Persistency of Poverty, Financial Frictions, and Entrepreneurship

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Abstract

Do financial constraints have persistent effects on the creation of businesses, or should we expect that forward-looking individuals always save their way out of credit constraints? This paper studies the interaction between savings and the decision to become an entrepreneur in a multi-period model with borrowing constraints to answer this question. The model has a simple threshold property: able individuals who start with wealth above a threshold save to become entrepreneurs, while those who start below this threshold remain wage earners forever. The threshold wealth such that able entrepreneurs find it beneficial to save decreases with entrepreneurial ability, i.e., the magniture of individual poverty traps decreases with entrepreneurial ability. Still, calibrated examples show that the magnitude of individual poverty traps can be large, specially if returns to scale are calibrated to be large.

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1 Introduction

There is wide support for the view that financial frictions affect the dynamics of small firms. Investment by smaller firms is more sensitive to cash-flows and aggregate shocks, age and size of firms are positively correlated, and the exit hazard rate of firms is decreasing with size and age.[1] Recent theoretical literature has studied alternative models of firm dynamics in environments with financial constraints and has shown that those models produce implications that are consistent with the observed data.[2]

Despite the large amount of research in this area, an important question remains open. Do financial constraints and initial conditions have persistent effects on the creation of businesses, or should we expect that forward-looking individuals save their way out of credit constraints? This question lie at the heart of the debate on the causes of the persistency of poverty and underdevelopment (see Banerjee and Duflo (2005)). In this paper I study a model of occupational choice with forward-looking savings to answer this question.

In particular, I study the optimal saving decision of individuals facing a choice between working for a wage or starting a business in a continuous-time framework.[3] The optimal decision to save is shown to have a simple threshold property. Able individuals who start with wealth above the threshold but below what is needed to start a profitable business save to become entrepreneurs; able individuals who start with wealth below the threshold have low savings and remain workers forever. In this sense, individuals with wealth below the threshold are in a "poverty trap" and financial frictions can have persistent effects on the number of businesses. Furthermore, it is shown that the threshold wealth above which able individuals find it optimal to save to become entrepreneurs is a decreasing function of ability. Indeed, there is an ability such that individuals above it are never in a "poverty trap", regardless of their initial wealth. In this sense, the size of "poverty traps" is bounded and financial frictions have only a transient effect for sufficiently able individuals.

The size of poverty traps and the welfare cost of borrowing constraints turn out to be especially sensitive to the span of control and the capital intensity

[1] See Caves (1998), Hubbard (1998) and Stein (2001) for recent surveys of this literature.

[2] See Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (forthcoming), Cooley and Quadrini (2001) and Hopenhayn and Vereshachagina (2005).

[3] The continuous-time framework allows a simple characterization of the savings problem despite the non-convexity introduced by the occupational choice. I extend earlier work by Skiba (1978) and Brock and Dechert (1983) to study the dynamics of wealth and occupational choice.

of the entrepreneurial technology. They are larger the higher is the span of control or the more capital intensive is the entrepreneurial technology. These two parameters determine the returns to scale to the factors that need to be financed. If the entrepreneurial technologies are fairly linear with respect to these factors, then the scale at which entrepreneurs obtain the surplus from operating their technology will be very large. This implies that the savings needed to overcome borrowing constraints involve too large a sacrifice. Forward looking individuals will choose to have low savings and remain workers in the long run. Summing up, within the range of parameter values used in the literature,[4] "poverty traps" can be quantitatively important.

Literature Review By studying the interaction between financial frictions and the creation of businesses, this paper closely relates to the recent development literature (e.g., the earlier work by Banerjee and Newman (1993), Galor and Zeira (1993)).[5] In these papers, borrowing constraints affect productivity and the distribution of income by restricting agents from profitable occupations that require capital, such as entrepreneurship. Individuals that start poor are doomed to remain poor. A limitation of these analyses is that they rely on strong assumptions. Generations are assumed to live for a single time period and the evolution of wealth is determined by a warm-glow bequest motive that is not forward-looking. This paper sheds light on the robustness of these results to environments where savings are forward-looking.

As previously discussed, in a model with forward-looking savings, initial conditions can have a persistent effect for individuals that start sufficiently poor. At first glance, lessons drawn from myopic models of the evolution of wealth seem robust. There is an important caveat, though. In forward-looking models poverty traps are a function of the profitability of the business opportunities. More able entrepreneurs are less likely to be in a poverty trap and the most able are never in a "poverty trap", regardless of their wealth.

This paper is also motivated by and complements a recent literature that studies the quantitative implications of models of occupational choice and borrowing constraints. Numerical solutions of related models are studied by Quadrini

[4] In the case of the returns to scale, this set is unpleasantly large.

[5] See also the theoretical contributions by Aghion and Bolton (1997), Caselli and Gennaioli (2002), Ghatak, Morelli and Sjostrom (2001), Lloyd-Ellis and Bernhardt (2000), Matsuyama (2000, 2003), Mokeerjee and Ray (2002, 2003), Galor and Moav (forthcoming); and the recent quantitative evaluation of these models by Gine and Townsend (2004) and Jeong and Townsend (2005).

(2000) and Cagetti and De Nardi (2005). These authors have shown that models featuring entrepreneurship and financial frictions are important to explain the observed wealth distribution in the U.S economy. In a similar vein, Cagetti and De Nardi (2004), Li (2002), and Meh (forthcoming) quantify the effect of various policies in models featuring entrepreneurs and credit constraints for the US economy. This paper complements this literature by providing an analytical characterization of the dynamic of wealth in this type of models, by exploring the extent to which financial friction and initial conditions have persistent effects on the long term dynamics of individual wealth and entrepreneurship.

The rest of the paper is organized as follows. Section 2 describes the individual's dynamic occupational choice problem and Section 3 characterizes its solution. Section 5 examines a numerical examples to understand the importance of poverty traps and the welfare cost resulting from borrowing constraints implied by the model. Section 6 concludes and discusses directions for future research.

