n, forming a function $I = f(\rho)$ where $f'(\rho) < 0$, we could state the con-
tion of investment to economic growth as $I\rho_0 + \int_{\rho_0}^{\infty} f(\rho)\, d\rho$, which would
ed the amount assigned to investment by the expression $I\rho_{t-1}$ if we set
ρ_{t-1}. The difference would be that inframarginal investments would then
signed a greater contribution to economic progress. But, when we are talking
it increasing the growth rate by adding to investment, we are not concerned
inframarginal investments, but about those beyond what would otherwise
ie cut-off rate of return. There is nothing inconsistent between the following
e statements:

Each year some investments are undertaken which promise to yield, at prevailing prices, very high rates of return.
Each year significant amounts of investment are undertaken which promise yields in the neighborhood of the observed past rate of return.
Additional investments above and beyond those actually undertaken would likely produce rates of return which are at best equal to the observed past average rate of return.

ccording to the point of view just expressed, "embodiment" of technical
nges takes place principally in inframarginal investments. This sort of
nomenon could easily produce the results observed by Berglas, with the
rginal effect of investment being approximated by the observed past average
of return, but with a substantial time trend in the ratio of output to total
ut. It is also consistent with the observation that, across industries or time
iods, there is relatively little relationship between the rate of increase in
ital stock and the movements in total factor productivity.

This is not the place for a very detailed discussion of these points, so I will
ve the matter here. I hope that I have been able to provide some support
my view that we cannot expect great changes in the growth rate from plaus-
e changes in the mix of direct and indirect taxation.

feel impelled, however, to point out that in our recent preoccupation with
rate of growth, we, as a profession, may have lost sight of some old and
damental truths. We should distinguish between the rate of growth of
ome, the level of income, and the level of welfare as possible goals of economic
licy. Concentration on the rate of growth as an objective can lead one to
nimize the value of having, say, x percent more income each year as a result
a policy change — simply because income would not grow any faster except
ring a transitional period. On the other hand, concentration on the level of
l income (as conventionally measured) can lead one to neglect the costs of
inging about changes in that level. If, by tax changes, we increase labor and
duce leisure, or if we increase saving and reduce consumption, we should, I
ink, not just look at the pluses and neglect the minuses. When we try to take
th pluses and minuses into account, we come to grips with the measurement
the effects of policy changes on welfare, which was the subject of the earlier
ctions of this paper.

III

The next chapter, which was presented at the annual meeting of the American Economic Association in December, 1963, and published the following year in *The American Economic Review*, is substantially more sophisticated than "Taxation, Resource Allocation and Welfare," but to a considerable extent this gain was bought (given space limitations) at the expense of the slower-paced, more didactic presentation of that earlier effort. To help compensate the cryptic nature of the exposition, I take this opportunity to indicate what I consider to be its main points.

In the first place, expressions (3.4) and (3.5), which measure in terms of coefficients R_{ij} the change in welfare induced by a set of distortions, though they look exactly like their counterparts in Chapter 2, which express welfare change in terms of coefficients S_{ij}, are really quite different. On the assumptions laid out in Chapter 2, the coefficients S_{ij} could be interpreted as simply compensated demand coefficients, and the whole analysis could be handled in straightforward demand-theory language. The R_{ij} are no such simple animals — they are really reduced-form coefficients showing how the equilibrium levels of various activities $X_1, X_2, \ldots, X_n$ depend on the degrees of distortion $T_1, T_2, \ldots, T_n$ to which each of the activities is subject.

To explore a bit the nature of the R_{ij}, expressed in terms of more familiar supply and demand parameters, consider first the simplest of textbook cases:

$$\text{(3A)} \qquad X_1 = \underbrace{a_1 + b_1(P_1 + T_1)}_{\text{Demand}} = \underbrace{e_1 + f_1 P_1}_{\text{Supply}}.$$

The solution for P_1 is

$$\text{(3B)} \qquad P_1 = \frac{a_1 - e_1 + b_1 T_1}{f_1 - b_1}, \quad \text{and that for } X_1 \text{ is}$$

$$\text{(3C)} \qquad X_1 = \frac{f_1 a_1 - b_1 e_1 + f_1 b_1 T_1}{f_1 - b_1}.$$

The coefficient R_{11} is therefore given by

$$R_{11} = \frac{f_1 b_1}{f_1 - b_1}, \tag{3D}$$

which perhaps looks straightforward enough. Now, however, let us try just a slightly more complicated setup with demand and supply for two goods.

$$\begin{cases} X_1 = a_1 + b_1(P_1 + T_1) + C_1(P_2 + T_2) \\ \quad = e_1 + f_1 P_1 + g_1 P_2 \\ X_2 = a_2 + b_2(P_2 + T_2) + C_2(P_1 + T_1) \\ \quad = e_2 + f_2 P_2 + g_2 P_1 \end{cases} \tag{3E}$$

Solving this, we find

$$\begin{cases} P_1 = \dfrac{[a_1 - e_1 + b_1 T_1 + c_1 T_2](f_2 - b_2) - [a_2 - e_2 + b_2 T_2 + c_2 T_1](g_1 - c_1)}{\Delta} \\ P_2 = \dfrac{[a_2 - e_2 + b_2 T_2 + c_2 T_1](f_1 - b_1) - [a_1 - e_1 + b_1 T_1 + c_1 T_2](g_2 - c_2)}{\Delta} \end{cases}, \tag{3F}$$

where $\Delta = (f_1 - b_1)(f_2 - b_2) - (g_2 - c_2)(g_1 - c_1)$. Using these equations we can solve explicitly for, say, G_{11}

$$G_{11} = \frac{f_1 b_1 (f_2 - b_2) - f_1 c_2 (g_1 - c_1) + g_1 c_2 (f_1 - b_1) - g_1 b_1 (g_2 - c_2)}{(f_1 - b_1)(f_2 - b_2) - (g_2 - c_2)(g_1 - c_1)}. \tag{3G}$$

Now (3.7) can already be described as a reasonably messy expression, but imagine what G_{11} would be like when the system had three or four or five interdependent demand and supply equations, let alone an arbitrarily large N!

