Lecture 1

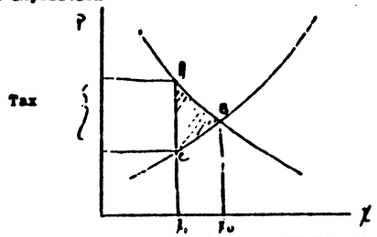
Measurement of Triangles

Consumer surplus and triangle analysis are useful tools in applying theory to the problems in public finance

A brief defense of the three basic postulates:

1) Demand price measures the value of the marginal unit of something to the demander
2) Supply price measures the value of the marginal unit of something to the supplier
3) Adding up—we can add up benefits and costs algebraically among members of the group in question

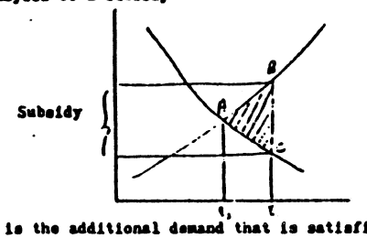
Example: Tax Imposition



Demanders forego Xo-X1 whose value is area AEXoX1 which is the gross cost because those units stop being consumed and produced. The benefit is the value of the freed resources—CBXoX1. Therefore the net cost (the efficiency cost) is ABC triangle.
Important to remember is that the demand and supply curves measure indifference between X and other alternatives.

When the tax is small than people divert less demand; when the tax gets larger, people divert more demand. The demand price reflects the critical point of indifference. The same argument can be made for the supply price of suppliers. We don't really care about indifference vis-a-vis what—motives for tastes are not analyzed here .

Example: Analysis of a subsidy



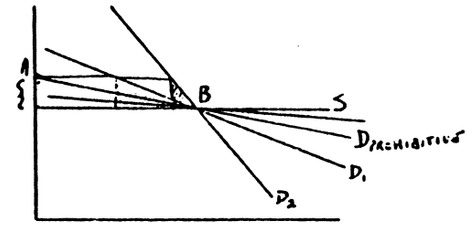
The benefit is the additional demand that is satisfied—ACXoX1

The cost is the value of the incrementally used resources—XoX1BA Therefore the net cost is the triangle ABC.

If the curves are linear, a 10¢ tax or a 10¢ subsidy will have exactly the same efficiency cost.

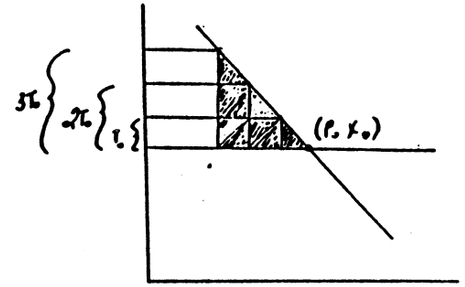
How welfare cost varies with the elasticity of the demand curve:

Assume infinitely elastic supply curves. The more elastic is the demand the greater the welfare cost of a given tax. Reason: Resources are reluctant to be misallocated when demand is not elastic. As demand gets more elastic resources become easier to be misallocated.



Point A is the point at which the tax is prohibitive. ABC is as big as the triangle can get. If demand is still more elastic, the triangle gets smaller because people switch out very easily due to the extreme prohibitiveness of the tax. We recognize all prohibitive taxes to be equivalent, by convention. For purposes of comparison, we use the tax which barely arrives at prohibiting the good as the representative for all prohibitive taxes—this way we can still say the more elastic the demand, the bigger the welfare cost.

How the welfare cost varies with the size of the tax:



T_o has a welfare cost of X
$2T_o$ " " $4X$
$3T_o$ " " $9X$

The Welfare cost varies with the square of the tax
The cost of $nT = n^2T$

Demand: $X = a + bT$ (Because of constant costs "a" includes the βp^o of the demand formulation $X = \alpha + \beta (p^o + T)$
When supply is perfectly elastic: $X = bT$

The triangle's area is $-\frac{1}{2}T\Delta X = -\frac{1}{2}bT^2$

Working with elasticities: $\eta_o = \frac{P^o}{X^o} \cdot \frac{dX}{dP} = \frac{P^o}{X^o} \cdot b$

Therefore, $b = \frac{\eta_o X^o}{P^o}$ and the welfare cost becomes: $= -\frac{1}{2}\eta_o X^o p^o / p^{o2} \cdot T^2$

When we express the tax as a percentage of the net of tax price:

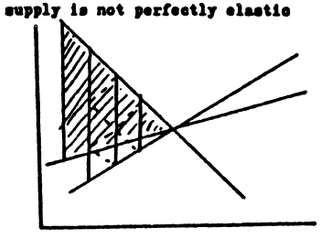
$t = T/p^o$

And when V^o, the value, $= X^o p^o$,

the welfare cost becomes $= -\frac{1}{2}\eta_o V t^2$ (I)

(Here we interpret prices as being initial prices. Everything is evaluated at P^o. Otherwise you can use another price and define "t" accordingly)

Example: When supply is not perfectly elastic



The more elastic the supply the greater the welfare cost.
Δ area $= \frac{1}{2}$ Base x Height : the base is now larger and the height is the same.

Demand $X = a + bP^d$

Supply $X = g + f P^s$

Tax $T = P^d - P^s = dP^d - dP^s$

$P^d = X/b - a/b$ $dP^d = \Delta X/b$

$P^s = X/f - g/f$ $dP^s = \Delta X/f$

$T = \Delta X (1/b - 1/f) = \Delta X(f - b/ fb)$ $\Delta X = fbT/f - b$

The area of the triangle is then $-\frac{1}{2}T\Delta X = -\frac{1}{2} fb/f - b\, T^2$

Since the elasticity of supply is $\epsilon = P^o/X^o \cdot f$

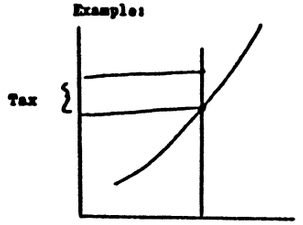
and the area of the triangle becomes: $\frac{-\frac{1}{2}(X^o/p^o \cdot \epsilon)(X^o/p^o \cdot \eta)\, T^2}{X^o/P^o \cdot \epsilon - X^o/P^o \cdot \eta}$

$= -\frac{1}{2}(\epsilon\eta / \epsilon - \eta) X^o p^o T^2 / p^{o2}$

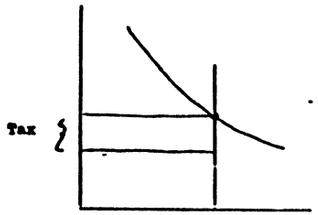
$= -\frac{1}{2}(\epsilon_o \eta^o / \epsilon^o - \eta) V^o t^2$ (II)

When $\epsilon \neq \infty$ this formula does not reduce any further, however when $\epsilon = \infty$ it reduces to formula (I).
ϵ and η enter symmetrically in the expression for the welfare cost. If either one is zero, the welfare cost is zero:

Example:



Tax goes from the consumers' surplus and there is no resource misallocation



Tax goes from the producers' surplus and there is no resource misallocation

Up to now we have seen taxes and subsidies which are price distortions. $p^d - p^s = T$ Distortions which produce a difference between prices faced by consumers and prices faced by producers.

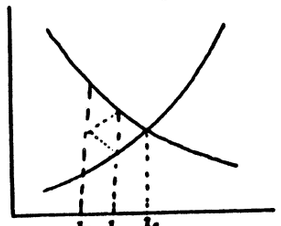
Quantity distortions

Imposing quotas of varying strengths, we get a welfare cost which increases with the square of the quantity dostortion.

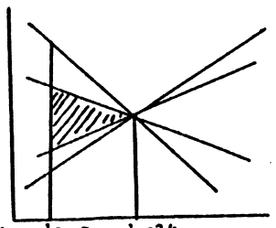
In this analysis we assume that when quantity is restricted, whatever was produced was produced in the most efficient way—i.e. that we remain on the supply curve.

The difference between price and quantity distortions is the way the elasticities affect their welfare costs: the higher elasticities reduce the welfare cost of quotas. The difference is that the base of the triangle doesn't increase as the elasticity increases since $\Delta X \Rightarrow \bar{Q}$ and ΔP is what will be dependent on the elasticity.