2 The Model Economy

The model is set in continuous time. Households are endowed with entrepreneurial ability, e, and initial wealth, a_0. In each instant of their life, they have the option to work for a wage, w, and invest their wealth at a constant interest rate, r, or to work and invest in an individual specific technology with productivity e, i.e., to become entrepreneurs. If households decide to be entrepreneurs they must devote all their labor endowment to run their businesses, i.e. occupations are indivisible. This captures a fundamental non-convexity: households are more productive by specializing in one activity. Households are only allow to borrow up to a fraction of their wealth.

2.1 Preferences

Agents' preferences over consumption profiles are represented by the time separable utility function

$$U(c) = \int_0^\infty e^{-\rho t} u(c(t))\, dt \tag{1}$$

where t is the age of the individual and ρ is the rate of time preference. The utility function over consumption, $u(c)$, is strictly increasing and strictly con-

cave.[6]

The infinite horizon is a convenient analytical assumption. The theory should be understood as describing the life-cycle of an individual. Under this interpretation, $\rho = \rho^* + p$, where ρ^* is the rate of time preferences and p is the constant rate at which agents die.

2.2 Resource Constraints and Technologies

Agents start their lives with wealth a_0. At any time $t \geq 0$, their wealth, $a(t)$, evolves according to the following law of motion

$$\dot{a}(t) = y(a(t)) - c(t) \quad t \geq 0, \tag{2}$$

where $y(a(t))$ is the income of the agent with wealth $a(t)$, and $\dot{a}$ refers to $\frac{\partial a(t)}{\partial t}$.[7] The shape of the income function depends on occupational choices as follows.

If agents choose to be wage earners, they will sell their labor endowment for a wage w and invest their wealth at a rate of return r. In this case, their income $y(a)$ is

$$y^W(a) = w + ra, \tag{3}$$

where ra is the return on their wealth a. I refer to w as *the wage*, but it should be understood that wages are individual-specific. Formally, $w = \bar{w}l$, where l are the efficiency units that an individual can supply and $\bar{w}$ is the price of an efficiency unit of labor.

If individuals run a business they must devote their entire labor endowment to operate the business. Their revenue is given by the function, $f(e,k)$, where e is the agent-specific ability and k is the amount of capital invested in the business. [8] $f(e,k)$ is assumed to be strictly increasing in both arguments, homogeneous of degree 1, and strictly concave in capital, $f_e(e,k) > 0$, $f_k(e,k) > 0$, $f_{kk}(e,k) < 0$. Inada conditions are assumed to hold, $\lim_{k\to 0} f_k(e,k) = \infty$

[6] In a life-cycle interpretation of the model t is the age of the agents and $\rho = \rho^* + p$, where ρ^* is the rate of time preferences and p is the constant rate at which agents die. This interpretation of the model is used when studying the quantitative implications of the theory.

[7] For simplicity of exposition, I drop the time as an argument of the different functions.

[8] The production function should be interpreted as the reduced form of a more general technology requiring capital and labor,

$$f(e,k) = \max_n \tilde{f}(e,k,n) - \bar{w}n,$$

where n are the efficiency units of labor employed and $\bar{w}$ is the price of an efficiency unit of labor. When calibrating the model and when discussing the predictions of the model for technologies with different capital intensities, the more general notation will be used.

and $\lim_{k\to\infty} f_k(e,k) = 0$. A higher entrepreneurial ability is associated with a higher marginal product of capital, $f_{ek}(e,k) > 0$, also $f(0,k) = 0$ and $\lim_{e\to\infty} f(e,k) = \infty$.

The amount of capital that agents can invest in their businesses is constrained by their wealth. To focus the analysis on the interaction between individual savings and occupational choice, I choose a simple specification of borrowing constraints. In particular, I assume that the value of an individual's business assets, k, must be less than or equal to the value of their wealth, $k \leq a$. If wealth exceeds the value of desired business assets, the remaining wealth is invested at the rate r.[9]

Therefore, the income of an entrepreneur solves the following static profit maximization problem:

$$y^E(e,a) = \max_{k\leq a} \{f(e,k) + r(a-k)\}. \tag{4}$$

Note that the scale of the business equals the individual's wealth, a, as long as wealth is lower than the unconstrained scale of the business, $k_u(e)$. The unconstrained scale is the solution to the unconstrained profit maximization problem, i.e.,

$$k_u(e) = \arg\max_k \{f(e,k) - rk\}.$$

This function is strictly increasing. Inada conditions are necessary to guarantee that this function is well defined for all e.

2.3 Consumer's Problem

Agents choose profiles for consumption, $c(t)$, wealth, $a(t)$, occupational choice, and business assets, $k(t)$, to solve

$$\begin{aligned}
&\max_{c(t),a(t),k(t)\geq 0} \int_0^\infty e^{-\rho t} u(c(t))\, dt \\
&s.t. \\
&\dot{a}(t) = y(a(t)) - c(t) \\
&y(e,a(t)) = \max\left\{y^E(e,a(t)), y^W(a(t))\right\}.
\end{aligned}$$

[9] When studying the quantitative implications of the theory, I allow for entrepreneurs to invest up to a fraction of their wealth, i.e., $k \leq \lambda a$ with $\lambda \geq 1$, and for capital to depreciate at the rate δ.

As is implicitly recognized in the statement of the problem, the occupational decision is a static one. That is, given current wealth, a, agents choose to be entrepreneurs if their income as entrepreneurs, $y^e(e,a)$, exceeds their income as wage earners, $y^w(a)$, i.e., $y^e(e,a) \geq y^w(a)$.

This can be expressed as a simple policy function. Define $\underline{e}$ to be the ability at which individuals are just indifferent between being wage earners and being entrepreneurs conditional on being able to borrow at the interest rate r.[10] Relatively able individuals (individuals with ability above $\underline{e}$) decide to be entrepreneurs if their current wealth is higher than the threshold wealth $\underline{a}(e)$, $a \geq \underline{a}(e)$,where $\underline{a}(e)$ solves

$$f(e, \underline{a}(e)) = w + r\underline{a}(e).$$

Intuitively, agents of a given ability choose to become entrepreneurs if they are wealthy enough to run their businesses at a profitable scale. Alternatively, agents of a given wealth a choose to become entrepreneurs if their ability is high enough. Both ability and resources determine the occupational decision.

