In a function $I = f(\rho)$ where $f'(\rho) < 0$, we could state the condition of investment to economic growth as $I_{\rho} \rho + \int_{\rho} f(\rho) \, d\rho$, which would then be multiplied by the amount assigned to investment by the expression $I_{\rho} \rho + \int_{\rho} f(\rho) \, d\rho$. The difference would be that inframarginal investments would then have been assigned a greater contribution to economic progress. But, when we are talking about increasing the growth rate by adding to investment, we are not concerned with inframarginal investments, but about those beyond what would otherwise be the cut-off rate of return. There is nothing inconsistent between the following two statements:

Each year some investments are undertaken which promise to yield, at prevailing prices, very high rates of return.

Each year significant amounts of investment are undertaken which promise yields in the neighborhood of the observed past rate of return.

Additional investments above and beyond those actually undertaken would likely produce rates of return which are at best equal to the observed past average rate of return.

According to the point of view just expressed, "embodiment" of technical change takes place principally in inframarginal investments. This sort of phenomenon could easily produce the results observed by Berglas, with the marginal effect of investment being approximated by the observed past average rate of return, but with a substantial time trend in the ratio of output to total input. It is also consistent with the observation that, across industries or time periods, there is relatively little relationship between the rate of increase in capital stock and the movements in total factor productivity.

This is not the place for a very detailed discussion of these points, so I will leave the matter here. I hope that I have been able to provide some support for my view that we cannot expect great changes in the growth rate from plausible changes in the mix of direct and indirect taxation.

I feel impelled, however, to point out that in our recent preoccupation with the rate of growth, we, as a profession, may have lost sight of some old and fundamental truths. We should distinguish between the rate of growth of some, the level of income, and the level of welfare as possible goals of economic policy. Concentration on the rate of growth as an objective can lead one to minimize the value of having, say, a percent more income each year as a result of a policy change — simply because income would not grow any faster except during a transitional period. On the other hand, concentration on the level of real income (as conventionally measured) can lead one to neglect the costs of swinging about changes in that level. If, by tax changes, we increase labor and reduce leisure, or if we increase saving and reduce consumption, we should, I think, not just look at the pluses and neglect the minuses. When we try to take both pluses and minuses into account, we come to grips with the measurement of the effects of policy changes on welfare, which was the subject of the earlier sections of this paper.

The next chapter, which was presented at the annual meeting of the American Economic Association in December, 1963, and published the following year in *The American Economic Review,* is substantially more sophisticated than "Taxation, Resource Allocation and Welfare," but to a considerable extent this gain was bought (given space limitations) at the expense of the slower-paced, more didactic presentation of that earlier effort. To help compensate the cryptic nature of the exposition, I take this opportunity to indicate what I consider to be its main points.

In the first place, expressions (3.4) and (3.5), which measure in terms of coefficients $R_{ij}$ the change in welfare induced by a set of distortions, though they look exactly like their counterparts in Chapter 2, which express welfare change in terms of coefficients $\lambda_{ij}$, are really quite different. On the assumptions laid out in Chapter 2, the coefficients $\lambda_{ij}$ could be interpreted as simply compensated demand coefficients, and the whole analysis could be handled in straightforward demand-theory language. The $R_{ij}$ are no such simple animals — they are really reduced-form coefficients showing how the equilibrium levels of various activities $X_1, X_2, \ldots, X_n$ depend on the degrees of distortion $T_1, T_2, \ldots, T_n$ to which each of the activities is subject.

To explore a bit the nature of the $R_{ij}$ expressed in terms of more familiar supply and demand parameters, consider first the simplest of textbook cases:

\[
(3A) \quad X_i = \underbrace{a_i + b_i (P_i + T_i)}_{\text{Demand}} + \underbrace{c_i P_i}_{\text{Supply}}.
\]

The solution for $P_i$ is

\[
(3B) \quad P_i = \frac{a_i - c_i + b_i T_i}{b_i - b_i}, \quad \text{and that for } X_i \text{ is}
\]

\[
(3C) \quad X_i = \frac{a_i c_i - b_i c_i + b_i c_i T_i}{b_i - b_i}.
\]
The coefficient $R_{11}$ is therefore given by

$$R_{11} = \frac{f_1 b_1}{f_1 - b_1},$$

which perhaps looks straightforward enough. Now, however, let us try just a slightly more complicated setup with demand and supply for two goods.

$$\begin{align*}
X_1 &= a_1 + b_1(P_1 + T_1) + c_3(P_2 + T_2) \\
&= e_1 + f_1 P_1 + g_3 P_2 \\
X_2 &= a_2 + b_2(P_2 + T_2) + c_3(P_1 + T_1) \\
&= e_2 + f_2 P_2 + g_3 P_1
\end{align*}$$

(S3E)

Solving this, we find

$$\begin{align*}
P_1 &= \frac{[a_1 - e_1 + b_1 T_1 + c_3 T_2](f_1 - b_2) - [a_2 - e_2 + b_2 T_2 + c_3 T_1](g_1 - c_2)}{\Delta} \\
P_2 &= \frac{[a_2 - e_2 + b_2 T_2 + c_3 T_1](f_2 - b_1) - [a_1 - e_1 + b_1 T_1 + c_3 T_2](g_2 - c_2)}{\Delta}
\end{align*}$$