Example: Welfare cost increases with the square of the restriction



Example: Welfare cost under differing elasticities



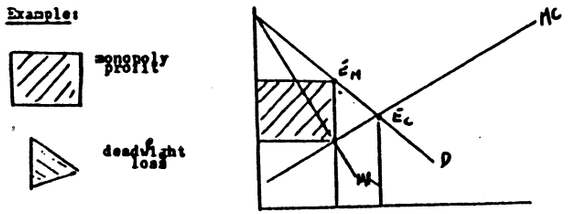
The area of the triangle is $-\frac{1}{2}Q\,\Delta P = -\frac{1}{2}\,Q^2/b$

As the elasticity increases, Δ P declines. The tax which corresponds (in terms of revenue) to the same quota will have to be smaller. If the elasticity of demand is high, people can easily get along without this good. (They replace it very quickly at almost no cost) So, if we take it away, they don't care very much. There is not much welfare loss. With a tax, the opposite happens: if they can replace it easily, they'll react very strongly misallocating resources. If people can't do without it(elasticity of demand is low), they will not move out and just pay the tax out of their consumers' surplus. With a quota, no matter how much they are willing to pay, they can't get it and the welfare loss becomes great.

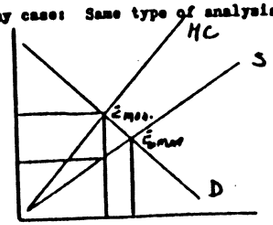
The Monopoly Tax

Movement from a competitive equilibirum to a monopoly equilibrium is the same as the movement brought about by a tax. This tax is privately imposed and collected.

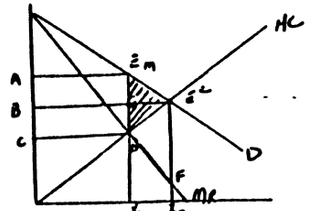
Example:



The Monopsony case: Same type of analysis



From the monopolist's view: Analyzing the tax looking from the price rather than quantity axis



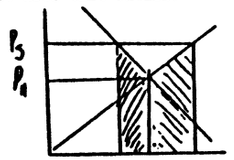
Consumers lose ABE^{c}E^{m}by paying more; producers' lost surplus is BE^{c}DC; the monopoly gets AE^{m}DC
The consumers are losing E^{m}E^{c}X^{o}X^1 but get back in terms of resources to spend elsewhere DFX^{o}X^1 so their net loss is E^{m}E^{c}FD .
The monopolist is gaining back DFE^c which is equivalent to AE^{m}BG minus GE^{c}D . AE^{m}GB is associated with the exercise of monopoly power. BGDC is associated with monopsony power.

Therefore the net loss to society remains the triangle DE^{m}E^c

Lecture 2

Agricultural price support

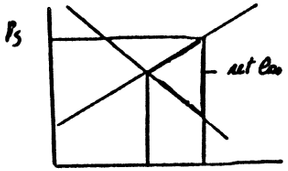
Objectives: to keep price received by producers higher than equilibrium price
What cost does that entail?



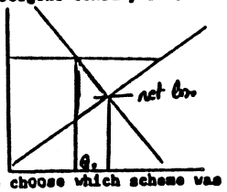
Cost in terms of additional resources

Cost in terms of foregone consumption

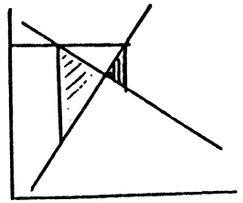
The government is accumulating stocks. It can do something with the stocks. Either it can sell them at what the market will bear...

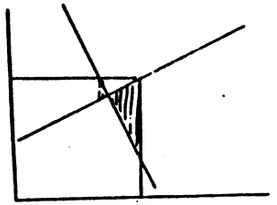


(then it produces a gain to consumers to offsetthe cost in terms of additional resources used. This amounts to a demand subsidy)
or the governemtn can restrict productions: hold producers down to production at Q^0, producing a sellers' gain to offset the cost in terms of forgone consumption. This is a supply subsidy.



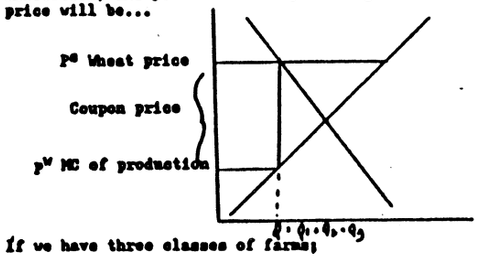
If we had to choose which scheme was better, we would have to look at the elasticities:
With an elastic supply we would prefer working with supply subsidies





With an elastic demand we would prefer working with demand subsidies

If we give farmers marketing coupons (assuming an arbitrary and free allocation) and permit that they be bought and sold: the coupon price will be...