Given the optimal static decision, the dynamic program is equivalent to a standard capital-accumulation problem subject to a production function of the form

$$y(e,a) = \begin{cases} w + ra & \text{if } a \in [0, \underline{a}(e)) \\ f(e,a) & \text{if } a \in [\underline{a}(e), k_u(e)) \\ f(e, k_u(e)) + r(a - k_u(e)) & \text{if } a \in [k_u(e), \infty) \end{cases}.$$

This technology is given by the upper envelope of the "wage earner technology," $y^w(a)$, and the "entrepreneurial technology," $y^e(e,a)$. Figure 1 describes these technologies. Notice that this production function is not concave. The return to capital increases if individuals invest more than $\underline{a}(e)$.

Necessary conditions for the wealth accumulation problem are given by

[10] $\underline{e}$ solves

$$\max_k f(\underline{e}, k) - rk = w.$$

The left hand side of this equation is well define, increasing, continuous and take the value zero for $e = 0$ and goes to infinity as e goes to infinity.

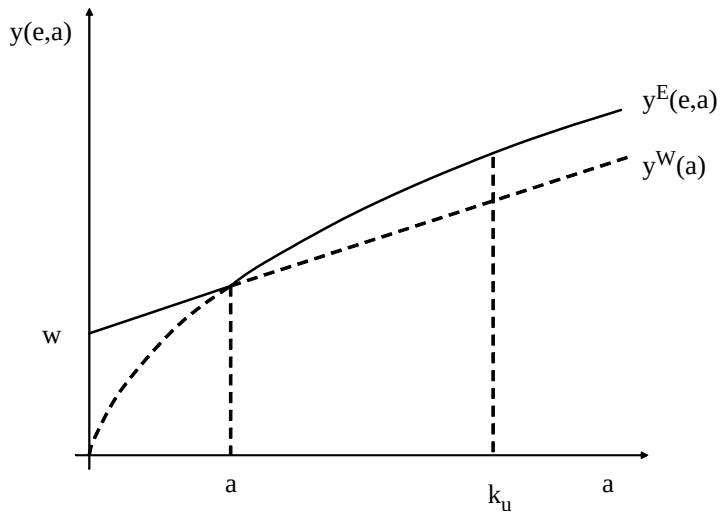

Figure 1: Technologies Available to Households

1. the Euler equation,

$$\frac{u''(c)\,c}{u'(c)}\frac{\dot{c}}{c} = \begin{cases} r-\rho & \text{if } a \in [0, \underline{a}(e)) \\ f_k(e,a)-\rho & \text{if } a \in [\underline{a}(e), k_u(e)) \\ r-\rho & \text{if } a \in [k_u(e), \infty) \end{cases}, \tag{5}$$

   stating that the marginal rate of substitution should equal the marginal rate of transformation;

2. the law of motion for wealth,

$$\dot{a} = y(e,a) - c, \tag{6}$$

   describing the evolution of the individual wealth;

3. and the tranversality condition,

$$\lim_{t\to\infty} e^{-\rho t} u'(c(t))\, a(t) = 0, \tag{7}$$

stating that value of wealth should converge to zero.

In the present case, these conditions are only necessary. As in any non-convex problem, there are solutions to the first order conditions that correspond

to global minima or to local maxima that are not global maxima. Also, there can be multiple maxima. In particular, provided that $r \leq \rho$, there exist to steady state solutions to the first order conditions, a low wealth worker steady state $(0, w)$, and a high wealth entrepreneuria steady state (a_{ss}, c_{ss}). In section 3, I analyze the optimal accumulation path under this technology.

I conclude this section by noting that:

Remark 1: The model is homogeneous of degree 1 in (a, w, e).

Exploiting this property, I normalize all the variables in the model by the wage. When studying the behavior of entrepreneurs in the data, this also suggests that wealth to wage ratios are the relevant measure of resources available to individuals and that the relevant notion of entrepreneurial ability to the model is relative ability, i.e., entrepreneurial ability relative to the ability as a worker e/w.

3 The Evolution of Individual Wealth

This section characterizes the evolution of individual wealth. The main results are: (a) There exists a threshold wealth level, $a_s(e)$, such that individuals with initial wealth below the threshold, $a_0 < a_s(e)$, follow a path associated with decreasing wealth, converging to a zero-wealth steady state where they are wage-earners. Meanwhile, households with initial wealth above the threshold, $a_0 \geq a_s(e)$, save to become entrepreneurs and converge to a high-wealth entrepreneurial steady state. (b) The function $a_s(e)$ is strictly decreasing in entrepreneurial ability and there exists a minimum ability, e_{high}, such that individuals with ability above e_{high} save to become entrepreneurs regardless of their initial wealth. (c) The threshold $a_s(e)$ is increasing in the discount rate and decreasing in the intertemporal elasticity of substitution.

Proposition 1 contains the main result of this section: given an ability level e, households with low initial wealth will follow a path converging to a zero wealth worker steady state, and households with high initial wealth will follow a path converging to a high wealth entrepreneurial steady state.

Proposition 1: *There exits a strictly positive ability level, e_{low} and a finite ability level, e_{high} such that:*

1. *For $e \leq e_{low}$ it is optimal for agents to follow the trajectory converging to the $(0, w)$ steady state for all levels of initial wealth ;*

2. *For* $e \in (e_{low}, e_{high})$ *there is a single initial wealth,* $a_s(e)$*, such that individuals starting with wealth level,* $a_s(e)$*, will be indifferent between following the trajectory converging to the* $(0, w)$ *steady state or the trajectory converging to the* (a_{ss}, c_{ss}) *steady state. Agents with initial wealth to the left of* $a_s(e)$ *prefer to follow the trajectory converging to the* $(0, w)$ *steady state. The converse holds for agents starting with wealth to the right of* $a_s(e)$*.*

3. *For* $e \geq e_{high}$ *it is optimal for agents to follow the trajectory converging to the* (a_{ss}, c_{ss}) *steady state for all levels of initial wealth .*

Intuitively, households with low initial wealth require a larger investment in terms of forgone consumption to save up toward the efficient scale. Thus, they prefer to have a lower but smoother consumption profile as wage earners. Figure 2 illustrates the optimal trajectories in the intermediate ability case[11].

