

II

2. Foster, E., and Sonnenschein, H. "Price Distortion and Economic Welfare," *Econometrica*, March 1970, 38(2), pp. 281-297.
3. Harberger, A. C. "Taxation, Resource Allocation and Welfare" in National Bureau of Economic Research and The Brookings Institution, *The Role of Direct and Indirect Taxes in the Federal Revenue System*. Princeton: Princeton University Press, 1964, pp. 25-75. [This volume, Chapter 2.]
4. ———. "The Measurement of Waste," *Amer. Econ. Assoc. Pap. and Proc.*, May 1964, 54, pp. 58-76. [This volume, Chapter 3.]
5. ———. "On Measuring the Social Opportunity Cost of Public Funds" in *The Discount Rate in Public Investment Evaluation* (Conference Proceedings of the Committee on the Economics of Water Resources Development, Western Agricultural Economics Research Council, Report No. 17, Denver, Colorado, December 17-18, 1968), pp. 1-24.
6. ———. "Professor Arrow on the Social Discount Rate" in *Cost-Benefit Analysis of Manpower Policies*, edited by G. G. Somers and W. D. Wood. Kingston, Ontario: Industrial Relations Centre, Queen's University, 1969, pp. 76-88.
7. Hicks, J. R. "The Rehabilitation of Consumers' Surplus," *Rev. Econ. Stud.*, February 1941, pp. 108-16. Reprinted in American Economic Association, *Readings in Welfare Economics*. Homewood, Ill.: Richard D. Irwin, Inc., 1969, pp. 325-335.
8. ———. *Value and Capital*. Second edition. Oxford: The Clarendon Press, 1946.
9. ———. *A Revision of Demand Theory*. London: Oxford University Press, 1956, chapters X and XVIII.
10. Hotelling, H. "The General Welfare in Relation to Problems of Railway and Utility Rates," *Econometrica*, July 1938, 6(3). Reprinted in American Economic Association, *Readings in Welfare Economics*. Homewood, Ill.: Richard D. Irwin, Inc., 1969.
11. Johnson, H. G. "The Cost of Protection and the Scientific Tariff," *J. Polit. Econ.*, August 1960, 68(4), pp. 327-345.
12. ———. "The Economic Theory of Customs Unions" in *Money, Trade and Economic Growth*. London: George Allen and Unwin, 1962, pp. 48 ff.
13. Kaldor, N. *An Expenditure Tax*. London: George Allen and Unwin, 1955.
14. Lange, O. "The Foundations of Welfare Economics," *Econometrica*, July-October 1942, 10, pp. 215-228. Reprinted in American Economic Association, *Readings in Welfare Economics*. Homewood, Ill.: Richard D. Irwin, Inc., 1969, pp. 26-38.
15. Lipsey, R. G., and Lancaster, K. "The General Theory of Second Best," *Rev. Econ. Stud.*, 1956-57, 25(63), pp. 11-32.
16. Lipsey, R. G. *The Theory of Customs Unions: A General Equilibrium Analysis*. London: Weidenfeld and Nicholson, 1970.
17. McKenzie, L. W. "Ideal Output and the Interdependence of Firms," *Econ. J.*, December 1951, 61, pp. 785-803.
18. Meade, J. E. *Trade and Welfare*, Vol. II, Mathematical Supplement. London: Oxford University Press, 1955.
19. Organization for Economic Cooperation and Development. *Manual of Industrial Project Analysis in Developing Countries*. Paris, Vol. I, 1968; Vol. II (by I. M. D. Little and J. A. Mirrlees), 1969.
20. United States Inter-Agency Committee on Water Resources. *Proposed Practices for Economic Analysis of River Basin Projects*. Washington: Government Printing Office, 1958.

The following chapter was first presented in 1963 at a conference sponsored by the National Bureau of Economic Research and The Brookings Institution and published jointly by them in a volume titled, *The Role of Direct and Indirect Taxes in the Federal Revenue System* the following year. Together with Chapter 3 of this volume, it deals in some detail with the analysis of the welfare costs of taxation in a general-equilibrium context. The initial problem posed was how, in the analysis of a particular tax or set of taxes, to account for the money (or command over resources) raised by the government in that way. On the one hand it could be assumed that the government spent the tax proceeds in some specific way, but unfortunately there is no limit to the number of alternative patterns of additional spending that could thus be assumed. Similarly, it could be postulated for the purposes of the analysis that other taxes were reduced to compensate the extra revenue generated by the tax(es) being studied. But again there is an infinite range of alternative taxes or combinations of taxes that could play this compensating role. On reflection, it becomes obvious that the problem requires some sort of "conventional" solution.

At the end of section I, I proposed the solution that in analyzing the effects of a particular set of taxes we consider as its counterpart a reduction of income taxes of similar incidence by income bracket (see Table 2.1). This solution has the possible virtue of "sounding practical," and it was with this in mind that I introduced it. But it has the theoretical flaws (a) of entailing distributional changes within income brackets (see footnote 5), and (b) of making it difficult to analyze changes in the income tax itself (indeed, I bypass this assumption when dealing with income taxation as such). And in any case its aura of "practicality" is false, in the sense that in no plausible real-world context would it work out that the

introduction of a new excise tax (or set of taxes) would be just counterbalanced by a reduction in income taxes neatly designed so as to leave the distribution of the tax burden by income bracket unchanged.

In retrospect, I would prefer to have proposed a more purist convention with respect to "where the tax money goes" — which indeed is the one I have always used in my own thinking on the subject. This solution uses the concept of neutral taxes and transfers, which can be conceived of as classical lump-sum taxes and subsidies. When a new tax or set of taxes is imposed, we conceive of it as being counterbalanced by a pattern of lump-sum subsidies (and possibly lump-sum taxes) which keeps the relative distribution of income unchanged. We thus rule out both (first-order) income effects and income-distribution effects as we study the influence of particular taxes on economic efficiency. In a sense this convention fully isolates the efficiency aspects of the problem under review, so that readers can feel on safer ground with it as the backdrop of the analysis, rather than the "practical convention" mentioned in the text.

As alluded to earlier (see Chapter 1), I find it surprising how many people have an instinctive negative reaction to an analytical apparatus which conceptually separates efficiency from distribution criteria in the analysis of a particular problem. One may perhaps caricature this frequent reaction as saying that if one separates out the efficiency aspect of a problem, it must mean that one implicitly favors the status quo as far as income distribution is concerned. Here I would simply like to introduce some arguments as a counterpoise to that reaction.

Consider first the standard textbook separation of the income and substitution effects of a change in price. As

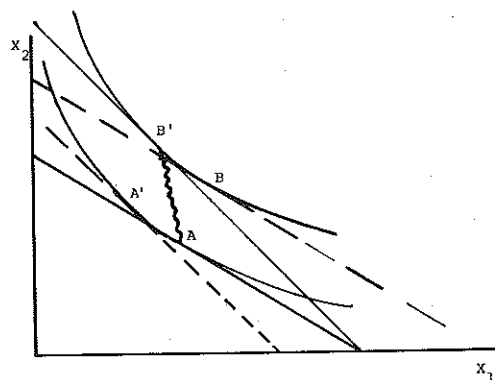


FIGURE 2A

depicted in Figure 2A, the total move is from A to B' . Are we to consider the substitution effect as the move from A to A' and the income effect that from A' to B' , or should we instead view the income effect as being the move from A to B , and the substitution effect that from B to B' ? Obviously there is no ground for choice here — indeed there is no reason in principle why we could not make the separation between income and substitution effects continuously as one varies price from its initial to its final level, as suggested by the zigzag line AB' , where "upward" moves represent income effects and "leftward" moves substitution effects.

It is similar with respect to distribution effects. If we let X represent the vector of outputs, D that of the distribution of income, and T that of policy variables (taxes, etc.), and express $X = F(D, T)$, we have four possible points, labeled to correspond to those in Figure 2A.

$$X_a(D_0, T_0)$$

$$X_a'(D_0, T_1)$$

$$X_b(D_1, T_0)$$

$$X_b'(D_1, T_1).$$

The moves from X_a to X_a' , and from X_b to X_b' , are two alternative measures of the efficiency effects of the policy shift from T_0 to T_1 , and there is no way to choose between them analytically. And since the same methodology for measuring efficiency effects (welfare costs) would be applied in either case, it is idle to attribute to the methodology or its users a bias in favor of the status quo.

Section II is an effort to derive analytically but at the same time expound intuitively the basic methodology of welfare-cost measurement. In order to keep the analysis simple and amenable to graphical treatment, it is limited to the case of a linear resource constraint. (However, the counterpart of this analysis for the case of a nonlinear constraint is presented in "The Measurement of Waste.") Toward the close of section II there is a direct derivation of the basic expressions for welfare cost from a general utility function. This is very similar to that appearing in equations (1.1) to (1.5') in Chapter 1, but there are differences. Basically the derivation in Chapter 2 uses the linear resource constraint to eliminate problems connected with possible changes in the marginal utility of income, whereas the derivation in Chapter 1 works without making any assumptions regarding resource constraints and contains a more sophisticated handling of possible changes in the marginal utility of

income. It is important to note, however, that for the problems treated in the following chapter, both derivations come to the same result.

Subsequent sections deal with the theoretical derivation of a measure of the welfare cost of income taxation, with an attempt to get a rough empirical estimate of the order of magnitude of this cost for the United States, with an assessment, in theoretical terms, of the comparative welfare costs of income and excise taxation, with explorations concerning the welfare costs related to the tax treatment of saving and with ruminations about the influence of taxation on economic growth.

Chapter 2

Taxation, Resource Allocation, and Welfare

I. INTRODUCTION¹

During the past decade the traditional solution to the issue of direct vs. indirect taxation has been subjected to serious challenge. Whereas earlier the theoretical superiority of direct over indirect taxation was accepted as virtually axiomatic, now it must be regarded as something that cannot be "proved," and that in principle has to be investigated for each particular case. The traditional case for direct taxation rested on its alleged "neutrality" among the alternatives open to consumers. Whereas an indirect tax on one item of consumption made that item artificially expensive to consumers, relative to other goods, and thus consumers were faced with relative prices that did not reflect the relative marginal costs of production of different goods, an income tax was generally held to be free of this defect. The income tax, it was held, was similar to a system of excise taxes striking all commodities with an equal percentage tax rate. Whatever might be the ratios between the marginal costs of producing different goods, the same ratios would (under competition) apply to the prices facing consumers. Marginal rates of substitution in consumption (given by the ratios

¹ This introductory section is meant to serve, in addition to the usual functions of an introduction, as a guide to readers who might otherwise be put off by the amount of mathematics that appears in the main body of the paper. The principal motivation for and conclusions from each section are set out here in nonmathematical terms, so that readers may skip any offending section of the main text without substantial loss of continuity. For the further guidance of readers, let me here point out that section IV contains the most mathematical manipulation, and can be skipped with the aid of the summary in the introduction without serious loss. Section II, on the other hand, develops the fundamental framework that is used throughout the paper. Skipping this section therefore entails substantial loss of content. I have tried, in section II, to develop the basic argument with the aid of graphical analysis, so as to ease the burden on less mathematically oriented readers, and I hope, accordingly, that most readers will be able to work through the derivations presented there. In comparison with sections II and IV, the remaining sections contain only a modest amount of mathematical manipulation.

of gross-of-tax prices between pairs of goods) would therefore be equal to marginal rates of substitution in production (given by the corresponding ratios of net-of-tax prices).

Many economists realized, long before 1951, that this proposition would have to be modified in a situation in which the supply of labor was other than completely inelastic. But the extent of the required modification was not appreciated. In point of fact, the required modification is so great that it destroys any theoretical presumption of the superiority of direct over indirect taxation. This was pointed out by Little in his fundamental paper written in 1951.² He presented a simple example in which labor was the only factor of production and in which there were only three "goods": say, bread, wine, and leisure. In this case, an excise tax on wine would distort the choices between wine and bread and between wine and leisure, but would leave the choice between bread and leisure undistorted. Similarly, an excise tax on bread would distort the choices between bread and wine and between bread and leisure, but would leave the choice between wine and leisure undistorted. However, an income tax (interpreted as an equal-rate excise tax on both bread and wine) would distort the choices between bread and leisure and between wine and leisure, leaving the choice between bread and wine undistorted. Each of the three cases distorts two of the three possible choices, while leaving the third choice undistorted. There is thus no qualitative difference between the nature of the effect of direct as against indirect taxation. Any preference for direct taxation over indirect must therefore be based on quantitative rather than qualitative comparisons, or else on grounds (like equity or political feasibility) that are not strictly economic.

This challenge to the traditional preference for direct taxation provides the focus for most of the present paper. Once Little's argument is appreciated, we can no longer resolve the issue by saying that distortions are present in the indirect-tax case but absent in the direct-tax case. We must recognize that distortions are present in both cases, obtain relevant measures of their effects, and then compare these measures. The relevant measure, in this case, is the cost to the economy of the inefficiencies resulting from tax-induced distortions of choices. This cost, which I like to call the welfare cost of a tax system, has traditionally been labeled the "excess burden" of taxation.

Section II of the paper is an attempt to expound the principles of measuring the welfare costs of a set of taxes in a case where all goods are produced at constant cost. The case chosen is a particularly simple one, because, with the simplification of constant costs, the argument can be carried out in diagrammatic terms.³ From this exercise a general expression is derived, from which we can determine the welfare cost of any given pattern of taxes on final goods and services when production is governed by constant costs, when distortions other than taxes are absent, and when the amount of labor supplied by each individual to the market does not vary as a consequence of changes in tax rates.

² I. M. D. Little, "Direct Versus Indirect Taxes," *Economic Journal*, September 1951, pp. 577-584.

³ For a more general mathematical treatment, see my paper, "The Measurement of Waste," *American Economic Review*, May 1964. [This volume, Chapter 3.]

In section III we generalize the result of section II, incorporating the possibility of tax-induced changes in the supply of labor. Thus we face up directly to the problem posed by Little, for the formulation of section III shows how it is possible to measure the welfare cost of an income tax and to compare this with the welfare costs induced by alternative patterns of indirect taxation. To emphasize the possibility of extracting useful numerical results from analyses of this type, an attempt is made at the conclusion of section III to estimate the welfare cost of the U.S. personal income tax. It should be borne in mind that this exercise is intended mainly as an example of how the job might be done, and not as a definitive analysis of the U.S. income tax. Among other things, the measurement effort of section III takes account only of the costs of the personal income tax arising from the labor-leisure choice, and not those that affect occupational choice, investment decisions, etc. Moreover, the measurement is based on crude and scanty evidence of a highly aggregative nature, whereas a more complete analysis would take into account the differential responses of different classes of labor supply to tax incentives (e.g., distinguishing between the effects of taxation on labor-force participation rates of women and men). Finally, the measurement in section III (as distinct from the theory presented) is based on the assumption that the income tax is the only tax present in the system, or, perhaps more plausibly, on the assumption that the welfare costs of the personal income tax are not altered by the many other taxes that are present in the U.S. system.