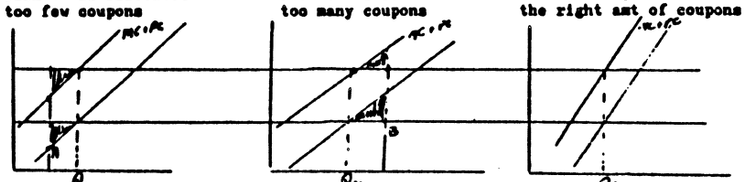


If we have three classes of farms:

Farm 1 was allocated too few coupons

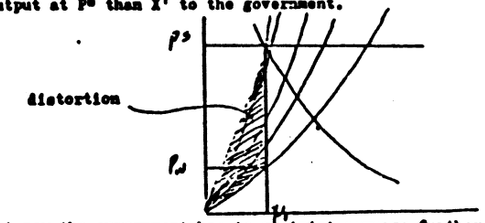
Farm 2 was allocated too many coupons

Farm 3 had just the right amt of coupons



If the farmer is at A because he has not enough coupons, he pays for the coupons in order to bring him to critical level.
If the farmer is at B, he sells his coupons because he would have more coupons than necessary to reach the critical level

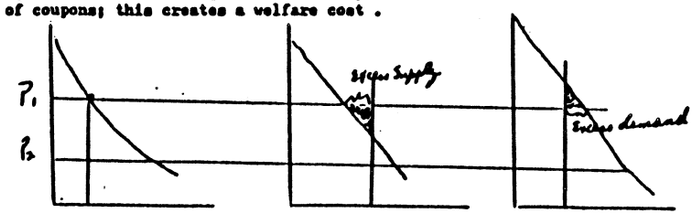
In the U.S., they allocated acreage instead of coupons. Restricting acreage creates problems because acreage is not the only input in agriculture. It will pay for the farmers to use the other inputs beyond the socially optimal efficiency level in order to sell more output at P^0 than X^1 to the government.



But now the government has to restrict acreage further to keep output at X_1 until finally the marginal cost of a unit of other factors reaches P^0. This imposes an additional cost to society.

Rationing Situations

Many of the additional costs due to particular schemes can be avoided by allowing further transactions. If it is decided that everybody would get the same rationing and it is a capital offense for transactions of coupons; this creates a welfare cost .

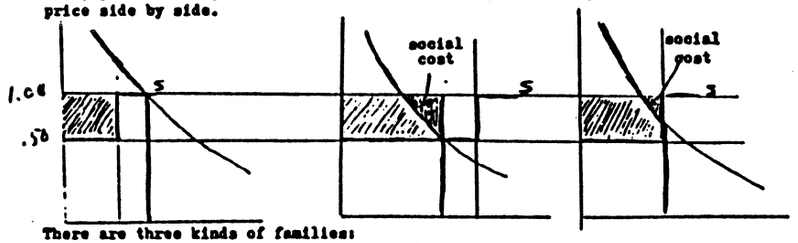


P1 is the price at which excess demand equals excess supply . the price of the ration coupon will be the excess between the rationed price P2 and the market clearing price P1.

A more interesting problem,

Example

Milk is being partly subsidised. Each family will be allocated a coupon for one gallon per child which will allow them to buy one gallon at the cheap price. The government thus creates a cheap and an expensive price side by side.



There are three kinds of families:

The ration is less than they would want to buy	Ration is more than they would want to buy	They start perceiving a subsidy
Family A	Family B	Family C

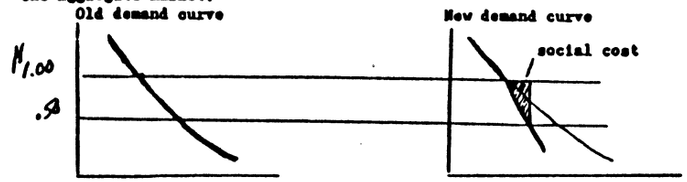
For family A, the rationing is an income transfer, an inframarginal subsidy --there is no social cost

For family B, they perceive the subsidy but not the rationing —there is a positive social cost

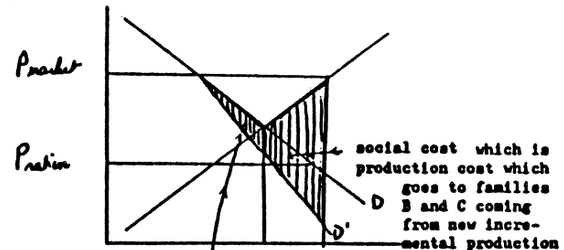
For family B, they start perceiving a subsidy and the social cost is positive

The new demand curves fall short of the old. The inefficiency and social cost in this example comes from the normal subsidy cost plus the inefficiency of giving some people more than what they would otherwise buy at the expense of people who would buy more—This is due to the inability to transact with coupons.