This proposition tells us that the typical policy function for consumption will be discontinuous. For agents with low initial wealth, it is optimal to start with relatively high, but decreasing, consumption. For agents with high initial wealth it is optimal to start with a relatively low, but increasing, consumption. Moreover, there is a unique threshold on initial wealth that divides individuals into these two groups. I refer to this threshold as the poverty trap threshold. The poverty trap threshold is a function of entrepreneurial ability.

This characterization implies the following corollary.

Corollary to Proposition 1: (a) *The saving rate of individuals who eventually become entrepreneurs is higher than the saving rate of individuals who remain wage earners. (b) The growth rate of consumption increases after individuals become entrepreneurs.*

This suggests two obvious tests for the model (see Buera, 2008, for tests of these predictions).

The next result states that the threshold $a_s(e)$ is decreasing in the agent's entrepreneurial ability. It also tells us that there is a minimum ability e_{low} and a maximum ability e_{high} such that nobody with ability lower than e_{low} is an entrepreneur in the long run and everybody with ability higher than e_{high} is an entrepreneur in the long run.

[11] For low enough ability e it will the case that $a_s > \underline{a}$, implying that there are individuals that start as entrepreneurs, but choose to eat their wealth an eventually become workers.

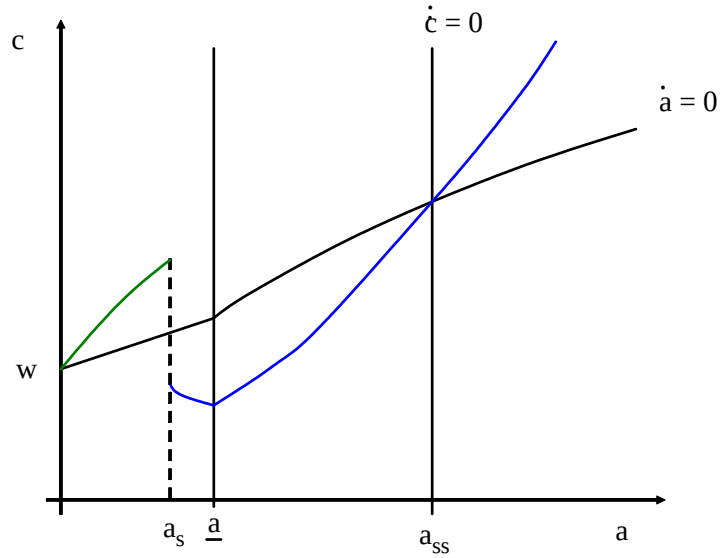

Figure 2: Optimal Trajectories (Intermediate Ability)

Proposition 2: *If $a_s(e) \leq \underline{a}(e)$, the "poverty trap", $a_s(e)$, is strictly decreasing in the agent's ability, i.e.,*

$$\frac{\partial a_s(e)}{\partial e} < 0.$$

The intuition of this result is straightfoward. For individuals that are still workers, entrepreneurial ability only affects those who plan to follow the entrepreneurial path. In other words, the more profitable it is to be an entrepreneur, the less likely an individual will be stuck in a poverty trap. The ability to save gives an upper bound on the importance of borrowing constraints.

Next, the relationship between various parameters of the model and the "poverty trap" threshold is discussed.

Proposition 3: *The following is true in a neighborhood of $r = \rho$ such that $r < \rho$: the poverty trap, a_s, is increasing in the discount rate, ρ, and decreasing in the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. Moreover, for e close to e_{low}, the poverty trap, a_s, increases with the interest rate, while for e close to e_{high}, it decreases with the interest rate.*

Poverty traps are more likely the lower the intertemporal elasticity of substitution and the higher the discount rate, since these two parameters affect the

cost of a high savings plan. Interestingly, the effects of the interest rate and a given worker's wage rate are ambiguous. A higher interest rate and a higher wage make it easier for able individuals to save the capital needed to start a profitable business, but, at the same time, a higher interest rate and higher wage raise the cost of being in business and make the option of being a wage earner relatively more attractive.

In the next section, I give a quantitative assessment of the ability of savings to overcome borrowing constraints to entrepreneurship.

4 Numerical Examples

This section studies a calibrated version of the dynamic model to understand the potential of borrowing constraints to generate significant poverty traps and welfare cost when foward-looking saving decisions are incorporated into the analysis.

4.1 Parametrization

I choose standard CES preferences

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ is the reciprocal of the intertemporal elasticity of substitution.

Following Lucas (1978), I propose a constant return to scale technology on the entrepreneurial ability, capital and labor,

$$\tilde{f}(\tilde{e}, k, n) = \tilde{e}^{\nu} \left(k^{\alpha} n^{1-\alpha}\right)^{1-\nu}$$

where α represents the share of payments going to the variable factors, capital and labor, that are paid to capital, and ν the share of payments going to the entrepreneur (it is also referred to as the span of control parameter (Lucas 1978)). Note that I only need to specify the reduced form for output net of labor cost as a function of capital and entrepreneurial ability, once labor is chosen optimally,

$$\begin{aligned} f(\tilde{e}, k) &= \max_{l} \left\{ \tilde{e}^{\nu} \left(k^{\alpha} n^{1-\alpha}\right)^{1-\nu} - \bar{w} n \right\} \\ &= A \tilde{e}^{1-\phi} k^{\phi}, \end{aligned}$$

where $\bar{w}$ is the price of an efficiency unit of labor,
$A = (1-(1-\nu)(1-\alpha))\left[\frac{(1-\nu)(1-\alpha)}{\bar{w}}\right]^{\frac{(1-\nu)(1-\alpha)}{1-(1-\nu)(1-\alpha)}}$, $\phi = \frac{(1-\nu)\alpha}{1-(1-\nu)(1-\alpha)}$.

