UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2001)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Constant returns to scale economy

In a standard model, inputs 1 and 2 are used to produce products A and B. The production functions are as follows.

$$x_A = (2z_{1A})^{1/2} z_{2A}^{1/2} \qquad x_B = z_{1B}^{1/2}(2z_{2B}^{1/2})$$

The economy has an endowment of $(\bar{z}_A, \bar{z}_B) = (200, 400)$.

For each of the following, explain carefully:

(a) What is the range of possible Walrasian equilibrium input price ratios r_1/r_2 if both commodities are produced?

(b) As the demand for commodity A rises, what happens to the relative price of the two inputs?

(c) For what output price ratios will commodity B be produced?

2. Extracting a natural resource

There are $\bar{Q}$ units available of a natural resource. It can be extracted at zero cost. There are T periods. The demand price for the commodity in period t is $p(q_t)$ and the interest rate is r.

(a) Write down the optimization problem if the objective is to maximize the present value of social surplus over the T periods.

(b) What can you say about the ratio of prices in succeeding periods?

(c) Compare this with the outcome of a monopolist who extracts the commodity and sells it at a price $p_t = p(q_t^m)$, $t = 1, \ldots, T$.

(d) In the special case in which $p(q) = q^{-1/\epsilon}$, $\epsilon > 1$, show that $\mathrm{MR}(q) = (1 - 1/\epsilon)p(q)$. Then compare the price sequence for the monopolist with the socially optimal price sequence.

(e) Suppose ownership of the resource is widely distributed and the good is sold "competitively." Compare the outcome to the monopoly outcome.

3. Long-run short-Run with noise

A long-run government can either choose a tight or expansionary monetary policy. With an expansionary policy, the probability of inflation is 75%. With a tight monetary policy the probability of inflation is 25%. A representative consumer in the economy can either anticipate inflation or not. The consumer gets a utility of 1 for guessing correctly about inflation and 0 for guessing incorrectly. The government has payoffs

	Guess no inflation	Guess inflation
tight	3	1
expansionary	4	2

Find the best perfect public equilibrium for the government (of the infinitely repeated game with public randomization) as a function of the discount factor, first, assuming that the consumer can observe whether or not policy is expansionary, and second assuming that the consumer can only observe whether or not there is inflation.

4. Centipede

Consider a game of "grab-a-dollar" in which player 1 may either earn \$1 or pass to player 2. Player 2 may either grab \$2 or pass back to player 1. Player 1 moves last and may either grab \$4 or give a gift to player 2 of \$8. (Note that one player always earns \$0.)

(a) Find all Nash equilibria of this game.

(b) Now suppose that there is a probability of 30% that player 1 is a generous type who will pass or give a gift at every opportunity. Find all sequential equilibria of this game.

5. Price-taking or price-making

There are N consumers indexed by i. Each has a utility function for N commodities plus a (numeraire) money commodity of the form $u_i(x_{i1}, \ldots, x_{iN}, m) = N^{-1}[\sum_{j=1}^{N} \phi(x_{ij})] + m_i$, where $x_{ij} \geq 0$. Each commodity can be produced by only one supplier, with supplier j having a cost function in terms of the numeraire commodity $c_j(x_{jj})$, where $x_{jj} \geq 0$ is j's supply of commodity j. Assume $\phi(x) = 2x - x^2/2$ and $c(x) = x^2/2$.

(a) Suppose $N = 1$. Find price-taking equilibrium. Alternatively, if the supplier acts as a price-maker while the consumer acts as a price-taker, find the monopoly equilibrium.

(b) Suppose $N = 2$. Find the price-taking equilibrium, Alternatively, find the (Bertrand) duopoly equilibrium when each seller sets price to maximize profits given the price set by the other (while the consumers continue to act as price-takers).

(c) Suppose $N = 100$. Find price-taking equilibrium and monopolistically competitive equilibrium (i.e., the N-seller version of Bertrand duopoly).

(d) Does monopolistic competition converge to price-taking equilibrium as N increases without bound? Explain.

6. Delivery/privacy problems

The cost of effort to agent a is the convex function $c_a(e_a)$. The agent can use his effort and the technology owned by the principal p to produce output according to the concave function $y_p(e_a)$. The principal's technology does not depend on the agent using it and the agent's cost does not depend on the principal's technology she is using. [In (a) and (b) there is only one principal and one agent, while in (c) and (d) there are many.]

(a) Think of y_p as expected output: actual output also depends on randomness and randomness implies that effort cannot be inferred from realized output; hence, contracts cannot be based on effort. Suppose the principal chooses a reward scheme in which he gives the worker one-half of the output. Show that the agent's effort will not achieve what is jointly optimal.

(b) Assuming the principal knows c_a, find a reward scheme that simultaneously maximizes the principal's benefit while also inducing the agent to expend optimal effort?

(c) Assume that output is not random, effort can be monitored, principals' technologies are common knowledge, but agents costs are known only to themselves. Consider a mechanism in which agents are assigned to principals to maximize total gains based on the agents' announcements of their costs. Describe a reward scheme inducing each agent to reveal her true cost of effort.

(d) With the same information conditions as (c), suppose there is only one principal, but there are many agents who bid for the right to use the principal's technology. Would the agent with the winning bid be the most efficient user of the technology, and would that agent expend the optimal amount of effort in using it?