

## ON THE THEORY OF TAX INCIDENCE

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THE aim of this article is to review and extend some of the general equilibrium propositions of incidence theory. Primary emphasis is placed on the calculation of the differential incidence of a number of general taxes, partial commodity taxes, and partial factor taxes, and on the analysis of the factors which determine whether a substitution of one type of tax finance for another type increases or decreases the real income of a particular group. Few of the results presented here are particularly novel, and most of them can be found in the writings of Meade (1935), Wells (1955), Johnson (1956), Musgrave (1959), and especially Harberger (1962), or in related literature on the theory of international trade. However, differences in analytical technique and differences in emphasis by various writers on one or a limited range of taxes have impeded understanding of the differential incidence of a relatively wider range of taxes. This article presents a unified treatment of qualitative (deductive) knowledge in this area of tax analysis.

The basic assumptions in the analysis are as follows. Price flexibility insures full employment at all times in an economy where two commodities  $X$  and  $Y$

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are produced under constant returns to scale through the inputs of labor ( $L$ ) and capital ( $K$ ). The total supplies of each of the two factors are assumed to be fixed. Capital and labor are perfectly shiftable between sectors, and both factors of production are paid the value of their marginal products. In equilibrium the after-tax rates of return to labor and capital are the same in both industries. Capital goods are durable producers' goods, and the depreciation of capital takes place in a declining balance form at a fixed rate. It is also assumed that the rate of depreciation of capital is the same in both industries.

In the analysis no reference is made to a monetary system, changes in absolute prices are ignored, and the discussion of incidence is carried out in terms of relative prices. Although the direction of adjustment to the imposition of indirect taxes and the induced response of the monetary authority may be important for some questions, changes in absolute prices are of secondary importance for incidence, since income will change to the same extent if the imposition of a general sales tax results in an increase in commodity prices at unchanged money incomes or in a decrease in factor incomes at unchanged commodity prices.

### THE DIFFERENTIAL INCIDENCE OF GENERAL TAXES

The main results about the incidence of general taxes, such as a proportional income tax, a general sales tax, or a gen-

eral production (value-added tax), can be summed up by the two following propositions. First, an income tax on gross income (gross of depreciation) is equivalent to a general sales tax that is imposed on gross value and applies to capital goods as well as consumption goods. It is also equivalent to a general production tax that makes no allowance for the depreciation of capital. Second, a value-added tax that is imposed at the same rate on the production of consumption and capital goods and under which the depreciation of capital is subtracted from the tax base is equivalent to a proportional income tax on net income (net of depreciation). From these two propositions it follows that a general sales tax is not equivalent to a proportional income tax imposed on net income. If we abstract from the distributional effects that may result from a change in the composition of output, the burden of a general sales tax falls more heavily on profits than does a proportional income tax on net income.

A rigorous proof of the equivalence between general taxes on gross value added is presented in the Mathematical Appendix of this paper. Yet algebraic manipulation is not really needed to show that a general sales tax is equivalent to a proportional income tax on gross income. There is a very simple explanation of this result. An indirect tax drives a wedge between commodity prices and factor prices. In response to the imposition of an indirect tax, factor incomes will fall proportionately at unchanged commodity prices, or, if factor incomes remain unchanged in money terms, commodity prices will rise by the amount of the tax and real factor earnings will be decreased proportionately.

Since both Shoup (1953) and Rolph (1964) have demonstrated the equivalence

between a value-added tax of the income type and a proportional income tax on net income, it is unnecessary to reproduce the argument in detail here. In essence, it is very similar to the one above for the equivalence of taxes on gross value added.<sup>1</sup>

The equivalence between different general taxes on gross value added and the equivalence between different general taxes on net value added seem intuitively obvious and are generally accepted in the literature. The only writer who reaches different conclusions is John Due (1963, 1965). Due takes the position that a sales tax or production tax on capital goods is borne in relation to consumption spending. Since he also believes that the burden of a sales tax on consumer goods likewise falls on consumers, he argues that a general sales

<sup>1</sup> Consider a two-sector economy where wages comprise 50 per cent of gross value added in each of the two sectors, and one-half of gross capital rentals is depreciation. For tax purposes, depreciation is on an original cost basis and is equal to true economic depreciation at the original set of commodity and factor prices that by convention are equal to one. GNP is equal to 200 units and NNP to 150 units. A proportional income tax of 10 per cent on net income decreases after tax wages to 90 and after tax profits to 15. The substitution of a value-added tax of 11.11 per cent will yield the same revenue of fifteen units and will leave after-tax incomes unchanged if commodity prices remain unchanged. The marginal revenue product of labor,  $w$ , is equal to  $1/1+t$ , where  $t$  is the tax rate, and the marginal revenue product is given by the relationship  $r+t$  ( $r - 0.5 = 1$ , where  $r$  is the capital rental and .5 is the depreciation allowance).

More realistically, if prices rise in response to a substitution of a value-added tax for a proportional income tax, the equivalence between the two types of taxes is broken if depreciation is allowed on an original cost basis. If this case, the owners of existing (old) capital will suffer a capital loss. For if old and currently produced capital goods are equally efficient, they will earn the same gross capital rental. However, since depreciation is on an original cost basis, the real value of the depreciation allowance is decreased, the real return on old capital falls, and the price of the old capital, measured in terms of consumption goods, falls.

tax is not equivalent to a general income tax but is equivalent to a general consumption tax or to a sales tax on consumption goods. He similarly concludes that an income form of the value-added tax is equivalent to a retail sales tax on consumption goods. Due's position is inconsistent with the theory of marginal product pricing for, as indicated above, this theory implies that a general sales tax on final goods is not equivalent to a general sales tax on final sales of consumption goods, since a general sales tax is equivalent to the latter tax *plus* a general sales tax on final sales of investment goods.<sup>3</sup>

<sup>3</sup> Due's conclusions are based on two considerations. The first is that capital goods are only another form of intermediate input, and when the production of this intermediate input is taxed, the tax will be reflected in the price of the final commodity. In Due's words, "it is actually largely immaterial for incidence whether a sales tax is collected entirely at the point of final sale of the consumption goods or in part on goods whose costs enter into the prices of the consumption goods" (Due, 1963, p. 1083).