It convenient to normalize the ability index to correspond to the profits an entrepreneur would make if unconstrained, $e = \max_k \left\{A\tilde{e}^{1-\phi}k^{\phi} - (r+\delta)k\right\}$. Note that when holding fix the unconstrained profits, e, and the level of capital, k, the profits are decreasing in the parameter ϕ. The larger the share of capital the lower the profits of an entrepreneur holding constant the scale and the unconstrained surplus. This implies that the larger the parameter ϕ the larger will be the initial wealth required by an individual of ability e to prefer saving in order to become an entrepreneur, i.e. $\frac{\partial a_s(e;\phi)}{\partial \phi} > 0$. Also, the scale at which an individual finds it profitable to start a business will be larger, $\frac{\partial \underline{a}(e;\phi)}{\partial \phi} > 0$. The bite of borrowing constraints will be larger the larger is ϕ (see Figure 5 for an illustration of this result).

4.2 Calibrated Individual Poverty Traps and Welfare Cost of Borrowing Constraints

I need to specify five parameters, σ, ρ, r, δ and ϕ. The first four parameters can be set with little controversy. I choose $\sigma = 1$, a reasonable value for the reciprocal of the intertemporal elasticity of substitution. I set the time period to be 1 year, and, correspondingly, I let $r = \rho = 0.04$ to reflect the average market return to wealth. The depreciation of business assets, δ, is set to 0.06. We take our benchmark economy to be one where individual have no access to credit, $\lambda = 1$.

It is less obvious how to choose a reasonable value for the curvature of the entrepreneurial technology, ϕ. On the one hand, recent evidence from micro data suggests to use a relatively low value for this parameter: Evans and Jovanovic (1989) estimate a static version of the above model using household data on self-employment and find $\phi = 0.39$; Cooper and Haltiwanger (2000) estimate $\phi = 0.6$ using panel data on plants from the Longitudinal Research Dataset. On the other hand, recent numerical exercises evaluating the effect of tax policies using models of entrepreneurship have calibrated their model using values of ϕ above $0.7 - 0.75$ (e.g. Quadrini (2000), Cagetti and De Nardi (2003), Li (2002)).[12] I set $\phi = 0.6$ as the benchmark case, but strees the sensitivity of

[12] Most of these studies follow Atkeson et. at. (1996) who advocate that a value of $\nu = 0.15$ is consistent with the cross-country evidence on the effect of labor markets frictions on worker reallocation. Given a capital share of 1/3, this choice of v implies $\phi = 0.7$.

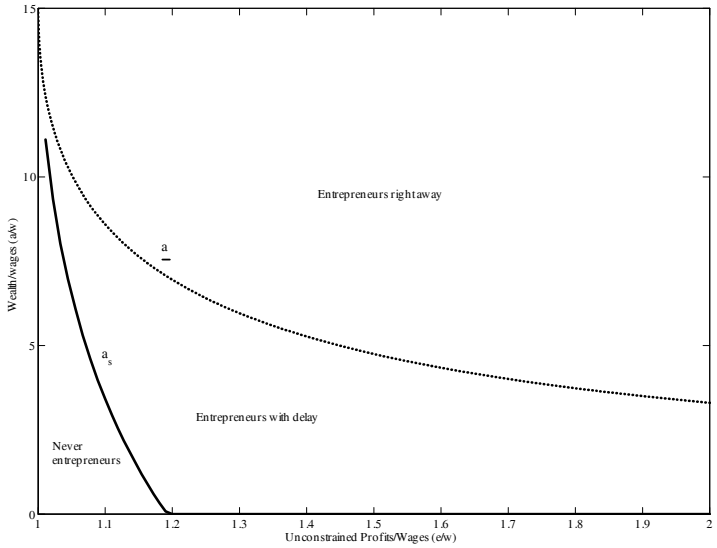

Figure 3: Individual Poverty Traps, Benchmark Economy

results to alternative specifications of the entrepreneurial technology.

Figure 3 illustrates the poverty trap threshold, $a_s(e)$, as a function of the ability of the entrepreneur. On the horizontal axis, I measure ability as the profits an individual would make if operating at the unconstrained scale relative to her wage, a measure of the returns to entrepreneurship. In the vertical axis, I measure wealth relative to the wage.[13] Individuals with ability and initial wealth to the southwest of this curve, but with relative ability higher than one, are in a "poverty trap". Even though they could run profitable businesses if operating at a unconstrained scale, the cost in terms of an uneven consumption profile is too large. I also plot the current wealth and returns combinations such that individuals are indifferent today between starting a business and working for a wage, i.e., the function $\underline{a}(e)$. This curve would be the poverty trap threshold if individuals were not allowed to save. The difference between the two curves gives a measure of the role of savings.[14] Initial wealth no longer fully determines whether individuals become entrepreneurs. Rather, even individuals who start

[13] As discussed earlier, this is a natural normalization of the data since the parametrized model is homogeneous of degree 1 in the opportunity cost, w, the wealth, a, and the entrepreneurial ability.

[14] Notice that for low ability individuals, $\underline{a}(e) < a_s(e)$, implying that these individuals will never transit from being workers to being entrepreneurs. This is an illustration of the first case of Proposition 1.

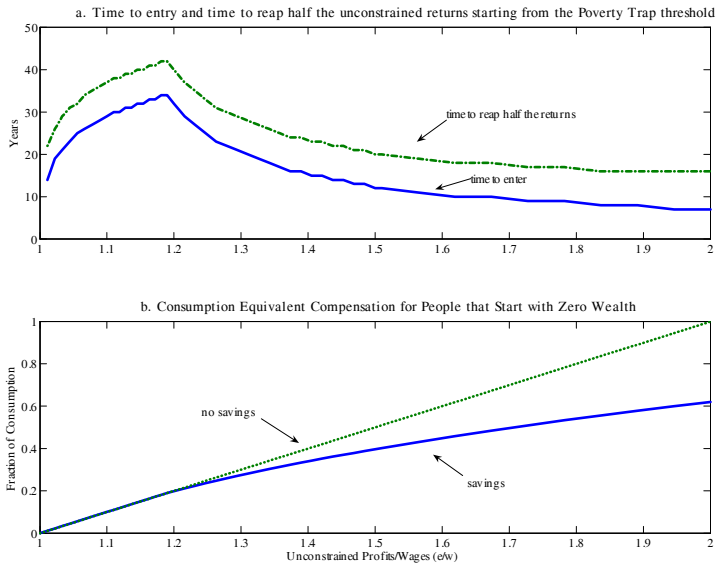

Figure 4: Time to Enter and Wealfare Cost

with low wealth could save to become entrepreneurs. Nevertheless, individuals that could earn up to 20% more as unconstrained entrepreneurs remain workers if they start with zero wealth!