How lucky we are, then, to find that the G_{ij} behave just like Slutsky-Hicks compensated demand coefficients, so that we can transfer all our accumulated understanding and intuition about demand phenomena to this much more complicated world. Or, to put it another way, is it not nice to know that if one works out a problem in the comparatively simple constant-cost framework of Chapter 2, the solution thus found will carry over completely to a world of interdependent demands and supplies for all commodities, simply by replacing the demand-defined S_{ij}'s by the reduced-form R_{ij}'s, which have the same simple and useful properties in spite of the massive amount of clockwork that each of them represents? And finally, even more so, is it not useful to know that similar well-behavedness prevails among the four sets of reduced-form coefficients which show how the amount of capital or labor in activity i responds to the degree of distortion applying to the use of labor or to that of capital in activity j? In short, this way of being able to set out what would otherwise be enormously complex problems in simple, potentially manageable terms is to my mind probably the most important result of the "Measurement of Waste."

But there is at least one other range of issues that might merit comment. It is connected (a) with the consolidation of a whole set of taxes into an equivalent set striking with different rates at the incomes from labor and capital in different activities, and (b) with the question of when is a tax or tax system neutral, and when is it not. With respect to consolidation, I point out that it is not likely to be always possible to find a set of factor taxes on the incomes from labor and capital in an activity that will be equivalent to a given tax on the product of that activity. Before one can even approach this equivalence, two assumptions must be made — that the relationship of material inputs to output is fixed, at least in the sense that it is not influenced by which of the two allegedly "equivalent" tax packages is chosen, and that the pattern of economic depreciation of the assets of an activity is similarly uninfluenced.

Table 3A starts in column (1) with an ordinary excise tax of 20 percent on a given product. In column (2) the consequences of a 20 percent tax on all components of costs are shown. Taxes paid at any given stage are shown in parentheses. The equivalence between tax patterns (1) and (2) is obvious; taxing all components at a given rate (col. 2), or taxing the sum of the components at the same rate (col. 1) must produce the same result. As we pass from column (2) to column (3), however, the assumption of fixed proportions between material inputs and final product is crucial. If substitution were possible between materials and final products, the shift from (2) to (3) would induce a substitution toward a more material-intensive method of production, with less use of labor and capital, but with fixed proportions the material-intensiveness of production cannot change. Under this assumption, putting the tax of 20 on gross value added (col. 3) rather than on all inputs (col. 2) is like imposing a tax of \$1 per shoe on all left shoes in place of a tax of \$1 per pair. As long as shoes are always sold in pairs, there can be no difference whatsoever between the two alternatives.

Now suppose that the tax setup were that indicated in column (4) — a 25 percent tax on wage payments together

TABLE 3A

Components of cost and price	*20% tax on final product* (1)	*20% tax on all inputs* (2)	*25% tax on gross value added* (3)	*25% tax on wages; 41 2/3% tax on capital's net earnings* (4)	*33 1/3% tax on net value added* (5)
Market price of final product	120	120	120	120	120
Cost of final product	100(+20)[a]	120	120	120	120
Depreciation	20	20(+4)[a]	20(+5)[a]	20	20
Capital	30	30(+6)	30(+7.5)	30(+12.5)[a]	30(+10)[a]
Labor	30	30(+6)	30(+7.5)	30(+7.5)	30(+10)
Materials	20	20(+4)	20	20	20

[a] Figures in parentheses in each case represent the tax to be added to the cost item in question.

with a 41 2/3 percent tax on the net earnings of capital. This, too, would be equivalent to the other three systems, so long as the shift from system (3) to system (4) did not alter choices in regard to length of asset life (i.e., so long as the depreciation component of costs remained invariant under the tax shift). Finally, consider the tax setup shown in column (5) — a tax raising the same amount, but on the basis of equal taxation of net value added by capital and labor. Clearly, the systems given by columns (4) and (5) would be equivalent only if there were no possibilities of substitution between labor and capital, as the relative weight of taxation on these two factors is altered as one moves from one tax scheme to the other.

The above establishes that a tax on gross value added (col. 4) is different in its effects from a tax on net value added (col. 5). What now can be said about the circumstances under which *general* taxes of these types will be neutral with respect to resource allocation? Obviously neutrality will not exist if the labor-leisure or the consumption-savings choice is affected, so this analysis will be carried out on the assumption of fixed supplies of labor and capital, or in the dynamic case on the assumption that the time paths of these supplies are independently determined and therefore unaffected by the tax changes being analyzed. Once this assumption is made, it is self-evident that a uniform tax on all labor earnings will be totally neutral in an allocative sense since the tax will affect only the economic rent earned by labor and not the wage paid by users of labor services. Similarly, a uniform tax taking a certain amount per year per dollar of capital, or (what amounts to the same thing when the net-of-depreciation rate of return to capital is equalized across uses) a uniform tax on the net-of-depreciation yield from all capital assets will reduce the net rate of return received by owners of capital but will not affect the net-of-depreciation cost of capital to its users.

Since a uniform tax on gross value added is definitionally equivalent to a uniform tax on all labor earnings plus a uniform tax on all net earnings of capital plus a uniform tax on all depreciation of capital assets, the issue of whether the gross-value-added tax is neutral boils down to whether a uniform tax on capital consumption (depreciation) would be neutral. The user cost of a capital asset can be represented by $(r + \delta)V$, where r is the net-of-depreciation rate of return, δ is the annual percentage rate of depreciation, and V is the value of the asset. If r, which we presume will tend to be equalized across uses, is 6 percent, the user cost of a building with a depreciation rate of 2 percent per year will be 8 percent of its value, while that of a machine depreciating at 10 percent per year will be 16 percent of its value. Thus taxing depreciation on all forms of capital at, say, 20 percent will raise the user cost of buildings from 8 to 8.4 percent of their value, but will raise the user cost of machines from 16 to 18 percent of theirs. The effect on user cost is obviously not uniform and the depreciation tax is therefore distorting, except in the totally theoretical extreme case where δ is the same for all types of capital assets. By the same token, a uniform tax on gross value added (or its equivalent, a uniform tax on the market value of all final products) will be nonneutral, even with fixed supplies of capital and labor, except in the extreme case just cited.

Chapter 3

The Measurement of Waste

I

The subject of this paper might be called "The Economics of the *n*th Best." This would distinguish the approach taken here from that taken by Lipsey and Lancaster in their fine article, "The General Theory of Second Best" [7], as well as from the conventional concern of economic analysis with the characteristics of fully optimal situations. To state the differences briefly, the conventional approach is concerned with how to get to a Pareto-optimal position, the Lipsey-Lancaster approach is concerned with how to make the best of a bad situation (i.e., how to get to a position which is optimal subject to one or more constraints which themselves violate the conditions of a full optimum), while this paper is concerned with measuring the deadweight loss associated with the economy's being in any given nonoptimal position.