In spite of these qualifications, however, I feel that the exercise of section III provides us with some useful insights. The estimated welfare cost of the U.S. personal income tax is about \$1 billion per year — hardly a negligible figure. Yet the simplifying assumptions just alluded to probably have the effect of understating the true cost. It is hard indeed, in the light of this result, to hold to the traditional position that the personal income tax is truly neutral.

In section IV we attempt a direct comparison of the welfare costs of income and excise taxation. This entails nothing more than a straightforward application of the general formulation derived in section III, but some relevant insights are obtained by carrying out the steps. As the first step in section IV, we try to find the "best" way of raising a given amount of revenue by commodity taxation. We find, as others⁴ have previously shown, that an income tax (i.e., a tax striking all goods and services other than leisure at an equal rate) will be the "best" way only in very unusual circumstances. If, as seems to be the case in the real world, we cannot tax leisure as such, then the "best" tax system we can achieve under this constraint is one that strikes at higher-than-average rates those goods that are complements to or poorer-than-average substitutes for leisure, and that strikes at lower-than-average rates those goods that are better-than-average substitutes for leisure. An income tax is "best," given the constraint that leisure cannot be taxed, only if all goods are equally

⁴ See Little in *Economic Journal*, September 1951; W. J. Corlett and D. C. Hague, "Complementarity and the Excess Burden of Taxation," *Review of Economic Studies*, No. 54, 1953-54, pp. 21-30; J. E. Meade, *Trade and Welfare*, Vol. II, Mathematical Supplement, London, 1955; R. G. Lipsey and K. Lancaster, "The General Theory of Second Best," *Review of Economic Studies*, No. 63, 1956-57, pp. 11-32.

good substitutes for leisure (i.e., only if a tax on leisure would lead to an equi-proportional change in the consumption of each and every good and service).

This conclusion could indeed be shattering to the principle of direct taxation if it meant that the welfare costs of taxation could be greatly lowered by having widely different rates of taxation on different goods and services as against the equal rates implied by income taxation. It does not appear, however, that this is the case. One way of determining the answer to the question just posed is to compare the welfare cost of a particular indirect tax with the welfare cost of an income tax yielding equal revenue. This is the second step taken in section IV. It recognizes that the relevant substitute for an indirect tax is not "no tax at all" but rather some other manner of obtaining the same yield, and it considers the particular alternative represented by income taxation. The result of this exercise is that, as long as the excise tax in question is not itself a very general tax (striking a large fraction of all goods and thus approaching an income tax in coverage), and as long as the elasticity of demand for the commodity or commodities subject to excise tax is not very small, the substitution of an income tax for the excise tax in question will very likely result in a reduction of welfare cost.

The third step in section IV attempts to make the preceding comparison a bit more realistic. One of the key propositions that emerges from the study of welfare costs in a general-equilibrium framework (such as that taken in this paper) is that the effects of a particular tax depend not only on that tax itself but on the other taxes that are present along with it. The comparison made in step two of section IV considers an excise tax that is being used as the sole tax in the system, and is replaced by and compared with an income tax of equal yield. In step three of section IV we alter this comparison, and instead consider an excise tax which, when present, stands side by side with an income tax of a given size. We compare this setup with an alternative one in which the excise tax is taken off and replaced by a sufficient rise in the rate of income tax to produce the same total yield as was previously obtained from the excise and income taxes combined. This comparison is slightly more complicated than that undertaken in step two, but does not greatly alter the conclusion. Broadly speaking, there is still a strong presumption in favor of replacing excise taxation with income taxation, as long as the excise taxation itself is not very general and as long as the taxed commodities do not have very low elasticities of demand. When one comes to consider excise taxes that have very low rates and/or very broad bases, however, the result is no longer so clear. In these cases, considerations of the complementarity (or low substitutability) of an excise-taxed commodity with leisure might weigh the final judgment in favor of retaining the excise tax rather than replacing it by an adjustment of income tax rates. One might say, in short, that the theoretical case in favor of direct taxation has been forced into full retreat by the considerations raised by Little and others in recent years; but that the practical case remains comparatively unscathed, particularly where the excise taxes in question have relatively high rates and do not strike a very high proportion of consumer expenditures.

In section V the effects of taxation on saving are considered. The treatment, far briefer than the subject deserves, focuses on the nonneutrality of ordinary

income taxation with respect to the saving decision, and on the measurement of the welfare costs engendered by this nonneutrality. It is shown that income taxation has the effect of reducing the rate of saving, when the alternative to the income tax is a consumption tax yielding the same amount each year. A simple formula for estimating the welfare costs of the distortions introduced by income taxation through its influence upon the savings decision is then presented. In terms of this formula, a very rough judgment is reached on the possible order of magnitude of this class of welfare costs in the U.S. economy. Finally, it is suggested that the welfare costs stemming from the differential tax treatment of different kinds of income from capital in the United States are likely to be substantially greater than the welfare costs arising from the influence of taxation on the saving decision as such. That is to say, the U.S. economy very likely suffers greater costs from tax-induced misallocations of its given capital stock than from the influence of taxation on the over-all size of that capital stock.

Finally, in section VI, an effort is made to draw some conclusions from the preceding analysis on the influence of taxation on economic growth. Since previous sections of this paper are concerned mainly with the labor-leisure choice and with the saving decision, the discussion of taxation and growth is focused in these terms. The conclusion is reached that the effects of taxation on the labor-leisure choice have a truly negligible effect on the economy's rate of growth, but that the effect of taxation on the saving decision may reduce the economy's annual rate of growth by as much as one-tenth or two-tenths of a percentage point. For those who are surprised by the small effect of taxation via the saving decision, section VI reviews the evidence supporting this result.

To set the stage for the treatment which follows, I should first like to discuss certain key assumptions that will be used throughout this paper.

1. It is assumed that, under any two situations being compared, the economy's productive resources are fully employed. This assumption is the conventional one in studies of the allocative effects of taxation. The resource-allocation question can be framed as "how will our resources be used?" as distinct from the alternative income-and-employment question of "will some of our resources be idle?" The effect of this assumption, for the problems treated in this paper, is to restrict the set of possible resource allocations to that determined by the production frontier of the economy. Thus in Figure 2.1, the alternative bundles of goods X_1 and X_2 produced in any of the cases to be compared will lie somewhere along the production frontier AB . A production point like C , indicating that the economy is not producing up to its potential, is ruled out by assumption.

2. It is assumed that, under any two alternative taxes to be compared, the government will obtain just that amount of purchasing power needed to buy a given bundle of goods. One need not belabor the point that if we are to discuss the effects of a given tax on resource allocation, we must implicitly or explicitly compare the situation in which that tax exists with some alternative situation. Once this is recognized, we have a variety of choices. We can compare the "tax situation" with a "no-tax situation" or with any possible combination of alternative taxes. But this is not the worst of it. If the total

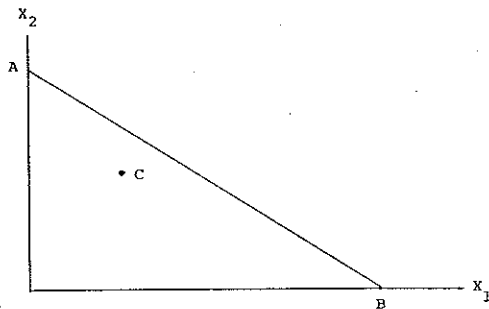


FIGURE 2.1

revenues of the government have different purchasing power in the cases being compared, we must either have differing amounts of deficit or surplus in the two cases, or differing amounts of government expenditure. If surpluses or deficits differ, deflationary or inflationary forces are set in motion by the tax change, and we are taken from the realm of allocative theory into that of income-and-employment analysis. If government expenditures differ in the two cases, we must inquire into how the government spends its added revenue or reduces its prior expenditures. This opens a Pandora's box of possibilities in an analysis of resource-allocation effects, for there are an infinity of ways in which the government could allocate any increase or reduction in expenditures. To avoid having to guess what the government would do with its money, we simply assume that the government would do the same thing in both the situations being compared. This, together with the previous assumption, enables us to conceive of a "consumption frontier" or, perhaps better, a private-sector expenditures frontier, obtained by deducting the fixed pattern of government purchases from the economy's production frontier. Thus, in Figure 2.2, every point on the "consumption frontier" CD is obtained by deducting the fixed amounts of government purchases G_1 and G_2 from the corresponding point on the production frontier AB . As long as the government is getting the bundle (G_1, G_2) of X_1 and X_2 , and as long as the economy is operating on the production frontier, production will take place somewhere within the segment

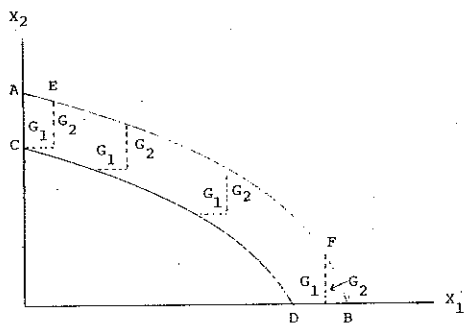


FIGURE 2.2

EF of AB , and the "consumption" of the private sector must lie somewhere along CD in any two cases being compared.

3. It is assumed that the changes in output dictated by the changes in tax policy being considered are accomplished at (approximately) constant unit cost. Taken strictly, this assumption requires that the production frontier be linear within the relevant range. Taken less strictly, it requires only that the production frontier be reasonably well approximated by a linear function within the relevant range. I am prepared to argue that this is likely to be the case for the changes in output which would be induced by plausible changes in the taxes now in force in the United States. But I must confess that this assumption is mainly dictated by considerations of convenience. The mathematical parts of subsequent sections of this paper would be greatly complicated if the assumption of constant costs were abandoned, and yet the principal qualitative conclusions of the analysis would be unaffected.

4. It is assumed that, where "welfare costs" are being measured, the tax setup being considered has as its alternative a tax system of similar overall incidence. The alternatives being compared are assumed to have similar overall incidence in order to isolate the allocative effects of taxation from its redistributive effects. For example, if we are concerned with the effects of eliminating a tax on jewelry, whose incidence among income brackets is as indicated in column 1 of Table 2.1, we might so adjust income-tax rates as to increase income-tax collections from each bracket by approximately these same amounts. Thus if income-tax collections, by bracket, were originally as given by column 2 of Table 2.1, we would consider an income tax collecting the amounts given in column 3 to be an appropriate alternative to the tax setup given by columns 1 and 2.

TABLE 2.1
Tax Collections by Income Bracket, per Taxpayer

Income brackets	Alternative 1		Alternative 2
	Excise tax on jewelry (1)	Income tax (2)	Income tax (3)
1	20	50	70
2	30	100	130
3	40	200	240
4	50	400	450
5	60	1,000	1,060

This is not to say that the real alternatives faced by policy-makers are likely to be as neatly comparable as those in Table 2.1. But if the practical alternative (say, 3) to alternative 1 is an income tax which is different in its incidence from alternative 2, we would simply analyze the move from 1 to 3 as the sum

of a move from 1 to 2, which we would evaluate using purely allocative criteria,⁵ plus a move from 2 to 3, which we would evaluate using purely distributive criteria.

II. MEASURING THE WELFARE COST OF EXCISE TAXES

In this section, we begin by outlining the "textbook" treatment of welfare cost in the simplest terms, and proceed to extend the analysis to more complicated cases. The textbook case is illustrated in Figure 2.3.

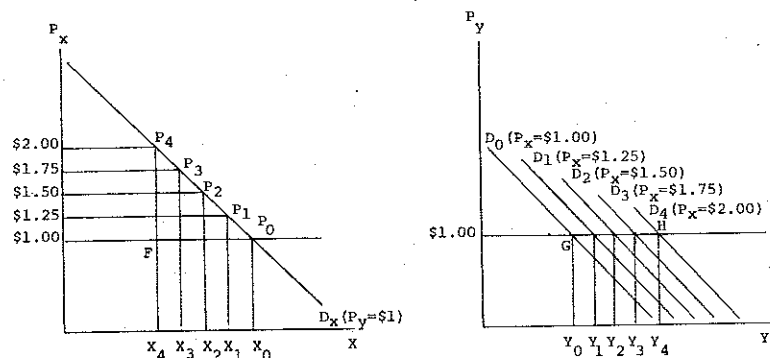


FIGURE 2.3

We are to compare a situation in which there is an excise tax of \$1 per unit of X ($X = X_4$; $P_x = \$2.00$) with a situation in which this tax does not exist, but in which the government is raising equivalent revenue by an income tax ($X = X_0$; $P_x = \$1$). It is important to recognize at the outset that D_x is defined as holding government revenues constant. Thus at P_4 we have government revenues equal to $(\$1)(X_4)$, all obtained from the excise tax on X . As we move to P_3 , the excise tax must be lowered to \$.75, and government revenues from the excise tax alone are only $(\$0.75)(X_3)$. However, since government revenues must be constant all along the curve, there must be, at point P_3 , income taxation to yield $(\$1)(X_4) - (\$0.75)(X_3)$. At P_2 income taxes must yield $(\$1)(X_4) - (\$0.50)(X_2)$; at P_1 they must yield $(\$1)(X_4) - (\$0.25)(X_1)$; and at P_0 they must yield $(\$1)(X_4)$.

⁵ Strictly speaking, one can eliminate distributive criteria in evaluating the move from 1 to 2 only when each and every individual bears the same burden of tax in both cases. Actually, the move from 1 to 2 as described above would entail some redistribution of income in favor of jewelry purchasers and away from those who do not purchase jewelry or purchase only small amounts relative to the average of their income bracket. But this redistribution is entirely within income brackets, and thus is not likely in itself to significantly alter the pattern of demand. Moreover, if the equal treatment of equals is accepted as a goal of tax policy, one would have to regard the move from 1 to 2 as having beneficial effects on distribution (differing tastes of people with similar incomes should not make these people "unequal"). Thus, as long as there would be an allocative gain on this move, we can consider it as being slightly reinforced by distributive considerations.

The simplest exposition of the measurement of the welfare cost of excise taxation can be made by looking only at the left-hand diagram in Figure 2.3. As between the situation (X_0, P_0) and the situation (X_4, P_4) , consumers have given up $(X_0 - X_4)$ units of the good X . The value placed by them on the first unit given up was \$1, but subsequent units had higher values. If we think of the tax being gradually raised from zero to \$1, this is clearly evident. When the tax is raised from zero to \$.01, the units of X given up have values between \$1 and \$1.01; when the tax is raised from \$.01 to \$.02, the units of X given up have values between \$1.01 and \$1.02. Finally, when the tax is raised from \$.99 to \$1.00, consumers value the units of X given up at between \$1.99 and \$2.00. The sum total of the value placed by consumers on all the X they have given up in the process of raising the tax from zero to \$1 can be measured by the area $X_0 P_0 P_4 X_4$.

But in the process of demanding less X as a consequence of the tax, consumers have released resources from industry X and increased their demand for other goods (Y). The measure $X_0 P_0 P_4 X_4$ therefore overstates their real loss; from it we have to deduct the value of the extra Y they consume in the excise-tax case. Where there is no tax (or other divergence between the value placed by consumers on the marginal product of productive factors and the rewards received by those factors) on Y , the value to consumers of the extra Y they have in case (X_4, P_4) compared to case (X_0, P_0) can simply be measured by the area $X_0 P_0 F X_4$. This represents the amount which was originally paid in industry X to the factors transferred to Y as a result of the tax. Deducting $X_0 P_0 F X_4$ from $X_0 P_0 P_4 X_4$ leaves the triangle $P_0 P_4 F$ as the measure of the welfare cost or "excess burden" of a \$1 per unit excise tax on X .