In Figure 4.a, I plot the time required to start a business starting from the poverty trap threshold, a_s, as a function of ability. There, I also plot the time required to earn half the unconstrained returns starting from the poverty trap threshold, a_s. Even people that could earn 100% more income as unconstrained entrepreneurs require more that five years to save the capital required to become entrepreneurs and above fifteen years to operate at a scale at which they make half the unconstrained returns. The delay to entry and to operate at a profitable scale suggest important welfare losses.

These welfare losses are illustrated in Figure 4.b. There I plot the fraction by which the path of consumption must be increased to make an individual of a given ability, who is born with zero wealth, indifferent between living in the economy with no credit and in the economy with perfect capital markets. The lower curve is the welfare cost associated with an economy where individuals are allowed to save, while the upper ones are the welfare costs in a static model. As

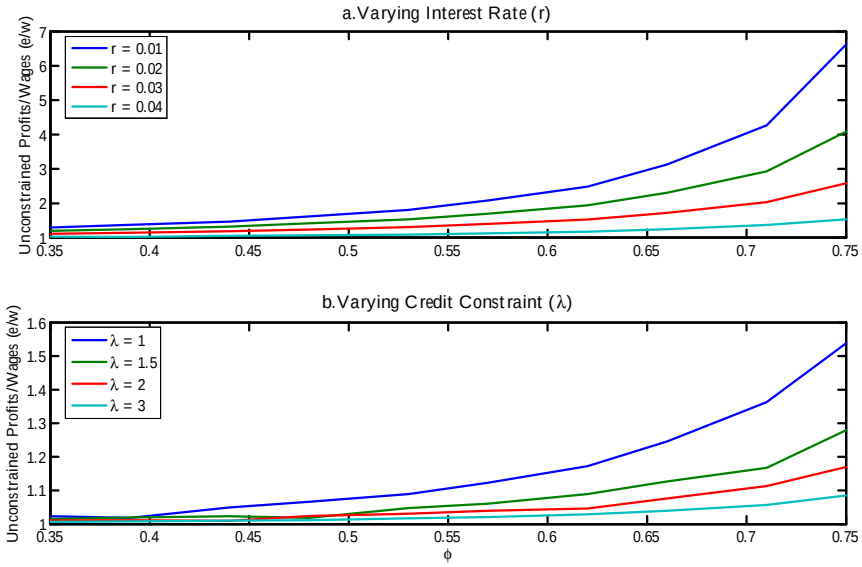

Figure 5: Sensitivity to ϕ, r, and λ

the time to entry suggests (Figure 4.a), there are potentially enormous welfare losses due to borrowing constraints.

As I mentioned when discussing the choice of parameter values, there is no consensus regarding the value for ϕ. It is therefore important to understand the sensitivity of these numerical results to the choice of ϕ. This is done in figures 5.a and 5.b. In these figures, I show how the minimum ability required for an individual that is borned with zero wealth to decide to save in order to eventually start a business, i.e., the ability e_s that solves $a_s(e_e) = 0$. In Figure 5.a The size of poverty traps and the welfare cost of borrowing constraints increase dramatically if we choose a value of ϕ above 0.6 while they tend to be unimportant if the entrepreneurial technology is characterized by strong decreasing returns to scale, ϕ below 0.5. Furthermore, the magnitude of individual poverty traps decreases with the interest rate (Figure 5.a) and the availability of credit (5.b).

For an economy with well-developed credit markets, e.g., the U.S., individual poverty traps are not substantial, and would be describe by the lowest curve in Figure 5.b. Conversely, in an economy with poorly working credit markets, we expect a low equilibrium interest rate (see Buera and Shin, 2007) and therefore, they would be better described by the upper most curve in Figure 5.a.

5 Conclusions

The motivation of this paper can be summarize by the question: Are borrowing constraints limiting entrepreneurship of any significance given that individuals can save to overcome them? Theoretically, this paper provides a qualified affirmative answer to this question. Able, but not too able entrepreneurs might end up refraining from starting profitable ventures. As argued in the paper, a definite answer depends on the parameters of the model: the tightness of credit constraints, the returns to scale of the entrepreneurial technology and the distribution of ability.

This analysis, by abstracting from general equilibrium effects, has ignored the potential for aggregate poverty traps (Piketty, 1996). Presumably, some of the results in the earlier literature on aggregate poverty traps could be sensitive to assumptions about myopic savings and one period-lived generations. At the same time, in models with forward-looking savings and heterogeneous returns, the wealth distribution might have an important role in determining the evolution of aggregates. These are important questions that are the subject of current research (Buera and Shin, 2007).

A Analysis of the Phase Diagram

As is standard in the analysis of deterministic dynamic models in continuous time, this section proceeds by analyzing the phase diagram of the system of ordinary differential equations (5) and (6) given the agent's first order conditions. The analysis is restricted to the case of $r < \rho$.

By setting $\dot{c} = 0$ in (5) an equation that describes the set of wealth-consumption pairs, (a, c), for which consumption is constant over time is obtained. In the case $f_k(e, \underline{a}) > \rho$, there exists a unique solution to this equation.[15] In particular, there exists a unique value of wealth, $a_{ss} \in (\underline{a}, k_u)$, solving the equation

$$f_k(e, a) - \rho = 0.$$

This gives the rightmost vertical curve in the (a, c) space. I label it $\dot{c} = 0$ in Figure 6. To the left of this curve, the return to capital exceeds the rate of time preference, therefore consumption increases over time. To the right of this curve the opposite is true.

Additionally, note that the locus $a = \underline{a}$ divides the space between points for which consumption decreases (to the left) and points for which consumption increases (to the right). At $\underline{a}$ agents switch occupations. Thus, the relevant return to their wealth changes from being low, $r < \rho$, to being high, $f_k(e, \underline{a}) > \rho$ (to the right of $\underline{a}$). This can we seen by inpecting the Euler equation (see Equation (5)).

