Leverage and Disagreement

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September 15, 2015
In this paper, I develop a model of:

- **Endogenous Leverage**
- Interest Rates on Collateralized Bonds

among competitive investors with **heterogenous beliefs**.

Geanakoplos (1997) and subsequent:

- **Only one** leverage ratio (simplifying assumption on the structure of beliefs / or on the number of agents).
- Counterfactual. **Many** leverage ratios, even for same asset: homebuyers, entrepreneurs, hedge funds, investment banks...

Relaxing the hypotheses leading to one leverage ratio, the model yields two key predictions.
1) When disagreement goes to 0, the upper tail of the distribution of leverage ratios goes to a Pareto with endogenous tail coefficient $2$, for any smooth and bounded away from zero density of beliefs.

- Cross section of Hedge Funds (TASS Lipper, 2006)

- Pareto in the upper tail ($l \in [150, 3000]$)

- Point estimate for tail coefficient: $\alpha = 1.95$ (std: 0.2).
Cross-section of homowners’ initial leverage ratios (Dataquick, for example October 1989).

Pareto of leverage ratios found also for:
- Entrepreneurs in the SCF.
- Firms in Compustat.

⇒ Pareto for borrowers’ expected / realized returns, however small belief heterogeneity:
- Pareto Returns to entrepreneurship.
- Pareto Returns to speculation in general.
2) Distribution of interest rates adjusts so that borrowers and lenders are matched assortatively: **interest rates are assignment / hedonic prices**, disconnected from expected and true default probability:

- New determinant for pricing fixed income securities. (⇒ Credit Spread Puzzle? / CDS-Bond Basis)
- Investing in high yield not necessarily risk shifting.
- High customization / fragmentation of the market = Endogenous OTC structure. ⇒ OTC versus exchanges debate.
Model Ingredients:

- Disagreement on mean rather than on default probabilities.

Key Results:

- **Pareto** distributions for leverage ratios / expected and realized returns. Also gives information on:
  - Representativeness of marginal buyer/ Elements of the belief distribution. (⇒ monitoring systemic risk?)
  - Underlying financial structure.
- Credit spreads as *hedonic interest rates*.

Other Theoretical / Methodological contributions:

- Pyramiding Lending Arrangements.
- Endogenous Short-sales:
  - Endogenous rebate rates, without transactions costs / risk aversion.
  - Endogenous short interest.
Literature

- **Heterogeneous Priors.** Miller (1977), Harrison, Kreps (1978), Ofek, Richardson (2003), Hong, Scheinkman, Xiong (2006), Hong, Stein (2007), Hong, Sraer (2012).


- **Credit Spread Puzzle.** Chen, Colling-Dufresne, Goldstein (2009), Buraschi, Trojani, Vedolin (2011), Huang and Huang (2012), Albagli, Hellwig, Tsyvinski (2012), McQuade (2013).

Model with Borrowing Contracts Only

Setup
Equilibrium Definition
Equilibrium Solution
Equilibrium Properties

Extension 1: "Pyramiding" Lending Arrangements

Extension 2: Short-Sales

Conclusion
Model with Borrowing Contracts Only

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Set-up

- Two Periods: 0 and 1.
- Continuum of agents. Measure 1.
- Wealth 1.
- Consume in period 1.
Assets

- Storage’s Return $R = 1. \rightarrow \text{Cash}.


- **Borrowing Contracts** collateralized by the Real Asset.
  - No-recourse.
  - Normalization: 1 unit of Real Asset in Collateral.
  - $\phi$: **Face Value** - promised payment in period 1.
  - Notation for contract: $(\phi)$.
  - Competitive Markets (Anonymous). Price: $q(\phi)$. "Loan amount”. Implicit interest rate: $r(\phi) = \phi / q(\phi)$.
  - Payoff: $\min\{\phi, p_1\}$. 
Beliefs

- Agents agree to disagree on $p_1$.
- Agent $i$: point expectations $p^i_1 \in [1 - \Delta, 1]$.

Key difference with Geanakoplos (1997), where agents agree on value upon default.

Generalization:

- Agents agree on a probability distribution around mean.
- Risk neutral.

Density $f(.)$, c.d.f $F(.)$ on $[1 - \Delta, 1]$.

Exogenously given.