The right-hand diagram in Figure 2.3 may help readers to verify that $X_0 P_0 F X_4$ does in fact measure the value to consumers of the extra Y they demand as a consequence of the tax on X . Again, considering a sequence in which the tax on X is gradually raised from zero to \$1, we have first a slight displacement of the demand curve for Y as the price of X is raised from \$1.00 to \$1.01, and a little extra Y is bought at the price of \$1.00. As the price of X is raised from \$1.01 to \$1.02, another slight displacement of the demand curve for Y occurs, and again a little extra Y is bought at the price of \$1. Summing up the values to consumers of the successive increments of Y generated by raising P_x all the way to \$2.00, we obtain the area $Y_0 G H Y_4$, which is equal to the area $X_0 P_0 F X_4$ in the left-hand diagram.

There is no contribution to welfare cost (corresponding to the triangle $P_0 P_4 F$) coming from the market for Y because, by assumption, no distortions are present in that market. The resource cost of an extra unit of Y is \$1.00, and the moment consumers find that it is worth \$1 to buy an extra unit of Y , they can freely do so. Thus the value to consumers of each extra unit of Y they take is equal to the price they pay for it.

To summarize the result obtained so far, when a tax on X of T_x per unit is the only distortion present, the welfare cost of that tax can be measured by $-1/2 T_x \Delta X$, where ΔX is the change in the consumption of X induced by the tax. If we define $t_x (= T_x/P_{x0})$ as the percentage rate of tax, we can note that (with constant costs) it also represents the percentage change in the price

of X . We then can derive $\Delta X = \eta_{xx} X t_x$, where η_{xx} is the own-price elasticity of demand for X . Substituting this expression for ΔX , and $P_{x_0} t_x$ for T_x in the cost measure $(-1/2 T_x \Delta X)$, we obtain as an alternative expression for the welfare cost of an excise tax $-1/2 X P_{x_0} \eta_{xx} t_x^2$. This expresses the welfare cost of a tax in terms of $(X P_{x_0})$, the total net-of-tax receipts from sales of the product in question; η_{xx} , the own-price elasticity of demand for the product (a negative number); and t_x , the effective percentage rate of tax. Since the total tax yield, R_x , can be expressed as $X P_{x_0} t_x$, we can also express the welfare cost of the tax as $-1/2 R_x \eta_{xx} t_x$. Since data on R_x and t_x are usually available for any existing tax, only η_{xx} need be estimated to obtain a measure of the welfare cost of such a tax.

In the course of this paper we shall show that this simple textbook measure of welfare cost is not a bad approximation in the cases of most existing taxes. But in order to reach this conclusion, we must modify the above analysis to take account of the main ways in which the real-world situation diverges from that depicted in Figure 2.3.

By far the most disquieting assumption underlying Figure 2.3 and the analysis of it is the assumption that no excise taxes or other distortions exist in the "rest" of the economy (industry Y). It is possible, however, to modify the analysis so as to avoid making this assumption.

Figure 2.4 differs from Figure 2.3 only in that the initial price of Y is taken to be \$1.50, consisting of a unit cost of \$1 plus a tax of \$.50 per unit. Once again, as we envision the tax on X being raised in small steps from 0 to \$1.00 per unit, we obtain the area $X_0 P_0 P_4 X_4$ as measuring the value to consumers of the $(X_0 - X_4)$ units of X that they are induced to forego as a consequence of the tax.

The difference between this and the earlier case appears in the right-hand diagram of Figure 2.4. Whereas, in Figure 2.3, consumers were initially paying \$1 per unit of Y , and continued paying that price after the tax was imposed on X , now, in Figure 2.4, consumers are initially paying \$1.50 per unit of Y , and continue to pay that price throughout the exercise. Hence the value to consumers of the extra Y that they are induced to consume by the tax on X is

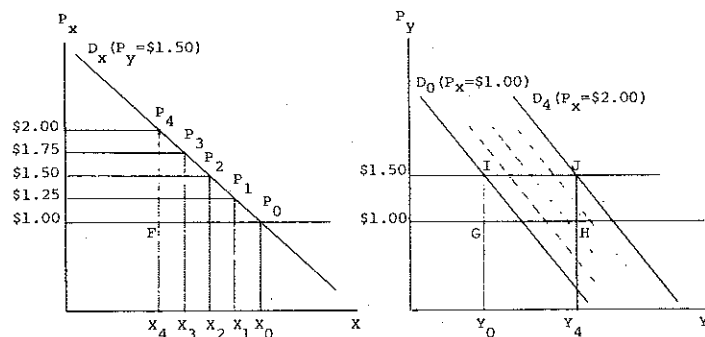


FIGURE 2.4

$Y_0 J J Y_4$, rather than $Y_0 G H Y_4$. In the present case, therefore, the net change in the welfare of consumers as a result of the tax on X is measured by the difference between a loss of $X_0 P_0 P_4 X_4$ (on account of $[X_0 - X_4]$ units of X being foregone), and a gain of $Y_0 J J Y_4$ (on account of $[Y_4 - Y_0]$ more units of Y being consumed). Since the areas $X_0 P_0 P_4 X_4$ and $Y_0 G H Y_4$ are equal as before (the cost of production of the addition to Y is the same as the reduction in production costs of X), the net loss in the welfare of consumers can be measured by $P_0 P_4 F$ minus $G I J H$. Since $G I J H = (t_y) \times (Y_0 G H Y_4)$, and $Y_0 G H Y_4 = X_0 P_0 P_4 X_4$, we can express $G I J H = (t_y) \times (X_0 P_0 P_4 X_4) = t_y P_{x_0} \Delta X$. This, in turn, can be expressed as $G I J H = t_y X P_{x_0} \eta_{xx} t_x$. Now $P_0 P_4 F = -1/2 X P_{x_0} \eta_{xx} t_x^2$, so that the net loss in welfare from placing a tax of t_x on X in the presence of an already existing tax of t_y on Y can be expressed as

$$-\frac{1}{2} X P_{x_0} \eta_{xx} [t_x - 2t_y] = -\frac{1}{2} R_x \eta_{xx} [t_x - 2t_y].$$

From this it can be seen that there is a clear gain from placing a tax on X equal to the pre-existing tax on Y . The expression $-1/2 R_x \eta_{xx}$ is always positive, and the expression $t_x - 2t_y$ is negative when $t_x = t_y$. Therefore the "cost" of moving t_x from zero to t_y is negative, indicating a gain in welfare.

In the example chosen for Figure 2.4, t_x was set at twice t_y . Thus the expression above indicates that there is no gain or loss in welfare. This can be verified by looking at Figure 2.4. The loss measured by the triangle $P_0 P_4 F$ is just counterbalanced by the gain measured by the rectangle $G I J H$.⁶

The extension of this result to the case of three or more commodities is self-evident. In Figure 2.5, we assume that a tax of \$.50 per unit on X_2 , and one of \$.25 per unit on X_3 were already in existence before a tax of \$1.00 per unit was levied on X_1 . The reduction in welfare of consumers as a consequence of the tax on X_1 is measured by the area $A B C$ minus the sum of the areas $D E F G$ and $H I J K$.

We can now turn to the derivation of a general expression for the welfare cost of a set of excise taxes. Assume there are three commodities, X_1, X_2, X_3 , all of which are taxed. The welfare cost of the whole set of taxes can be measured by taking first the welfare cost of a tax on X_1 , with no other taxes in existence;

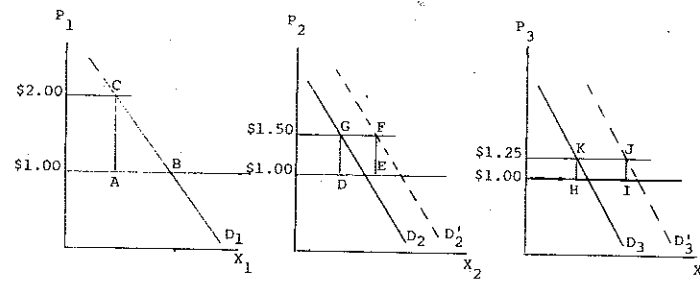


FIGURE 2.5

⁶ Both the rectangle and the triangle have equal bases, but the height of the triangle is just twice that of the rectangle.

adding to this the addition to the welfare cost obtained by imposing a tax on X_2 , given that the tax on X_1 already exists; and finally adding the increment to the welfare cost incurred by imposing a tax on X_3 , given that the taxes on X_1 and X_2 already exist.

Since the effects of the taxes will be analyzed in three steps, let us denote by Δ_1 those changes taking place in the first step (when a tax is imposed on X_1), and by Δ_2 and Δ_3 , respectively, those changes taking place in the second and third steps.

When a tax of T_1 is imposed on X_1 , with no taxes on X_2 and X_3 , the welfare cost is simply

$$(a) \quad -\frac{1}{2}T_1 \Delta_1 X_1,$$

i.e., the triangle P_0P_4F in Figure 2.3. When a tax of T_2 is imposed on X_2 in the presence of a tax of T_1 on X_1 , the addition to welfare cost is

$$(b) \quad -\frac{1}{2}T_2 \Delta_2 X_2 - T_1 \Delta_2 X_1.$$

The first component is the triangle under the demand curve for X_2 (corresponding to ABC in Figure 2.5), and the second is the offset (corresponding to $DEFG$ in Figure 2.5) arising from the fact that a tax on X_1 already exists. (There is no offset corresponding to $HIJK$ in Figure 2.5, because at this step we have not yet imposed any tax on X_3 .)

When a tax of T_3 is imposed on X_3 in the presence of taxes of T_1 and T_2 on X_1 and X_2 , the addition to welfare cost is

$$(c) \quad -\frac{1}{2}T_3 \Delta_3 X_3 - T_1 \Delta_3 X_1 - T_2 \Delta_3 X_2.$$

The sum of (a) + (b) + (c) gives us the welfare cost of the set of taxes (T_1 , T_2 , T_3) taken together. Let me here introduce S_{ij} as a general notation for $\partial X_i / \partial P_j$, defined to include just the substitution effect. We can then write:

$$\Delta_1 X_1 = \frac{\partial X_1}{\partial P_1} T_1 = S_{11} T_1$$

$$\Delta_2 X_1 = \frac{\partial X_1}{\partial P_2} T_2 = S_{12} T_2$$

$$\Delta_3 X_1 = \frac{\partial X_1}{\partial P_3} T_3 = S_{13} T_3$$

$$\Delta_2 X_2 = \frac{\partial X_2}{\partial P_2} T_2 = S_{22} T_2$$

$$\Delta_3 X_2 = \frac{\partial X_2}{\partial P_3} T_3 = S_{23} T_3$$

$$\Delta_3 X_3 = \frac{\partial X_3}{\partial P_3} T_3 = S_{33} T_3.$$

Substituting into (a), (b), and (c), and summing, we obtain as the expression for the welfare cost of the set of taxes

$$-\frac{1}{2}S_{11}T_1^2 - \frac{1}{2}S_{22}T_2^2 - \frac{1}{2}S_{33}T_3^2 - S_{12}T_1T_2 - S_{13}T_1T_3 - S_{23}T_2T_3,$$

or alternatively,

$$-\frac{1}{2}[S_{11}T_1^2 + S_{22}T_2^2 + S_{33}T_3^2 + 2S_{12}T_1T_2 + 2S_{13}T_1T_3 + 2S_{23}T_2T_3].$$

Recognizing that $S_{ij} = S_{ji}$,⁷ we can express this as

$$-\frac{1}{2} \sum_i \sum_j S_{ij} T_i T_j.$$

This is one of three fundamental forms in which the welfare cost of a set of taxes may be written. The second fundamental form may be derived by breaking up (a) + (b) + (c) into expressions reflecting changes in the quantity of each commodity. Thus we have, from commodity 1,

$$-\frac{1}{2}T_1 \Delta_1 X_1 - T_1 \Delta_2 X_1 - T_1 \Delta_3 X_1.$$

Denoting the total change in X_1 by $\Delta X_1 = \Delta_1 X_1 + \Delta_2 X_1 + \Delta_3 X_1$, we may express this as

$$(a') \quad -\frac{1}{2}T_1 \Delta X_1 - \frac{1}{2}T_1 \Delta_2 X_1 - \frac{1}{2}T_1 \Delta_3 X_1.$$

Likewise, we have for commodity 2,

$$-\frac{1}{2}T_2 \Delta_2 X_2 - T_2 \Delta_3 X_2,$$

which we can express as

$$(b') \quad -\frac{1}{2}T_2 \Delta X_2 - \frac{1}{2}T_2 \Delta_3 X_2 + \frac{1}{2}T_2 \Delta_1 X_2.$$

Finally, we have for commodity 3,

$$-\frac{1}{2}T_3 \Delta_3 X_3,$$

which can be expressed as

$$(c') \quad -\frac{1}{2}T_3 \Delta X_3 + \frac{1}{2}T_3 \Delta_1 X_3 + \frac{1}{2}T_3 \Delta_2 X_3.$$

The sum of (a') + (b') + (c') will give us the welfare cost of the set of taxes under consideration. But six of the nine terms in the sum cancel each other out. Consider the two terms $-1/2 T_1 \Delta_2 X_1 + 1/2 T_2 \Delta_1 X_2$. We know that $\Delta_2 X_1 = S_{12} T_2$, and $\Delta_1 X_2 = S_{21} T_1$. Thus these terms can be expressed as

$$-\frac{1}{2}T_1 T_2 S_{12} + \frac{1}{2}T_2 T_1 S_{21},$$

⁷ The equality of $\partial X_i / \partial P_j$ with $\partial X_j / \partial P_i$ is one of the fundamental properties of demand relationships reflecting the substitution effect only. See John R. Hicks, *Value and Capital*, p. 311. Hicks uses the notation X_{ij} to refer to what we have called S_{ij} . It is not easy to give an intuitive interpretation of why S_{ij} should equal S_{ji} . The best I can do is as follows: If the prices of both X_1 and X_2 rise by 1 percent, no substitution effects will take place between them. This means that the substitution effect between X_1 and X_2 which takes place because of a rise of 1 percent in P_1 is annulled by the substitution effect taking place as a result of a 1 percent rise in P_2 . Now the transfer of purchasing power from X_1 to X_2 taking place as a result of a 1 percent rise in the price of X_1 is given by $P_2(\partial X_2 / \partial P_1)P_1$. And the transfer of purchasing power from X_2 to X_1 resulting from a 1 percent rise in the price of X_2 is equal to $P_1(\partial X_1 / \partial P_2)P_2$. In order for the second effect to annul the first, $(\partial X_1 / \partial P_1)$ must equal $(\partial X_1 / \partial P_2)$, or $S_{21} = S_{12}$.