With respect to this first point, it is undeniable that capital goods are used in further production and that, if a tax is imposed on an intermediate input such as cloth or steel, the burden of the tax will fall on the purchaser of the final commodity. However, capital goods are final products that are purchased by individuals or firms. Moreover, machines are used to produce machines. What Due seems to overlook is that these are net profits associated with capital, and to say that the price of a commodity adds up to its costs of production is not particularly enlightening as it tells us nothing about the distribution of income among the three primary factors, land, labor, and waiting (net returns on capital).

Whatever its shortcomings, marginal productivity theory does supply one answer to this basic question. Since each factor receives the value of its marginal revenue product net of indirect taxes, the imposition of tax on capital goods will not change factor earnings but will increase the price of capital goods relative to consumer goods. In order to purchase one unit of the asset that earns the same rental, more consumption has to be sacrificed. In other words, the net rate of return per unit of consumption has decreased. Due believes that this decrease will not occur because the tax on capital goods will be reflected in the price of consumer goods. We do not understand this argument, since marginal productivity theory implies that commodity prices depend on

#### THE DIFFERENTIAL INCIDENCE OF PARTIAL COMMODITY AND FACTOR TAXES

While general taxes have no effect on the allocation of resources and relative commodity prices except insofar as the pattern of government spending differs from the pattern of private expenditures, commodity taxes imposed at differential rates on different commodities and taxes on wages and/or profits in particular sectors affect consumption choice and the demand for factors of production. The incidence of these taxes depends on a complicated interaction between use of income (demand) factors and source of income (production) factors. A commodity tax or a tax on factor earning in  $X$  will increase the price of this good and will be relatively more burdensome to the group which spends a high proportion of its income on this commodity. Furthermore, as the demand for  $X$  ( $Y$ ) will fall (increase) as a result of the tax, the relative factor intensities of the two sectors will affect the distribution of after-tax income. If  $X$  is capital (labor) intensive, the output effect of a tax on

factor price (returns) and not on the prices of durable factors of production that were produced sometime in the past.

Due's second point, that firms in oligopolistic markets will pass on the tax on producer's goods, is also not persuasive—primarily because it is not developed and is little more than a *deus ex machina*. It is sometimes argued in discussions of the incidence of the corporate profits tax that certain industries possess sufficient monopoly power so that they can set their product prices so as to achieve some arbitrary, fixed after-tax rate of return on investment capital. Even if this were correct (we do not believe that it is), a general sales tax would not be equivalent to a consumption tax because large sectors of the economy are competitive, and this fact would have to be accounted for in a full analysis of incidence. Furthermore, if monopoly power makes possible the shifting of an indirect tax, it likewise makes possible the shifting of a proportional income tax and/or a general profits tax. Thus monopoly power does not upset the equivalence between general direct and general indirect taxes.

$X$  will tend to decrease (increase) the price of capital relative to the price of labor as the contracting sector,  $X$ , will release relatively more capital (labor) than the expanding sector is willing to absorb at factor prices that existed before the tax was introduced.

The imposition of a tax on capital or labor in one of the two sectors also results in a factor-substitution effect in that sector, for under our assumptions factors receive the same after-tax payments in both industries. If a capital tax is imposed in  $X$ , the before-tax price of capital must rise in that sector in order to restore equilibrium in capital markets. This increase in the price of capital to that sector will lead to the substitution of labor for capital in the tax industry, and this effect will tend to increase wages relative to profits. Incidence, then, depends on three separable effects: the source of income (demand) effect, the output (factor-intensity) effect, and the factor-substitution effect. One or more of these effects may be equal to zero for the same taxes and under certain conditions. The factor-substitution effect is always equal to zero for a commodity tax. The demand effect also has no bearing on incidence when all groups have the same spending propensities, and in this case the relative tax burden on different groups can be measured in terms of changes in money income alone. There is no factor-substitution effect when a taxed industry is characterized by fixed coefficients of production, and, finally, there is no output effect when the two industries have the same factor intensities.

Initially, we shall follow Harberger (1962) and assume that all individuals have the same average and marginal

spending propensities. We shall also assume that when the government imposes a tax it spends the proceeds in such a way that if initial prices continue to prevail public expenditures will just counterbalance the reduction in private expenditures on each of the two goods. This assumption permits us to write the change in the demand for  $X$  as a function of changes in relative prices alone. Since by assumption all individuals have the same spending propensities, the incidence of the various taxes can be measured by changes in the after-tax distribution of money income. The expressions  $p_1$  and  $p_2$  stand, respectively, for the after-tax prices of capital and labor; and the price of labor is the numeraire, that is,  $dp_2 = 0$ .

As all variables are measured in terms of the after-tax price of labor, the measurement of incidence depends on information about the level of tax proceeds and the change in the after-tax price of capital,  $dp_1$ . The deviation of the general expressions of  $dp_1$  for several partial taxes is presented in the Mathematical Appendix. These expressions are derived by solving a system of eight equations in eight unknowns. The equations are a demand relation which determines the change in the demand for one of the commodities,  $X$ ; a supply equation which determines changes in the output of  $X$ ; two price equations that relate changes in commodity prices to changes in factor prices and taxes; two factor-demand equations that indicate how factor proportions in each of the two sectors change in response to changes in factor prices and taxes; and two factor-endowment relations which specify the total supplies of each of the two factors. The solution for  $dp_1$  for a tax on profits in  $X$  expressed as a per unit tax,  $T_{22}$ , is equal to