No learning.
Agents’ Problem

Given \((p, q(.)\)), agent \(i\) chooses \((n^i_A, n^i_B(.), n^i_C)\) to max. expected wealth \((W)\) in period 1 under:

- **Budget Constraint (BC).**
- **Collateral Constraint (CC).**

\[
\max_{(n^i_A, n^i_B(.), n^i_C)} \left( n^i_A p^i_1 + \int n^i_B(\phi) \min\{\phi, p^i_1\} d\phi + n^i_C \right) \quad \text{(W)}
\]

\[
\text{s.t.} \quad n^i_A p + \int n^i_B(\phi) q(\phi) d\phi + n^i_C \leq 1 \quad \text{(BC)}
\]

\[
\text{s.t.} \quad \int \max\{-n^i_B(\phi), 0\} d\phi \leq n^i_A \quad \text{(CC)}
\]

\[
\text{s.t.} \quad n^i_A \geq 0, \quad n^i_C \geq 0
\]
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Equilibrium

Definition (Competitive Equilibrium for Economy $\mathcal{E}^B$)

A competitive equilibrium is a price system $(p, q(.))$, and portfolios $(n_i^A, n_i^B(.), n_i^C)$ for all $i$ such that:

- Given $(p, q(.))$, agent $i$ chooses $(n_i^A, n_i^B(\phi), n_i^C)$ maximizing $(W)$ under (BC) and (CC),

- Markets clear:

\[
\int_i n_i^A di = 1,
\]

and $\forall \phi$, \[
\int_i n_i^B(\phi) di = 0.
\]
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Agents’ Types

Agents split into three types depending on optimism:

Cash Investors  Lenders  Borrowers

1-Δ  1

Cash Investors ("Homeowners", "Hedge Funds", "Entrepreneurs").

Lenders.

Borrowers.
Agents’ Types

Agents split into three types depending on optimism:

Cash Investors  Lenders  Borrowers

$1 - \Delta$  $\xi$  $p$  $\tau$  $1$

Cash Investors  Lenders  Borrowers

$\mathbf{\triangleright}$ $p^i_1 \in [\tau, 1] \rightarrow$ Borrowers (”Homeowners”, ”Hedge Funds”, ”Entrepreneurs”).

$n^i_A > 0 \quad \exists \phi, \quad n^i_B(\phi) < 0.$
Agents’ Types

Agents split into three types depending on optimism:

- **Cash Investors**
- **Lenders**
- **Borrowers**

\[ 1 - \Delta \quad \xi \quad \rho \quad \tau \quad 1 \]

- **Cash Investors** \( p^i_1 \in [\tau, 1] \) → Borrowers ("Homeowners", "Hedge Funds", "Entrepreneurs").
  \[ n^i_A > 0 \quad \exists \phi, \quad n^i_B(\phi) < 0. \]

- **Lenders** \( p^i_1 \in [\xi, \tau] \) → Lenders ("Banks", "Money-Market Fund").
  \[ \exists \phi, \quad n^i_B(\phi) > 0. \]
Agents’ Types

Agents split into three types depending on optimism:

- $p_1^i \in [\tau, 1] \rightarrow$ Borrowers ("Homeowners", "Hedge Funds", "Entrepreneurs").
  \[ n_A^i > 0 \quad \exists \phi, \quad n_B^i(\phi) < 0. \]

- $p_1^i \in [\xi, \tau] \rightarrow$ Lenders ("Banks", "Money-Market Fund").
  \[ \exists \phi, \quad n_B^i(\phi) > 0. \]

- $p_1^i \in [1 - \Delta, \xi] \rightarrow$ Cash Investors.
  \[ n_C^i = 1. \]
Borrowers’ Problem

Lemma

A borrower $p_1^i$ chooses $(\phi)$ s.t.: $\phi = \arg \max_\phi \frac{p_1^i - \phi}{p - q(\phi)}$.

- Coll. Const. binds: 1 Real asset $\Rightarrow$ 1 Borrowing Contract.
- Number: $1/(p - q(\phi))$ of Real assets / Borrowing Contracts.

- Leverage ratio of $(\phi)$: $l(\phi) = p/(p - q(\phi))$.

\[
\frac{1}{p - q(\phi)}(p_1^i - \phi) = \frac{p_1^i}{p} l(\phi) - \frac{\phi}{q(\phi)} (l(\phi) - 1) \\
= \frac{p_1^i}{p} + \left( \frac{p_1^i}{p} - r(\phi) \right) (l(\phi) - 1).
\]

- Promise $\phi \nearrow$ $\Rightarrow$ $q(\phi) \nearrow$ $\Rightarrow$ $q'(\phi) > 0$ $\Rightarrow$ $l'(\phi) > 0$ $\Rightarrow$ Leverage rises with face value $\phi$.
- Trade-off between higher $\phi$ but higher $r(\phi)$ $\Rightarrow$ $r'(\phi) > 0$. 