(3F)

where $\Delta = (f_1 - b_1)(f_2 - b_2) - (g_2 - c_2)(g_1 - c_1)$. Using these equations we can solve explicitly for, say, $G_{11}$

$$G_{11} = \frac{f_1 b_1 (f_2 - b_2) - f_1 e_1 (g_1 - c_1) + g_3 e_2 (f_1 - b_2) - g_3 b_1 (g_2 - c_2)}{(f_1 - b_1)(f_2 - b_2) - (g_2 - c_2)(g_1 - c_1)}.$$

(3G)

Now (3.7) can already be described as a reasonably messy expression, but imagine what $G_{ij}$ would be like when the system had three or four or five interdependent demand and supply equations, let alone an arbitrarily large $N$!

How lucky we are, then, to find that the $G_{ij}$ behave just like Slutsky-Hicks compensated demand coefficients, so that we can transfer all our accumulated understanding and intuition about demand phenomena to this much more complicated world. Or, to put it another way, it is not nice to know that if one works out a problem in the comparatively simple constant-cost framework of Chapter 2, the solution thus found will carry over completely to a world of interdependent demands and supplies for all commodities, simply by replacing the demand-defined $S_{ij}$'s by the reduced-form $R_{ij}$'s, which have the same simple and useful properties in spite of the massive amount of clockwork that each of them represents? And finally, even more so, is it not useful to know that similar well-behavedness prevails among the four sets of reduced-form coefficients which show how the amount of capital or labor in activity $i$ responds to the degree of distortion applying to the use of labor or to that of capital in activity $j$? In short, this way of being able to set out what would otherwise be enormously complex problems in simple, potentially manageable terms is to my mind probably the most important result of the "Measurement of Waste."

But there is at least one other range of issues that might merit comment. It is connected (a) with the consolidation of a whole set of taxes into an equivalent set striking with different rates at the incomes from labor and capital in different activities, and (b) with the question of when a tax or tax system neutral, and when is it not. With respect to consolidation, I point out that it is not likely to be always possible to find a set of factor taxes on the incomes from labor and capital in an activity that will be equivalent to a given tax on the product of that activity. Before one can even approach this equivalence, two assumptions must be made — that the relationship of material inputs to output is fixed, at least in the sense that it is not influenced by which of the two allegedly "equivalent" tax packages is chosen, and that the pattern of economic depreciation of the assets of an activity is similarly uninfluenced.

Table 3A starts in column (1) with an ordinary excise tax of 20 percent on a given product. In column (2) the consequences of a 20 percent tax on all components of costs are shown. Taxes paid at any given stage are shown in parentheses. The equivalence between tax patterns (1) and (2) is obvious; taxing all components at a given rate (col. 2), or taxing the sum of the components at the same rate (col. 1) must produce the same result. As we pass from column (2) to column (3), however, the assumption of fixed proportions between material inputs and final product is crucial. If substitution were possible between materials and final products, the shift from (2) to (3) would induce a substitution toward a more material-intensive method of production, with less use of labor and capital, but with fixed proportions the material-intensiveness of production cannot change. Under this assumption, putting the tax of 20 percent on gross value added (col. 3) rather than on all inputs (col. 2) is like imposing a tax of $1 per shoe on all left shoes in place of a tax of $1 per pair. As long as shoes are always sold in pairs, there can be no difference whatsoever between the two alternatives.

Now suppose that the tax setup were that indicated in column (4) — a 25 percent tax on wage payments together
<table>
<thead>
<tr>
<th>Components of cost and price</th>
<th>20% tax on final product (1)</th>
<th>20% tax on all inputs (2)</th>
<th>25% tax on gross value added (3)</th>
<th>25% tax on wages; 41 2/3% tax on capital's net earnings (4)</th>
<th>33 1/3% tax on net value added (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market price of final product</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Cost of final product</td>
<td>100(+20)*</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Depreciation</td>
<td>20</td>
<td>20(+4)*</td>
<td>20(+5)*</td>
<td>20(+)12.5*</td>
<td>20(+10)*</td>
</tr>
<tr>
<td>Capital</td>
<td>30</td>
<td>30(+6)</td>
<td>30(+7.5)</td>
<td>30(+12.5)</td>
<td>30(+10)</td>
</tr>
<tr>
<td>Labor</td>
<td>30</td>
<td>30(+6)</td>
<td>30(+7.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>20</td>
<td>20(+4)</td>
<td>20</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

* Figures in parentheses in each case represent the tax to be added to the cost item in question.
Chapter 3

The Measurement of Waste

I

The subject of this paper might be called "The Economics of the nth Best." This would distinguish the approach taken here from that taken by Lipsey and Lancaster in their fine article, "The General Theory of Second Best" [7], as well as from the conventional concern of economic analysis with the characteristics of fully optimal situations. To state the differences briefly, the conventional approach is concerned with how to get to a Pareto-optimal position, the Lipsey-Lancaster approach is concerned with how to make the best of a bad situation (i.e., how to get to a position which is optimal subject to one or more constraints which themselves violate the conditions of a full optimum), while this paper is concerned with measuring the deadweight loss associated with the economy's being in any given nonoptimal position.