$$dp_1 = \frac{[E'_{12}(K_0/K_1 - L_0/L_1) + S_0(f_{12}K_0/K_1 + f_{11}L_0/L_1)]T_{22}}{E(g_2 - f_2)(K_0/K_1 - L_0/L_1) - S_2 - S_0(f_{12}K_0/K_1 + f_{11}L_0/L_1)} \quad (1)$$

where  $E$  is the price elasticity of demand for  $X$ ;  $K_x, K_y, L_x, L_y$  are the original amounts of capital and labor employed in industries  $X$  and  $Y$ , respectively;  $S_x$  and  $S_y$  are the elasticities of substitution between labor and capital, respectively, in industries  $X$  and  $Y$ ;  $f_L, f_K$  are the initial share of labor and capital, respectively, in the total cost of producing  $X$ ;  $g_x$  is the share of capital in  $Y$ ; and  $T_{x,c}$  is a tax on capital earning in  $X$  expressed as a per unit tax.<sup>3</sup>

If  $dp_x$  turns out to be zero, we conclude that the burden of the tax on capital in  $X$  is proportional to the factors' original contribution to national income. Since  $dp_x$  by assumption is equal to zero, the change in national income resulting from the imposition of the tax is equal to the amount of the tax proceeds,  $T_{x,c}K_x$ , plus the change in after-tax profit income,  $dp_x(K_x + K_y)$ . If  $dp_x = 0$ , and national income measured in the terms of the numeraire rises by the amount of the tax proceeds, and the shares of profits and wages do not change from the pretax situation, then the result is that the incidence of the tax is equivalent to a proportional income tax. When  $dp_x$  is negative (positive), capital bears a proportionately larger (smaller) share of the tax burden. For each tax it is also possible to find the value of  $dp_x$  at which one of the two factors bears the whole burden of the tax.<sup>4</sup>

<sup>3</sup> It should be noted that the treatment of taxes as per unit taxes, rather than as ad valorem, in no way limits the generality of the analysis. The ad valorem equivalent to a per unit tax can always be calculated. By appropriate choice of units, original commodity and factor prices are equal to one. Hence to a first-order approximation a per unit tax of 0.10 on commodity  $X$  is equivalent to a 10 per cent ad valorem tax on  $X$ . Another simplification that we use is to measure tax yield on the basis of original quantities of commodities and factors. That is, when a tax  $T_x$  is imposed on capital in  $X$ , we calculate the yield as  $T_{x,c}K_x$  rather than  $T_{x,c}(K_x + dK_x)$ .

As  $(g_x - f_x)$  and  $(K_x/K_y - L_x/L_y)$  are always of opposite sign and  $E, S_x, S_y$  are negative, the denominator of expression 1,  $D$ , is unambiguously positive. The output effect, equal to  $f_x E (K_x/K_y - L_x/L_y)$ , which is  $f_x A$  in the shorthand notation we adopt to conserve space, is negative when  $X$  is capital intensive relative to  $Y$  and is positive when  $X$  is labor intensive. The factor-substitution effect,  $S_x (f_L K_x/K_y + f_L L_x/L_y)$ , which is written  $S_x B$ , is always negative. We see, therefore, that when  $X$  is capital intensive, the numerator is unambiguously negative, and capital is unambiguously worse off relative to labor than it would be under a proportional income tax of equal yield. If  $X$  is labor intensive, the output effect works in favor of capital while the factor-substitution effect works against it, and it is not clear whether  $dp_x$  is negative or positive.

We analyzed above the differential incidence of a tax on capital in  $X$  and a proportional income tax by solving for the sign of  $dp_x$ . The comparison of the incidence of this tax with other partial taxes imposed in  $X$  and/or  $Y$  is somewhat more complicated. The solution of  $dp_x$  for a commodity tax on  $X$ ,  $T_x$ , is equal to

$$\frac{AT_x}{D}, \quad (2)$$

and  $dp_x$  for a tax on labor in  $X$  is equal to

$$\frac{(f_L A - S_x B)}{D} T_{x,L}, \quad (3)$$

where  $T_{x,L}$  is a tax on labor expressed as a per unit tax. The term  $A = E (K_x/K_y - L_x/L_y)$

<sup>4</sup> For a commodity tax on  $X$ , the change in national income will be  $(K_x + L_x)T_x + (K_y + L_y)dp_x$ . Capital will bear the full burden of this tax if  $dp_x = -T_x(K_x + L_x)/(K_x + K_y)$ , for in this case labor's share in national income remains unchanged. Labor bears the entire burden of a tax when the percentage change in the price of capital is equal to the percentage change in national income.

$K_x - L_x/L_y$  is negative when  $X$  is capital intensive. Thus from (2) follows the familiar result that when a tax is imposed on one of the two commodities the factor which is used more intensively in the taxed industry will bear a larger proportion of this tax than its original share in national income. Another familiar result, that equal-rate taxes on capital and labor imposed in the same sector are equivalent to a commodity tax in this sector, can be derived by putting  $T_{xL}$  and  $T_{xK}$  equal to each other and summing expressions (1) and (3). The summation yields expression (2).<sup>5</sup>

The most meaningful approach to differential incidence is the comparison of two taxes of equal yield. The yield from a tax on capital in  $X$  is  $T_{xK}K_x$ , the yield from a tax on labor in  $X$  is  $T_{xL}L_x$ , and the yield from a commodity tax on  $X$  is  $T_x(L_x + K_x)$ . In comparing the differential incidence of  $T_{xK}$  and  $T_{xL}$ , we impose the equal-yield condition and so substitute  $T_{xK}[(K_x + L_x)/K_x]$  equal to  $T_x(1/f_k)$  for  $T_{xK}$  and  $T_x[(K_x + L_x)/L_x] = T_x(1/f_L)$  for  $T_{xL}$ . When this substitution is made, the term  $f_k AT_{xK}$  in the numerator of (1) and the term  $f_L AT_{xL}$  in the

numerator of (3) will be equal to  $AT_x$ , the numerator of (2). In other words the output effect for these three taxes is the same when the equal-yield condition is imposed. Thus there follows the very plausible result that of the three equal-yield partial taxes that can be imposed in one of the sectors, a tax on labor in  $X$  results in the smallest (largest) tax burden for capital (labor), and a tax on capital results in the largest (smallest) tax burden for capital. As the output effect cancels (is the same for the three taxes), the only difference results from the factor-substitution effect. This effect works against capital when a tax on capital is imposed and against labor in the case of the tax on labor. For a commodity tax there is no such effect. When  $S_x$  (the elasticity of substitution between labor and capital in the taxed sector) is equal to zero, the three taxes are equivalent.