The measurement of deadweight losses is not new to economics by any means. It goes back at least as far as Dupuit; and more recently Hotelling [4], Hicks [3], Debreu [2], Meade [10], and H. Johnson [5, 6] have made important contributions. Nonetheless I feel that the profession as a whole has not given to the area the attention that I think it deserves. We do not live on the Pareto frontier, and we are not going to do so in the future. Yet policy decisions are constantly being made which can move us either toward or away from that frontier. What could be more relevant to a choice between policy A and policy B than a statement that policy A will move us toward the Pareto frontier in such a way as to gain for the economy as a whole, say, approximately $200 million per year, while policy B will produce a gain of, say, about $30 million per year? What could be more useful to us as a guide to priorities in tax reform than the knowledge that the deadweight losses stemming from the tax loopholes (percentage depletion and capital gains) open to explorers for oil and gas are probably greater in total magnitude than the deadweight losses associated

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h all the other inefficiencies induced by the corporation income tax? What ld be more tantalizing than the possibility (which I believe to be a real one) t the U.S. tariff, whose indirect effect is to restrict the equilibrium value of S. exports, produces by this route a gain for the U.S. from a partial exploi-ion of U.S. monopoly power in world markets which nearly offsets (or per-ps fully or more than fully offsets) the efficiency-losses produced by tariff-luced substitution of more expensive domestic products for cheaper imports? ese and similar questions seem to me so interesting, so relevant, so central our understanding of the economy we live in, that I find it hard to explain y the measurement of deadweight losses should be the province of only a ndful of economists rather than at least the occasional hobby of a much ger group. Let me simply suggest four possible reasons for the apparent popularity of the loss-measurement game:

1. Even the simplest attempts to measure the deadweight loss (or, as I efer to call it, the welfare cost) associated with particular distortions involve e use of numerical values for certain key parameters (elasticities of demand, substitution, etc.), which may be impossible to obtain at all, or which may be timated but with substantial error. Workers in this field must be ready to ntent themselves with results that may be wrong by a factor of 2 or 3 in many ses. But, on the other hand, it is a field in which our professional judgment so poorly developed that the pinning down of an answer to within a factor of can be very helpful. Be that as it may, one cannot expect the field to attract lleagues who prefer their results to be meticulously exact.

2. While it is relatively easy to measure the welfare costs of a particular stortion when one assumes other distortions to be absent, it is much more fficult to carry through the measurement in a way which takes account of the resence of other distortions. One of the profound lessons taught us by earlier orkers in this field (Hotelling [4], Viner [11], Lipsey and Lancaster [7], orlett and Hague [1], Little [8], and others) is that an action (i.e., imposing tax of T_1 per unit on good X_1) which would take us away from a Pareto ptimum if we were starting from that position can actually bring us toward ich an optimum if we start from an initially distorted situation. Crude easures can thus mislead us, while correct measures are hard to come by.

3. Many people find it difficult to isolate the measurement of efficiency sses due to particular distortions from the changes in the distribution of in-ome that they conceive would ensue if the distortions were actually removed. f these, some are undoubtedly not willing to make the kind of assumptions ey have to make in order to compare the changes in welfare of different dividuals or groups.

4. Consumer surplus, in spite of its successive rehabilitations, is still looked pon with suspicion by many economists. In spite of the fact that it is possible formulate measures of welfare cost which do not directly involve the use of he consumer surplus concept, the most convenient and most frequently cited easures of welfare cost do involve this concept. Thus, I venture to guess, nother group of potential workers (or at least tasters) in the vineyard do not enture to enter.

II

The main purpose of this paper is to explore a variety of possible ways of formulating measures of deadweight losses. All the ways considered are members of a single family. This section begins by expounding a widely accepted approach to the problem and then proceeds to extend this approach to what I believe are new areas.

Let us begin by assuming that the only distortions present in the economy are taxes. Monopoly elements, externalities, and other market imperfections will be introduced at a later stage. We shall assume that the economy will seek and find a unique full employment equilibrium once its basic resource endowments, the distribution of income, the quantities of goods purchased by the government, and the set of distortions (taxes) are known. Letting X represent the vector of equilibrium quantities, D be a vector representing the proportion of total income received by each spending unit, G be a vector representing the quantities of the different goods and services purchased by government, and T be a vector representing the tax levied per unit of the different goods and services produced in the economy, we have $X = f(D, G, T)$.

Now to isolate the efficiency effects of distortions, we must hold D and G constant. Thus, with respect to D, we conceive of the possibility of keeping the percentage share of each spending unit in the total national income constant by means of neutral taxes and transfers. With respect to G, we assume that, in any pair of situations being compared, the government buys the same bundle of goods and services. Even though the comparison of two actual situations might be between $X = f(D, G, T)$ and $X' = f(D', G', T')$, we split up the move from X to X' into a minimum of two steps. The first step is from $X = f(D, G, T)$ to $X^* = f(D, G, T')$. This step isolates the efficiency aspects of the change. The move from X^* to X' entails no change in the distortions affecting the economy, and involves only shifts in the distribution of income and in the level of government expenditures. To the extent that the tax yield produced by the vector T is insufficient or more than sufficient to finance the expenditure vector G, we assume that neutral taxes or transfers will be called upon to make up the difference. (Should fiscal policy measures be necessary to provide full employment, neutral taxes and transfers would be the instruments used to bring the total tax take to the required level. Government expenditures, on our assumptions, would be held fixed.)

The above assumptions have the effect of setting first-order income effects (whether caused by redistributions or changes in the size of government purchases) to one side so as to isolate the efficiency effects of alternative tax patterns. They put us in a world of substitution effects and of relative prices. When dealing with relative price phenomena, it is customary to treat a single product as the numeraire. This procedure is, however, not essential. One could normalize by holding any desired index of prices constant, or in a variety of different ways. For our purposes, it is convenient to normalize by holding the money national income constant as among all possible situations being compared. We could alternatively hold constant money net national product, gross national oduct, gross national product less excise taxes, or any of a variety of other ssible aggregates. But, as will be seen, holding money national income nstant is exceedingly convenient for the problems with which we shall deal.

Let us consider first a case that has been frequently dealt with in the literature. sume that the production function of the economy is linear and that only e factor of production, in fixed supply, is involved in production.