The measurement of deadweight losses is not new to economics in any means. It goes back at least as far as Dupuit; and more recently Hotelling [4], Hicks [3], Debreu [2], Meade [10], and H. Johnson [5, 6] have made important contributions. Nonetheless I feel that the profession as a whole has not given to the area the attention that I think it deserves. We do not live on the Pareto frontier, and we are not going to do so in the future. Yet policy decisions are constantly being made which can move us either toward or away from that frontier. What could be more relevant to a choice between policy A and policy B than a statement that policy A will move us toward the Pareto frontier in such a way as to gain for the economy as a whole, say, approximately $200 million per year, while policy B will produce a gain of, say, about $30 million per year? What could be more useful to us as a guide to priorities in tax reform than the knowledge that the deadweight losses stemming from the tax loopholes (percentage depletion and capital gains) open to explorers for oil and gas are probably greater in total magnitude than the deadweight losses associated

with all the other inefficiencies induced by the corporation income tax? What
would be more tantalizing than the possibility (which I believe to be a real one)
at the U.S. tariff, whose indirect effect is to restrict the equilibrium value of U.
exports, produces by this route a gain for the U.S. from a partial explo-
dition of U.S. monopoly power in world markets which nearly offsets (or per-
haps fully or more than fully offsets) the efficiency-losses produced by tariff-
substituted substitution of more expensive domestic products for cheaper imports?
These and similar questions seem to me so interesting, so relevant, so central
to our understanding of the economy we live in, that I find it hard to explain
why the measurement of deadweight losses should be the province of only a
handful of economists rather than at least the occasional hobby of a much
larger group. Let me simply suggest four possible reasons for the apparent
popularity of the loss-measurement game:

1. Even the simplest attempts to measure the deadweight loss (or, as
I refer to call it, the welfare cost) associated with particular distortions involve
the use of numerical values for certain key parameters (elasticities of demand,
substitution, etc.), which may be impossible to obtain at all, or which may be
estimated but with substantial error. Workers in this field must be ready to
tent themselves with results that may be wrong by a factor of 2 or 3 in many
cases. But, on the other hand, it is a field in which our professional judgment
so poorly developed that the pinning down of an answer to within a factor of
can be very helpful. Be that as it may, one cannot expect the field to attract
colleagues who prefer their results to be meticulously exact.

2. While it is relatively easy to measure the welfare costs of a particular
distortion when one assumes other distortions to be absent, it is much more
difficult to carry through the measurement in a way which takes account of
the presence of other distortions. One of the profound lessons taught us by earlier
workers in this field (Hotelling [4], Viner [11], Lipsey and Lancaster [7],
Rotte and Hagedoorn [1], Little [8], and others) is that an action (i.e., imposing
a tax of $T_p$ per unit on good $X_p$) which would take us away from a Pareto
optimum if we were starting from that position can actually bring us toward
such an optimum if we start from an initially distorted situation. Crude
measures can thus mislead us, while correct measures are hard to come by.

3. Many people find it difficult to isolate the measurement of efficiency
losses due to particular distortions from the changes in the distribution of in-
ome that they conceive would ensue if the distortions were actually removed.
Of these, some are undoubtedly not willing to make the kind of assumptions
they have to make in order to compare the changes in welfare of different
individuals or groups.

4. Consumer surplus, in spite of its successive rehabilitations, is still looked
upon with suspicion by many economists. In spite of the fact that it is possible
to formulate measures of welfare cost which do not directly involve the use of
the consumer surplus concept, the most convenient and most frequently cited
measures of welfare cost do involve this concept. Thus, I venture to guess
another group of potential workers (or at least tasters) in the vineyard do not
venture to enter.

II

The main purpose of this paper is to explore a variety of possible ways of
formulating measures of deadweight losses. All the ways considered are members
of a single family. This section begins by expounding a widely accepted approach
to the problem and then proceeds to extend this approach to what I believe
are new areas.

Let us begin by assuming that the only distortions present in the economy
are taxes. Monopoly elements, externalities, and other market imperfections
will be introduced at a later stage. We shall assume that the economy will
seek and find a unique full employment equilibrium once its basic resource
endowments, the distribution of income, the quantities of goods purchased by
the government, and the set of distortions (taxes) are known. Letting $X$ represent
the vector of equilibrium quantities, $D$ be a vector representing the propor-
tion of total income received by each spending unit, $G$ be a vector representing
the quantities of the different goods and services purchased by government, and
$T$ be a vector representing the tax levied per unit of the different goods and
services produced in the economy, we have $X = f(D, G, T)$.

