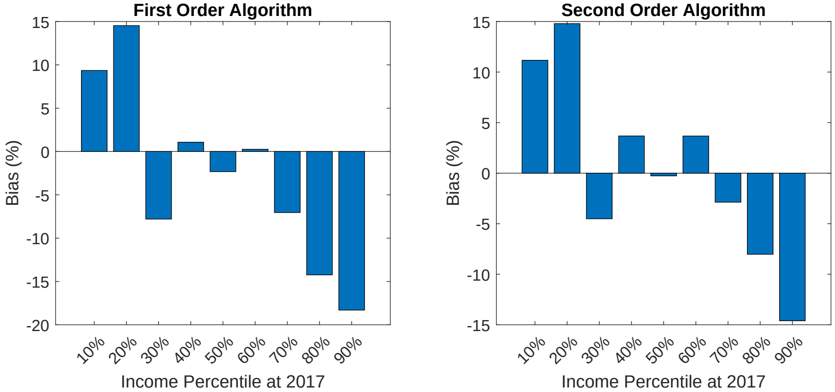


Figure: Results of the first/second order algorithm of Jaravel and Lashkari (2022) with K=1 to the disaggregated UK household data: Log difference between real consumption and the money-metric

Non-homothetic CES

- Suppose that $e(e^X(\boldsymbol{p}^X, u), e^Y(\boldsymbol{p}^Y, u), u)$ is

$$e(x, y, u) = \left(\omega_X u^{\xi_X} x^{1-\gamma(u)} + \omega_Y u^{\xi_Y} y^{1-\gamma(u)}\right)^{\frac{1}{1-\gamma(u)}}.$$

  In this case, $\sigma(b_X, u)$ varies as a function of utility but not as a function of relative prices.

- Hence $\sigma(\cdot)$ is determined by:

$$\sigma(I) = 1 - \frac{1 - \varepsilon_k(I, t_0) + B_k(I, t_0) \sum_{i \in X} (\eta_i(I, t_0) - 1) B_{Xi}(I, t_0)}{(1 - B_X(I, t_0)) B_{Xk}(I, t_0)},$$

  where $1 - \varepsilon_k(I, t_0)$ is the uncompensated elasticity of B_X with respect to the price of k and $\eta_k(I, t_0)$ is the income elasticity for $k \in X$ at time t_0 for households with income I.

back

Decomposition