The fact that the only distortions present in our system are per unit excise xes assures us that when the vector $T = 0$, we are at a Pareto optimum. (In is case the government is raising all its revenue by taxes that are by definition utral; e.g., head taxes.) Thus if we set up an index of welfare W as a function the tax vector T, we have that $W_{max} = W(0)$. We can take money national come, $\bar{Y}$, as the measure of $W(0)$. We can, therefore, indicate the level of elfare associated with any tax vector T by $\bar{Y}$ plus a deviation ΔW, depending T and expressed in the same units as Y. The relevant expression for ΔW, a wide class of cases, is

$$\Delta W = \sum_{i=1}^{n} \int_0^{T_i} \sum_{j \leq i} T_j \frac{\partial X_j}{\partial T_i}\, dT_i. \qquad (3.1)$$

wo basic rules underlie this expression.

First, if as a result of an increment dT_i in the unit tax on X_i, there is an crement or decrement dX_j in the equilibrium quantity of a good X_j in the arket for which no distortion exists, the change dX_j carries with it no direct conribution to the measure of ΔW. For each successive minute increment of X_j, emand price is equal to marginal cost, and the gain to demanders of having ore of X_j is just offset by the costs of producing the extra amount.

Second, if as a result of an increment dT_i in the unit tax on X_i there is an crement dX_j in the equilibrium quantity of good X_j, in the market for which distortion T_j already exists, there is a social gain associated with the change X_j equal to $T_j\, dX_j$. Here demand price exceeds marginal cost, on each unit crement of X_j, by the amount T_j. Likewise if dX_j is negative, there is a social oss involved equal in magnitude to $T_j\, dX_j$.[1]

Obviously, the second rule given above contains the first, but I have set them out as two rules to emphasize the neutrality of changes taking place in undistorted sectors. Once this fact is appreciated, the rest of the road is easy.

Let me emphasize at this point that up to now there is nothing new in what has been said. Expression (3.1), and the rules behind it, say only that

$$\frac{\partial W}{\partial T_i} = \sum_j T_j \frac{\partial X_j}{\partial T_i}. \qquad (3.2)$$

[1] Another way of looking at this problem is to consider that consumers, in transferring their demand to X_j are indifferent between what they get and what they give up for each marginal unit of purchasing power transferred; and that suppliers of factor services are likewise, for each marginal unit of services transferred, on the borderline of indifference. But if X_j goes up by dX_j, the government will obtain an increase in tax receipts of $T_j\, dX_j$, which (under our assumptions) will permit either a corresponding reduction in associated lump-sum taxes or a corresponding increase in lump-sum transfers. In short, "the people" gain to the tune of $T_j\, dX_j$.

This expression pops up in one form or another all through the literature on the measurement of welfare costs, the economics of second best, the theory of customs unions, etc. It appears, or can be derived from what appears, in Corlett and Hague [1], Hotelling [4], H. Johnson [5, 6], Meade [10], and Lipsey and Lancaster [7], among others.

Let us now linearize expression (3.1) by setting

$$\frac{\partial X_j}{\partial T_i} = R_{ji}.$$

With this substitution, (3.1) evaluates at

$$\Delta W = \frac{1}{2}\sum_{i=1}^{n} R_{ii} T_i^2 + \sum_{i}^{n} \sum_{j<1} R_{ji} T_j T_i. \qquad (3.3)$$

Expression (3.3) can be simplified, however, using the integrability condition

$$\frac{\partial X_i}{\partial T_j} = \frac{\partial X_j}{\partial T_i},$$

which translates in the linearized form into $R_{ij} = R_{ji}$. In economic terms, this same condition derives from the fact that the welfare cost of a set of taxes should not, in a comparative static framework such as this, depend on the order in which those taxes are conceived to be imposed. Thus if we impose T_1 first and follow it by T_2, we have $\Delta W = 1/2\, R_{11} T_1^2 + 1/2\, R_{22} T_2^2 + R_{12} T_1 T_2$. If on the other hand we impose T_2 first and follow it by T_1, we have $\Delta W = 1/2\, R_{22} T_2^2 + 1/2\, R_{11} T_1^2 + R_{21} T_2 T_1$. Hence if the linearized expression (3.3) is to be invariant with respect to order of imposition of taxes, R_{12} must equal R_{21}, and in general R_{ij} must equal R_{ji}. This enables (3.3) to be simplified to

$$\Delta W = \frac{1}{2} \sum_i \sum_j R_{ij} T_i T_j. \qquad (3.4)$$

For each $R_{ji} (j < i)$ appearing in (3.3), we simply substitute $1/2\, R_{ji} + 1/2\, R_{ij}$, to obtain (3.4).

A further condition on the R_{ij} can be established by noting that a set of taxes with some $T_i \neq 0$ can at best produce an equal level of welfare as an undistorted situation. This yields

$$\Delta W = \frac{1}{2} \sum_i \sum_j R_{ij} T_i T_j \leq 0 \quad \text{for all possible values of } T_i,\ T_j. \qquad (3.5)$$

As a special case of (3.5) we have

$$R_{ii} \leq 0 \quad \text{for all } i. \qquad (3.6)$$

This is obtained when $T_i \neq 0$, while $T_j = 0$ for all $j \neq i$.

We are by now quite close to establishing the Hicksian substitution conditions by the back door, so to speak. What we need to finish the job is the adding-up property. Suppose it to be true that a proportional tax at the rate t on all the X_i would indeed be a neutral tax. We can define $T_i = t_i c_i$, where $c_i =$ marginal it, and t_i = percentage rate of tax on X_i, to obtain

7) $$\Delta W = \frac{1}{2} \sum_i \sum_j c_i c_j R_{ij} t_i t_j.$$

an equal percentage tax on all commodities is neutral, we have

8) $$\sum_i \sum_j c_i c_j R_{ij} = 0.$$

it we actually have much more than this. If a proportional tax at the rate t truly neutral, then, given our assumptions about the constancy of income stribution and of government purchases, it simply substitutes for the head x that would have to exist if all the T_i were zero. It must produce the same uilibrium quantity for each and every commodity. Thus we have that

.10) $$\frac{\partial X_i}{\partial t} = \sum_j c_j R_{ij} = 0 \qquad \text{for all } i.$$

his is the counterpart of the Hicksian adding-up property.

However, a tax at the rate t on all X_i will be neutral only in certain cases.