The differential incidence of taxes imposed in different sectors is much less clear-cut, except for the two commodity taxes,  $T_x$  and  $T_y$ , which depend only on the relative factor intensities of the two industries. For example, the differential incidence of tax on capital in  $X$  and on equal-yield tax on capital in  $Y$  depends not only on the relative factor intensities of the two industries but also on the relative sizes of the factor-substitution effects which are determined by the relative sizes of the elasticities of substitution  $S_x$  and  $S_y$ .<sup>6</sup>

<sup>5</sup> This result can be used to simplify the analysis of a complicated tax system where all six partial taxes are imposed at the same time at different rates. Since  $T_x$  is equivalent to a tax on the earnings of capital and a tax on the earnings of labor at the rate  $T_x$ , we can first reduce a system of six partial taxes to a system of four partial factor taxes,  $T_x + T_{xK}$ ,  $T_x + T_{xL}$ ,  $T_y + T_{yK}$ , and  $T_y + T_{yL}$ . Next we subtract the smallest capital taxes from the larger one and the smallest partial labor tax from the larger one, thus reducing the system to a general profit tax, a general wage tax, and a partial tax on labor and a partial tax on capital. For example, suppose  $(T_x + T_{xK}) - (T_y + T_{yK})$  is positive while  $(T_x + T_{xL}) - (T_y + T_{yL})$  is negative. Then we would say that the original system of six partial taxes is equivalent to general profit tax imposed at the rate  $T_x + T_{xK}$ , a general wage tax imposed at the rate  $T_x + T_{xL}$ , a partial tax on capital in  $X$  imposed at the rate  $(T_x + T_{xK}) - (T_y + T_{yK})$ , and a partial tax on labor in  $Y$  imposed at the rate  $(T_x + T_{xL}) - (T_y + T_{yL})$ .

<sup>6</sup> This possibility may be elaborated on by referring to two special cases dealt with by Harberger. On the assumption that national income is divided equally between two sectors and that one of the production functions is a Cobb-Douglas, he first examines the case where the taxed industry is characterized by fixed proportions, that is, for  $T_{xK}$ ,  $S_x = 0$ . As the  $X$  sector contracts, labor and capital are emitted in some fixed proportion. If  $X$  is capital intensive relative to  $Y$  (the Cobb-Douglas sector), capital will bear a proportionately larger amount of

Thus capital may be better off under a tax on capital in a capital-intensive sector than under a tax on capital in a labor-intensive sector if the elasticity of substitution is relatively small in the capital-intensive industry.<sup>7</sup>

The possibility that the output (factor-intensity) effect and the factor-substitution effect may work in opposite directions also explains why capital may be worse off under a partial tax on labor than under a partial tax on capital. When the capital tax is imposed in a labor-intensive sector characterized by a very low elasticity of substitution between labor and capital, capital will benefit from the output effect and lose little from the factor-substitution effect.

It is clear from the above results that many questions of incidence depend on the absolute and the relative sizes of the

the tax burden than labor. For a tax on capital in  $Y$  (the non-taxed sector is characterized by variable proportions), Harberger finds that capital will bear more than the full burden of the tax—and labor will enjoy an increase in its real income. The reason for this outcome is that, since  $X$  is characterized by fixed factor proportions, it must absorb some labor as well as capital. The tax on capital in  $Y$ , the industry with the high elasticity of substitution, will drive up the price of labor in that sector and will increase the demand for labor in that industry. The upward pressure on the price of labor will be compounded by the demand for labor by the non-taxed sector. The outcome of these two forces is a considerable decrease in the price of capital relative to the price of labor. The same considerations apply in explaining the results on differential incidence of the capital taxes presented in the text above.

<sup>7</sup> Actually the comparison is more complicated. The two relevant expressions for  $dp_c$  are  $(f_{cd} + S_{cd})T_{cd}/D$  for a tax on capital in  $X$  and  $(-g_{cd} + S_{cd})/D/T_{cd}$ . The expression  $B = f_{cd}(K_c/K_x) + f_{cd}(L_c/L_x)$  may be written  $[X/(f_{cd}f_x)/Y(g_{cd})]$  or  $(K_c L_c)/(K_x L_x)$ . The equal-yield condition requires that  $T_{cd} = T_{cd}K_c/K_x$ . Hence  $S_{cd} Y_{cd}$  need not equal  $S_{cd} T_{cd}$  even when  $S_{cd} = S_{cd}$ . This ambiguity is not surprising, for the relative magnitudes of the factor-substitution effects depend on the size of the taxes as well as  $S_{cd}$  and  $S_{cd}$ . The sums of the tax rates in turn depend on the relative factor intensities of the two industries and the relative size of the two sectors.

two elasticities of substitution,  $S_{cd}$  and  $S_{cd}$ . In expressions (1), (2), and (3) the elasticity of substitution in the untaxed sector,  $S_{cd}$ , appears only in the denominator. Consequently, the larger the value of this parameter, the smaller the value of  $dp_c$  will be, and more nearly proportional to the factor's original contribution in national income will be the incidence of any partial tax.