which must equal zero since $S_{12} = S_{21}$. By the same token, each of the pairs of terms $(-1/2 T_1 \Delta_3 X_1 + 1/2 T_3 \Delta_1 X_3)$ and $(-1/2 T_2 \Delta_3 X_2 + 1/2 T_3 \Delta_2 X_3)$ must equal zero. Thus the second fundamental expression for welfare cost reduces to

$$-\frac{1}{2}T_1 \Delta X_1 - \frac{1}{2}T_2 \Delta X_2 - \frac{1}{2}T_3 \Delta X_3 = -\frac{1}{2} \sum_i T_i \Delta X_i.$$

To derive the third form of the expression for welfare cost, consider the expression (first form), letting $C_i =$ unit cost of commodity i , t_i the percentage rate of tax on commodity i , and $C_i t_i = T_i$, the unit tax on i ,

$$-\frac{1}{2}[C_1^2 S_{11} t_1^2 + C_2^2 S_{22} t_2^2 + C_3^2 S_{33} t_3^2 + 2C_1 C_2 S_{12} t_1 t_2 + 2C_1 C_3 S_{13} t_1 t_3 + 2C_2 C_3 S_{23} t_2 t_3].$$

Now we make use of the well-known relationship among the S_{ij} , $\sum_i C_i S_{ij} = \sum_j C_j S_{ji} = 0$.⁸

$$\begin{aligned} C_1^2 S_{11} t_1^2 &= -C_1 C_2 S_{12} t_1^2 - C_1 C_3 S_{13} t_1^2 \\ C_2^2 S_{22} t_2^2 &= -C_1 C_2 S_{12} t_2^2 - C_2 C_3 S_{23} t_2^2 \\ C_3^2 S_{33} t_3^2 &= -C_1 C_3 S_{13} t_3^2 - C_2 C_3 S_{23} t_3^2 \\ 2C_1 C_2 S_{12} t_1 t_2 &= 2C_1 C_3 S_{13} t_1 t_3 \\ 2C_2 C_3 S_{23} t_2 t_3 &= 2C_2 C_3 S_{23} t_2 t_3 \\ 2C_1 C_2 S_{12} t_1 t_2 &= 2C_1 C_2 S_{12} t_1 t_2. \end{aligned}$$

On the left-hand side of the equalities in the six equations above, we have the terms appearing within the bracket in the expression for welfare cost (first form). On the right-hand side, we have the expressions which will be used in deriving the third form. To do so, we simply collect terms in S_{12} , S_{13} , and S_{23} , yielding

$$\begin{aligned} -S_{12} C_1 C_2 [t_1^2 - 2t_1 t_2 + t_2^2] &= -S_{12} C_1 C_2 (t_1 - t_2)^2 \\ -S_{13} C_1 C_3 [t_1^2 - 2t_1 t_3 + t_3^2] &= -S_{13} C_1 C_3 (t_1 - t_3)^2 \\ -S_{23} C_2 C_3 [t_2^2 - 2t_2 t_3 + t_3^2] &= -S_{23} C_2 C_3 (t_2 - t_3)^2. \end{aligned}$$

⁸ This relationship can be interpreted intuitively in two ways. First, when there is a linear constraint of the form $\sum_i C_i X_i = K$, then, as long as the constraint is satisfied, $\sum_i C_i (\partial X_i) / (\partial z) = 0$, for any definition of z . Thus setting $z = P_j$, we have $\sum_i C_i (\partial X_i) / (\partial P_j) = 0$, or $\sum_i C_i S_{ij} = 0$. Alternatively, we may start from the general property of substitution effects that "only relative prices count." This says that the effect on X_i of a 1 percent rise in its price would be annulled by a 1 percent rise in all other prices. The effect on X_i of a 1 percent rise in its own price is given by

$$\frac{\partial X_i}{\partial P_i} P_i = S_{ii} P_i.$$

The effect on X_i of a 1 percent rise in the price of X_j is $(\partial X_i) / (\partial P_j) P_j = S_{ij} P_j$. The effect on X_i of a 1 percent rise in all other prices is $\sum_{j \neq i} S_{ij} P_j$. In order for the first effect, $S_{ii} P_i$, to be offset by the second, we must have

$$S_{ii} P_i + \sum_{j \neq i} S_{ij} P_j = 0, \text{ or } \sum_j S_{ij} P_j = 0.$$

Now if, as we do when measuring the total welfare cost of a set of distortions, we are measuring this cost as against an undistorted initial situation, we can characterize the undistorted situation as one in which, for all j , $P_j = kC_j$; that is, prices bear the same relationship to unit costs for all commodities. Thus we have $\sum_j S_{ij} kC_j = 0$, or $\sum_j S_{ij} C_j = 0$. [See Hicks, *Value and Capital*, pp. 310-311.]

Summing the three terms to the right of the equalities, we have an expression equivalent to that in brackets in the first form measure of the welfare cost. Since the bracket is multiplied by $(-1/2)$, we have for our third fundamental form of the measure for the welfare cost:

$$\frac{1}{2} \sum_j \sum_{i < j} S_{ij} (t_i - t_j)^2 C_i C_j.$$

The expression for the welfare cost of a set of taxes can be derived directly from utility functions as follows.

$$(2.1) \quad U = U(X_1, X_2, \dots, X_n)$$

Taking a Taylor expansion up to quadratic terms, we have

$$(2.2) \quad \Delta U = \sum_i U_i \Delta X_i + \frac{1}{2} \sum_i \sum_j U_{ij} \Delta X_i \Delta X_j, \text{ where } U_i = \frac{\partial U}{\partial X_i} \text{ and}$$

$$U_{ij} = \frac{\partial^2 U}{\partial X_i \partial X_j}.$$

Now U_i is itself a function of the X_j , so

$$(2.3) \quad U_i = U_i(X_1, X_2, \dots, X_n), \text{ and}$$

$$(2.4) \quad \Delta U_i = \sum_j U_{ij} \Delta X_j.$$

Thus we can rewrite (2.2) as

$$(2.5) \quad \Delta U = \sum_i U_i \Delta X_i + \frac{1}{2} \sum_i \Delta U_i \Delta X_i.$$

If consumers maximize utility subject to their money income and the prices facing them in any situation, we have, as a first-order condition of maximum, that

$$(2.6) \quad U_i = \lambda P_i,$$

where λ is a Lagrange multiplier representing the marginal utility of money. Obviously, from (2.6)

$$(2.7) \quad \Delta U_i = \lambda \Delta P_i + P_i \Delta \lambda.$$

Substituting (2.6) and (2.7) into (2.5), we have

$$(2.8) \quad \Delta U = \lambda \sum_i P_i \Delta X_i + \frac{1}{2} \lambda \sum_i \Delta P_i \Delta X_i + \frac{1}{2} (\Delta \lambda) \sum P_i \Delta X_i.$$

Interpreting the P_i 's in (2.8) as representing the prices in an undistorted situation (i.e., as being proportional to unit costs), we have $\sum_i P_i \Delta X_i = \sum_i kC_i \Delta X_i$, which must equal zero under our assumption of constant costs (linear transformation functions). Thus (2.8) reduces to

$$(2.9) \quad \Delta U = \frac{1}{2} \lambda \sum_i \Delta P_i \Delta X_i.$$

To translate ΔU from utility terms into money terms, we must divide by the marginal utility of money. Thus money value of change in utility equals

$$(2.10) \quad \frac{\Delta U}{\lambda} = \frac{1}{2} \sum_i \Delta P_i \Delta X_i,$$

which expressed as a cost is $-1/2 \sum_i \Delta P_i \Delta X_i$. When the only disturbances between the two situations being compared are taxes, we have $\Delta P_i = T_i$. Thus from (2.10) we can derive $-1/2 \sum_i T_i \Delta X_i$ as the measure of the welfare cost of a set of taxes (T_1, \dots, T_n) . Moreover, since we require that changes in the X_i all take place along a linear constraint, without first-order income effects, we can express $\Delta X_i = \sum_j S_{ij} \Delta P_j = \sum_j S_{ij} T_j$. Substituting this into $-1/2 \sum_i T_i \Delta X_i$, we obtain

$$-\frac{1}{2} \sum_i \sum_j S_{ij} T_i T_j,$$

as an alternative (equivalent) measure of welfare cost. This derivation is essentially the same as that given by H. Hotelling.⁹ Hicks derives the expression $1/2 \sum_i \sum_j S_{ij} T_i T_j$ (in his notation $1/2 \sum_r \sum_s X_{rs} dP_r dP_s$) by a somewhat different route.¹⁰

III. MEASURING THE WELFARE COST OF INCOME TAXATION

In this section we apply the apparatus derived in the previous section to the problem of direct taxation with particular reference to the choice between labor and leisure. We shall disregard problems connected with the choice between consumption and saving, which will be dealt with separately below, and shall accordingly assume that all goods are direct consumption goods, and that all income is spent on such goods. The income tax will be represented as a flat-rate excise tax striking all goods (other than leisure) at the same percentage rate.

To begin, let us briefly review the traditional case for preferring direct to indirect taxation. Here leisure is not considered as a good, and the supply of work is assumed to be unaffected by alterations in the pattern of taxation. The third fundamental form of the expression for welfare cost, i.e.,

$$\frac{1}{2} \sum_j \sum_{i < j} S_{ij} (t_i - t_j)^2 C_i C_j,$$

shows that an income tax has no welfare cost under the assumptions stated above. Since, under an income tax (with no concomitant excise taxes) $t_i = t_j$, for all i and j , every term in the summation is equal to zero.

On the other hand, no pattern of taxes on individual commodities can have a negative welfare cost, since by definition the welfare cost of a total set of taxes is measured against a neutral alternative.¹¹ The best one could do, then, with unequal taxation of different goods, would be to have a zero welfare cost (such as would be the case if all goods but one were taxed equally, and the last good — taxed at a different rate — had a zero own-price elasticity of demand). But except in such curious and unrealistic cases, the welfare cost of a system of unequal excise taxes will be positive.

⁹ "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates," *Econometrica*, July 1938.

¹⁰ *Value and Capital*, p. 331.

¹¹ Formally, the expression $1/2 \sum_i \sum_j S_{ij} T_i T_j$ is negative or zero for any values of the T 's. So $-1/2 \sum_i \sum_j S_{ij} T_i T_j$ must be positive or zero. See Hicks, *Value and Capital*, p. 311.

When leisure is introduced as a separate good, however, these statements no longer hold. Now an income tax must be interpreted as an equal-rate tax on all goods except leisure. Hence the income tax is no longer a truly general tax, and cannot be presumed to have a zero welfare cost. As Little [1951] has pointed out, in a world of three goods, X_1 , X_2 , and X_3 , where X_3 is leisure, an equal percentage tax on X_1 and X_2 at the rate of $t_1 = t_2$ leaves undistorted consumers' choices between X_1 and X_2 , but distorts choices between X_1 and X_3 and between X_2 and X_3 . On the other hand, taxing only X_1 distorts the choices between X_1 and X_2 and between X_1 and X_3 , while leaving undistorted the choice between X_2 and X_3 . Similarly, a tax on X_2 introduces distortions between X_2 and X_1 , and between X_2 and X_3 , but leaves choices between X_1 and X_3 undistorted.¹² In each case a distortion is introduced into choices in two pairs of goods, and choice in the third pair is left undistorted. In this sense, therefore, an income tax is on a par with an excise tax. Any preference for income taxation, from a welfare point of view, must be based on empirical considerations, not on a clear qualitative superiority of income taxation in all conceivable circumstances. We now turn, therefore, to the development of some propositions which will be of help in evaluating alternative patterns of taxation when the labor-leisure choice is explicitly taken into account.

First, considering the case in which there are three goods, of which the third is leisure, we may inquire into the effect of an income tax (i.e., a tax on X_1 and X_2 at the percentage rate t). For convenience, let us introduce at this juncture a harmless convention — we choose our units of commodities in such a way that their unit cost, C_i , is unity for every commodity. Thus we may say that the unit of leisure, for a given individual, is that amount of time in which he would earn a dollar, if working. The units of X_1 and X_2 are amounts which sell for a dollar net of taxes. This enables us to leave out the C in our expressions for welfare cost. The welfare cost of a tax at the rate t on X_1 and X_2 is therefore

$$\frac{1}{2} [S_{12}(t - t)^2 + S_{13}(t - 0)^2 + S_{23}(t - 0)^2] = \frac{1}{2} [S_{13}t^2 + S_{23}t^2].$$

When the C 's are equal to 1, the restriction on the S_{ij} 's is simply $\sum_i S_{ij} = \sum_j S_{ij} = 0$. Thus $S_{13} + S_{23} = -S_{33}$, and the welfare cost of a proportional income tax at the rate t can be expressed as $-1/2 S_{33}t^2$. This expression is directly analogous to that for an excise tax on a single commodity. An excise tax on X_1 alone has a welfare cost of $-1/2 S_{11}t_1^2$, and one on X_2 alone has a welfare cost of $-1/2 S_{22}t_2^2$. This symmetry exists because a proportional income tax has the same effect on relative prices as a subsidy to leisure, and the expression $-1/2 S_{33}t^2$ is the same for a given absolute value of t , regardless of whether t is negative (subsidy) or positive (tax).

In the above formula, taxes are expressed as percentages of the net prices (unit costs) of the corresponding commodities. Income-tax rates, however, are normally expressed as percentages of income before tax. It is worthwhile to establish at an early stage the relationships involved in the comparison of income taxes with "equivalent" excise taxes, so as to avoid confusion later on.

¹² I. M. D. Little, "Direct Versus Indirect Taxes," *Economic Journal*, September 1951, pp. 577-584.

A proportional income tax of r percent has as its counterpart an excise tax on the net prices of all goods other than leisure of $r/(1-r)$ percent. That is, a 20 percent income tax corresponds to an excise tax of 25 percent on X_1 and X_2 , for this creates a situation in which the gross prices of X_1 and X_2 exceed their net prices by 20 percent of the gross price. Correspondingly, an income tax has the same effect on the allocation of resources as a subsidy at the rate of $r/(1-r)$ percent on the net price of leisure. (Recall here that in this entire analysis we are holding the purchasing power of government receipts constant, so that if leisure were in fact to be subsidized directly there would have to exist a neutral — say, lump-sum — tax side by side with the subsidy on leisure so as to produce the required level of government receipts.) But a subsidy at the rate of $r/(1-r)$ percent on the net price of leisure is equal to a subsidy of r percent on the gross price of leisure. Using P_n and P_g to represent net and gross price, respectively, we have

$$P_n \left(1 + \frac{r}{1-r}\right) = P_g; \quad P_n \left(\frac{1}{1-r}\right) = P_g.$$

This yields $P_g(1-r) = P_n$.