Case A: Suppose that, as was assumed above, the production frontier of the conomy is linear —

$$\sum_i c_i X_i = \text{a constant.}$$

his means that total production is in inelastic supply, and therefore that a tax hich strikes the value of all production at a constant rate will be neutral. In nis case all the X_i must be final products; the R_{ij} here turn out to be precisely ne Hicksian substitution terms.

Case B: Suppose that all the X_i are final products, and that the production ontier of the economy is convex from above. Suppose, moreover, that all basic ictors of production are fixed in total supply. So long as a tax at the rate t on nal products is in effect a tax at a fixed rate on the net earnings of all factors of roduction, it will be neutral, and condition (3.10) will hold. In this case, the $_{ij}$, while obeying the properties of the Hicksian substitution terms, are actually uite different from them. Here the R_{ij} are really the "reduced form" coefficients howing how the equilibrium value of X_i (with supply and demand equal for ll commodities) depends on T_j.

Case B presents no problem when capital is not among the basic factors of roduction, or when the relation between gross and net earnings of capital is he same in all uses. However, when capital is among the basic factors and vhen the relationship between gross and net earnings does (because of different lepreciation patterns) differ among uses, then an equal tax on all final products vill not be neutral, even though capital and other factors of production are ixed in total supply. This is because increases in the rate of proportional tax, , will create incentives which would relatively favor the longer-lived applications of capital. An equal tax on value added in all industries, however, would be neutral in these circumstances, because we assume the net rate of return on capital to be equalized among all uses of capital. (This, of course, assumes that

the stock of capital and the supplies of other basic factors of production are fixed.)

Problems quite similar to those presented by different depreciation patterns in different applications of capital arise when the possibility of taxing intermediate products is introduced. As McKenzie [9] has forcefully pointed out, an equal percentage tax on all products will generally be nonneutral if any of the products in question are intermediate or primary products not in fixed supply. A tax at an equal percentage rate on value added in every activity will, on the other hand, be neutral so long as the basic factors of production are in fixed supply.

Case C: When considering taxes on value added we let X_i represent the volume of final product of industry (or activity) i, v_i represent value added per unit of the product of activity i, and T_i represent the tax per unit of final product in industry i. (Although the tax is levied on value added, T_i is here expressed per unit of product.) Once again letting $R_{ij} = \partial X_i / \partial T_j$, we have (3.5) as the expression for ΔW. To reflect the neutrality of an equal percentage tax on value added everywhere, we require that the response of any X_i to such a tax be zero; i.e., that

$$\sum_j v_j R_{ij} t_j = 0 \quad \text{when} \quad t_j = t \qquad \text{for all } j.$$

Here v_j = value added per unit of the product X_j, t_j = percentage rate of tax on value added in industry j, $v_j t_j = T_j$. Hence we have

$$\sum_j v_j R_{ij} = 0; \quad R_{ij} = R_{ji}; \quad \sum_i \sum_j R_{ij} T_i T_j \leq 0 \text{ for all } T_i, T_j; \quad R_{ii} \leq 0 \text{ for all } i$$

as before.

Case C deals rather neatly with problems of differential depreciation and taxes on nonfinal products. However, case C assumes that indirect taxes are levied on value added, whereas most frequently in the real world they are levied on the final product.

Fortunately, it is possible to translate product taxes into value-added taxes, and still stay within the framework of case C so long as inputs other than labor and capital enter their respective products in fixed proportions. The reason for this is obvious. All the effects of a tax at the rate t_i on product i can be replicated by a tax at the same rate on all factor shares (including materials input) entering into the production of product i. These are simply two ways of imposing the same tax. Suppose that with a tax of 10 percent on all factor shares in the ith industry an equilibrium is reached in which materials inputs account for half the value of product and labor and capital the other half. So long as materials inputs must be used in fixed proportions per unit of product, a shift from a 10 percent tax on all factor shares in the ith industry to a 20 percent tax on value added in the ith industry would introduce no incentive to change the equilibrium reached with a 10 percent tax on all factor shares. Purchasers could pay the same price for the product; labor, capital, and materials sellers could get the same net reward; and the government could get the same tax take. Moreover, since the taxes on labor and capital shares would still be at equal rcentage rates, there would be no inducement for substitution between them. short, so long as materials are used in fixed proportions to output, we can nslate any given tax on output into a tax on value added that is equivalent all respects relevant for this analysis.

We now turn to a broader set of problems — all of which take into account e possibility of different rates of tax on the return to capital and to labor in y given activity. Consider first the set of possible taxes B_i per unit of capital activity i. The change in welfare associated with such taxes can be written, suming no other nonneutral taxes in the system, as

$$(3.11)\qquad \Delta W = \frac{1}{2}\sum_i\sum_j G_{ij}B_iB_j,^2$$

here

$$G_{ij} = \frac{\partial K_i}{\partial B_j},$$

nd K_i represents the number of units of capital employed in activity i. Corre-ondingly, if we consider the set of possible taxes E_i per unit of labor in activity and assume no other nonneutral taxes, we can write:

$$(3.12)\qquad \Delta W = \frac{1}{2}\sum_i\sum_j M_{ij}E_iE_j,$$

here

$$M_{ij} = \frac{\partial L_i}{\partial E_j},$$

nd L_i represents the number of units of labor employed in activity i. The G_{ij} nd the M_{ij} will obey the following properties:

$$G_{ij} = G_{ji};\qquad \sum_i\sum_j G_{ij}B_iB_j \leq 0 \text{ for all } B_i, B_j;\qquad G_{ii} \leq 0 \text{ for all } i$$

$$M_{ij} = M_{ji};\qquad \sum_i\sum_j M_{ij}E_iE_j \leq 0 \text{ for all } E_i, E_j;\qquad M_{ii} \leq 0 \text{ for all } i.$$

Ioreover, with fixed supplies of capital and labor we have

$$\sum_i G_{ij} = 0,\qquad \sum_i M_{ij} = 0.$$

When nonneutral taxes are levied only on capital in different activities, (3.11) neasures the cost of the distortions involved; when only labor is affected by onneutral taxes, (3.12) is the relevant measure. But when nonneutral taxes re levied on both labor and capital in different activities, the interaction

We could here have explicitly set out an equation corresponding to (3.1); i.e.,

$$\Delta W = \sum_{i=1}^{n}\int_0^{B_i}\sum_{j\leq i} B_j\frac{\partial X_j}{\partial B_i}\,dB_i,$$

inearized this expression as in (3.3), and then used the symmetry property to obtain (3.11) or its counterpart. These steps are not presented explicitly in this and the other cases treated in this section.