The Aggregate market:



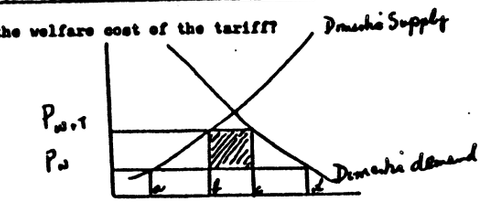
We plotted a new Aggregate demand curve following only the relevant segments reflected in the market.(Dark black lines in the previous diagrams). There are two costs present now: a) the normal cost of a subsidy and b) the inefficiency cost incurred by giving some people more than what they'd buy otherwise at the expense of people who would buy more.



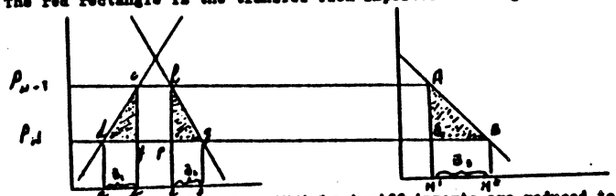
Lecture 3

Problems connected with international trade

What is the welfare cost of the tariff?



Production goes up from "a" to "b"
Consumption is reduced by "cd"
The red rectangle is the transfer from importer to the government



M^o are the initial imports. With the tariff imports are reduced to M'.

abcd has an area equal to $\frac{P^w + (P^w + T)}{2} \cdot B1$

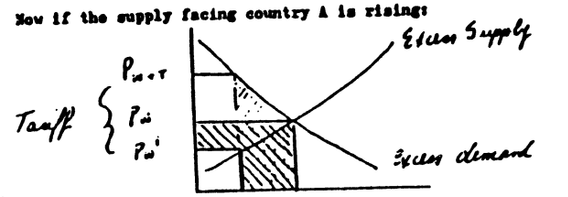
efgh has an area equal to $\frac{P^w + (P^w + T)}{2} \cdot B2$

$M'M^oAB$ has an area equal to that of abcd+ efgh because $B3 = B1 + B2$

abqd and efgp is the value of released resources to be used for producing other things and it is equivalent to area $M'M^oBC$ because $(B1 + B2)\ P^w = B3\ P^w$
So the triangles of net cost are qcd (producer cost) + pgh (consumer cost) = which are equivalent to triangle ABC

This last example assumed the world price was given

Now if the supply facing country A is rising:



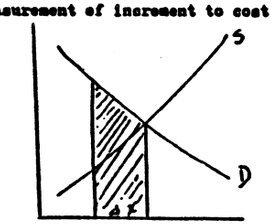
When country A puts a tariff, the world price falls. The area under

the excess supply curve has no meaning for country A. The country which imposes the tariff has lost the value of the product formerly imported therefore the cost is measured by the blue trapezoid. We measure the resource saving under the marginal cost curve because the saving is greater than the area under the excess supply curve. They used to pay P^w for M^o, now they pay $P^{w'}$ for M^o. The saving now may be greater than the cost.i.e. the red hatched area might be greater than the blue hatched area.

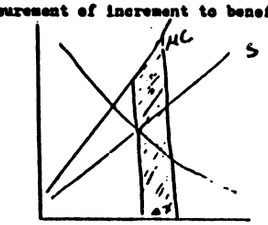
The optimum tariff

When would a small increase in tariff cause an increment to benefit equal to an increment of cost?

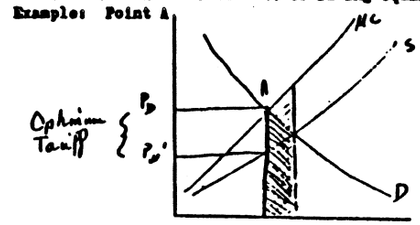
Measurement of increment to cost



Measurement of increment to benefit



The optimaltariff is when the increment to saving equals the increment to cost. Example: Point A

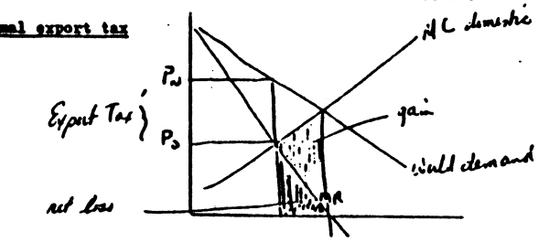


$P^w\ (1 + 1/\epsilon) = P^w\ (1 + t)$ i.e. marginal cost = demand price

$t = 1/\epsilon$ elasticity of supply of imports by world

The optimum tariff is exploitation of monopsony position--the single justification for a tariff as a first best solution from the standpoint of the country. This is so only when the country has an upward rising supply curve: when its demand can affect the supply price.