We can now render in diagrammatic form the expression $-1/2 S_{33}t^2$ (Figure 2.6). Let LL be the supply curve of labor (defined so as to contain the substitution effect only). Let w be the prevailing wage, gross of income tax. Then, where r is the tax rate on gross income, $rw (= AC)$ is the amount of tax per unit of labor and DA is the net-of-tax income per unit of labor. Now S_{33} is the derivative of the demand function for leisure with respect to the wage rate, so the derivative of the supply function of labor with respect to the wage rate will be $-S_{33}$. Hence the reduction in the amount of labor performed as a consequence of an income tax at the rate r will be $-S_{33}rw (= BC)$. The over-all reduction in gross money income to the worker will be $DEBC$. But the worker will have gained leisure valued at $DEBA$. This leaves as the net welfare cost of the tax the triangle ABC , or $-1/2 S_{33}(rw)^2$. Recalling that $-S_{33} = \partial L/\partial w$, this expression for welfare cost can be written

$$\frac{1}{2} \frac{\partial L}{\partial w} (rw)^2 = \frac{1}{2} \left(\frac{w}{L} \frac{\partial L}{\partial w} \right) r^2 w L = \frac{1}{2} \epsilon r^2 w L,$$

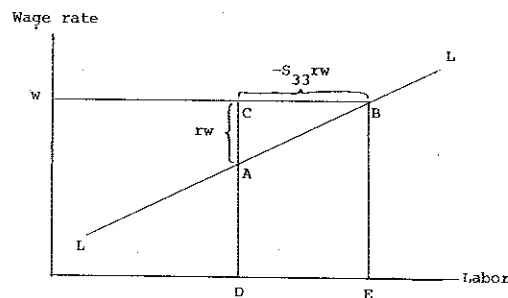


FIGURE 2.6

where ϵ is the elasticity of supply of labour (expressing the substitution effect only), wL represents the earnings of labor gross of income tax, and r represents the rate of income tax expressed in the usual way as a percentage of gross income.

In order to apply this result to the United States case, we must first obtain an estimate of ϵ , and then take account of the progressivity of the actual U.S. income tax. As far as ϵ is concerned, we have fairly reliable estimates on the secular pattern of response of hours worked to changes in real wages. Broadly speaking, a 1 percent rise in real wages per hour has been associated with a reduction of .25 percent in average hours worked.¹³ Unfortunately for our purposes, this historical relationship reflects both the income effect and the substitution effect of wage increases. The problem is illustrated in Figure 2.7. In response to a rise in real wages, we observe an increase in the amount of leisure taken equal to $A'C'$. But this is composed of an income effect $A'B'$ (such as would arise if the lower wage rate remained in effect and the worker inherited an annuity providing him with GE of income per period) plus a substitution effect $B'C'$ (such as would take place if simultaneously the worker's annuity were canceled and the wage rate rose as depicted). Our problem is to guess at the possible magnitude of the income effect so as to be able to conclude something about the size of the substitution effect.

¹³ Wages per hour of work in the manufacturing industry rose from \$.502 in 1900 to \$2.24 in 1957, expressed in 1957 prices (Albert Rees, "Patterns of Wages, Prices, and Productivity," in Charles Myers, ed., *Wages, Prices, Profits, and Productivity*, New York, 1957, pp. 15-16). Average hours of work per week fell from 55 to 37.8 during the same period.

	Log Hours Worked	Log Real Wages per Hour
1900	1.74036	9.70070-10
1957	1.58771	0.35025
Difference	0.15265	0.64955

The elasticity of hours with respect to real hourly wages derived from these data is $-.235$. The elasticities obtained from the regression of annual data on these two variables for the period 1900-1957 (excluding the depression and war years) were $-.26$ for manufacturing, $-.26$ for railroads, $-.19$ for bituminous coal, and $-.21$ for anthracite coal. (See Ethel B. Jones, "Hours of Work in the United States, 1900-1957," unpublished Ph.D. dissertation, University of Chicago, 1961, p. 44.)

Obviously, using these figures to measure the elasticity of labor supply implicitly assumes that average hours tended to be those "desired" by workers, i.e., that workers were in some sense "on" their labor supply function. One need not labor the point that individual workers are constrained to some extent by the norms adopted in the particular places they work, so that a worker who desired to work only 30 hours a week would either have to find a job requiring this amount of work or be "off" his labor supply function. But it seems reasonable to suppose that the main force behind the historical downward trend in hours was the desire for additional leisure on the part of the typical worker. Unions would probably not have negotiated, and employers would probably not have conceded, reductions in normal hours of work if these reductions were not desired by the workers themselves. By the same token, there can be little doubt that wage rises were the principal force motivating the desire for greater leisure on the part of workers. Certainly, legislative changes do not account for a significant part of the reduction in hours. Laws imposing maximum hours affected only 4.1 percent of workers in manufacturing by 1929 (*ibid.*, p. 56). Jones concludes that only the Fair Labor Standards Act of 1938 is likely to have had a significant independent effect on hours worked in manufacturing, but finds that by the postwar period, actual hours in manufacturing were slightly above those which would be predicted by the pre-1929 relationships between average hours and real wage rates (*ibid.*, p. 70). Actually, the major reductions in hours in the postwar period have come through increases in vacation and holiday time, which are unaffected by legislation and which seem to be a highly plausible form for workers to take increases in their "desired" leisure time.

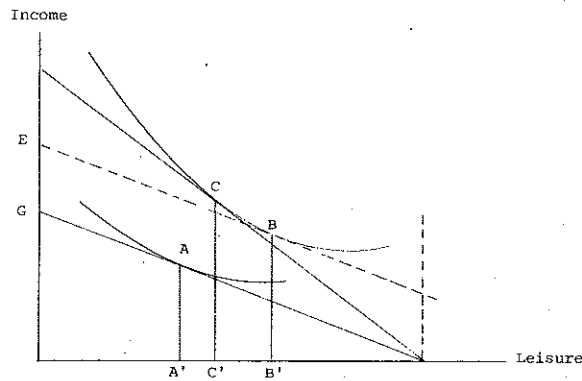


FIGURE 2.7

To put the problem in terms of the elasticity of supply of labor directly, let us define $\delta L/\delta w$ as the partial derivative of labor supply with respect to the wage rate, when the income effect is included, and $\partial L/\partial w$ as the corresponding partial derivative when the income effect is excluded. Let $\partial L/\partial y$ be the response of labor supply to a change in income (such as would come with the receipt of an annuity), holding the wage rate constant. Then we can write:

$$\frac{\delta L}{\delta w} = \frac{\partial L}{\partial w} + \frac{\partial L}{\partial y} \frac{\partial y}{\partial w}$$

Now with no other changes taking place, income will change by the number of hours worked times the change in the wage rate (i.e., $\Delta Y = L \Delta w$) so we can set $(\partial y/\partial w) = L$. Hence $(w/L)(\delta L/\delta w) = (w/L)(\partial L/\partial w) + w(\partial L/\partial y)$ or $e' = e + w(\partial L/\partial y)$, where e' represents the elasticity of supply of labor including income and substitution effects and e represents the elasticity of supply of labor including the substitution effect only. $w(\partial L/\partial y)$ is simply the negative of the fraction of an increase in nonlabor income which is taken out in the form of leisure. Thus if, for example, a worker on receiving an annuity of \$1,000 a year would reduce his labor earnings by \$250 a year, $w(\partial L/\partial y)$ would for him be $-.25$. If he would reduce his labor earnings by \$500 a year, $w(\partial L/\partial y)$ would for him be $-.50$.

Now, from historical observations, we take e' to be $-.25$. Hence if $w(\partial L/\partial y)$ equaled $-.25$, e must equal zero, and if $w(\partial L/\partial y) = -.50$, e must equal $+.25$. Since e cannot be negative, and the available evidence indicates that e' is about $-.25$, we cannot "guess" that $w(\partial L/\partial y)$ is less in absolute magnitude than $.25$. We can guess, however, that it is substantially less than $.5$ in absolute value. In order to have a specific number to play with, I shall assume that $w(\partial L/\partial y) = -.375$, indicating that a typical worker inheriting an annuity of \$1,000 will tend to reduce his labor earnings by \$375. Readers need not take this guess too seriously, however, for the results that will be obtained using it can easily be adjusted for alternative assumptions.

When measuring the welfare cost of a proportional income tax, we can use

TABLE 2.2
Welfare Cost per Dollar of Labor Income, by Adjusted Gross
Income Class

Adjusted gross income class (thousands)	Total adjusted gross income in class (millions) (1)	Taxable income per return with adjusted gross income in class (2)	Marginal tax rate in class (m_i) (3)	Implied reduction in labor supply within class (e_i) ($e = .125$) (4)	Welfare cost per dollar of labor income in class ($\frac{1}{2}em_i^2$) (5)
Less than \$5	\$84,278	\$887	20%	2.50%	\$.0025
\$5- 10	144,984	3,750	20	2.50	.0025
10- 15	48,553	7,865	22	2.75	.0030
15- 20	15,151	12,400	30	3.75	.0056
20- 25	7,738	16,906	34	4.25	.0072
25- 50	16,594	26,496	43	5.38	.0116
50- 100	7,268	53,659	62	7.75	.0240
100- 150	2,015	96,775	72	9.00	.0324
150- 200	936	136,488	78	9.75	.0380
200- 500	1,750	223,392	89	11.13	.0495
500- 1,000	663	528,092	91	11.38	.0518
Over 1,000	806	1,480,450	91	11.38	.0518
Total	330,936				

Source: [U.S. Treasury Department, Internal Revenue Service] *Statistics of Income: Individual Income Tax Returns for 1961* [Washington, D.C.: Government Printing Office, 1963], pp. 14-16.

the expression $1/2 er^2wL$ without further ado. But when the tax system is progressive, we must recognize that the choices of an individual at the margin are governed by his marginal tax rate, which we can call m . Letting m_i be the marginal tax rate in the i th tax bracket, and $W_i = w_iL_i$ be the total wage income earned by people in the i th tax bracket, we may express the total welfare cost of the personal income tax as $1/2 \sum_i e_i m_i^2 W_i$. This is what is done in Tables 2.2 and 2.3. It is assumed that e_i is equal to $.125$ for all tax brackets, on the basis of the value of $-.375$ for $w(\partial L/\partial y)$ selected above. Column 1 of Table 2.2 is merely for the information of the reader. The \$331 billion of reported income on returns with adjusted gross income compares with \$427 billion reported as total personal income in the national accounts for 1961. Column 2 of Table 2.2 presents the average adjusted gross income per return with adjusted gross income in each class. This figure was used to determine the marginal tax rate that would be assumed for each class in the calculations to follow. In obtaining the marginal tax rate from the tax table, the rates applying to joint returns were used. Thus a downward bias was introduced into the calculations of welfare cost. Column 4 of Table 2.2 presents the percentage reduction in labor effort within each class which is implied by the assumptions of this analysis. It is meant to be a check on the plausibility of the results. As can be seen, the implied reductions in labor effort range from 2.5 percent in the 20 percent bracket to around 11.4 percent in the 91 percent bracket. This says, in effect, that if it were possible to extract out of each income class the same tax as was in fact

TABLE 2.3
Welfare Cost on Wages and Salaries and Business and Professional
Income, by Adjusted Gross Income Class

Adjusted gross income class (thousands)	Welfare cost per dollar of labor income in class Table 2, col. 5 (1)	Wage and salary income in taxable returns in class (millions) (2)	Welfare cost on wage and salary income (cols. 1 × 2) (millions) (3)	Business and professional income on taxable returns in class (millions) (4)	Potential welfare cost on business and professional income in class (cols. 1 × 4) (millions) (5)
Less than \$5	\$.0025	\$58,418	\$146	\$4,371	\$11
\$5- \$10	.0025	129,695	324	6,630	17
10- 15	.0030	40,448	121	3,434	10
15- 20	.0056	9,837	55	2,166	12
20- 25	.0072	4,145	30	1,468	11
25- 50	.0116	6,864	80	3,338	39
50- 100	.0240	2,452	59	991	24
100- 150	.0324	546	18	115	4
150- 200	.0380	185	7	33	1
200- 500	.0495	234	12	30	1
500- 1,000	.0518	39	2	6	—
Over 1,000	.0518	19	1	5	—
Total		252,882	855	22,587	130

Source: [U.S. Treasury Department, Internal Revenue Service] *Statistics of Income: Individual Income Tax Returns for 1961* [Washington, D.C.: Government Printing Office, 1963], pp. 14-16.

obtained, but in such a way that tax incentives did not distort the choice between labor and leisure at the margin, we would get 11.4 percent more work out of our top income brackets and 2.5 percent more work out of people presently in the lowest range of taxable income. These figures do not imply that top-bracket people work less than low-bracket people, but only that they work 11 or so percent less than they would in the absence of the income-tax incentive for leisure.

Much work has still to be done to enrich the theory underlying this analysis and to check its implications in detail. However, I would suggest the following principal sources of the reduction in labor effort, particularly in the higher brackets:

1. Longer vacations (winters in Florida, etc.).
2. A higher incidence of early retirement.
3. Less labor force participation of women.
4. Less supplemental labor income from sources other than main employment.

And I find it quite plausible that these factors might account for a 5 percent reduction in labor effort in the \$25,000-\$50,000 bracket, and for an 11 percent reduction in the highest brackets, as against what it would be in the absence of tax incentives at the margin, but in the presence of the existing total weight of tax. Needless to say, I do not expect everybody to agree with this judgment,

but shall proceed on this basis in the absence of any alternative that seems clearly better to me.

Column 5 of Table 2.2 (reproduced as column 1 of Table 2.3) gives the estimated welfare cost per dollar of labor income in each class. In column 2 of Table 2.3 we have the amount of wage and salary income (by far the major category of labor income) declared on taxable returns in each class, and in column 3 the estimated welfare cost associated with the tax incentive to wage and salary earners. In column 4 we have the amount of business and professional income declared on taxable returns in each class. Obviously not all of this income comes from labor effort, but the estimates of welfare cost in column 5 are based on the extreme assumption that business and professional income is all labor income. Since the welfare costs estimated in column 5 are not large compared with those in column 3, I elected not to attempt an adjustment to exclude that fraction of business and professional income which might be attributable to property.

The results emerging from Table 2.3 suggest that the welfare costs associated with the distortion of the labor-leisure choice by the personal income tax may be of the order of \$1 billion per year. My own reaction to this is twofold. First of all, \$1 billion is not a very large amount in comparison to the \$42 billion per year raised by the personal income tax in 1961. If this were a necessary cost to obtain the kind of progression that the personal income tax contributes to our tax system, I would judge it to be a worthwhile cost. But the roughly \$1 billion per year cost estimated in Table 2.3 is clearly not a necessary cost to obtain the existing degree of progression in the personal income tax structure. Actually, the average tax rate (income-tax adjusted-gross-income) does not reach 45 percent even for the highest brackets, and for most brackets is only about half the marginal tax rate.¹⁴ This means that there are real possibilities for obtaining about as much revenue from each class as at present, but with lower marginal rates. This can be done by broadening the tax base (reducing exemptions and deductions, eliminating or modifying a host of special provisions such as percentage depletion and capital gains, etc.). Most tax experts have advocated such a reform of our tax system, and the present analysis simply adds support to the proposal. If, for example, the marginal rate in each class of adjusted gross income could be reduced to seven-tenths of its present value, the welfare cost estimated above would be cut about in half. And it is likely that this could be done without loss of either revenue or effective progressivity.