Now to isolate the efficiency effects of distortions, we must hold $D$ and $G$
constant. Thus, with respect to $D$, we conceive of the possibility of keeping the
percentage share of each spending unit in the total national income constant by
means of neutral taxes and transfers. With respect to $G$, we assume that, in any
pair of situations being compared, the government buys the same bundle of
goods and services. Even though the comparison of two actual situations might
be between $X = f(D, G, T)$ and $X' = f(D', G', T')$, we split up the move from
$X$ to $X'$ into a minimum of two steps. The first step is from $X = f(D, G, T)$ to
$X^* = f(D, G, T^*)$. This step isolates the efficiency aspects of the change. The
move from $X^*$ to $X'$ entails no change in the distortions affecting the economy,
and involves only shifts in the distribution of income and in the level of govern-
ment expenditures. To the extent that the tax yield produced by the vector $T$
is insufficient or more than sufficient to finance the expenditure vector $G$, we
assume that neutral taxes or transfers will be called upon to make up the
difference. (Should fiscal policy measures be necessary to provide full employ-
ment, neutral taxes and transfers will be the instruments used to bring the
total tax take to the required level. Government expenditures, on our assump-
tions, would be held fixed.)

The above assumptions have the effect of setting first-order income effects
(whether caused by redistribution or changes in the size of government pur-
chases) to one side so as to isolate the efficiency effects of alternative tax patterns.
They put us in a world of substitution effects and of relative prices. When
dealing with relative price phenomena, it is customary to treat a single product
as the numeraire. This procedure is, however, not essential. One could normalize
by holding any desired index of prices constant, or in a variety of different
ways. For our purposes, it is convenient to normalize by holding the money
national income constant as among all possible situations being compared. We
could alternatively hold constant money net national product, gross national
This expression pops up in one form or another all through the literature on the measurement of welfare costs, the economics of second best, the theory of customs unions, etc. It appears, or can be derived from what appears, in Corlett and Hague [1], Hotelling [4], H. Johnson [5,6], Meade [10], and Lipsey and Lancaster [7], among others.

Let us now linearize expression (3.1) by setting

$$ \frac{\partial X_i}{\partial T_i} = R_{ii}. $$

With this substitution, (3.1) evaluates at

$$ \Delta W = \frac{1}{2} \sum_{i=1}^{n} R_{ii} T_i^2 + \sum_{i,j}^{n} R_{ij} T_i T_j. $$

Expression (3.3) can be simplified, however, using the integrability condition

$$ \frac{\partial X_i}{\partial T_j} = \frac{\partial X_j}{\partial T_i}, $$

which translates in the linearized form into $R_{ij} = R_{ji}$. In economic terms, this same condition derives from the fact that the welfare cost of a set of taxes should not, in a comparative static framework such as this, depend on the order in which those taxes are conceived to be imposed. Thus if we impose $T_1$ first and follow it by $T_2$, we have $\Delta W = 1/2 R_{11} T_1^2 + 1/2 R_{22} T_2^2 + R_{12} T_1 T_2$. If on the other hand we impose $T_2$ first and follow it by $T_1$, we have $\Delta W = 1/2 R_{22} T_2^2 + 1/2 R_{11} T_1^2 + R_{21} T_2 T_1$. Hence if the linearized expression (3.3) is to be invariant with respect to order of imposition of taxes, $R_{ij}$ must equal $R_{ji}$, and in general $R_{ij}$ must equal $R_{ji}$. This enables (3.3) to be simplified to

$$ \Delta W = \frac{1}{2} \sum_{i,j}^{n} R_{ij} T_i T_j. $$

For each $R_{ii}(j < i)$ appearing in (3.3), we simply substitute $1/2 R_{ii} + 1/2 R_{jj}$ to obtain (3.4).

A further condition on the $R_{ij}$ can be established by noting that a set of taxes with some $T_i \neq 0$ can at best produce an equal level of welfare as an undistorted situation. This yields

$$ \Delta W = \frac{1}{2} \sum_{i,j}^{n} R_{ij} T_i T_j \leq 0 \quad \text{for all possible values of } T_i, T_j. $$

As a special case of (3.5) we have

$$ R_{ii} \leq 0 \quad \text{for all } i. $$

This is obtained when $T_i \neq 0$, while $T_j = 0$ for all $j \neq i$.

We are by now quite close to establishing the Hicsian substitution conditions by the back door, so to speak. What we need to finish the job is the adding-up property. Suppose it to be true that a proportional tax at the rate $t$ on all the $X_i$ would indeed be a neutral tax. We can define $T_i = tX_i$, where $X_i$ is marginal...
and, and \( t_i \) = percentage rate of tax on \( X_i \) to obtain

\[
\Delta W = \frac{1}{2} \sum_i c_i c_i R_i t_i.
\]

An equal percentage tax on all commodities is neutral, we have

\[
\sum_i c_i c_i R_i t_i = 0.
\]

What we actually have much more than this. If a proportional tax at the rate \( t \) truly neutral, then, given our assumptions about the constancy of income distribution and of government purchases, it simply substitutes for the head tax that would have to exist if all the \( T_i \) were zero. It must produce the same equilibrium quantity for each and every commodity. Thus we have that

\[
\frac{\partial X_i}{\partial t} = \sum_j c_j R_{ij} = 0 \quad \text{for all } i.
\]

This is the counterpart of the Hicksian adding-up property.