between them must be taken into account. Let us define

$$H_{ij} = \frac{\partial K_i}{\partial E_j}\quad\text{and}\quad N_{ij} = \frac{\partial N_i}{\partial B_j}.$$

Here symmetry exists between H_{ij} and N_{ji}. Suppose for example, we impose first a tax of B_1 and then one of E_2. We obtain $1/2\ G_{11}B_1^2 + M_{22}E_2^2 + H_{12}B_1E_2$ as our measure of ΔW. If we conceive of E_2 being imposed first, and then B_1, we obtain $1/2\ M_{22}E_2^2 + 1/2\ G_{11}B_1^2 + N_{21}E_2B_1$. If we think of imposing a set of taxes B_i first and then a set of taxes E_i, we have

$$(3.13)\qquad \Delta W = \frac{1}{2}\sum_i\sum_j G_{ij}B_iB_j + \frac{1}{2}\sum_i\sum_j M_{ij}E_iE_j + \sum_i\sum_j H_{ij}B_iE_j.$$

If we think of it the other way around, we have

$$(3.14)\qquad \Delta W = \frac{1}{2}\sum_i\sum_j M_{ij}E_iE_j + \frac{1}{2}\sum_i\sum_j G_{ij}B_iB_j + \sum_j\sum_i N_{ji}E_jB_i.$$

For a reason that will be apparent later, it is most convenient to write:

$$(3.15)\qquad \Delta W = \frac{1}{2}\sum_j\sum_i M_{ji}E_jE_i + \frac{1}{2}\sum_j\sum_i N_{ji}E_jB_i + \frac{1}{2}\sum_i\sum_j G_{ij}B_iB_j + \frac{1}{2}\sum_i\sum_j H_{ij}B_iE_j.$$

Now, when labor is in fixed supply, a tax on capital in industry i can only redistribute the existing amount of labor. Hence

$$\sum_j N_{ji} = 0.$$

Likewise, when capital is in fixed supply, a tax on labor in industry j can only redistribute the available capital, so that

$$\sum_i H_{ij} = 0.$$

The interaction terms disappear for neutral taxes because in this case $E_i = \bar{E}$ for all i, $B_i = \bar{B}$ for all i. (Since the wage is assumed to be equalized in all uses of labor and since the net rate of return is assumed to be equalized in all uses of capital, an equal tax per unit of labor is also an equal percentage tax on value added by labor in different activities, and likewise for capital.)

Thus we have:

Case D: When labor and capital are in fixed supply, expression (3.15) measures the change in welfare due to any pattern of taxes on labor and capital in different activities. The M_{ji} and the G_{ij} obey the Hicksian conditions, with the adding-up property in this case

$$\sum_j M_{ij} = 0 = \sum_i G_{ij}.$$

All terms vanish for taxes on labor that are equal in all uses together with taxes capital that are equal in all uses. The interaction terms can in general be itive or negative, but the whole expression (3.15) must always be ≤ 0. In case the coefficients reflect not only conditions of final demand and supply also conditions of factor substitution.

Ve now attempt to allow for the fact that the supply of labor in the market y itself be a function of the pattern of taxation. This question has been dealt h in the literature of second-best by Little, Corlett, and Hague, Lipsey and ncaster, and Meade, among others. The key to at least the last three of se treatments is the substitution of the assumption (!) that the number of urs in the year is fixed for the assumption that the number of man-hours ered in the market is fixed. We can do this simply by adding another activity labor — labeled "leisure" or "nonmarket activity." If there are n market ivities, we add an $n+1$st, and have

$$\sum_{j=1}^{n+1} L_j = L.$$

is does not change the form of equation (3.12) but it does alter the definition a neutral tax. Now an equal tax on all labor in market activities is not utral, because it neglects the $n+1$st activity. However, a tax that struck hours equally (including leisure hours) would be neutral. Hence we have

$$\sum_{j=1}^{n+1} M_{ji} = 0 = \sum_{i=1}^{n+1} M_{ij}.$$

measure the welfare cost of an equal tax of $\bar{E}$ on all activities except leisure take

$$\Delta W = \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} M_{ji}\bar{E}^2 \tag{3.16}$$

t

$$\sum_{i=1}^{n+1} M_{ji} = 0, \quad \text{so} \quad \sum_{i=1}^{n} M_{ji} = -M_{j,n+1}.$$

ence (3.16) reduces to

$$\Delta W = -\frac{1}{2}\sum_{j=1}^{n} M_{j,n+1}\bar{E}^2. \tag{3.17}$$

it

$$\sum_{j=1}^{n+1} M_{j,n+1} = 0, \quad \text{so that} \quad \sum_{j=1}^{n} M_{j,n+1} = -M_{n+1,n+1}.$$

hus (3.17) reduces to

$$\Delta W = \tfrac{1}{2} M_{n+1,n+1}\bar{E}^2, \tag{3.18}$$

here $M_{n+1,n+1}$ represents the responsiveness of leisure to a change in the tax te on leisure (or to the negative of a change in the tax rate on work). This ercise illustrates, I think, the usefulness of properties of the kind that we have een establishing in the various cases examined. (3.16) taken by itself looks

hard to interpret; with the aid of the adding-up properties, however, it can be reduced to (3.18), which is easy to interpret and perhaps even to measure.

The general expression for ΔW, for a fixed capital stock and for a fixed amount of labor-plus-leisure, is

$$\Delta W = \frac{1}{2}\sum_{j=1}^{n+1}\sum_{i=1}^{n+1} M_{ji}E_jE_i + \frac{1}{2}\sum_{j=1}^{n+1}\sum_{i=1}^{n} N_{ji}E_jB_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n} G_{ij}B_iB_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n+1} H_{ij}B_iE_j. \tag{3.19}$$

Its properties are basically the same as those of (3.15), modified only to take account of the fact that labor has $n+1$ activities available to it while capital has only n. Thus, in the interaction terms we have

$$\sum_{j=1}^{n+1} N_{ji} = 0; \qquad \sum_{i=1}^{n} H_{ij} = 0.$$

Hence we have

Case E: Where capital is in fixed supply to market activities, but labor is in fixed supply only to market-plus-nonmarket activities, and where taxes are considered which strike labor and capital differentially in different activities, (3.19) measures the change in welfare stemming from any set of such taxes. Neutral taxes in this case are taxes striking each unit of capital (or each dollar of net return from capital) equally, and taxes striking each hour of a worker's day equally. This last set of taxes could equivalently be called head taxes, but, as was shown above, convenient results can be obtained using properties derived from the neutrality of an equal tax per hour.