IV. COMPARISON OF THE WELFARE COSTS OF DIRECT AND INDIRECT TAXATION

In this section we attempt to assess the relative merits of direct and indirect taxation taking explicit account of the issues raised by the labor-leisure choice. Throughout this section we shall set aside the problem of progressivity and

¹⁴ The average tax rates in 1961 by bracket are: 7.2 percent for 0-\$5,000; 11.0 percent for \$5,000-10,000; 14.3 percent for \$10,000-15,000; 17.0 percent for \$15,000-20,000; 19.5 percent for \$20,000-25,000; 24.5 percent for \$25,000-50,000; 34.2 percent for \$50,000-100,000; 40.1 percent for \$100,000-150,000; 42.4 percent for \$150,000-200,000; 43.7 percent for \$200,000-500,000; 44.8 percent for \$500,000-1,000,000; and 42.4 percent for over \$1,000,000.

consider the income tax as a proportional tax, and as equivalent to a set of excise taxes striking all goods other than leisure at the same percentage rate. The first question we ask is: Granted that the income tax has a certain welfare cost, under what circumstances does it have a smaller welfare cost than a set of excise taxes at unequal rates which yields the same revenue? Second, we turn to the theoretical question of how to measure the difference in welfare costs between an excise tax and an income tax yielding the same revenue. Third, we turn to the more interesting question of how we would measure the change in welfare costs resulting from eliminating an existing excise tax and substituting for it an increment to an already existing income tax.

To answer our first question, let us again postulate a three-commodity world, in which the third good is leisure. We are allowed to tax only goods 1 and 2, and we are required to raise a given amount of revenue. The expression for welfare cost, when commodity units are chosen so that unit costs equal 1, is

$$-\frac{1}{2}[S_{11}t_1^2 + S_{22}t_2^2 + S_{33}t_3^2 + 2S_{12}t_1t_2 + 2S_{13}t_1t_3 + 2S_{23}t_2t_3].$$

Because our problem restricts t_3 to be zero, this reduces to

$$-\frac{1}{2}[S_{11}t_1^2 + S_{22}t_2^2 + 2S_{12}t_1t_2].$$

We seek to minimize this expression by our choice of t_1 and t_2 , subject to the constraint that $t_1X_1 + t_2X_2 = K$, i.e., that a given amount of total revenue, K , is obtained. Minimizing

$$W = -\frac{1}{2}[S_{11}t_1^2 + S_{22}t_2^2 + 2S_{12}t_1t_2] + \lambda[t_1X_1 + t_2X_2 - K],$$

we obtain

$$\frac{\partial W}{\partial t_1} = -S_{11}t_1 - S_{12}t_2 + \lambda X_1 + \lambda t_1 \frac{\partial X_1}{\partial t_1} + \lambda t_2 \frac{\partial X_2}{\partial t_1} = 0.$$

Since

$$\frac{\partial X_1}{\partial t_1} = S_{11} \quad \text{and} \quad \frac{\partial X_2}{\partial t_1} = S_{21} = S_{12},$$

the above simplifies to

$$\frac{\partial W}{\partial t_1} = -S_{11}(1 - \lambda)t_1 - S_{12}(1 - \lambda)t_2 + \lambda X_1 = 0.$$

Similarly,

$$\frac{\partial W}{\partial t_2} = -S_{12}(1 - \lambda)t_1 - S_{22}(1 - \lambda)t_2 + \lambda X_2 = 0.$$

If we set $\mu = \lambda/(1 - \lambda)$, we obtain

$$\frac{\partial W}{\partial t_1} = -S_{11}t_1 - S_{12}t_2 + \mu X_1 = 0$$

$$\frac{\partial W}{\partial t_2} = -S_{22}t_2 - S_{12}t_1 + \mu X_2 = 0.$$

The solution for t_1 and t_2 for minimum welfare cost is

$$\hat{t}_1 = \frac{\begin{vmatrix} \mu X_1 & S_{12} \\ \mu X_2 & S_{22} \end{vmatrix}}{\begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{vmatrix}}; \quad \hat{t}_2 = \frac{\begin{vmatrix} S_{11} & \mu X_1 \\ S_{12} & \mu X_2 \end{vmatrix}}{\begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{vmatrix}}.$$

Obviously, \hat{t}_1 will equal \hat{t}_2 when the numerators of the above expressions are equal, i.e., when

$$X_1S_{22} - X_2S_{12} = X_2S_{11} - X_1S_{12}.$$

Using the fact that, when unit costs = 1, $S_{11} = -S_{12} - S_{13}$; $S_{22} = -S_{12} - S_{23}$, this equality can be rewritten

$$-X_1S_{12} - X_2S_{12} - X_1S_{23} = -X_2S_{12} - X_1S_{12} - X_2S_{13}.$$

Hence the equality requires that X_1S_{23} be equal to X_2S_{13} , or that $S_{13}/X_1 = S_{23}/X_2$. But $S_{13} = \partial X_1/\partial P_3$; and with $P_3 = 1$, η_{13} , the cross elasticity of demand for commodity 1 with respect to the price of leisure can be written $(1/X_1)(\partial X_1/\partial P_3) = (S_{13}/X_1)$. Likewise, $S_{23}/X_2 = \eta_{23}$. Hence the condition for equal excise taxation of both goods being the best pattern of excise taxation by which to raise a given sum of money is that both goods have the same cross elasticity of demand with respect to the price of leisure.

To see under what circumstances a minimum welfare cost is obtained with a lower (or a higher) tax on X_1 than X_2 , we can express the ratio \hat{t}_1/\hat{t}_2 as

$$\frac{\hat{t}_1}{\hat{t}_2} = \frac{X_1S_{12} + X_2S_{12} + X_1S_{23}}{X_1S_{12} + X_2S_{12} + X_2S_{13}}$$

Obviously, when X_1S_{23} is smaller than X_2S_{13} , \hat{t}_1 is less than \hat{t}_2 . This can happen when S_{23} is negative, i.e., when X_2 is complementary to leisure, or when, S_{23} and S_{13} both being positive, $X_1S_{23} < X_2S_{13}$. This last inequality is equivalent to $\eta_{23} < \eta_{13}$. Thus the tax on X_1 "should" be lower than the tax on X_2 whenever X_2 is less substitutable for leisure than is X_1 .¹⁵

This result should not be taken to suggest, however, that it is likely that minimum-welfare-cost patterns of excise taxation are likely to have very different rates. Dividing both numerator and denominator in the above expression by X_1X_2 , and recalling that $S_{12} = S_{21}$, we obtain

$$\frac{\hat{t}_1}{\hat{t}_2} = \frac{\eta_{21} + \eta_{12} + \eta_{23}}{\eta_{21} + \eta_{12} + \eta_{13}}$$

If, as is likely, the cross elasticity of both goods with respect to the price of leisure is low in comparison to the cross elasticities of the goods with respect to each other's prices, the ratio of \hat{t}_1 to \hat{t}_2 will not be "very" different from unity.

¹⁵ For an early analysis arriving at essentially the same conclusion, though by a somewhat more complicated argument, see W. J. Corlett and D. C. Hague, "Complementarity and the Excess Burden of Taxation," *Review of Economic Studies*, 1953-54, pp. 21-30.

We now turn to the second question mentioned at the beginning of this section — measuring the difference in welfare cost between an excise tax and an income tax of equal yield. The welfare cost of an excise tax at the rate t_1 is simply

$$C_1 = -\frac{1}{2}S_{11}t_1^2.$$

A proportional income tax at the rate t^* will have equal yield when $t^*(X_1 + X_2) = t_1X_1$, i.e., where $t^* = t_1X_1/(X_1 + X_2)$. The welfare cost of this income tax will be

$$C_2 = -\frac{1}{2}S_{33}(t^*)^2.$$

Hence we may write

$$C_1 - C_2 = -\frac{1}{2}[S_{11}t_1^2 - S_{33}(t^*)^2], \text{ or}$$

$$C_1 - C_2 = -\frac{1}{2}S_{11}t_1^2 \left[1 - \frac{S_{33}}{S_{11}} \left(\frac{t^*}{t_1} \right)^2 \right].$$

This equation expresses the difference in welfare cost between an excise tax and an equal-yield income tax as the welfare cost of the excise tax taken by itself times one minus a "correction factor" $[(S_{33}/S_{11})(t^*/t_1)^2]$. Now t^*/t_1 is simply $X_1/(X_1 + X_2)$, or the fraction of money income accounted for by the taxed commodity. Calling this measure a_1 , we may express the correction factor as $[S_{33}/(X_1 + X_2)][X_1/S_{11}]a_1$. Now X_1/S_{11} is simply $1/\eta_{11}$. Now, as indicated in the preceding section $-S_{33}$ is equal to the derivative of the supply of labor with respect to the wage (P_3). Where all income is produced by labor, and all prices are unity, $(X_1 + X_2)$ represents the quantity of labor supplied. Therefore $-S_{33}/(X_1 + X_2)$ is equal to ϵ , the elasticity of supply of labor. Hence $[(S_{33}/S_{11})/(t^*/t_1)^2]$ can be expressed as $-a_1\epsilon/\eta_{11}$. Since η_{11} is necessarily negative, the correction factor is positive. What is more important, however, is the fact that the correction factor is likely to be very small, as long as the excise tax is not so general as to be nearly an income tax itself. In the preceding section we judged a plausible value for ϵ to be .125. Suppose we consider t_1 to be an excise tax which strikes 20 percent of all goods and services other than leisure, i.e., $.2 = X_1/(X_1 + X_2) = a_1$. The correction factor will then be equal to $-(.2)(.125)/\eta_{11}$, or $-.025/\eta_{11}$. Thus if the own-price elasticity of demand for the taxed good were as small in absolute value as .25, the correction factor would only be .1; and if the elasticity were in the range between $-.5$ and -1.0 , the correction factor could, for all practical purposes, be neglected. Only when the elasticity of demand for the excise-taxed commodity is very small indeed would the correction factor become of sizable magnitude. And this elasticity would have to be virtually zero (in our example, less than .025) in order for the correction factor to exceed unity, and thus for the excise tax to have lower welfare cost than an equal-yield income tax.

The result just obtained is heartening to those who felt all along that direct taxation is better than indirect taxation. But the case just covered is one in which the excise tax and the income tax are strictly alternatives, and as such it is far removed from reality. We shall now modify the preceding example to allow for an income tax already in existence, and shall inquire about the size of the difference between the increments to welfare cost incurred by either

adding a given amount to total revenue by way of an excise tax on X_1 , or adding the same amount to total revenue by increasing the rate of income tax.

Let t_0 be the existing rate of income tax, and, as before t_1 and t^* be the alternative increments to tax rates. The situations we are comparing then are

	Tax Rate on X_1	Tax Rate on X_2
Situation 1	$t_0 + t_1$	t_0
Situation 2	$t_0 + t^*$	$t_0 + t^*$

where, as before, $t^* = a_1t_1$.

The expressions for the welfare cost are in this case most conveniently expressed in terms of the third fundamental form $1/2 \sum_i \sum_{i < j} S_{ij}(t_i - t_j)^2$.

$$C_1 = \frac{1}{2}[S_{12}t_1^2 + S_{13}(t_0 + t_1)^2 + S_{23}t_0^2]$$

$$C_2 = \frac{1}{2}[S_{13}(t_0 + t^*)^2 + S_{23}(t_0 + t^*)^2].$$

Expanding, we have

$$C_1 = \frac{1}{2}[S_{12}t_1^2 + S_{13}t_0^2 + 2S_{13}t_0t_1 + S_{13}t_1^2 + S_{23}t_0^2]$$

$$C_2 = \frac{1}{2}[S_{13}t_0^2 + 2S_{13}t_0t^* + S_{13}(t^*)^2 + S_{23}t_0^2 + 2S_{23}t_0t^* + S_{23}(t^*)^2].$$

Subtracting, we obtain

$$C_1 - C_2 = \frac{1}{2}[S_{12}t_1^2 + 2S_{13}t_0t_1 + S_{13}t_1^2 - 2S_{13}t_0t^* - S_{13}(t^*)^2 - 2S_{23}t_0t^* - S_{23}(t^*)^2].$$

Since $S_{12} + S_{13} = -S_{11}$, and $-S_{13} - S_{23} = S_{33}$, this reduces to

$$C_1 - C_2 = \frac{1}{2}[-S_{11}t_1^2 + S_{33}(t^*)^2 + 2S_{13}t_0t_1 - 2S_{13}t_0t^* - 2S_{23}t_0t^*].$$

Substituting a_1t_1 for t^* in the last two terms, we have

$$C_1 - C_2 = \frac{1}{2}[-S_{11}t_1^2 + S_{33}(t^*)^2 + 2(1 - a_1)S_{13}t_0t_1 - 2a_1S_{23}t_0t_1].$$

$$C_1 - C_2 = -\frac{1}{2}S_{11}t_1^2 \left[1 - \frac{S_{33}}{S_{11}} \left(\frac{t^*}{t_1} \right)^2 - 2 \frac{(1 - a_1)S_{13}t_0 - a_1S_{23}t_0}{S_{11}t_1} \right].$$

Once again we are expressing the difference in the welfare cost of an excise tax and (in this case) an increment to the income tax rate as being equal to the welfare cost of the excise tax by itself times one minus a correction factor. For simplicity, let us say one minus two correction factors. The first of these, $(S_{33}/S_{11})(t^*/t_1)^2$, has already been shown to be equal to $-a_1\epsilon/\eta_{11}$, and likely to be small.

We therefore turn to an evaluation of the second correction factor, $2 \cdot \{(1 - a_1)S_{13} - a_1S_{23}\}/S_{11}(t_0/t_1)$. First let us convert this relationship into elasticity form, using $S_{13} = X_1\eta_{13}$; $S_{23} = X_2\eta_{23}$ and $S_{11} = X_1\eta_{11}$. This yields

$$2 \cdot \frac{(1 - a_1)X_1\eta_{13} - a_1X_2\eta_{23}}{X_1\eta_{11}} \left(\frac{t_0}{t_1} \right).$$

Now let us define $a_2 = X_2/(X_1 + X_2) = (1 - a_1)$. Dividing numerator and denominator by $(X_1 + X_2)$, we obtain

$$2 \cdot \frac{a_2(\eta_{13} - \eta_{23})}{\eta_{11}} \left(\frac{t_0}{t_1} \right).$$

The correction factor will be positive if the taxed commodity (X_1) is complementary with leisure ($\eta_{13} < 0$), or if, both goods being substitutes for leisure, the taxed commodity is the poorer substitute ($0 < \eta_{13} < \eta_{23}$). This simply reflects the fact that the minimum-welfare-cost pattern of taxing X_1 and X_2 calls for higher taxation of the poorer substitute for leisure. But it is to be emphasized that the sign of the second correction factor could be either positive or negative, and that neither sign is more likely than the other. The sign depends only on whether the commodity chosen for excise taxation happens to be a better-than-average or a worse-than-average substitute for leisure.

It is to be presumed that in most cases the second adjustment factor will be of small magnitude, at least where the tax rate t_1 is not very small in relation to the existing income tax rate, t_0 , and where the own-price elasticity of demand for X_1 is not itself very small.