However, a tax at the rate \( t \) on all \( X_i \) will be neutral only in certain cases.

Case A: Suppose that, as was assumed above, the production frontier of the economy is linear —

\[
\sum_i c_i X_i = \text{a constant}.
\]

This means that total production is in inelastic supply, and therefore that a tax which strikes the value of all production at a constant rate will be neutral. In this case all the \( X_i \) must be final products; the \( R_{ij} \) here turn out to be precisely the Hicksian substitution terms.

Case B: Suppose that all the \( X_i \) are final products, and that the production frontier of the economy is convex from above. Suppose, moreover, that all basic factors of production are fixed in total supply. So long as a tax at the rate \( t \) on final products is in effect a tax at a fixed rate on the net earnings of all factors of production, it will be neutral, and condition (3.10) will hold. In this case, the \( R_{ij} \), while obeying the properties of the Hicksian substitution terms, are actually quite different from them. Here the \( R_{ij} \) are really the “reduced form” coefficients showing how the equilibrium value of \( X_i \) (with supply and demand equal for all commodities) depends on \( T_j \).

Case B presents no problem when capital is not among the basic factors of production, or when the relation between gross and net earnings of capital is the same in all uses. However, when capital is among the basic factors and when the relationship between gross and net earnings does (because of different depreciation patterns) differ among uses, then an equal tax on all final products will not be neutral, even though capital and other factors of production are fixed in total supply. This is because increases in the rate of proportional tax, \( t_i \), will create incentives which would relatively favor the longer-lived applications of capital. An equal tax on value added in all industries, however, would be neutral in these circumstances, because we assume the net rate of return on capital to be equalized among all uses of capital. (This, of course, assumes that

the stock of capital and the supplies of other basic factors of production are fixed.)

Problems quite similar to those presented by different depreciation patterns in different applications of capital arise when the possibility of taxing intermediate products is introduced. As McKenzie [9] has forcefully pointed out, an equal percentage tax on all products will generally be nonneutral if any of the products in question are intermediate or primary products not in fixed supply. A tax at an equal percentage rate on value added in every activity will, on the other hand, be neutral so long as the basic factors of production are in fixed supply.

Case C: When considering taxes on value added we let \( X_i \) represent the volume of final product of industry (or activity) \( i \). \( v_i \) represent value added per unit of the product of activity \( i \), and \( T_i \) represent the tax per unit of final product in industry \( i \). (Although the tax is levied on value added, \( T_i \) is here expressed per unit of product.) Once again letting \( R_{ij} = \partial X_j / \partial T_i \), we have (3.5) as the expression for \( \Delta W \). To reflect the neutrality of an equal percentage tax on value added everywhere, we require that the response of any \( X_i \) to such a tax be zero; i.e., that

\[
\sum_j v_j R_{ij} t_j = 0 \quad \text{when } t_j = t \quad \text{for all } j.
\]

Here \( v_j \) = value added per unit of the product \( X_j \), \( t_j \) = percentage rate of tax on value added in industry \( j \), \( v_j t_j = T_j \). Hence we have

\[
\sum_j v_j R_{ij} = 0; \quad R_{ij} = R_{ij}; \quad \sum_i R_{ij} T_i T_j \leq 0 \quad \text{for all } T_i, T_j; \quad R_{ij} \leq 0 \quad \text{for all } i
\]

as before.

Case C deals rather neatly with problems of differential depreciation and taxes on nonfinal products. However, case C assumes that indirect taxes are levied on value added, whereas most frequently in the real world they are levied on the final product.

Fortunately, it is possible to translate product taxes into value-added taxes, and still stay within the framework of case C so long as inputs other than labor and capital enter their respective products in fixed proportions. The reason for this is obvious. All the effects of a tax at the rate \( t_i \) on product \( i \) can be replicated by a tax at the same rate on all factor shares (including materials inputs) entering into the production of product \( i \). These are simply two ways of imposing the same tax. Suppose that with a tax of 10 percent on all factor shares in the 7th industry an equilibrium is reached in which materials inputs account for half the value of product and labor and capital the other half. So long as materials inputs must be used in fixed proportions per unit of product, a shift from a 10 percent tax on all factor shares in the 7th industry to a 20 percent tax on value added in the 7th industry would introduce no incentive to change the equilibrium reached with a 10 percent tax on all factor shares. Purchasers could pay the same price for the product; labor, capital, and materials sellers could get the same net reward; and the government could get the same tax take. Moreover, since the taxes on labor and capital shares would still be at equal
between them must be taken into account. Let us define

$$H_{ij} = \frac{\partial K_i}{\partial E_j} \text{ and } N_{ij} = \frac{\partial N_i}{\partial B_j}.$$ 