The size of the elasticity of substitution between labor and capital in the taxed sector,  $S_{cd}$ , determines the differential incidence of these three partial taxes. When  $S_{cd}$  is large, a tax on capital will be particularly burdensome for profits relative to the burden on capital earnings for an equal-yield commodity tax or tax on labor. When  $S_{cd}$  is zero, the three taxes are equivalent.

There is no clear-cut relation between the size of the elasticity of substitution in the taxed sector and the size of  $dp_c$  for any particular factor tax. This is to say that it is difficult to determine how a change in this parameter will affect the incidence of a tax relative to a proportional income tax. For example, when (1), the expression for  $dp_c$  for a tax on capital, is evaluated for a very large value for  $S_{cd}$ , the after-tax price of capital falls by the amount of the tax, that is,  $dp_c = T_{cd}$ . Capital bears more than the full burden of the tax as the fall in capital income  $T_{cd}(K_c + K_x)$  exceeds tax collections which equal  $K_c T_{cd}$ . On the other hand, it is not necessarily true that when  $S_{cd}$  increases capital bears a larger amount of the burden of this tax. The partial differentiation of expression (1) with respect to  $S_{cd}$  shows that  $J dp_c / J S_{cd}$  is unambiguously negative only if  $X$  is capital intensive. This ambiguity arises from the fact that although a large  $S_{cd}$  will accentuate the factor-substitution effect re-

sulting from the imposition of a tax on capital, large changes in the relative after-tax prices of the two factors will be dampened by the increased sensitivity of relative factor demands in the taxed sector that result from a large value of  $S_p$ . This means in effect that the incidence of a partial tax relative to a proportional tax may be quite insensitive to a wide range of values of the elasticity substitution in the taxed sector, which simplifies possible empirical applications of the above analysis.

To this point we have assumed that all individuals have the same spending propensities and have analyzed incidence solely in terms of production (source of income) effects. The introduction of different spending patterns for the two groups adds one further complication to the analysis and makes the differential incidence of various taxes even less definite. For example, it may be that capitalists gain from the substitution of a tax on capital in a *capital-intensive* sector for a proportional income tax if capitalists spend a high proportion of their income on the commodities produced by the untaxed sectors. The unfavorable source of income effects of the tax substitution may be outweighed by the favorable use of income effects. As we continue to assume workers and capitalists receive only one type of income, the basic change resulting from the more general approach is in the specification of demand. Above, the change in the demand for  $X$  was written as a function of relative prices. Now as the two groups have different spending propensities, the change in the demand for  $X$ ,  $dX$ , is equal to the income effect of a change in the price of capital,  $\bar{m} K dp_c$ , where  $\bar{m}$  is the marginal propensity to spend on  $X$  by capitalists plus the income effect of a change in the

price of  $X$  ( $\bar{m}mK + \bar{n}nL$ )  $dp_x$ , where  $\bar{m}$  and  $\bar{n}$  are workers' marginal and average propensities to spend on  $X$ , respectively, plus the income effect of a change in the price of  $Y$ ,  $[(\bar{m})(1-m)K + (\bar{n})(1-n)L]$   $dp_y$ , plus the change in the government's demand for  $X$ ,  $g_x T$ , where  $g_x$  is the government's propensity to spend on  $X$ , and  $T$  is total tax collections plus the substitution effect of a change in relative prices  $H(d\bar{p}_x - d\bar{p}_y)$ , where  $H$  is the Hicksian substitution term which is assumed to be the same for both capitalists and workers.

It is possible to analyze differential incidence in terms of the change in the real income of one of the two groups, for when a change in the tax system is unambiguously favorable for capitalists it will be unambiguously unfavorable for workers, and conversely.<sup>3</sup>

The first-order measure of change in real income  $dR$ , for capitalists, resulting from the imposition of a tax is equal to

$$dR = p_x dX_c + p_y dY_c, \quad (4)$$

where  $p_x$  and  $p_y$  are the original prices of  $X$  and  $Y$ , respectively; and  $dX_c$ ,  $dY_c$  are the change in the consumption of  $X$  and  $Y$ , by capitalists, respectively.

Using the budget identity  $K dp_c = p_x dX_c + dp_x X_c + p_y dY_c + dp_y Y_c$ , and the convention that original commodity

<sup>3</sup> It is not generally true that the substitution of one tax for another tax will always increase the real income of one group and decrease the income of the second. Because of the excess burden of some taxes, some tax changes can make both groups better off or both groups worse off. However, here we are primarily concerned with the derivation of conditions under which one of the two groups is unambiguously worse off or better off as the result of a change in the type of tax finance. Whenever the change in real income is unambiguously positive (negative), on a qualitative basis, the change in real income for workers will be unambiguously negative (positive). Of course, within the limitations of the measure of real income that is used, the change in real income can be calculated for both groups.



prices are equal to one, the change in real income is equal to

$$K dp_x - X_c dp_x - Y_c dp_x. \quad (5)$$

For a tax on capital in  $X$ , the change in prices is equal to

$$dp_x = f_x(dp_x + T_x);$$

$$dp_y = g_y(dp_x).$$

As the original prices of capital and labor are equal to one, we can rewrite (5) as

$$K[1 - mf_x - (1 - m)g_x]dp_x - mKf_xT_x, \quad (6)$$

where  $m$  is the capitalist's average propensity to spend on  $X$ .

