# Misspecified Wage Profiles and Estimates of Labor Supply Elasticities of Older Workers

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#### Abstract

Structural labor supply models rely on accurate estimates of the age profile of offered wages. This paper explores the consequences of misspecifying the wage profile for estimates of labor supply elasticities. I show that misspecification of the offered wage profile for older workers leads to structural estimates of the intertemporal elasticity of substitution of labor supply that are biased upwards by 30 to 130 percent, depending on the specification.

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# 1 Introduction

Structural estimation and calibration of labor supply models have become more prevalent in recent years. These models rely on estimates for the laws of motion of exogenous state variables that are often obtained outside the model or taken from other studies in the literature. A key input to structural models of labor supply is the age profile of offered wages, also known as the *deterministic* or *predictable* component of wages. The most common specification for the wage profile used in the literature is hump-shaped over the life cycle. Moreover, participation decisions at older ages are often modeled as a response to the declining wage profile implied by the hump-shaped specification.

Casanova (2013) shows that the offered wage profile does not trend downwards at older ages, as implied by the hump-shaped specification. Instead, it is flat for full time worker, and drops discretely for those who transit into part-time work. Moreover, Casanova (2013) shows that wage and hours declines at older ages are determined simultaneously at the time of transition into part-time work for a majority of workers. Hence, retirement transitions are not a response to a exogenously-declining wage profile.

This paper investigates the implications of these results for structural estimates of the intertemporal elasticity of substitution of labor supply (*i.e.s.*). The magnitude of this parameter determines participation and hours responses to lifetime wages, and is crucial in studies of issues ranging from business cycles to tax or welfare reform. The *i.e.s.* has attracted considerable interest from both the labor and macro fields for the last 40 years (Lucas and Rapping (1969), Ghez and Becker (1975)). I show that estimates of the parameter that drives labor supply responses to lifetime wage changes, namely the intertemporal elasticity of substitution of labor supply (*i.e.s.*), are severely biased when a smoothly declining wage profile is used as a proxy for the flat offered wage path at older ages. I use artificial data generated from a structural model of consumption and labor supply choices of older workers to show that this type of misspecification leads to an upward bias in estimates of the *i.e.s.* of 30 to 130 percent, depending on the specification.

#### 2 Overview

Structural estimation and calibration have a long tradition in economics, and have become more prevalent with the advent of more powerful computers. When estimating or calibrating a structural model, it is common practice to rely on estimates for some of the model primitives that are obtained outside the model. Needless to say, if those externals estimates are biased, this will affect all subsequent estimates and predictions.

The age profile of offered wages is an example of a model primitive that is often estimated outside structural models of labor supply, or taken from other studies in the literature. The age-wage profile of interest for a model that endogenizes labor supply is the one measuring the wage increase or decrease anticipated by a worker, should he remain employed for an extra period. As has been discussed in previous sections, it is common to proxy this *of*-*fered* wage profile with the *average* wage profile, even though the two series will generally not overlap, particularly for older workers. In this section, I consider the consequences of such misspecification of the age-wage profile for the estimation of labor supply models. In particular, I study how it affects estimates of the *i.e.s.*.

Because of its crucial role determining intertemporal labor supply allocations, the estimation of the *i.e.s.* has long been a topic of interest in both the labor and macro literatures. Early studies identified the parameter from the covariation in observed hours and wages over the life-cycle (e.g., MaCurdy (1981), Browning, Deaton and Irish (1989) and Altonji (1986)) and concluded that its value was close to zero. More recently, several papers have argued that this covariation does not allow to determine whether labor supply responses (or lack thereof) are driven by the *i.e.s*, or by alternative factors such as liquidity constraints (Domeij and Flodén (2006)), human capital accumulation (Imai and Keane (2004)), or precautionary saving motives (Low(2005)). These papers have structurally estimated the *i.e.s.*, and argued that its value is considerably higher than suggested by the earlier literature.