The optimal export tax



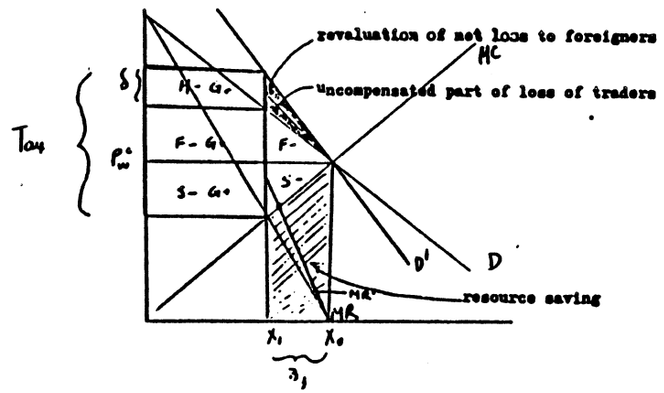
The condition for the optimal export tax is when the gain and net loss are equal. This will be satisfied when MR = MC

$P^w (1 + 1/\eta) = P^w (1 - t_x)$

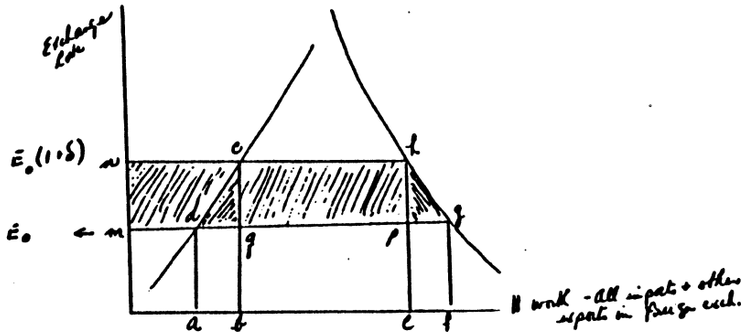
$t_x = 1/\eta_i$ (η elasticity of world demand for export i) (the tax is defined as a percentage of gross price)

The tax comes into the picture when the market organization is competitive or else it won't do any good. It would lead to a monoptimal position if there were a monopoly firm which would react to the tax by restricting production by more.

When the export good is a major item in foreign trade of country A: (for example coffee in Brazil)
We look at the benefits and costs from the point of view of the country which puts on the tax. In the process of putting on the tax, the foreign exchange price rises and the new demand curve is expressed in terms of the new exchange rate.
The tax imposition reduces the supply of foreign exchange and bids up the price of the exported good. The government is getting an extra revenue from the change in the exchange rate—the top rectangle whose area is $\delta \times X_1$. δ is the amount of the devaluation due to the imposition of the tax-- $\delta = f(\tilde{t})$.



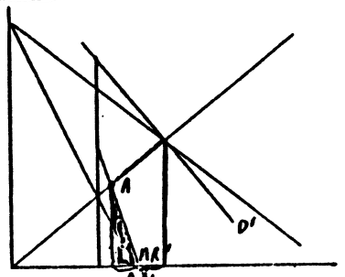
G—government
F—foreigners
S—suppliers
M— importers



In the initial equilibrium, area afgd is equal to what the country got from foreigners in local currency
Area bqpe is the cruzeiro value of new exports valued at E^0
Area qchp is the extra amount of cruzeiros value of new exports due to the rise of the exchange rate
Area mnhg is the loss to buyers due to the rise in the exchange rate
Area mncd is the gain to sellers due to the rise in the exchange rate
Areas qdc and pgh are the loss which is uncompensated part of the loss of other traders
Area qchp is what the government gains—the compensated loss

Lecture 4

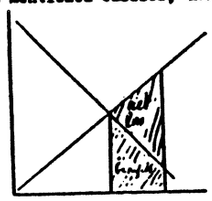
The optimal export tax (cont.)