Here symmetry exists between $H_{ij}$ and $N_{ij}$. Suppose for example, we impose first a tax of $B_1$ and then one of $E_1$. We obtain $1/2 G_{11} B_1^2 + M_{21} B_2 E_2 + H_{11} B_1 E_2$ as our measure of $\Delta W$. If we conceive of $E_2$ being imposed first, and then $B_1$, we obtain $1/2 M_{21} B_2 E_2^2 + 1/2 G_{11} B_1^2 + N_{11} E_1 B_1$. If we think of imposing a set of taxes $B_i$ first and then a set of taxes $E_i$, we have

$$\Delta W = \frac{1}{2} \sum \sum G_{ij} B_i B_j.$$ 

If we think of it the other way around, we have

$$\Delta W = \frac{1}{2} \sum \sum M_{ij} E_i E_j + \sum \sum N_{ij} E_i B_j.$$ 

For a reason that will be apparent later, it is most convenient to write:

$$\Delta W = \frac{1}{2} \sum \sum M_{ij} E_i E_j + \frac{1}{2} \sum \sum G_{ij} B_i B_j + \frac{1}{2} \sum \sum N_{ij} B_i E_j.$$ 

Now, when labor is in fixed supply, a tax on capital in industry $i$ can only redistribute the existing amount of labor. Hence

$$\sum_i N_{ij} = 0.$$ 

Likewise, when capital is in fixed supply, a tax on labor in industry $j$ can only redistribute the available capital, so that

$$\sum_j H_{ij} = 0.$$ 

The interaction terms disappear for neutral taxes because in this case $E_i = E$ for all $i$, $B_i = B$ for all $i$. (Since the wage is assumed to be equalized in all uses of labor and since the net rate of return is assumed to be equalized in all uses of capital, an equal tax per unit of labor is also an equal percentage tax on value added by labor in different activities, and likewise for capital.)

Thus we have:

Case D: When labor and capital are in fixed supply, expression (3.15) measures the change in welfare due to any pattern of taxes on labor and capital in different activities. The $M_{ij}$ and the $G_{ij}$ obey the Hicksonian conditions, with the adding-up property in this case

$$\sum_i M_{ij} = 0 = \sum_i G_{ij},$$

All terms vanish for taxes on labor that are equal in all uses together with taxes.
capital that are equal in all uses. The interaction terms can in general be positive or negative, but the whole expression (3.15) must always be \( \leq 0 \). In this case the coefficients reflect not only conditions of final demand and supply but also conditions of factor substitution.

We now attempt to allow for the fact that the supply of labor in the market by itself is a function of the pattern of taxation. This question has been dealt with in the literature of second-best by Little, Corlet, and Hague, Lipsey and Lancaster, and Meade, among others. The key to at least the last three of these treatments is the substitution of the assumption (1) that the number of hours in the year is fixed for the assumption that the number of man-hours worked in the market is fixed. We can do this simply by adding another activity — labeled "leisure" or "nonmarket activity." If there are \( n \) market activities, we add an \( n + 1 \)st, and have

\[
\sum_{j=1}^{n+1} L_j = L.
\]

This does not change the form of equation (3.12) but it does alter the definition of a neutral tax. Now an equal tax on all labor in market activities is not neutral, because it neglects the \( n + 1 \)st activity. However, a tax that struck hours equally (including leisure hours) would be neutral. Hence we have

\[
\sum_{j=1}^{n+1} M_{ij} = 0 = \sum_{j=1}^{n+1} M_{ij}.
\]

To measure the welfare cost of an equal tax of \( E \) on all activities except leisure we take

\[
\Delta W = \frac{1}{2} \sum_{j=1}^{n} \sum_{j=1}^{n+1} M_{ij} E^a
\]

so that

\[
\sum_{i=1}^{n+1} M_{ij} = 0, \quad \text{so} \quad \sum_{j=1}^{n} M_{ij} = -M_{j,n+1}.
\]

Hence (3.16) reduces to

\[
\Delta W = -\frac{1}{2} \sum_{j=1}^{n} M_{j,n+1} E^a.
\]

Thus (3.17) reduces to

\[
\Delta W = \frac{1}{2} M_{n+1,n+1} E^2,
\]

where \( M_{n+1,n+1} \) represents the responsiveness of leisure to a change in the tax rate on leisure (or to the negative of a change in the tax rate on work). This exercise illustrates, I think, the usefulness of properties of the kind that we have been establishing in the various cases examined. (3.16) taken by itself looks hard to interpret; with the aid of the adding-up properties, however, it can be reduced to (3.18), which is easy to interpret and perhaps even to measure.

The general expression for \( \Delta W \), for a fixed capital stock and for a fixed amount of labor-plus-leisure, is

\[
\Delta W = \frac{1}{2} \sum_{j=1}^{n+1} \sum_{j=1}^{n+1} M_{ij} E_i E_j + \frac{1}{2} \sum_{j=1}^{n} N_{ij} E_j + \frac{1}{2} \sum_{j=1}^{n} G_{ij} B_i B_j + \frac{1}{2} \sum_{j=1}^{n+1} H_{ij} B_i E_j.
\]

Its properties are basically the same as those of (3.15), modified only to take account of the fact that labor has \( n + 1 \) activities available to it while capital has only \( n \). Thus, in the interaction terms we have

\[
\sum_{j=1}^{n+1} N_{ij} = 0; \quad \sum_{j=1}^{n} H_{ij} B_i = 0.
\]

Hence we have

Case E: Where capital is in fixed supply to market activities, but labor is in fixed supply only to market-plus-nonmarket activities, and where taxes are considered which strike labor and capital differentially in different activities, (3.19) measures the change in welfare stemming from any set of such taxes. Neutral taxes in this case are taxes striking each unit of capital (or each dollar of net return from capital) equally, and taxes striking each hour of a worker's day equally. This last set of taxes could equivalently be called head taxes, but, as was shown above, convenient results can be obtained using properties derived from the neutrality of an equal tax per hour.