In what follows, I discuss how sensitive structural estimates of the *i.e.s.* are to misspecification of the lifetime profile for offered wages. I do this in the context of a life-cycle model of consumption and labor supply choices that is consistent with the *voluntary retirement* model described in Casanova (2013).<sup>1</sup> I generate artificial data from the model, and use them to estimate the *i.e.s.* for different specifications of the offered wage profile.

#### 3 Model

Individuals consumption and leisure to maximize expected discounted utility:

<sup>&</sup>lt;sup>1</sup>Involuntary retirements are ignored to focus on those workers whose retirement decision responds most directly to wages, as opposed to labor supply constraints.

$$\max_{\{c_t\}_{t=t_0}^T, \{h_t\}_{t=t_0}^{R < T}} E_{t_0} \sum_{t=t_0}^T \beta^{(t-t_0)} \left\{ \frac{c_t^{(1-\rho)}}{1-\rho} + B_t \frac{l_t^{(1-\frac{1}{\gamma})}}{1-\frac{1}{\gamma}} \right\},\tag{1}$$

where  $c_t$  is consumption and  $l_t$  is leisure. Leisure is a linear function of hours of market work  $(h_t)$  that will be specified below. There are three possible choices for work hours: full-time work  $(h^F)$ , part-time work  $(h^P)$ , and zero. This coarse classification is sufficient to capture the choice between full-time work, partial and full retirement faced by older workers. The utility of consumption is a standard CRRA. Consumption and leisure are assumed separable for simplicity, but the marginal rate of substitution between leisure and consumption is allowed to vary with age through the parameter  $B_t$ .

Since I will want to compare my results to previous analysis of the *i.e.s.*, I have chosen a standard parameterization for the utility of leisure that follows Chang et al. (2011) and Rogerson and Wallenius (2009 and forthcoming), among others. The parameter  $\gamma$  measures the intertemporal elasticity of substitution of *leisure*. Notice that the intertemporal elasticity of substitution of labor supply is equal to  $\gamma$  multiplied by a non-negative factor. Hence all results regarding biases in estimates of  $\gamma$  translate directly into biases in the *i.e.s.* 

Individuals start their life at  $t_0$  and die with certainty in period T. They make consumption and hours decisions until age R < T. After age R they become permanently retired, and only choose consumption.

Lifetime utility is maximized subject to the following borrowing constraint:

$$A_{t+1} + c_t = \exp(w_t)h_t + SS_t + (1+r)A_t,$$
(2)

where  $w_t$  is the log offered hourly wage,  $A_t$  are asset holdings, r is the interest rate -assumed constant-, and  $SS_t$  represents retirement benefits. The latter are modeled as a fixed proportion of the deterministic component of wages, including the fixed effect. For simplicity, I abstract from issues related to pension eligibility and assume that retirement benefits are available at any age.<sup>2</sup> Individuals cannot borrow against future labor income or retirement income; that is, the borrowing constraint  $A_t \geq 0$  is imposed in all periods.

Full-time log wages evolve according to the following process:

 $<sup>^2\</sup>mathrm{Because}$  I only model the behavior of older workers, this assumption implies that retirement coverage is available from age 50 onwards.

$$w_{it} = f_i + W(t) + u_{it},$$
  
$$u_{it} \sim \text{Normal}(0, \sigma_u), \qquad (3)$$

where  $f_i$  is an individual fixed effect and W some function of age.<sup>3</sup> This specification is consistent with the equations estimated in Casanova (2013), and is also common in the literature (see, for example, French (2005)).<sup>4</sup> There is a wage penalty associated to part-time work, which is specified as in Chang et al. (2011): part-time workers' hourly wage is equal to  $(1 - \alpha)w_{it}$ .

So far, the model closely resembles others that have been used to study the sensitivity of estimates of the *i.e.s.* to different modeling assumptions. Chang et al. (2011), for example, use a similar model -with infinitely-lived households- to match the average parttime participation rate of prime-age men and women. When focusing on prime-age workers, a steady-state equilibrium concept is appropriate. In this case, instead, the focus is on the labor supply changes that take place over the retirement period. In order to capture these changes, I add two additional features to the model.