Let  $[1 - mf_x - (1 - m)g_x]$  be equal to  $M$ . This term is necessarily positive. For a tax on labor in  $X$ ,  $dR$  becomes

$$KM dp_x - mKf_xT_x; \quad (7)$$

for a commodity tax in  $X$ ,

$$KM dp_x - mKT_x; \quad (8)$$

for a general commodity tax imposed at the same rate in  $X$  and  $Y$ ,

$$KM dp_x - KT; \quad (9)$$

for a tax on capital in  $Y$ ,

$$KM dp_x - (1 - m)Kg_yT_y; \quad (10)$$

for a tax on labor on  $Y$ ,

$$KM dp_x - (1 - m)Kg_yT_y; \quad (11)$$

for a commodity tax in  $Y$ ,

$$KM dp_x - (1 - m)KT_y. \quad (12)$$

Each of these expressions contains a term in  $dp_x$  and a tax term. It is convenient to refer to the former as the source of income effect and the latter as the use of income effect. When  $dp_x$  is zero, or is the same for taxes of equal

yield, differential incidence depends on one simple condition, namely, whether capitalists or workers spend a higher or lower proportion of their income on the product of the taxed sector. Consider the substitution of a commodity tax on  $X$  for a proportional income tax when  $dp_x$  is equal to zero. The equal condition requirement is that  $T = (X/X + Y)T_x$ , where  $X$  and  $Y$  are the total amounts purchased of the two commodities. Substituting this expression in (9), it follows (8) will exceed (9) when  $m > (X/X + Y)$ . This condition will be fulfilled when  $m > \pi$ , that is, when capitalists spend a higher proportion of their income on  $X$  than workers do. The same sort of result holds for any tax substitution when the relative price of labor and capital does not change. A tax of any type on  $X$  will decrease (increase) the real income of capital (labor) more than a tax  $Y$  if  $m > \pi$ , and conversely. The three partial taxes in a particular sector are equivalent.

These straightforward results hold when  $dp_x$  is zero or is the same for different taxes. Of course, in most cases relative factor prices will change when the type of tax finance is changed. Although the expressions for the change in the price of capital relative to the price of labor are more complicated because of the more general specification of the demand side of the model, the principle qualitative results on the change in the price of capital are the same as those results given above for the simpler model.<sup>9</sup>

<sup>9</sup> A number of paradoxical results are possible when the two groups have different spending and/or when the average propensity to spend on one of the two commodities differs substantially from the marginal propensity. When the elasticities of substitution,  $S_x, S_y$ , are small, the possibility arises that a tax on capital in a capital-intensive sector may lead to an increase in the demand for the commodity (quite apart from the influence of government de-

The results on the differential incidence of the taxes under consideration are unambiguous only if the factor-intensity (output) effect, the demand effect, and the factor-substitution effect operate in the "same direction." For example, the substitution of a tax on labor in  $X$  for a tax on capital in  $Y$  will be certain to increase the real income of capital if  $X$  is labor intensive relative to  $Y$  and if capitalists spend a higher proportion

of their income on  $Y$  than workers do. If  $X$  is capital intensive, the result of the tax substitution is ambiguous, qualitatively, as the factor-substitution effect and the demand effect operate in favor of capital while the output effect works against it.

Since there is no factor-substitution effect for commodity taxes, these taxes and a proportional income tax can be ranked by capitalists (or workers) on the basis of the demand effect and the output effect. For these taxes there will be no clear-cut answer unless the effects reinforce one another. The only clear-cut results that carry over from the simpler model are those for the three partial taxes imposed in one of the two sectors. The output and demand effects are identical in this case, and the only difference is the factor-substitution effect. This effect works against capital when capital is taxed, increases the price of capital when labor is taxed, and is equal to zero for a commodity tax.

mand) and to an increase in the price of capital relative to the price of labor. This result arises from the possibility that redistribution of income and varying spending propensities will outweigh the tax-induced increases in price. For example, if capitalists have a high propensity to spend on  $X$ , and if  $X$  is capital intensive, a shock (that is, the imposition of a tax) may lead to an excess demand for  $X$ , which leads to an increase in the output of  $X$ , which in turn leads to a redistribution of income in capital's favor, which results in an increase in the excess demand for  $X$ , and so on. In short, if interactions between income distribution and spending patterns are taken into account, an economy may be characterized by multiple equilibria, some of which may be unstable. Harry G. Johnson (1959) derived this result and showed that, because of changes in income distribution, an increase in the price of commodity  $X$  may increase the imports of  $X$ .

Also it is possible that the substitution of a general tax for a partial tax in  $X$  will decrease the demand for  $X$  if the marginal propensity to spend on  $X$  for one of the two groups is very high relative to its average propensity to spend on  $X$ . The size of the income effect of an increase in the price of  $X$  which occurs when the tax on  $X$  is eliminated depends on the size of the marginal as well as average propensity to spend. It can be shown that the income effect on the demand for  $X$  of the removal of an excise tax is equal to  $(\epsilon + \epsilon_s - \eta)(K/X) - \eta(L/X)$ , while the income effect of the imposition of a general commodity or income tax, of equal yield, is  $(\epsilon_s - \eta)(X/(X + Y)) - (\eta X/(X + Y))L/X$ .  $\epsilon_s$ , the income-compensated price elasticity of demand, is negative, yet the demand for  $X$  may fall if  $\eta$  is high relative to  $\epsilon$  and if  $\eta$  is low relative to  $(X/(X + Y))$ . Hence, if the group with the relatively low average propensity to spend on  $X$  has a high marginal propensity to spend on this good relative to the marginal propensity to spend by the other group, the income effect of the imposition of general income tax, which depends on income, will outweigh the income effect of the removal of a commodity tax on  $X$ , which depends on the original amount spent on  $X$ .

#### CONCLUDING REMARKS

In this paper I have presented an integrated analysis of differential incidence of general taxes and a number of partial taxes. Hopefully the approach developed here will facilitate much needed empirical work in this area of tax analysis and will make it easier to shake off Marshallian partial equilibrium methods. A classic example of the way in which these methods lead to half-truths, or worse, is the conventional analysis of the property tax which treats the tax as a number of partial excises that are shifted to consumers and, by emphasizing the impact of the tax on housing, loses sight of the fact that the tax is imposed on industrial property as well and is actually a tax on profit income. Even if the property

tax were imposed only on residential real estate, the fact that the price of housing services would increase as a result of the tax is probably less important for incidence than the decrease in return on capital that would result from a shift of capital out of the extremely capital-intensive housing sector.