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Lenders

Lemma

A lender with beliefs $p^i_1$ chooses contract $(p^i_1)$.

▶ For lenders: Face value of the loan = Beliefs about the Real Asset.
  ▶ Why not a higher $\phi$? Default for sure.
  
  Return: $\frac{\min\{p^i_1, \phi\}}{q(\phi)} = \frac{p^i_1}{q(\phi)} \searrow \phi$.

▶ Why not a lower $\phi$?

Return: $\frac{\min\{p^i_1, \phi\}}{q(\phi)} = \frac{\phi}{q(\phi)} = r(\phi) \nearrow \phi$.

▶ Leverage rises with $\phi$, and $\phi = p^i_1$ of lenders $\Rightarrow$ Leverage rises with beliefs of lenders.

▶ Lenders think they trade perfectly safe contracts.
Positive Sorting

- Supermodularity of Expected Wealth of a Borrower with respect to his Beliefs $p_1^i$ and the face value $\phi$:

$$\frac{p_1^i - \phi}{p - q(\phi)} = \frac{p_1^i}{p} \left(1 + l(\phi)\right) - \frac{\phi}{q(\phi)} l(\phi)$$

$$\Rightarrow \frac{\partial^2}{\partial \phi \partial p_1^i} (.) = \frac{1}{p} l'(\phi) > 0.$$ 

- **Complementarity** between leverage ($\phi$) and expected return on each asset ($p_1^i$).

- $\phi = p_1^i$ of lenders $\Rightarrow$ **Positive Sorting** of borrowers and lenders. Empirically: Over-The-Counter (OTC) Markets.

- $\Gamma(.)$: Belief of borrower $\rightarrow$ Belief of lender. Sorting: $\Gamma'(.) > 0$. 
2 first-order ODE for $\Gamma(.)$ and $q(.)$

- $p_1^j = y$ chooses $\phi$ s.t. lender choosing same $\phi$ is $\Gamma(y)$:
  \[
  \Gamma(y) = \arg\max_{\phi} \frac{y - \phi}{p - q(\phi)} \quad \Rightarrow \quad q'(\phi) \frac{y - \phi}{p - q(\phi)} = 1
  \]
  \[
  \Rightarrow \quad (y - \Gamma(y)) q'(\Gamma(y)) = p - q(\Gamma(y)).
  \]

- Market clearing for contract $(x)$:
  \[
  \int_{i} h_i^j(x) di = 0 \quad \Rightarrow \quad \frac{f(\Gamma(y)) d\Gamma(y)}{q(\Gamma(y))} = \frac{f(y) dy}{p - q(\Gamma(y))}
  \]
  \[
  \Rightarrow \quad (p - q(\Gamma(y))) f(\Gamma(y)) \Gamma'(y) = q(\Gamma(y)) f(y).
  \]
- Unknowns: \( q(.) (\equiv r(.)), \Gamma(.), \xi, p, \tau \).

- 2 First-Order ODEs ⇒ Need 5 algebraic equations.

- Indifference Cash / Lending: \( r(\xi) = 1 \).
Unknowns: \( q(.) (\equiv r(.)), \Gamma(.), \xi, p, \tau. \)

- 2 First-Order ODEs ⇒ Need 5 algebraic equations.

\[
\begin{align*}
1-\Delta & \quad \xi & \quad p & \quad \tau & \quad 1 \\
\text{Cash Investors} & \quad \text{Lenders} & \quad \text{Borrowers}
\end{align*}
\]

- Indifference Cash / Lending: \( r(\xi) = 1. \)
- Indifference Lending / Investing: \( r(\tau) = \frac{\tau - \xi}{p - \xi}. \)
- **Unkowns:** $q(.) (\equiv r(.))$, $\Gamma(.)$, $\xi$, $p$, $\tau$.

- **2 First-Order ODEs** $\Rightarrow$ Need 5 algebraic equations.

- **Indifference Cash / Lending:**
  \[ r(\xi) = 1. \]

- **Indifference Lending / Investing:**
  \[ r(\tau) = \frac{\tau - \xi}{p - \xi}. \]

- **Most pessimistic lenders & borrowers:**
  \[ \Gamma(\tau) \equiv \xi. \]
- Unknowns: $q(.) (\equiv r(.)), \Gamma(.), \xi, p, \tau$.