Similarly, by setting $\dot{a} = 0$ in equation (6), I obtain an equation that describes the locus of points where wealth is constant. These correspond to points for which consumption equals income. Above this curve consumption exceeds income and therefore wealth decreases over time. Below, consumption is less than income thus wealth decreases over time. This curve is labeled as $\dot{a} = 0$ in Figure 2.

Trajectories in Region III move to the northwest, those in Region IV move to the northeast, in Region V they move to the southwest, while those in Region VI travel to the southeast.

Combination of wealth and consumption (a, c) in Region I will follow trajectories going to the southwest. These are points for which consumption exceeds income, and wealth is not high enough for the entrepreneurial technology to be

[15] If $f_k(e, \underline{a}) < \rho$ there is no steady state with positive wealth. In this case, it is optimal for individuals to desacumulate wealth. Individuals with $a(0) > \underline{a}(e)$ start being entrepreneurs but they eventually become workers.

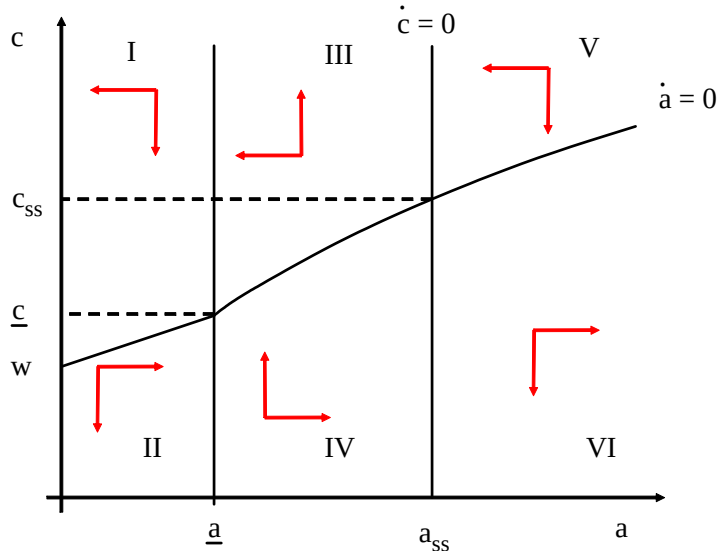

Figure 6: Phase Diagram of Households' Problem ($r < \rho$)

profitable, therefore the relevant return to savings is given by the interest rate that is lower than the rate of time preference. Wealth and consumption pairs in Region II also correspond to pairs with low wealth level, and low return to savings, therefore consumption will tend to decrease. But for pairs in Region II, since consumption is lower than income, wealth increases over time.

Of all these trajectories, only the ones converging to the points (a_{ss}, c_{ss}) and $(0, w)$ satisfy the transversality condition and do not exhibit jumps in finite time. For example, trajectories in region I above the one converging to the point $(0, w)$ will eventually hit the y axis above w and will be associated with a discontinuous consumption path in finite time.

To identify the trajectories that converge to the two steady states, it is helpful to view the phase diagram of this model as the combination of the phase diagram of the standard neoclassical growth model (regions III, IV, V and VI) and the phase diagram of the saving problem of a worker with no borrowing constraints and $r < \rho$ (regions I and II).

From the analysis of the standard growth model it is known that in a neighborhood of (a_{ss}, c_{ss}), there exists a single trajectory converging to this steady state (the stable path). In a similar fashion, from the savings problem with

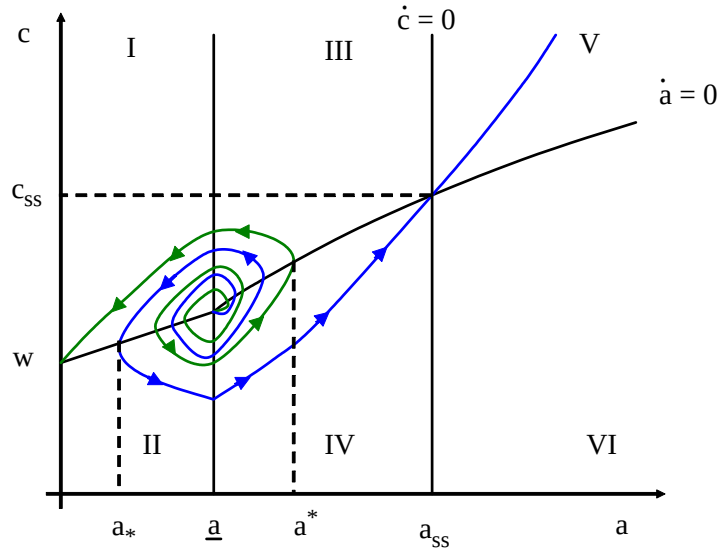

Figure 7: Trajectories Satisfying the Necessary Conditions (Intermediate Ability)

$r < \rho$, it is known that, locally, there exists a single trajectory passing through the point $(0, w)$.

Since the problem is not concave, there is not a unique path starting from a given level of initial wealth that satisfies the necessary conditions. Also, the necessary conditions are not sufficient: there may be many trajectories that start from a given level of initial wealth and satisfy the necessary conditions. But most of them are not optimal.

For instance, for levels of initial wealth close to $\underline{a}$ there exist at least two initial levels of consumption associated with trajectories that satisfy the transversality condition and do not exhibit jumps in finite time. The one with high consumption will lead in finite time to the low-wealth steady state. The trajectory associated with low initial consumption will eventually lead to the high wealth entrepreneurial steady state.

Moreover, for $a_0 = \underline{a}$ there exist many other initial consumption levels that eventually converge to one of these steady states after cycling around the point $(\underline{a}, \underline{c})$. Figure 7 illustrates the trajectories satisfying the necessary conditions for the case of intermediate ability (see Proposition 1).

In order to discriminate among the many trajectories satisfying the necessary

conditions, it is useful to introduce the Hamiltonian function of this problem:

$$H(a_0, \lambda_0) = \max_c \{u(c) + \lambda_0 (y(e, a_0) - c)\}.$$

This function gives the value of following a path that satisfies the necessary conditions (See Skiba, 1978). Note that the Hamiltonian is a strictly convex function of λ_0. This simple observation allows us eliminate all paths with the exception of the paths with the highest and lowest consumption. Define $V^e(a, e)$ ($V^w(a, e)$) to be the value associated with the lower (upper) trajectory. Similarly, define $c^e(a, e)$ and $\dot{a}^e(a, e)$ ($c^w(a, e)$ and $\dot{a}(a, e)$) be the consumption functions associated with the lower (upper) trajectory.