The formulation of case E is quite versatile. It can deal with proportional income taxation (equal percentage taxes on the income from labor and capital), and can recognize the nonneutrality of ordinary income taxation as regards the choice between labor and leisure. It can also cope with progressive income taxation, simply by using the effective marginal rate of tax to apply to income from labor and capital (here one has to assume that each individual's supply of capital is constant). It can cope with indirect taxes on intermediate as well as final products, provided that one is prepared to make the assumption that materials inputs bear fixed relationships to final products. And, most important of all, it can cope with property and corporation income taxes, which have widely differing burdens on the income from capital in different industries. Finally, it is possible to deal with situations in which all the above-mentioned taxes are simultaneously present, amalgamating those taxes (including allocations of excise tax receipts) falling on income from capital in each activity, and those falling on income from labor.

III

This section consists of three "appendix notes" to the earlier analysis. The first (A) reduces the expressions derived in section II to a common simplified form. The second (B) discusses how distortions other than taxes can be
measured, they will enable us to estimate the changes in welfare associated with any arbitrary combination of taxes.

In practice one cannot expect to measure all the relevant reaction coefficients, but one can place reasonable bounds on their orders of magnitude and thus get estimates of the order of magnitude of the welfare costs of a given set of taxes, or of particular changes in the existing tax structure. In dealing with practical problems, the presumptive dominance of the diagonal elements in the matrices of reaction coefficients can be put to good use. Consider, for example, the case of a tax of $T_1$ on $X_1$, in case A or B of section II. If there are no other taxes present in the system, the change in welfare associated with this tax will be $\Delta W = 1/2 \, R_{11} \, T_1^2$. If there are other taxes already present in the system, the effect on welfare of adding a tax of $T_1$ on $X_1$ will be

$$\frac{\partial W}{\partial T_1} = \frac{1}{2} R_{11} T_1^2 + \sum_{i=2}^{n} R_{i1} T_1 T_{i1},$$

(3.20)

Similarly, (3.20)' can be rewritten as

$$\frac{\partial W}{\partial T_1} = \frac{1}{2} \epsilon_1 T_1^2 + \sum_{i=2}^{n} \epsilon_i \epsilon_{i1} T_{i1} T_i.$$

(3.20)'

Since

$$\sum_{i=1}^{n} \epsilon_i R_{i1} = -\epsilon_1 R_{11},$$

(3.20) can be rewritten as

$$\frac{\partial W}{\partial T_1} = \frac{1}{2} \epsilon_1 T_1^2 + \sum_{i=2}^{n} \epsilon_i (R_{i1} - \epsilon_i R_{11}) T_i.$$

(3.20)'

Thus, $T_1$ has to be compared with a weighted average of the tax rates on other commodities. Even though we cannot measure the $R_{i1}$, so as to know the precise weights to apply, in many cases it is possible to set reasonable limits within which the true weighting pattern is likely to lie. We are likely to have a good idea of which, if any, of goods $X_k$ to $X_n$ are very close substitutes or complements to good $X_1$. After making allowance for the plausible degree of substitution or complementarity here, we are not likely to go far wrong if we assume that the remaining commodities are remote, "general" substitutes for $X_1$. Thus the procedure would be first to estimate $-\epsilon_1 R_{11}$; then to estimate $\epsilon_2 R_{21}$ and $\epsilon_3 R_{31}$, say, if goods 2 and 3 were particularly close substitutes or complements to good one; and finally to distribute the remaining total weights ($-\epsilon_1 R_{11} - \epsilon_2 R_{21} - \epsilon_3 R_{31}$) to commodities $X_4$ to $X_n$, say, in proportion to their relative importance in the national income. Obviously this procedure is not exact, but it is unlikely to lead to a result that is of an erroneous order of magnitude.

B. We now attempt to take account of distortions other than taxes. These can be treated as "autonomous" taxes or subsidies. If a monopoly is present in industry $i$, which prices its products at 20 percent above marginal cost, it is as if a 20 percent tax existed on the product of industry $i$, or, perhaps, a 40 percent tax on the value added by labor and by capital in industry $i$. If activity $j$ has
Obviously, however, this answer, though perhaps adequate for many practical applications, really begs the fundamental question. As I see it, there are three main roads to a solution.