- 2 First-Order ODEs $\Rightarrow$ Need 5 algebraic equations.

\begin{align*}
\text{Cash Investors} & & \text{Lenders} & & \text{Borrowers} \\
1-\Delta & & \xi & & p \quad \tau & & 1 \\
& & & & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad}
- **Unknowns:** \( q(.) (\equiv r(.)) \), \( \Gamma(.) \), \( \xi \), \( p \), \( \tau \).

- **2 First-Order ODEs \( \Rightarrow \) Need 5 algebraic equations.**

![Diagram showing cash investors, lenders, and borrowers with intervals](#)

- **Indifference Cash / Lending:** \( r(\xi) = 1. \)
- **Indifference Lending / Investing:** \( r(\tau) = \frac{\tau - \xi}{p - \xi}. \)
- **Most pessimistic lenders & borrowers:** \( \Gamma(\tau) = \xi. \)
- **Most optimistic lenders & borrowers:** \( \Gamma(1) = \tau. \)
- **Market clearing for the real asset:** \( 1 - F(\xi) = p. \)
Model with Borrowing Contracts Only
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Extension 1: "Pyramiding" Lending Arrangements

Extension 2: Short-Sales

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Illustrating examples: \( f \) uniform, \( f \) increasing

\[ f(.) \\
\begin{array}{c}
1/\Delta \\
1-\Delta \quad 1
\end{array}
\]

\[ p_i^1 \\
\]

\[ \Delta \\
\]

\[ \sqrt{1+\Delta} \\
\]

\[ \frac{1+\Delta}{1+\Delta^2} \]

\[ \frac{1+\Delta^2}{1+\Delta^4} \]

\[ p = \frac{1 + \Delta + 2\Delta^2 + 2\Delta^3 - \sqrt{(-1 + \Delta)^2 (1 + 2\Delta^2)}}{2\Delta + \Delta^2 + 4\Delta^3 + 2\Delta^4} = 1 - O(\Delta^2). \]

- Uniform: 2 first-order ODE \( \rightarrow \) second-order ODE:

\[ \Gamma'' (\Gamma - x) + \Gamma' + \Gamma'^2 = 0 \quad \Rightarrow \quad \Gamma(x) = -x - a + b\sqrt{x} + c. \]

- Closed form: \( p, \xi, \tau, r(.), q(.), L(.), a, b, c. \) Example:
Cutoffs as a function of $\Delta$ ($f$ uniform)

True across bounded away from zero density function:

$$p = 1 - O(\Delta^2), \quad \tau = 1 - O(\Delta^2), \quad \text{and} \quad \xi = 1 - O(\Delta).$$
Limiting Pareto Tail of Endogenous Tail Coefficient 2

- In uniform case, truncated Pareto with coeff 2:

\[
\frac{p}{p - Q(y)} = \frac{p}{\sqrt{2\xi}} \sqrt{\frac{p - \xi}{\tau - \xi}} \frac{1}{\sqrt{(p+\xi)(\tau - \xi)(p-\xi)}} - y.
\]

Proposition (Limiting Pareto Distribution for Leverage Ratios of Optimists for smooth \(f(.)\))

Let \(f(.)\) differentiable, \(f'\) continuous, \(f(.)\) bounded away from 0. \(G_\Delta(.)\) distribution function for the leverage of borrowers for \(f_\Delta(.)\):

\[
\exists A_\Delta, \quad \| f^2(1 - G_\Delta(l)) - A_\Delta \|_{\infty}^{[L_\Delta(1)/2,L_\Delta(1)]} \xrightarrow{\Delta \to 0} 0,
\]

- Heuristically:

\[
1 - G_\Delta(l) \sim \frac{A_\Delta}{f^2}.
\]

- Upper tail behavior: not dependent on \(f(.)\).
Pareto Distributions for Leverage Ratios, Uniform Distribution

Coefficient: 2.

Disagreement $\Delta = 10\%$
Disagreement $\Delta = 5\%$
Disagreement $\Delta = 2\%$

Log$_{10}$ Leverage Ratio
Log$_{10}$ Survivor Function
Pareto Distributions for Leverage Ratios, Increasing Distribution

Still Coefficient: 2.
Empirical Counterpart

TASS Hedge Fund Database, August 2006.

Slope: -1.95

Calibration: disagreement ≈ 1.8%.
Non Bounded away from 0.

- If $f(x) \sim (1 - x)^\rho \Rightarrow$ Pareto with coefficient $2 + \rho$.
- Scale Independence Remains.

![Graph showing the relationship between Log10 Leverage Ratio and Log10 Survivor Function for different values of $\Delta$ and $\rho$.](attachment:graph.png)
Returns to Entrepreneurship?