More generally, controversies, such as whether taxes are shifted forward onto consumer or shifted back onto factor earnings, are seen to be sterile when viewed in general equilibrium terms. For example, a commodity tax on a particular commodity is shifted forward only in the sense that the price of this commod-

ity will rise relative to other commodities, and this factor is of no interest if all groups spend the same proportions of their incomes on the taxed commodity. Furthermore, it is only meaningful to talk of a partial factor tax's being shifted to consumers to the extent that relative commodity prices change and under the condition that it is possible to ignore the factor-substitution effect and the factor-intensity effects of this tax. The point is, of course, that there are two sides to incidence, the use of income and the source of income, and there is no a priori reason why one side should be given precedence over the other.

#### MATHEMATICAL APPENDIX

For the Harberger model of incidence, the assumptions that each group has the same average and marginal propensities to spend and that government spends tax proceeds in such a way that if initial commodity prices prevail government spending will just counterbalance the production in private expenditures on each of the two goods,  $X$  and  $Y$ , allow us to write the change in the demand for  $X$  as a function of changes in relative prices:

$$dX/X = E(d\hat{p}_x - d\hat{p}_y), \quad (A.1)$$

where  $E$  is the income-compensated price elasticity of demand for  $X$ .<sup>10</sup>

On the assumption that the production

<sup>10</sup>The "private" demand for  $X$  is equal to  $k(I, A, p)$ , where  $I$  is after-tax income measured in terms of the price of labor. After differentiating this general demand function, we split the effects of price changes into income effects and pure substitution effects by making use of the Slutsky equation. Using the relations for changes in commodity prices, which are described in the text, and the convention  $d\hat{p}_L = 0$ , it can be shown that the over-all income effect on private demand is equal to  $T_x A_x$ , where  $T_x$  is tax revenues and  $A_x$  is the private sector's marginal propensity to spend on  $X$ . By assumption, this income effect is offset by the government's purchases of  $X$  so that the change in the over-all demand for  $X$ ,  $dX$ , is equal to  $H(d\hat{p}_x - d\hat{p}_y)$ , where  $H$  is the Hicksian substitution term.

functions are homogeneous of the first degree, we have

$$dX/X = f_L dL/L_0 + f_K dK/K_0, \quad (A.2)$$

where  $f_L$  and  $f_K$  are the initial shares of labor and capital, respectively, in the total cost of producing  $X$ , and  $L_0$ ,  $K_0$  are the initial amounts of labor and capital, respectively, employed in the production of  $X$ . Also because of the constant returns to scale assumption, we know that the ratio of factor inputs is uniquely related to relative factor prices, that is,  $K_0/L_0 = g(\hat{p}_x/\hat{p}_L)$  and  $K_0/L_0 = h(\hat{p}_x/\hat{p}_L)$ . Choosing units of labor and capital so that their initial prices are equal to unity and differentiating the two relations above, we have

$$dK_0/K_0 - dL_0/L_0 = S_x(d\hat{p}_x - d\hat{p}_L), \quad (A.3)$$

and

$$dK_0/K_0 - dL_0/L_0 = S_y(d\hat{p}_x - d\hat{p}_L), \quad (A.4)$$

where  $S_x$  and  $S_y$  are the elasticities of substitution between labor and capital, respectively, in industries  $X$  and  $Y$ . The other equations of the system are

$$dK_0 = -dK_0, \quad (A.5)$$

$$dL_0 = -dL_0, \quad (A.6)$$

$$d p_n = f_L d p_L + f_n d p_n, \quad (\text{A.7})$$

$$d p_n = g_L d p_L + g_n d p_n, \quad (\text{A.8})$$

and

$$d p_n = 0. \quad (\text{A.9})$$

Equations (A.5) and (A.6) come from the assumption of fixed factor supplies. Equations (A.7) and (A.8) come from the assumption of constant returns to scale and marginal product pricing. Equation (A.9) chooses the price of labor as the numeraire. As we study the differential incidence of six different partial taxes, we have written equations (A.1)-(A.9) in general form. The introduction of taxes is effected as follows: For a per unit tax on capital in  $X$ , the right-hand side of equation (A.3) becomes  $S_n(d p_n + T_{nX} - d p_L)$ , and the right-hand side of (A.7) becomes  $f_L d p_L + f_n(d p_n + T_{nX})$ . The other relations remain unchanged. For a per unit tax on labor  $T_{LX}$  in  $X$ , the right-hand side of equation (A.3) becomes  $S_n[d p_n - (d p_L + T_{LX})]$ , and the right-hand side of equation (A.7) becomes  $f_L(d p_L + T_{LX}) + f_n d p_n$ . For a per unit commodity tax on  $X$ , equation (A.7) becomes  $d p_n = f_L d p_L + f_n d p_n + T_{nX}$  and the other equations remain unchanged. The modifications of the equations for the taxes imposed on the  $Y$  sector are the same except that the tax terms are introduced in equations (A.4) and (A.8).