Another way of expressing the second correction factor is

$$2 \cdot \left[a_2 \left(\frac{S_{13}}{S_{33}} \right) - a_1 \left(\frac{S_{23}}{S_{33}} \right) \right] \left(\frac{S_{33}}{S_{11}} \cdot \frac{t_0}{t_1} \right).$$

Since $S_{13} + S_{23} = -S_{33}$, (S_{13}/S_{33}) and (S_{23}/S_{33}) must add up to -1 . In the normal case where X_1 and X_2 are both substitutes for leisure, S_{13}/S_{33} and S_{23}/S_{33} will both be negative fractions. Calling them f_1 and f_2 , respectively, the second correction factor can be rewritten as

$$2 \cdot (a_2 f_1 - a_1 f_2) \frac{S_{33} t_0}{S_{11} t_1}.$$

$(S_{33} t_0 / S_{11} t_1)$ is simply the ratio of the welfare cost per dollar of tax receipts of the existing income tax to the welfare cost per dollar of tax receipts of the excise tax under consideration, divided by a_1 , the share of the taxed commodity in total income. The welfare cost of the income tax is $-1/2 S_{33} t_0^2 = 1/2 (X_1 + X_2) e t_0^2$. The revenue from the income tax is $(X_1 + X_2) t_0$, as the welfare cost per dollar of revenue is $1/2 e t_0$. The welfare cost of the excise tax on X_1 , in turn, is $-1/2 S_{11} t_1^2 = -1/2 X_1 \eta_{11} t_1^2$. It yields $X_1 t_1$ in revenue, so the welfare cost per dollar of tax is $-1/2 \eta_{11} t_1$. Hence if we call h_0 and h_1 the welfare costs per dollar of revenue of the existing income tax and the excise tax under consideration, respectively, we can write $(S_{33} t_0 / S_{11} t_1) = h_0 / a_1 h_1$. Hence the correction factor is equal to

$$2([a_2 f_1 / a_1] - f_2)(h_0 / h_1).$$

Where the welfare cost per dollar of revenue is significantly higher for the excise tax than for the existing income tax, the correction factor is likely to be small. When $e = .125$, the welfare cost of an income tax, per dollar of revenue,

will range between \$.0125 and \$.0250 for tax rates between .2 and .4, for example. When $\eta_{11} = -1$, on the other hand, the welfare cost of an excise tax of 20 percent will be \$.10 per dollar of tax revenue. Thus where an excise tax is likely to look bad (i.e., to have a high welfare cost per dollar of tax revenue) when taken by itself, it is also likely to look bad in comparison with the alternative of adding to an existing income tax. When, on the other hand, an excise tax is contemplated which does not look bad by itself, we cannot be confident that an increase in income tax designed to produce an equal increment of revenue would be a preferable alternative.

The conclusion that I draw from this exercise is, therefore, that by and large the traditional preference for direct over indirect taxation is justified, and that the simple textbook measure of welfare cost (the triangle under the demand curve) yields a good first approximation in most cases.

V. TAXATION AND THE INCENTIVE TO SAVE

We now turn to a consideration of how taxation influences welfare through its effect on saving. Here we have another area where income taxation is non-neutral. The private incentives to save are governed by the after-tax yield on the sums involved, but the social rate of return is given by the before-tax yield. Clearly, in order to make the tax system neutral with respect to savings decisions, one would require a consumption tax of the type that Kaldor has proposed, rather than an income tax of the conventional type.

Having recognized the nonneutrality of income taxation with respect to savings, we now turn to the measurement of the welfare costs that it entails. The solution to this problem is not as clear-cut as that of labor-leisure choice. The difficulty arises because the introduction of savings into the picture injects a dynamic element into what would otherwise be a comparative static model. This can be handled neatly if the time path of savings is independent of the changes in tax policy under consideration, but in such a case income taxation would have no welfare cost arising from interference with the savings decision, and our problem would disappear. As long as we take our problem seriously, we must face up to a possible effect of taxation on the volume of savings and investment, and thus the problem of incorporating a dynamic element in what would otherwise be a static model must be faced.

I shall discuss here two alternative ways of dealing with the above problem, the first of which inquires into the purpose for which the saving in question is undertaken, and the second of which does not. The first approach starts by taking a particular time-horizon for a given act of saving — say, ten years. If \$1 is saved now, and if its full social yield (at the rate ρ) is reinvested, it will produce, ten years from now, a sum equal to $\$1(1 + \rho)^{10}$. If, however, the yield of this saving is annually taxed at the rate t , its private value ten years from now will be $\$1(1 + r)^{10}$, if $r = \rho(1 - t)$ is the private rate of return on savings. The marginal rate of substitution in consumption between present goods and goods ten years in the future will therefore be $1:(1 + r)^{10}$, while the social marginal rate of transformation will be $1:(1 + \rho)^{10}$. The percentage excess of $(1 + \rho)^{10}$ over $(1 + r)^{10}$ plays the same role as would a tax of this

percentage on savings made for the purpose of accumulation for ten years and for consumption at the end of the period. Similarly, the percentage excess of $(1 + \rho)^n$ over $(1 + r)^n$ plays the same role as a corresponding tax on savings made for the purpose of accumulation for n years and then for consumption. Since in the real world savings are not made for purposes as clearly specified as these, one may regard the effects of income taxation as being equivalent to a tax on savings at a rate equal to a weighted average of the expressions $[(1 + \rho)^n / (1 + r)^n - 1]$, the weights reflecting the probabilities attaching to the savings being used in particular future time periods.

The approach just outlined has the advantage of emphasizing the way in which the effects of tax incentives upon saving depend on what might be called the savings strategy of the affected individuals. The marginal rate of substitution between this year's goods and next year's goods is not likely to be much affected even by very heavy income taxation; while the marginal rate of substitution between this year's goods and goods a quarter century from now will be substantially affected, even by comparatively moderate income taxation. For example, taxing away 50 percent of the income from capital, in a case where ρ , the social marginal productivity of capital, is 6 percent, changes the marginal rate of substitution between present and future goods from 1.06 to 1.03 for one-year savings decisions, from 1.34 to 1.16 for five-year savings decisions, from 1.79 to 1.34 for ten-year savings decisions, and from 3.21 to 1.81 for twenty-year savings decisions. But by the same token, this approach runs into the difficulty of having to specify the savers' strategies with respect to reinvestment and consumption out of capital.

The second approach is simpler, neater, and more conventional. It simply regards each dollar saved as purchasing a perpetual income stream in the amount of $\$p$ per year for the society as a whole, and of $\$p(1 - t)$, or $\$r$ per year for the individual saver. The present value of this income stream to the saver, evaluated at the rate r , is $\$1$. But the present value to society is $\$p/r$. The social value of a marginal dollar of saving thus exceeds the private value by the percentage $t/(1 - t)$. If the elimination of this distortion would produce a change in savings equal to ΔS dollars, then we can assign as the welfare cost of the distortion of the savings decision the amount $1/2 (\Delta S) \times [t/(1 - t)]$.

I must confess that I have labored for some time under a prejudice against this second approach, mainly because the idea of persons considering saving only in terms of perpetuities did not seem particularly realistic. However, further reflection has led me to conclude that my original objection was not a serious one. The essential difference between the two approaches lies in their treatment of decisions taken at different times. The first approach tries to deal simultaneously with the saving that takes place now, with the reinvestment of the earnings of that saving for some period in the future, and with the disinvestment of the accumulated value at the end of the period. The second approach deals only with the savings decisions taken at a given period in time. This year's savings are considered when the analysis focuses on this year's actions; next year's savings are taken into account when the analysis focuses on next year's actions, etc. In particular, the second approach does not ask

what will be done next year with the earnings on this year's savings; it simply treats those earnings as part of the total income accruing next year, in terms of which the individual will reach a decision as to how much to save. Nor does the second approach treat the realization of the proceeds of a given investment as an act of dissaving; it looks instead at the total movement in the capital stock of an individual in a given year. If this capital stock has increased, the individual has saved, net, regardless of how many previous investments may have been liquidated during the year. It seems to me, therefore, that the second approach, in addition to being simpler and more convenient to handle in the analysis of welfare costs, also looks at the savings decision in a scientifically more sophisticated way.

Before leaving the subject of the effects of taxation on the rate of saving, we should inquire whether the distortions introduced by income taxation can lead to an increase in the rate of saving. It is well known that where changes in the rate of return on savings include an income effect, a reduction in the rate of return can produce an increase in the volume of savings. Does this possibility emerge in the case under discussion? The answer, I believe, is clearly negative. Income effects are kept out of the picture, in this analysis, by the requirement that the government be able, year by year, to buy the same bundles of goods under any alternative patterns of taxation being compared. If we were to consider the effects of reducing the rate of income taxation, without imposing any alternative tax to make up for the lost revenue, then an income effect would be present, and a decrease in saving could conceivably result. However, if we require that the government make up for the lost revenue, say, by increasing the rate of consumption taxation, then no first-order income effect will be present, and the change in the rate of savings must be in the same direction as the change in the net-of-tax rate of return.

Consumers in their role as savers see their future income reduced because of the tax that they have to pay on the return to capital under an income tax. But if the shift were made to a pure consumption tax, they would see their future income reduced because of the higher tax they would have to pay on their future consumption expenditures.¹⁶ Consumers will in fact get more future income under a consumption-tax setup than under an equal-yield income tax, but they will do so only because they save more as a consequence of the higher return they perceive on their capital. If they save exactly the same amounts under the two setups, the extra future income they get from their capital will be just counterbalanced by the extra future taxes they have to pay on their consumption when they face a consumption rather than an income tax.

In spite of a clear presumption that the taxation of income from property will reduce the rate of savings as against what it would be under neutral taxation of equal yield, it is very hard to build an empirical case for the proposition that the taxation of property income has significantly affected the savings rate. But one must recognize that it would not require a phenomenal responsiveness of savings to the net rate of return in order to generate a fairly significant welfare cost as a consequence of the distortion of the savings-consumption decision. If

¹⁶ See Martin J. Bailey, *National Income and the Price Level*, New York, 1962, pp. 180-182.

net savings, for example, were only 1 percent of the national income less than they would be under straight consumption taxation, the welfare cost of taxes striking the income from capital at a $33\frac{1}{3}$ percent rate would be roughly $1/2 (.01)(.50)$ times the national income. This is .25 percent of national income, or a sum of over \$1 billion per year at present levels.

What rankles me much more than the possible effect of our tax system on the total volume of savings, however, is the way in which our taxes influence the uses to which our savings are put. The heavy weight of taxes on income from capital can be justified on equity grounds in a number of ways, but the grossly unequal tax treatment of different types of income from capital cannot be seriously defended by appeals to equity considerations. Most risky corporate investments, which have little chance to be financed by debt capital, have the worst of it, while oil wells are powerfully subsidized to produce petroleum at real cost far above what it would take to obtain the oil from abroad. Income is freely transmuted into capital gains in a variety of activities (lumber, coal mining, rental housing, cattle feeding), inducing an excessive flow of resources into these fields. Perhaps the most blatant case is that of owner-occupied housing which is allowed to generate no taxable income, just tax deductions. The stimuli given by the tax treatment of owner-occupied housing are virtually nil for the needy, and increase directly with the opulence of the taxpayer. It is easy to establish that these tax incentives give rise to a substantial misallocation of investible funds, with a consequent cost to consumers throughout the economy. Elsewhere¹⁷ I have tried to establish the order of magnitude of the losses involved due to tax-induced misallocations of our capital stock. Roughly speaking, \$1 billion per year (say, between \$.5 and \$1.5 billion) seems to be the right order of magnitude for the distortions caused by the corporation income tax. Percentage depletion and related provisions appear to cost the economy between \$.5 and \$1.0 billion per year. David Laidler, in a study currently nearing completion, estimates the cost to the economy of the inefficiencies introduced by the special treatment of owner-occupied housing to be in the order of \$.5 to \$1.0 billion per year. Taking all these costs together, and recognizing that there are a number of important tax incentives to the allocation of investment which are not accounted for above, I think that a clear case can be made for rationalizing our manner of taxing income from capital so as not to discriminate so severely among different allocations of capital resources.

VI. TAXATION AND GROWTH

It is likely that the feature which best distinguishes the economic thinking (both professional and popular) of the postwar period from earlier decades is the emphasis placed upon economic growth — as a phenomenon to be explained, as a criterion of economic performance, and as an objective of policy. In

¹⁷ Arnold C. Harberger, "The Taxation of Mineral Industries," in *Federal Tax Policy for Economic Growth and Stability*, Washington, Joint Committee on the Economic Report, 1955, pp. 439-449 [this volume, Chapter 11], and "The Corporation Income Tax: An Empirical Appraisal," in *Tax Revision Compendium*, U.S. House of Representatives, Ways and Means Committee, Washington, 1959.

today's environment it is quite natural, therefore, to inquire into the likely effects of alternative policies upon the rate of growth. In the terms of reference of this conference, and of my paper, this boils down to the question of how significantly the rate of growth could be influenced by plausible changes in the mix of direct and indirect taxation. I think that the answer is not very much.

The modern approach to the empirical analysis of economic growth (used, with only slight variations, by Abramovitz, Denison, Fabricant, Kendrick, Schultz, and Solow, among others) is to split up observed growth into a part attributed to the increase in labor inputs, a part attributed to the increase in capital inputs, and a residual not explained by these factors. The part assigned to labor is this year's growth in man-hours times last year's average wage; this procedure uses last year's average wage as a measure of what the marginal product of the added labor would be in the absence of technical progress and of changes in the quality of labor. The part assigned to capital is this year's net investment times last year's average before-tax rate of return to capital; once again last year's observed rate of return is used as a measure of what added capital would contribute to national income in the absence of technical advance and other forces not taken directly into account. The breakdown of the change in output from one period (year) to the next can be summarized as

$$\Delta Y_t = w_{t-1} \Delta L_t + \rho_{t-1} I_t + T_t, \text{ where}$$

ΔY_t = change of national income (or net national product) between period $t - 1$ and period t (expressed in dollars of $t - 1$).

ΔL_t = change in labor input (man-hours) between period $t - 1$ and period t .

w_{t-1} = average wage per man-hour in period $t - 1$.

ρ_{t-1} = average (gross-of-tax) rate of productivity of capital (net of depreciation) in period $t - 1$.

I_t = net investment in period t .

T_t = the amount of growth of national income between $t - 1$ and t which is unexplained by changes in labor and capital inputs (expressed in dollars of $t - 1$).

The formula above can be divided by Y_{t-1} so as to express the dependency of the rate of growth of income on various factors:

$$y_t = S_{t-1} l_t + \rho_{t-1} i_t + r_t, \text{ where}$$

$y_t = \Delta Y_t / Y_{t-1}$ = rate of growth of income in year t .

$S_{t-1} = w_{t-1} L_{t-1} / Y_{t-1}$ = share of labor in the national income in year $t - 1$.

$l_t = \Delta L_t / L_{t-1}$ = rate of growth of labor input in year t .

$i_t = I_t / Y_{t-1}$ = net investment in year t as a fraction of Y_{t-1} .

$r_t = T_t / Y_{t-1}$ = percentage increase in income in year t which is unexplained by changes in labor and capital inputs.