- Expected Returns are Pareto from envelope condition:
  \[ R'(y) = \frac{1}{p - Q(y)} = \frac{\text{Leverage}(y)}{p}. \]

![Graph showing Log10 Survivor Function for different Disagreement levels (Δ=10%, Δ=5%, Δ=2%) on a Log10 Expected Returns scale.](image)
Hedonic Interest rates $r(.)$ on safe bonds for lenders. Can be substantial. Example with $f(x) = 2(1 - x)/\Delta$.

$\text{Corr}(r(.), l(.)) > 0$ from disagreement. But: no risk shifting $\Rightarrow$ Different regulatory implications.
Hedonic Interest Rates

- Non monotonic relationship between leverage and realized returns of borrowers, because of spreads.
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Pyramiding Lending Arrangements

- Allow Borrowing Contracts to be used as collateral.

- Hedonic interest rates ⇒ Lenders want to leverage into them!

- Example for houses, loans to SMEs: securitization. Orrehypothecation of collateral, repos of mortgage-backed securities, etc.

- Price $p$ increases even more.
Akin to tranching. The lender of type 2 is repaid until $\phi'$, then lender of type 1 is repaid on $\phi - \phi'$, then the borrower gets $p_1 - \phi$. 
Pyramiding Lending Arrangements

- Pareto Coefficients decrease (leverage distributions are multiplied) \(\Rightarrow\) Leverage Ratio distribution shifted to the right.

- Price expresses the opinion of superoptimists.

![Graph showing the comparison between Borrowing and Pyramiding Economies. The x-axis represents Log_{10} Leverage Ratio, and the y-axis represents Log_{10} Survivor Function. The graph shows two lines: one for Borrowing Economy and one for Pyramiding Economy. The slopes are labeled as \(-3.98556\) and \(-3.26585\) respectively.](image-url)
Empirics

- Leverage Ratios on New Loans. Source: Dataquick.

- \( \approx 100,000 - 500,000 \) new loans per month.
Empirics

- Leverage Ratios on New Loans. Source: Dataquick.
- \( \approx 100,000 - 500,000 \) new loans per month.
Empirics

- Leverage Ratios on New Loans. Source: Dataquick.

- ≈ 100,000 - 500,000 new loans per month.
Empirics

- Leverage Ratios on New Loans. Source: Dataquick.

- $\approx 100,000 - 500,000$ new loans per month.
Pyramiding Lending Arrangements

- Video: the leverage ratio distribution from 1987 to 2012.
The model allows to recover the corresponding increase in borrowers’ expected returns.

In a model with a little bit of risk aversion: more risk taking?
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Short-Sales

- Unlike existing disagreement models, the model allows the treatment of short-sales.

- Price = pessimists’ valuations ⇒ Systematic undervaluation - similar to noise trader risks in De Long et al. (1990), but risk neutrality. Equity premium, discount of closed-end funds, etc.

- **Endogenous rebate rates** - apparent short-selling costs not evidence of constraints: about 100 bps, larger with more disagreement.

- **Endogenous Short-interest** (a few percent).
Short-Sales

Disagreement $\Delta$

Percentile (Ranked by Degree of Optimism)

Lenders

Borrowers

Securities Lenders

Short-Sellers

Cutoff $\sigma$

Cutoff $\tau$

Price of Real Asset $p$

Cutoff $\xi$
Endogenous Rebate Rates and Cash Collateral

- No short-selling costs or costs of default.
Endogenous Short Interest

- Only a few percent of stocks are on loan in equilibrium, even though all are potentially available.
Larger Spreads on Bonds, even the safest (AAA)
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▶ Homeowners / Entrepreneurs’ / Hedge Funds data lend support to a very stylized model.
▶ New (static) source of Pareto distributions in returns independent from Gibrat’s law/ random growth.
▶ New intuitions on key financial prices / quantities:
  ▶ Returns on Bonds.
  ▶ Short-selling ”costs”.
  ▶ Short interest

Potential for future work:

▶ Empirical work on short interest, rebate rates, distributions of leverage ratios to recover disagreement.
▶ Financial regulation:
  ▶ Costs of moving OTC onto exchanges.
  ▶ Monitoring financial system through ultimate borrowers’ leverage ratio distribution ?
Thank you
Leverage Ratios of Entrepreneurs

Slope: -2.02

Log_{10} Survivor

Log_{10} Leverage Ratio

Fitted values

Leverage Ratio

Fitted values