The general expressions for  $d p_n$  for the six taxes under consideration are as follows: For a tax on capital employed in the production of  $X$  (the tax studied by Harberger),

$$d p_n = \frac{[E f_n (K_n/K_y - L_n/L_y) + S_n (f_L K_n/K_y + f_n L_n/L_y)] T_{nX}}{E (g_n - f_n) (K_n/K_y - L_n/L_y) - S_n - S_n (f_L K_n/K_y + f_n L_n/L_y)}. \quad (\text{A.10})$$

To conserve space, we use the following shorthand notation. Let  $A$  equal  $E(K_n/K_y - L_n/L_y)$ , let  $B$  equal  $S_n(f_L K_n/K_y + f_n L_n/L_y)$ , and let  $D$  equal the denominator. Expression (A.10) is rewritten as

$$d p_n = (f_n A + B) T_{nX} / D.$$

For a tax on labor in  $X$ ,

$$d p_n = (f_L A - B) T_{LX} / D; \quad (\text{A.11})$$

for a commodity tax on  $X$ ,

$$d p_n = A T_{nX} / D; \quad (\text{A.12})$$

for a tax on capital in  $Y$ ,

$$d p_n = (-g_n A + S_n) T_{nY} / D; \quad (\text{A.13})$$

for a tax on labor in  $Y$ ,

$$d p_n = (-g_L A - S_n) T_{LY} / D; \quad (\text{A.14})$$

for a commodity tax in  $Y$ ,

$$d p_n = -A T_{nY} / D. \quad (\text{A.15})$$

In analyzing differential incidence, we impose the equal-yield condition and write all of the tax rates in terms of the commodity tax on  $X$ ,

$$T_{nX} = \frac{X}{K_n} T_x; \quad T_{LX} = \frac{X}{L_n} T_x; \quad T_{nY} = \frac{X}{K_y} T_x;$$

$$T_{LY} = \frac{X}{L_y} T_x; \quad T_{nY} = \frac{X}{Y} T_x.$$

For example, in comparing the two commodity taxes,  $(X/Y) T_x$  is substituted for  $T_x$  in (A.15), and this expression is then compared with (A.12).

To see that a tax on capital in  $X$  and a tax on labor in  $X$  is equivalent to a commodity tax on  $X$ , let  $T_{nX} = T_{LX}$ , and add (A.10) and (A.11). If the two commodity taxes are set equal to each other, expressions (A.12) and (A.15) added, the result is equal to zero. This shows that a general sales tax is equivalent to a proportional income tax on gross income. Interpreting the two com-

modity taxes  $T_{nX}$ ,  $T_{LY}$  as taxes on net value added, it follows that a general value-added tax is equivalent to a proportional tax on net income.

In the more general case, when the two groups have different spending propensities, the change for the demand for  $X$  is given by the expression

$$\begin{aligned} \frac{dX}{X} = & \left\{ \bar{m} \frac{K}{X} - \left[ \bar{m} m \frac{K}{X} f_h \right. \right. \\ & \left. \left. + (\bar{m})(1-m) g_h \frac{K}{X} + \bar{n} n \frac{L}{X} f_h \right. \right. \\ & \left. \left. + (\bar{n})(1-n) \frac{L}{X} g_h + E(f_h - g_h) \right] \right\} d\tau_h \quad (\text{A.16}) \\ & + \left[ E f_h + g_h f_h - \bar{m} m \frac{K}{X} f_h \right. \\ & \left. - \bar{n}(n) f_h \frac{L}{X} \right] T_{sh}, \end{aligned}$$

where  $\bar{m}$  and  $m$  are the capitalists' marginal and average propensities to spend on  $X$ , respectively;  $\bar{n}$  and  $n$  are the workers' marginal and average propensity to spend on  $X$ ;  $g_h$  is the government's average propensity to spend on  $X$ ; and  $E$  is the income-compensated price elasticity of demand for  $X$ , which is assumed to be the same for both groups. In shorthand notation, (A.16) is rewritten

$$dX/X = Z d\tau_h + CT_{sh}. \quad (\text{A.16a})$$

The other eight relations in the solution for  $d\tau_h$  are (A.3)-(A.9). Then,  $d\tau_h$  for a tax on

capital in  $X$  is equal to

$$d\tau_h = \frac{[C(K_s/K_v - L_s/L_v) + B]}{D_1} T_{sh}. \quad (\text{A.17})$$

where

$$\begin{aligned} D_1 = & Z \left( \frac{K_s}{K_v} - \frac{L_s}{L_v} \right) - S_v \\ & - S_s \left( f_h \frac{L_s}{L_v} + f_L \frac{K_s}{K_v} \right). \end{aligned}$$

Note that  $C$  is not unambiguously negative, and the denominator  $D_1$ , unlike  $D$ , is not unambiguously positive. However, as explained in footnote 9 above,  $C$  will be positive and  $D_1$  negative only under very special conditions. Consequently, for qualitative analysis, (A.17) is "equivalent" to (A.10). The structure of the expressions for the other five taxes is the same as expressions (A.11)-(A.15). For example, for a commodity tax on  $X$ ,

$$d\tau_h = \frac{1}{f_h} C \frac{[(K_s/K_v) - (L_s/L_v)]}{D_1} T_s. \quad (\text{A.18})$$

#### REFERENCES

- Due, John. "Sales Taxation and the Consumer," *A.E.R.*, LIII (December, 1963), 1078-84.
- . "The Value Added Tax," *Western Econ. Rev.*, Vol. III (Spring, 1965).
- Harberger, A. C. "The Incidence of the Corporation Income Tax," *J.P.E.*, Vol. LXX (June, 1962).
- Johnson, H. G. "General Equilibrium Analysis of Excise Taxes: Comment," *A.E.R.*, XLVI (March, 1956), 151-56.
- . "International Trade, Income Distribution and the Offer Curve," *Manchester School Econ. and Soc. Studies*, Vol. XXXVII (September, 1959).
- Mende, James. *Mathematical Supplement to Trade and Welfare*. Oxford: Oxford Univ. Press, 1955.
- Musgrave, R. A. *Theory of Public Finance*. New York: McGraw-Hill Book Co., 1959.
- Ralph, Earl R. "The Economic Effects of a Federal Value Added Tax," in Committee on Ways and Means. *Excise Tax Compendium—Compendium of Papers on Excise Tax Structure*. Washington: Government Printing Office, 1964.
- Shoup, Carl S. "Theory and Background of the Value Added Tax," *Proc. Nat. Tax Assoc.* (October, 1955).
- Wells, Paul. "General Equilibrium Analysis of Excise Taxes," *A.E.R.*, XLV (June, 1955), 345-59.