First, the parameter  $B_t$ , which in steady-state models is assumed constant, is allowed to change with age. As in French and Jones (2011), I assume that preferences for leisure increase linearly with age, that is:

$$B_t = b_0 + b_1 t \tag{4}$$

The increasing taste for leisure is enough to fit the declining trend in full-time participation with age, but not to fit both the proportion of workers that are retired and those working part time. Hence I also introduce a cost of work that varies with age and with the number of hours worked, i.e., it is different for part- and full-time workers.<sup>5</sup> This is consistent with

<sup>&</sup>lt;sup>3</sup>In the model, age and time are indistinguishable.

<sup>&</sup>lt;sup>4</sup>An alternative representation of the error assumes a random walk process instead of a fixed effect (Meghir and Pistaferri (2004)), allowing for persistent shocks to wages. This model focuses on individuals approaching retirement, who will be employed for a short number of years. With a short time horizon, the effects of persistent and transitory shocks to wages are similar, and for simplicity I choose to focus on the latter only.

<sup>&</sup>lt;sup>5</sup>Chang et al. (2011) exploit a different nonconvexity to fit part-time participation rates. Specifically, they calibrate a different value for the part-time wage penalty for each of the baseline values of the parameter  $\gamma$ . Here the value of the part-time wage penalty is fixed at the level estimated in Casanova (2013). I use the differential cost of work for part- and full-time workers as an alternative way of varying the relative value of part- and full-time work.

plenty of evidence from the retirement literature suggesting that workers value the flexibility associated to part-time jobs increasingly as they age (Hurd, 1996, Abraham and Houseman, 2004). As in French (2005) and French and Jones (2011), the cost of work is modeled as a loss of leisure. The amount of leisure enjoyed in every period is given by:

$$l_t = L - h_t - \phi_t,\tag{5}$$

where L is the leisure endowment, and the cost of work  $\phi_t$  is parameterized as:

$$\phi_t = q_0 + q_1 t + q_2 h_t + q_3 h_t t, \quad \text{with } \phi_t \in [0, L - h_t]$$
(6)

# 4 Calibration

I consider 4 baseline economies characterized by the value of  $\gamma$ , assumed to be equal to 0.25, 0.5, 0.75, and 0.95 in each one of them, respectively.

Some parameter values are common to all baseline scenarios, and their values are taken from existing estimates. The set of external parameters is shown in table 1. The coefficient of relative risk aversion,  $\rho$ , is set equal to 2. This implies a value for the intertemporal elasticity of substitution that is consistent with the evidence provided by Attanasio and Weber (1995). The yearly interest rate is fixed at 3%. The discount factor is set at at 0.95, somewhat lower than estimates such as French (2005) because these papers account separately for mortality risk.

Since the model is intended to simulate the behavior of individuals approaching retirement, the initial age  $t_0$  is taken to be 50. The maximum retirement age R is set at 75, and the terminal age T is 90. Retirement benefits are assumed to replace 40% of the deterministic component of wages, and the part-time penalty  $\alpha$  is set at 0.3, consistent with the estimate obtained in Casanova (2013).

Consistent with the findings in the previous sections, the age wage profile of offered wages is taken to be constant in the baseline specifications. In order to match the average full-time hourly wage in the data, which is approximately \$20, I set  $W(t) = W = \ln(20) = 3$ .

This leaves 6 remaining parameters, the 2 taste for leisure parameters  $b_0$  and  $b_1$  and the 4 cost of work parameters  $q_0$ ,  $q_1$ ,  $q_2$  and  $q_3$ . They are calibrated to fit 2 sets of employment targets. The first one is the proportion of individuals that are fully retired at every age

between 51 and 69. The second one is the proportion of workers who are employed part time at every age between 51 and 69. The last set of moments is key in this context, because the increasing proportion of part-time workers with age is the driver of the declining wage profiles often estimated in the literature. Since there are more moment conditions (38) than parameters (6), I use a standard minimum distance criterion to obtain the calibrated values.