1. One could attempt to extend the “models” of section II to many time periods, building in all of the relevant dynamics. This, I think, would be scientifically the most satisfying approach to take. However, I am afraid that this approach is likely to complicate the analysis to the point where it will be hard to apply it to real-world problems. Nonetheless, I feel that this is a line worth pursuing.

2. One could attempt to separate the “dynamic” costs of alternative tax set-ups. Suppose that changing from tax vector $T$ to tax vector $T'$ leads to a change in the rate of saving from $s$ to $s'$. We could measure the change in welfare due to the change in taxes first on the assumption that the rate of saving was unaffected, and then attempt to measure the additional cost or benefit associated with the change in the rate of saving. This approach has a particular appeal because, given the assumption that the net return of return to capital is equalized in all uses, it is reasonable to assume the rate of saving depends only on the level of real income and the net rate of return.

One can go quite some distance with this approach without greatly complicating the analysis. The present value to the saver of a dollar of saving at the margin is $\$1.00 = p(1 - t)/r$, where $p$ is the social rate of marginal net productivity of capital and $r$ (which at least in uncomplicated situations should equal $p(1 - t)$) is the after-tax rate of discount which the individual uses to obtain present values, and $t$ is the expected future rate of tax on income from saving. The present value of the social yield of capital is simply $p/r$, so that a dollar’s worth of savings should have a social value of $\$1.00/(1 - t)$. The change in welfare due to the difference in this year’s savings stemming from a tax rate of $t$ rather than a tax rate of zero would then be $1/2 t \Delta s/(1 - t)$, where $\Delta s$ is the tax-induced change in the amount of this year’s savings. If we call this expression $\Delta_A W$, and expression (3.19), say, $\Delta_A W$, we can express $\Delta W$ as $\Delta_A W + \Delta_x W$. $\Delta_x W$ expresses the cost this year of misallocating the resources that would have been this year if the rate of savings were unaffected by tax changes. $\Delta_A W$ measures the present value of the future benefit foregone because the economy — for tax reasons — did not save “enough” this year. One could correspondingly estimate $\Delta W$ stemming from a particular tax structure for a series of future years, and estimate the present value of the future stream of welfare costs associated with that tax structure.

The principal difficulty with approach number 2 is, I believe, that it requires the assumption that $p$ will remain constant in the future. The approach could of course be modified so as to impose a particular nonconstant time-path for $p$ in the future, but the basic difficulty remains that the model does not itself tell us what that time-path should be. As a practical matter, however, I believe that changes in the marginal net productivity of capital are likely to be sufficiently slow so that the assumption of constancy will not introduce serious errors in the estimation of $\Delta W$. 

We now turn to a problem which was consciously avoided in section II. Here we maintained the assumption that the capital stock was given. Now we must investigate the possibilities of eliminating this restrictive assumption.

In the first place, we can recognize that, for the analysis of section II, we did not need to assume that the capital stock remains fixed through time. Both population and capital stock can change through time, and the analysis of section II can be modified to take account of these changes, so long as the changes (in population and in capital stock) are not dependent on tax rates and other distortions. The difficulties appear when we try to allow for the effects of changes in tax rates, etc., on the level of capital stock (and/or population).

Particularly since I have no really satisfactory solution to the problem posed, I am inclined to defend the assumption that the level of capital stock is reasonably independent of tax rate changes (at least of the sorts of tax rate changes that we have observed in the past). Here I rely on the secular constancy of the rate of net saving in the United States, in the face of substantial swings in the rate of return and in the face of significant alterations in the tax structure. I would not expect, given this historical experience, that the neglect of an effect of taxation upon savings would introduce large errors into the measures derived in section II.
3. One could attempt to incorporate tax-induced changes in capital stock directly into the analysis. This approach requires two changes in the analysis of cases D and E of section II. First the assumptions that

$$\sum G_{ij} = 0 \quad \text{and} \quad \sum H_{ij} = 0$$

must be abandoned; and second, we must eliminate the assumption of the neutrality of any tax striking equally the income from capital in all uses. In effect this means that the only neutral tax treatment of the income from capital would be not to tax it at all. These two adjustments could easily be incorporated into the framework developed in section II. One additional step could also be necessary. Since the savings-effects of a tax change are likely to go on indefinitely, one would have to decide on the specific time period over which one was measuring the effect of tax changes on the capital stock. This could enable one in principle to deal with specific values for

$$\sum G_{ij} \quad \text{and} \quad \sum H_{ij},$$

whereas otherwise these values could be almost anything, depending on the time period over which the reactions were being measured. This last requirement — of measurement over a specific time period — is to my mind the most serious disadvantage of approach number 3.

I shall not go into more detail here on the possible merits and disadvantages of the three approaches to the savings problem that I have suggested. This problem is, as I have indicated, the most serious “open end” in the analysis of section II, and I hope that further work in the field, following one or more of the approaches outlined above, will help close this important gap.

REFERENCES

6. ______. “The Economic Theory of Customs Unions,” in Money,

Though I find the nontaxation of income from capital repugnant as a policy prescription, there is no doubt that even proportional income taxation is nonneutral in respect of the decision to save. The social yield of saving is the gross of tax return to capital, while the private yield net of tax.