The earlier sections of this paper were devoted to the possible effects of direct and indirect taxation on the supply of labor and on the volume of savings. There is a clear presumption that greater resort to indirect taxation would increase the supply of labor, though the magnitude of the increase would not be very great. The rough guesses of column 4 of Table 2.2 suggest that approximately a

3 percent increase in the supply of labor could be obtained by eliminating the inducements to leisure implied by our present marginal personal income-tax rates. But to get to this result in practice would require constructing a tax system in which there were no inducements favoring leisure over labor *at the margin*. The more likely alternatives to our present income-tax structure, such as greater reliance on broad-based indirect taxes, would not go this far; they would only reduce, not eliminate, the incentives toward leisure at the margin. Thus we should probably think of plausible shifts toward more indirect taxation as leading to a substantially less than 3 percent increase in the supply of labor, under given conditions. This change in the supply of labor would increase the measured national output (assuming full employment), but would not necessarily contribute to the normal rate of growth of output. Any contribution of decreased leisure to the rate of growth of output would have to come from its influence either on the rate of growth of the labor supply or on the share of labor in total income. I see no reason to expect any significant change in either of these magnitudes as a consequence of shifting the mix of direct and indirect taxation,¹⁸ hence I would put down as negligible the possible contribution of such shifts to the overall rate of growth.

As distinct from their influence on growth via the labor-leisure choice, there are more significant possibilities for shifts between direct and indirect taxation to alter the rate of growth via their effects on saving. These can be conveniently represented if we make the assumption that, through time, the marginal rate of productivity of capital remains constant. If this rate is 10 percent, and net investment (= net savings) increases from 8 to 9 percent of the national income, the rate of growth of output should on that account change by .10 percent per annum $(.10) \times (.01)$. Increasing the rate of net saving from 8 to 10 percent of the national income will, in these circumstances, add .20 percent per annum to the rate of growth.

Influencing the rate of growth by .10 or .20 of a percentage point may seem small, but it is not negligible in the sense that I have labeled the impact on growth via the labor-leisure choice as negligible. In discussing the effects on growth via savings, I shall first consider what kinds of effects on savings we might plausibly expect to come even from drastic changes in tax policy, and then proceed to deal with the possibility that the formulation of the growth process presented above might understate the contribution of added savings.

Although there has been some debate as to the degree to which the marginal propensity to save increases with income, I believe that there can be little doubt that it does increase. Existing studies of saving by income class do not include accruing capital gains either as part of income or as part of saving, yet in principle they should do both. Thus studies measuring income and savings on the conventional basis can yield what purports to be a constant marginal propensity

¹⁸ It is clearly possible to influence the long-run rate of growth of the labor force by tax measures explicitly aimed at this objective. I have no doubt, for example, that the birth rate could be influenced by changes in income-tax exemption policy, or by indirect taxes on goods specific to child-raising. And any change in the birth rate would in the long run have its effect on the normal rate of growth of the labor force. But I do not believe that any of us considers specific measures aimed at influencing population growth to be within the terms of reference of this conference, and accordingly will not pursue the matter further.

to save, or only a mildly increasing one, and yet a correctly measured marginal propensity to save might be a significantly increasing function of income.

As far as I know, no study exists which incorporates accruing capital gains in both income and savings; hence I cannot bring evidence to bear on this subject. But making extreme assumptions often allows one to get an idea of the possibilities. Suppose, for example, that the marginal propensity to save were 50 percent for all incomes above \$20,000 per year, and 10 percent for all incomes below \$20,000. Suppose, moreover, that in a drastic tax revolution all incomes above \$20,000 were entirely relieved of income tax, and that the entire income-tax burden now borne by them were placed (via direct or indirect taxation) on income brackets below \$20,000. What would be the effect on savings? In 1961, adjusted gross incomes above \$20,000 paid less than \$11 billion of income tax. If these groups were relieved of this burden altogether, they would, on our absurd assumptions, save \$5.5 billion more. And if lower brackets were required to bear \$11 billion more of tax, they would, on our assumptions, save \$1.1 billion less. There would be a net increase in savings of \$4.4 billion, or just about 1 percent of the 1961 national income.

Now I cannot imagine that plausible redistributions in the tax burden would be nearly so drastic as that assumed above. Nor do I find it plausible that differences in marginal propensities to consume are as sharp as those assumed above. Hence I conclude that the redistributive effects associated with plausible tax changes are not likely to increase saving by more than, say, .50 percent of the national income.

The evidence we have on the substitution effects of taxation upon saving is even less secure than that on the redistributive effects. Here, appeal is often made to the observed secular constancy of the ratio of savings to income, and the conclusion is drawn, in the face of fairly substantial changes in the perceived rate of return, that the substitution effects in question are small. I adhere in general to this position, though it must be recognized that income effects were also present in the situations producing the observed data. Until we can isolate the magnitude of the income effects that were at work, we should not conclude that the substitution effects which operated to roughly offset the income effects were minor in importance. Having said this, however, let me add that I would be surprised if plausible tax changes (e.g., a shift from income to expenditures as the base of our major progressive tax) would increase national savings by more than 1, 1.5, or 2 percent of the national income.

If these rough judgments can be accepted, we must face the fact that tax changes are unlikely to increase the rate of growth of national income by more than .10 or .20 of a percentage point — unless the model set out at the beginning of this section has a serious flaw.

The flaw which would be the most likely to change the growth-rate implications of the preceding analysis would be an underestimation of the social marginal productivity of investment (= savings). If investment carries with it positive external effects on output, the average perceived gross-of-tax rate of return would understate the contribution of investment to the growth of national income, and our imputation would understate the part of observed growth due to capital accumulation.

Solow, in a highly original paper, has attempted to cope with this possibility requiring that technical advances be "embodied" in capital equipment before they produce increases in output.¹⁹ His method was to assume that the capital equipment produced in year t was 100λ percent "better" (i.e., more productive) than that produced in year $t - 1$. Working with this assumption, Solow reconstituted existing data making gross investment in year t "equivalent" that of the base year by taking $I_t' = I_t(1 + \lambda)^t$, where I_t is the conventional measure of gross investment (in constant dollars) and I_t' is the measure incorporating the improved productivity of investment in year t over the same constant-dollar volume of investment in year 0. Proceeding on this basis, Solow builds up a time series on capital stock from the adjusted gross investment data, together with retirement estimates based on Terborgh's figures. He estimates separate capital stocks for plant and equipment, introducing the possibility of separate λ 's for these two categories of capital. He ends up with one series for capital stock associated with λ 's of .02 for both plant and equipment; another associated with λ 's of .03 for both plant and equipment, and three more for other combinations (.02, .03), (.02, .04), (.03, .04) of λ 's for plant and equipment. He then introduces these alternative measures of capital stock along with variables measuring labor input and unemployment, into a production function of the Cobb-Douglas type (modified to incorporate the unemployment variables). His results improve as he moves from capital stock estimates based on lower values for λ to those based on higher values.

Solow's device appears to yield some mileage, as far as the effect of investment on growth is concerned. Whereas on our assumption of a 10 percent rate of productivity on capital, it takes a 10 percentage point increase in the fraction of income invested to produce a 1 percentage point change in the growth rate, Solow achieves the same effect by an increase of 2.5 percentage points in the fraction of income invested.

However, Solow's results depend critically on his assumption that there are no independent forces producing economic growth. Eitan Berglas has followed Solow's procedure for calculating capital stock, but has allowed for the production function to shift with time as a result of independent forces as well.²⁰ Berglas finds that the best explanation of observed changes in output, under this hypothesis, emerges when the λ 's for plant and equipment are both set at zero — i.e., when the conventional measure of capital stock is used. More important, as long as a trend shift of production functions is allowed for, the effect of investment on output is no greater when λ is high than when it is zero. That is, adjusting the capital stock to incorporate assumed rates of technical improvement does not add to the estimated effect of investment on economic growth — regardless of what positive values of λ are chosen.

It seems to me that Berglas' reformulation of Solow's procedure must be preferred to the original. In the first place, we know that there are forces which have had a trend influence on total factor productivity in the economy that

are not directly associated with the volume of investment — improvements in the quality of the labor force and advances in the art of management are two cases in point. This knowledge should dictate a procedure which would allow for a trend shift of production functions as one of the explanatory factors of changes in output. If, when trend shifts were allowed for, they proved to have an insignificant explanatory power, we could conclude that they have been unimportant. But this is not the result that emerges. In fact, the trend variables introduced by Berglas are highly significant statistically (with t ratios lying between 8 and 15 for various assumptions regarding λ). Moreover, they lead to plausible coefficients for the capital input in the Cobb-Douglas production function (around .25 for $\lambda = 0$, as opposed to 1.31 for $\lambda = 0$ when trend is not taken into account, and .52 for $\lambda = .03$ when trend is not taken into account). I want to emphasize that, though Berglas' study suggests that $\lambda = 0$ is the best hypothesis for explaining observed changes in output in the presence of a trend shift in production functions, this is not the real lesson to be learned from his experiments. For $\lambda = .03$, when trend is allowed for, Berglas explains 99.31 percent of variations in output per man-hour over the period 1929-1960; while for $\lambda = 0$ he explains 99.46 percent of the same variations. There is thus not much basis for choice among different plausible values of λ . The important result emerging from Berglas' study is that higher values of λ do not mean that investment has a greater influence on economic growth than is obtained when λ is assumed to be zero. In short, Berglas' work has put us back into the situation where an extra percentage point of national income devoted to investment will affect the rate of growth by around .10 of a percentage point.

Berglas' conclusions are fortified by additional experiments. In the first, an effort was made to see whether the rate of growth depended significantly on the "newness" of the capital stock. To test this hypothesis, the cumulated gross investment of a period, expressed as a percentage of initial capital stock, is used to explain changes in total factor productivity over the same period in different industries. This was done for thirteen two-digit industries, for the periods 1948-1957, 1948-1953, and 1953-1957. In none of these cases did a significant correlation appear, and in two of the three (1948-1957 and 1953-1957) the correlation was mildly negative. Similar results emerged when cumulated investment for 1948-1953, expressed as a fraction of capital stock, was used to explain changes in total factor productivity between 1953 and 1957. This last effort was an attempt to allow for a possible lag between investment and subsequent improvement in total factor productivity.

The limited power of added investment to improve the growth rate should not really be surprising. It does not deny that, each year, technical advances make particular investments highly productive. Nor does it imply that the allocations presented at the beginning of this section are correct. I suspect, in fact, that there is a substantial degree of "embodiment" of technical advances in particular types of capital goods with high prospective marginal productivity. Each year, I think, technical improvements create prospects of very high rates of return in certain investments, say, 30, 25, or 20 percent. The contribution of these investments to economic growth is greater than ρ , the average observed rate of return to capital. If we ranked investments according to prospective rates of

¹⁹ R. M. Solow, "Technical Progress, Capital Formation, and Economic Growth," *American Economic Review*, May 1962, pp. 76-86.

²⁰ Eitan Berglas, "Stimulating Investment: Costs and Effects," unpublished Ph.D. dissertation, University of Chicago, 1963, Chapter 5.

turn, forming a function $I = f(\rho)$ where $f'(\rho) < 0$, we could state the contribution of investment to economic growth as $I\rho_0 + \int_{\rho_0}^{\infty} f(\rho) d\rho$, which would exceed the amount assigned to investment by the expression $I\rho_{t-1}$ if we set $\rho_t = \rho_{t-1}$. The difference would be that inframarginal investments would then be assigned a greater contribution to economic progress. But, when we are talking about increasing the growth rate by adding to investment, we are not concerned with inframarginal investments, but about those beyond what would otherwise be the cut-off rate of return. There is nothing inconsistent between the following three statements:

1. Each year some investments are undertaken which promise to yield, at prevailing prices, very high rates of return.
2. Each year significant amounts of investment are undertaken which promise yields in the neighborhood of the observed past rate of return.
3. Additional investments above and beyond those actually undertaken would likely produce rates of return which are at best equal to the observed past average rate of return.

According to the point of view just expressed, "embodiment" of technical changes takes place principally in inframarginal investments. This sort of phenomenon could easily produce the results observed by Berglas, with the marginal effect of investment being approximated by the observed past average rate of return, but with a substantial time trend in the ratio of output to total capital stock. It is also consistent with the observation that, across industries or time periods, there is relatively little relationship between the rate of increase in capital stock and the movements in total factor productivity.

This is not the place for a very detailed discussion of these points, so I will leave the matter here. I hope that I have been able to provide some support for my view that we cannot expect great changes in the growth rate from plausible changes in the mix of direct and indirect taxation.

I feel impelled, however, to point out that in our recent preoccupation with the rate of growth, we, as a profession, may have lost sight of some old and fundamental truths. We should distinguish between the rate of growth of income, the level of income, and the level of welfare as possible goals of economic policy. Concentration on the rate of growth as an objective can lead one to minimize the value of having, say, x percent more income each year as a result of a policy change — simply because income would not grow any faster except during a transitional period. On the other hand, concentration on the level of real income (as conventionally measured) can lead one to neglect the costs of bringing about changes in that level. If, by tax changes, we increase labor and reduce leisure, or if we increase saving and reduce consumption, we should, I think, not just look at the pluses and neglect the minuses. When we try to take both pluses and minuses into account, we come to grips with the measurement of the effects of policy changes on welfare, which was the subject of the earlier sections of this paper.

III

The next chapter, which was presented at the annual meeting of the American Economic Association in December, 1963, and published the following year in *The American Economic Review*, is substantially more sophisticated than "Taxation, Resource Allocation and Welfare," but to a considerable extent this gain was bought (given space limitations) at the expense of the slower-paced, more didactic presentation of that earlier effort. To help compensate the cryptic nature of the exposition, I take this opportunity to indicate what I consider to be its main points.

In the first place, expressions (3.4) and (3.5), which measure in terms of coefficients R_{ij} the change in welfare induced by a set of distortions, though they look exactly like their counterparts in Chapter 2, which express welfare change in terms of coefficients S_{ij} , are really quite different. On the assumptions laid out in Chapter 2, the coefficients S_{ij} could be interpreted as simply compensated demand coefficients, and the whole analysis could be handled in straightforward demand-theory language. The R_{ij} are no such simple animals — they are really reduced-form coefficients showing how the equilibrium levels of various activities X_1, X_2, \dots, X_n depend on the degrees of distortion T_1, T_2, \dots, T_n to which each of the activities is subject.

To explore a bit the nature of the R_{ij} , expressed in terms of more familiar supply and demand parameters, consider first the simplest of textbook cases:

$$(3A) \quad X_1 = \underbrace{a_1 + b_1(P_1 + T_1)}_{\text{Demand}} = \underbrace{e_1 + f_1 P_1}_{\text{Supply}}$$

The solution for P_1 is

$$(3B) \quad P_1 = \frac{a_1 - e_1 + b_1 T_1}{f_1 - b_1}, \quad \text{and that for } X_1 \text{ is}$$

$$(3C) \quad X_1 = \frac{f_1 a_1 - b_1 e_1 + f_1 b_1 T_1}{f_1 - b_1}$$