The 6 parameters of interest are calibrated separately for each baseline value of  $\gamma$ . Results are presented in table 2, and the implied values for the taste of work and the difference between the cost of full-time and part-time work are plotted in figure 1. The graph shows that the taste for leisure and the relative cost of full-time versus part-time work increase with age in all baseline economies.

Figure 2 shows the predicted proportions of workers employed full time, part time and retired (solid lines) versus the corresponding ones in the data (dashed lines) for the 4 baseline economies. The figure shows that this coarse model is able to provide a good fit for the main participation trends.

#### 5 Results

I use the model to generate hours and wage data (if employed) for each individual at every age. Using the same definition of partial retirement, and the same age criterion for sample selection as in Casanova (2013), I re-run the wage regressions in all baseline scenarios. Figure 3 shows the age-wage coefficients for the baseline economy where  $\gamma = 0.5$  -the results are very similar for the others. As was the case when using HRS data, I find that the age-wage profile from a fixed-effects regression that does not control for participation status declines with age. Once the partial retirement indicator is included as an additional regressor, the estimated age-wage profile is completely flat. The first row of table 3 shows the estimated coefficients on the partial retirement indicators for each of the baseline economies. The wage drop associated to the transition into partial retirement is approximately 34%. The next row shows the equivalent coefficients from the hours regressions. The transition into partial retirement is associated with a 56% decline in hours. These results follow from the values assumed for the part-time penalty and the number of hours worked by full- and part-time workers.

The next step is to investigate how estimates of the parameter  $\gamma$  are affected by misspecification of the age-wage profile. In particular, I will use the smoothly declining wage profile obtained from regressions that pool observations of full- and part-time workers as a proxy for the offered wage profile. This is the procedure commonly used in the literature. To maintain the same average wage at all ages, in the counterfactual the part-time wage penalty is set to zero.<sup>6</sup> I fix all other model parameters except for  $\gamma$  to their baseline values.

I estimate  $\gamma$  by matching full-time and part-time participation rates generated in the counterfactual simulations to those obtained for the baseline simulations. Row (5) in table 3 shows the results. All the estimated values are higher than the baseline value of the parameter, and the bias increases from close to 30% in the first column to over 130% in the last one. Row (3) shows that in this scenario wages do not change upon entrance into partial retirement -all declines are attributed to age- and, given the maintained assumption of discrete labor supply, row (4) indicates that hours still change discretely upon transition out of full-time work. The coefficients in rows (3) and (4) provide the intuition for the upward bias in the counterfactual estimates of  $\gamma$ . In the counterfactual economies, the same drops in hours have to be generated in the absence of the large wage changes observed at baseline. These hour changes are attributed to a high elasticity of leisure that interacts with the increasing taste for leisure to produce exits from full-time work. Thus the estimates of  $\gamma$  obtained using the misspecified age-wage profile are biased upwards.

# 6 Policy Experiments

[INCOMPLETE]

# 7 Conclusions

This paper shows that the offered hourly wage profile does not decline at older ages. The hump-shaped profile often estimated in the literature results from pooling observations of full- and part-time workers, and is not a good proxy for the wage path followed by the typical individual.

This finding has important implications for structural models of labor supply, which often use the hump-shaped estimates found in the literature as a proxy for the age path of offered

<sup>&</sup>lt;sup>6</sup>Keeping the part-time penalty makes part-time wages lower, on average, in the counterfactual economies, which are then not directly comparable to their baseline counterparts. However, to show that my results are not driven by the absence of the part-time penalty, I have estimated  $\gamma$  in the counterfactual scenario maintaining the part-time penalty, and the direction and size of the bias is similar.

wages. I illustrate the consequences of misspecifying the offered wage profile by showing that it leads to biases of up to 130% in estimates of the intertemporal elasticity of substitution of labor supply.

My results have implications for all results in the labor supply literature that rely on the correct specification of the age-wage profile, including measures of income risk; studies of retirement decisions; computations of present discounted values of private and social security pensions; and results based on estimates of the i.e.s., from labor supply responses to business cycles to life-cycle patterns of work and leisure.