The next result characterizes the possible configurations of the trajectories in the phase diagram: for agents with high entrepreneurial ability only the trajectory converging to the low wealth worker steady state cycles around the point $(\underline{a}, \underline{c})$; for agents with low entrepreneurial ability only the trajectory converging to the high-wealth steady state cycles around the point $(\underline{a}, \underline{c})$; for individuals with intermediate entrepreneurial ability Figure 3 is the relevant case

Proposition A.1: *There are three possible configurations of the trajectories satisfying the necessary conditions:*

1. *Only the trajectory converging to the (a_{ss}, c_{ss}) steady state cycles around the point $(\underline{a}, \underline{c})$.*

2. *Both the trajectory converging to the $(0, w)$ steady state and the trajectory converging to the (a_{ss}, c_{ss}) steady state cycle around the point $(\underline{a}, \underline{c})$* (intermediate case).

3. *Only the trajectory converging to the $(0, w)$ steady state cycle around the point $(\underline{a}, \underline{c})$.*

Proof:. For sufficiently high e the first case arises since the trajectories to the left of $\underline{a}$ are not affected by ability, e. Similarly for sufficiently low e (e.g. $e = \underline{e}$) we have the third case. For intermediate value of ability the intermediate case arises. The only thing that need to be proved is that a case where neither the trajectory converging to the $(0, w)$ nor the trajectory converging to the (a_{ss}, c_{ss}) cycle around the point $(\underline{a}, \underline{c})$ is not possible. This is proven by contradiction.

Assume that a case where neither the trajectory converging to the $(0, w)$ nor the trajectory converging to the (a_{ss}, c_{ss}) cycle around the point $(\underline{a}, \underline{c})$ is

possible. Then for values of initial wealth close to $a_0 = 0$ the trajectory converging to the (a_{ss}, c_{ss}) steady state is optimal since for $a_0 = 0$ the plan that states forever at the point $(0, w)$ corresponds to a zero of the Hamiltonian function therefore the plan starting with lower consumption and converging eventually to the (a_{ss}, c_{ss}) steady state is preferred.[16] Furthermore, for all $a \leq a_{ss}$ $\partial V^e(a,e)/\partial a = u'(c^e(a,e)) > u'(c^w(a,e)) = \partial V^w(a,e)/\partial a$, as consumption for the path converging to (a_{ss}, c_{ss}) is always below consumption for the path converging to $(0, w)$. This implies $V^e(a,e) > V^w(a,e)$ for all $a \in [0, a_{ss}]$. But this contradicts the fact that for initial wealth close to $a_0 = a_{ss}$ choosing the lower path is a global minima of the Hamiltonian function. ■

The next step is to discriminate among all of the paths satisfying the necessary conditions for optima.

Proof of Proposition 1:. I first consider the intermediate case in Proposition A.1.

The first step is to rule out the trajectories that circle around $(\underline{a}, \underline{c})$. As was discussed before, that these trajectories are not optimal follows from the Hamiltonian function being strictly convex in the initial Lagrange multiplier, i.e. optimal trajectories are among the trajectories that start with extreme values for Lagrange multiplier and therefore consumption. Let a_* (a^*) be the first (last) point at which the lower (upper) trajectory crosses the $\dot{a} = 0$ locus. For $a_0 = a_*$, we have that $V^w(a_*, e) > V^e(a_*, e)$ since the lower path corresponds to a global minima of the Hamiltonean function. Similarly, for $a_0 = a^*$, we have that $V^w(a_*, e) < V^e(a_*, e)$. Furthermore, for $a \in [a_*, a^*]$ we have that $\partial V^e(a,e)/\partial a = u'(c^e(a,e)) > u'(c^w(a,e)) = \partial V^w(a,e)/\partial$, since consumption for the path converging to (a_{ss}, c_{ss}) is alway below consumption for the path converging to $(0, w)$. Thus, there exist a unique level of wealth such that $V^w(a_s, e) = V^e(a_s, e)$.

Clearly, for $e < \underline{e}$ $V^w(a,e) > V^e(a,e)$ for all a, since for $e < \underline{e}$ it is always prefer to be a worker than an entrepreneur. Therefore, there is an infimum ability $e_{low} \geq \underline{e}$ such that $V^w(a,e) \geq V^e(a,e)$ for all a. Similarly, since $V^w(0,e) = V^w(0)$, independent of e, and $lim_{c \to \infty} u(c) = \infty$, then there exist a supremum (finite) ability e_{high} such that $V^e(0, e_{high}) > V^w(0, e_{high})$. ■

Proof of Proposition 2. The threshold is implicitly defined by the

[16] The derivative of the Hamiltonean with respect to λ_0 equals $\frac{\partial H(a_0, \lambda_0)}{\partial \lambda_0} = \dot{a}$. Furthermore, the Hamiltonean function is strictly convex. Thus, a path for which $\dot{a} = 0$ corresponds to a global minima of the Hamiltonean function.

following equation

$$V^e(a_s, e) - V^w(a_s, e) = 0$$

where V^w is the value of following the upper trajectory and V^e is the value associated with the lower trajectory in Figure 3. For $a_s < \underline{a}$ V^w does not depend on e while V^e is a strictly increasing function of e therefore a_s is strictly decreasing in e. ■

Proof of Proposition 3. If $r = \rho$, the poverty traps threshold solve $w + ra_s = c^e(\underline{a})$, where $c^e(.)$ is the policy function for consumption associated with the stable path of the entrepreneurial problem. If both, ρ or σ, increases, then, $c^e(\underline{a})$ increases and, therefore, a_s locally increases. If e is close to e_{high}, a decrease in r has no effect on $V^w(e)$ while it decreases the value of $V^e(e)$ since individuals are saving to become entrepreneurs. For e close to e_{low} the opposite is true. ■

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