The formulation of case E is quite versatile. It can deal with proportional income taxation (equal percentage taxes on the income from labor and capital), and can recognize the nonneutrality of ordinary income taxation as regards the choice between labor and leisure. It can also cope with progressive income taxation, simply by using the effective marginal rate of tax to apply to income from labor and capital (here one has to assume that each individual's supply of capital is constant). It can cope with indirect taxes on intermediate as well as final products, provided that one is prepared to make the assumption that materials inputs bear fixed relationships to final products. And, most important of all, it can cope with property and corporation income taxes, which have widely differing burdens on the income from capital in different industries. Finally, it is possible to deal with situations in which all the above-mentioned taxes are simultaneously present, amalgamating those taxes (including allocations of excise tax receipts) falling on income from capital in each activity, and those falling on income from labor.

III

This section consists of three "appended notes" to the earlier analysis. The first (A) reduces the expressions derived in section II to a common simplified form. The second (B) discusses how distortions other than taxes can be corporated in the analysis. The third (C) discusses the problems that arise hen one eliminates the assumption of a constant capital stock.

The cases dealt with in the preceding section all have in common a simple operty. Since, in (3.4)

$$\sum_j R_{ij}T_j \text{ can be expressed as } \Delta X_i,$$

.4) itself can be rewritten:

$$\Delta W = \tfrac{1}{2}\sum_i T_i \Delta X_i. \qquad \text{.4')}$$

r cases A and B, T_i refers to taxes on final products only, and the X_i's are al products. For case C, (3.4′) might better be written

$$\Delta W = \tfrac{1}{2}\sum_i v_i t_i \Delta X_i, \qquad \text{.4'')}$$

here the X_i are now final or intermediate products, the v_i represent value lded per unit of product in activity i, and the t_i are percentage taxes on value lded in activity i. Since

$$\sum_j G_{ij}B_j \text{ can be expressed as } \Delta K_i,$$

.11) can be written:

$$\Delta W = \tfrac{1}{2}\sum_i B_i \Delta K_i. \qquad \text{.11')}$$

milarly, (3.12) can be written:

$$\Delta W = \tfrac{1}{2}\sum_i E_i \Delta L_i. \qquad \text{.12')}$$

n (3.15),

$$\sum_i M_{ji}E_i + \sum_i N_{ji}B$$

an be expressed as ΔL_j, while

$$\sum_j G_{ij}B_j + \sum_j H_{ij}E_j$$

an be expressed as ΔK_i, so that (3.15) can be written:

$$\Delta W = \tfrac{1}{2}\sum_j E_j \Delta L_j + \tfrac{1}{2}\sum_i B_i \Delta K_i. \qquad \text{3.15')}$$

3.15′) also serves as an alternative form for (3.19), with the index j going from to $n+1$ and the index i going from 1 to n. Thus all of the cases discussed ere are extensions of the "triangle-under-the-demand-curve" that emerges in extbook discussions of the excess burden of taxation. But I believe that for ctual work the simplified forms presented above are not as useful as those resented in section II of this paper, in which explicit account is taken of how he reaction coefficients R_{ij}, G_{ij}, H_{ij}, M_{ij} and N_{ij} enter into the determination f the result. One can conceive, at least hypothetically, of measuring these eaction coefficients by experimental movements in individual tax rates. Once

measured, they will enable us to estimate the changes in welfare associated with any arbitrary combination of taxes.

In practice one cannot expect to measure all the relevant reaction coefficients, but one can place reasonable bounds on their orders of magnitude and thus get estimates of the order of magnitude of the welfare costs of a given set of taxes, or of particular changes in the existing tax structure. In dealing with practical problems, the presumptive dominance of the diagonal elements in the matrices of reaction coefficients can be put to good use. Consider, for example, the case of a tax of T_1 on X_1, in case A or B of section II. If there are no other taxes present in the system, the change in welfare associated with this tax will be $\Delta W = 1/2\ R_{11}T_1^2$. If there are other taxes already present in the system, the effect on welfare of adding a tax of T_1 on X_1 will be

$$\text{(3.20)} \qquad \frac{\partial W}{\partial T_1}T_1 = \tfrac{1}{2}R_{11}T_1^2 + \sum_{i=2}^{n} R_{i1}T_iT_1, \quad \text{or}$$

$$\text{(3.20')} \qquad \frac{\partial W}{\partial T_1}T_1 = \tfrac{1}{2}c_1^2R_{11}t_1^2 + \sum_{i=2}^{n} c_1c_iR_{i1}t_it_1.$$

Since

$$\sum_{i=2}^{n} c_iR_{i1} = -c_1R_{11},$$

(3.20′) can be rewritten as

$$\text{(3.21)} \qquad \frac{\partial W}{\partial T_1}T_1 = \tfrac{1}{2}c_1^2R_{11}t_1\left[t_1 - 2\sum_{i=2}^{n}(c_iR_{i1}/-c_1R_{11})t_i\right].$$

Thus, t_1 has to be compared with a weighted average of the tax rates on other commodities. Even though we cannot measure the R_{i1}, so as to know the precise weights to apply, in many cases it is possible to set reasonable limits within which the true weighting pattern is likely to lie. We are likely to have a good idea of which, if any, of goods X_2 to X_n are very close substitutes or complements to good X_1. After making allowance for the plausible degree of substitution or complementarity here, we are not likely to go far wrong if we assume that the remaining commodities are remote, "general" substitutes for X_1. Thus the procedure would be first to estimate $-c_1R_{11}$; then to estimate c_2R_{21} and c_3R_{31}, say, if goods 2 and 3 were particularly close substitutes or complements to good one; and finally to distribute the remaining total weights $(-c_1R_{11} - c_2R_{21} - c_3R_{31})$ to commodities X_4 to X_n, say, in proportion to their relative importance in the national income. Obviously this procedure is not exact, but it is unlikely to lead to a result that is of an erroneous order of magnitude.

B. We now attempt to take account of distortions other than taxes. These can be treated as "autonomous" taxes or subsidies. If a monopoly is present in industry i, which prices its products at 20 percent above marginal cost, it is as if a 20 percent tax existed on the product of industry i, or, perhaps, a 40 percent tax on the value added by labor and by capital in industry i. If activity j has ɔsitive external effects, leading to an excess of 10 percent of social benefit over arginal cost (at the margin) it is once again as if a tax of 10 percent existed the value produced in industry j, or of an appropriately greater percentage the value added in industry j. Correspondingly, if an industry's product has egative external effects, it is as if a subsidy existed on the value produced or e value added in that industry (i.e., the economy, by itself, tends to produce o much of that industry's product).