Moreover, the results are also relevant to models of life-cycle consumption and savings. These models generally do not endogenize labor supply, and they use labor earnings, rather than hourly wages, as a model primitive. They often take estimates of earnings profiles from external sources. As was the case for wages, the paper shows that the smoothly declining earnings profile often estimated in the literature is not a good proxy for the earnings path of the typical older worker. Misspecification of the earnings profile is bound to affect the estimates and predictions obtained from these models.

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Parameters		
Risk aversion	$\rho$	2.0
Discount factor	$\beta$	0.95
Yearly interest rate	r	0.03
Initial age	$t_0$	50
Max. retirement age	R	75
Terminal age	T	90
Retirement benefits replacement rate		0.40
Part time penalty	$\alpha$	0.30
Age-Wage profile	$W_t \ (= W)$	3.00

 Table 1: External Parameters

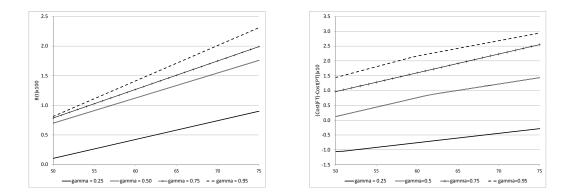
Table 2: Calibrated parameters

	$\gamma = 0.25$	$\gamma = 0.50$	$\gamma = 0.75$	$\gamma = 0.95$
$b_0$	-5.36E-4	4.88E-3	5.45E-3	5.12E-3
$b_1$	3.18E-4	4.24E-4	4.82E-4	5.99E-4
$q_0$	8.88E-2	-5.25E-2	-6.25E-2	-1.02E-1
$q_1$	3.02E-2	-3.29E-3	-8.17E-3	-1.13E-2
$q_2$	9.73E-2	5.08E-2	2.25E-1	3.75E-1
$q_3$	-9.35E-2	1.50E-2	2.22E-2	2.56E-2

	$\gamma = 0.25$	$\gamma = 0.50$	$\gamma = 0.75$	$\gamma=0.95$			
I. Baseline							
(1) $\Delta w_{it}$ upon partial retirement	-0.343	-0.341	-0.340	-0.340			
(2) $\Delta H_{it}$ upon partial retirement	-0.563	-0.559	-0.556	-0.555			
II. Declining age-wage profile, discrete labor supply							
(3) $\Delta w_{it}$ upon partial retirement	-0.005	-0.003	-0.004	-0.004			
(4) $\Delta H_{it}$ upon partial retirement	-0.579	-0.576	-0.579	-0.578			
$(5)$ $\hat{\gamma}$	0.321	1.00	1.551	2.203			
(6) GMM criterion	1.14	0.418	0.415	0.481			

Table 3: Simulation results

Figure 1: Calibrated taste for work (left) and cost-of-work difference between part time and full time (right)



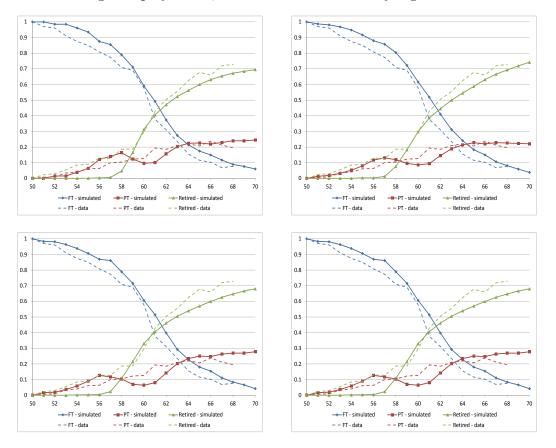


Figure 2: Percentage Employed FT, PT and Retired at Every Age. Data vs. Simulations.

Figure 3: Log Wage Profiles for Different Specifications Using Simulated Data.  $\gamma = 0.5$ 