To see how these other distortions would be taken into account, assume that monopoly exists in industry 1, such that price is $(1 + m_1)$ times marginal ɔst. Suppose, moreover, that a tax of t_1' percent exists (or is contemplated) on is product. To take account of the combined effect of the monopoly and the x, we would simply set $t_1 = [(1 + m_1)/(1 - t_1')] - 1$, and then use this value r t_1 in (3.7).

It seems to me that most distortions other than taxes can be taken into ccount in the way just indicated. There is no intrinsic difficulty, however, in ealing with more complicated cases in which the percentage excess of social alue over marginal cost is a function of output rather than a constant. Cases which the external effects of an industry or activity are independent of its ıtput or level, and depend only on the existence of the industry, need not be ealt with within the framework of this analysis. If the industry or activity is to kist in all situations being compared, external effects of this sort will be equal all such situations. If, on the other hand, one contemplates eliminating an ıdustry with a given negative external effect, one can calculate by an analysis the type used in this paper what would be the efficiency-cost of a tax which as just barely prohibitive of the activities of the industry, and see whether is cost outweighed the negative external effect or not.

. We now turn to a problem which was consciously avoided in section II. here we maintained the assumption that the capital stock was given. Now e must investigate the possibilities of eliminating this restrictive assumption.

In the first place, we can recognize that, for the analysis of section II, we o not need to assume that the capital stock remains fixed through time. Both opulation and capital stock can change through time, and the analysis of ection II can be modified to take account of these changes, so long as the hanges (in population and in capital stock) are not dependent on tax rates nd other distortions. The difficulties appear when we try to allow for the ffects of changes in tax rates, etc., on the level of capital stock (and/or opulation).

Particularly since I have no really satisfactory solution to the problem posed, am inclined to defend the assumption that the level of capital stock is reasonbly independent of tax rate changes (at least of the sorts of tax rate changes hat we have observed in the past). Here I rely on the secular constancy of the ate of net saving in the United States, in the face of substantial swings in the ate of return and in the face of significant alterations in the tax structure. I vould not expect, given this historical experience, that the neglect of an effect f taxation upon savings would introduce large errors into the measures derived n section II.

Obviously, however, this answer, though perhaps adequate for many practical applications, really begs the fundamental question. As I see it, there are three main roads to a solution.

1. One could attempt to extend the "models" of section II to many time periods, building in all of the relevant dynamics. This, I think, would be scientifically the most satisfying approach to take. However, I am afraid that this approach is likely to complicate the analysis to the point where it will be hard to apply it to real-world problems. Nonetheless, I feel that this is a line worth pursuing.

2. One could attempt to separate the "comparative static" from the "dynamic" costs of alternative tax set-ups. Suppose that changing from tax vector T to tax vector T'' leads to a change in the rate of saving from s to s'. We could measure the change in welfare due to the change in taxes first on the assumption that the rate of saving was unaffected, and then attempt to measure the additional cost or benefit associated with the change in the rate of saving. This approach has a particular appeal because, given the assumption that the net rate of return to capital is equalized in all uses, it is reasonable to assume the rate of saving depends only on the level of real income and the net rate of return.

One can go quite some distance with this approach without greatly complicating the analysis. The present value to the saver of a dollar of saving at the margin is $\$1.00 = \rho(1 - t)/r$, where ρ is the social rate of marginal net productivity of capital and r (which at least in uncomplicated situations should equal $\rho(1 - t)$) is the after-tax rate of discount which the individual uses to obtain present values, and t is the expected future rate of tax on income from saving. The present value of the social yield of capital is simply ρ/r, so that a dollar's worth of savings should have a social value of $\$1.00/(1 - t)$. The change in welfare due to the difference in this year's savings stemming from a tax rate of t rather than a tax rate of zero would then be $1/2\, t\Delta s/(1 - t)$, where Δs is the tax-induced change in the amount of this year's savings. If we call this expression $\Delta_2 W$, and expression (3.19), say, $\Delta_1 W$, we can express ΔW as $\Delta_1 W + \Delta_2 W$. $\Delta_1 W$ expresses the cost this year of misallocating the resources that would be present this year if the rate of savings were unaffected by tax changes. $\Delta_2 W$ measures the present value of the future benefit foregone because the economy — for tax reasons — did not save "enough" this year. One could correspondingly estimate the ΔW stemming from a particular tax structure for a series of future years, and estimate the present value of the future stream of welfare costs associated with that tax structure.

The principal difficulty with approach number 2 is, I believe, that it requires the assumption that ρ will remain constant in the future. The approach could of course be modified so as to impose a particular nonconstant time-path for ρ in the future, but the basic difficulty remains that the model does not itself tell us what that time-path should be. As a practical matter, however, I believe that changes in the marginal net productivity of capital are likely to be sufficiently slow so that the assumption of constancy will not introduce serious errors in the estimation of ΔW.

3. One could attempt to incorporate tax-induced changes in capital stock irectly into the analysis. This approach requires two changes in the analysis of ises D and E of section II. First the assumptions that

$$\sum_i G_{ij} = 0 \quad \text{and} \quad \sum_i H_{ij} = 0$$

ıust be abandoned; and second, we must eliminate the assumption of the eutrality of any tax striking equally the income from capital in all uses. In ffect this means that the only neutral tax treatment of the income from ıpital would be not to tax it all.[3] These two adjustments could easily be ıcorporated into the framework developed in section II. One additional step ould also be necessary. Since the savings-effects of a tax change are likely) go on indefinitely, one would have to decide on the specific time period over hich one was measuring the effect of tax changes on the capital stock. This ould enable one in principle to deal with specific values for

$$\sum_i G_{ij} \quad \text{and} \quad \sum_i H_{ij},$$

hereas otherwise these values could be almost anything, depending on the me period over which the reactions were being measured. This last requireıent — of measurement over a specific time period — is to my mind the most rious disadvantage of approach number 3.

I shall not go into more detail here on the possible merits and disadvantages the three approaches to the savings problem that I have suggested. This roblem is, as I have indicated, the most serious "open end" in the analysis of ction II, and I hope that further work in the field, following one or more of e approaches outlined above, will help close this important gap.

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