In this case the optimum position is the point at which the modified marginal revenue equals marginal cost—such as point A
If we have an exchange rate effect the optimum change in quantity is ΔX_1: there is less total restriction on the quantity than if we did not have exchange rate effects.
The optimum position is thus the point where the gain due to released resources—(the trapezoid outlined in blue)— balances off the cost due to marginal revenue forgone—(the black area)— plus the excess burden we impose on the importing group—(the red area).

Distributional weights in cost-benefit analysis

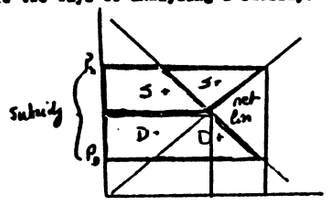
As mentioned earlier, there are two ways of analysing a subsidy:



From the standpoint of the quantity axis

The costs are represented by the large trpezoid and the benefits by the small trapezoid. The triangle is the net loss.

or



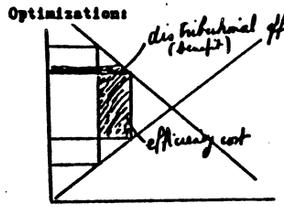
From the standpoint of the price axis

The government loses the whole amount of the subsidy. S+ is the producers' gain and D+ is the demanders' gain—The net loss is the triangle.

That dualism no longer applies when we use weights.
By convention, we assign the following weights:

The subsidy doesn't help the egg farmers. The policy to follow is to subsidize the farmers with the most inelastic supply—"more bang for the buck" on P+
If either ϕ_s or ϕ_d or both are less than one then a subsidy will work
If either ϕ or ϕ_d or both are greater then one then a tax will work

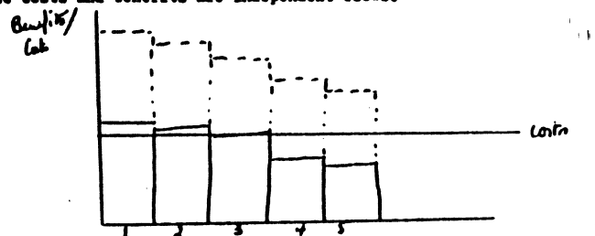
Optimization:



The optimal tax is where the marginal gain assigned to the transfer just barely counterbalances the efficiency cost

This leads to high prices being paid to transfers.

In the arena of project evaluation: Consider a sequence of projects whose costs and benefits are independent flows.



If the beneficiary has $\phi = 2$, then applying distributional weights all benefits must be raised by a factor of two. Now even project five will be approved.

However, suppose unskilled labor supply is infinitely elastic. Even with distributional weights the project would not be accepted. (Fig. I)

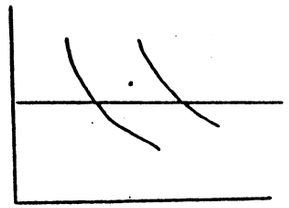


Fig I

Where the project pays the market wage and bids it up, the real benefit accrues mostly to people in the labor market who are not on the project's wage bill. (Fig II)

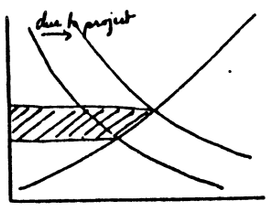
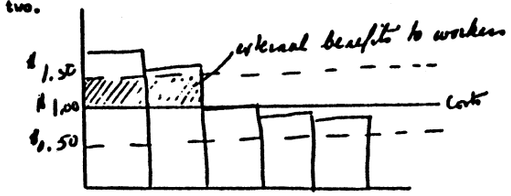


Fig II

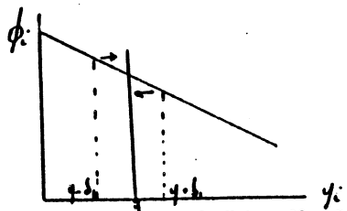
When we consider projects we deduct from costs the surplus given to workers and when we have higher weights then it is the surplus which is multiplied. The social opportunity cost is now lower and you could accept the project.
Suppose the project pays $1.50 for wages. Suppose also that workers have a weight of two.



If project wage is $1.50., the surplus is $.50 which is multiplied by the dist. weight of 2 to get a social cost of $.50. This will allow project four to be undertaken.
Our evaluation ends up depending on purely financial considerations and on the distributional weights of those we are taxing.

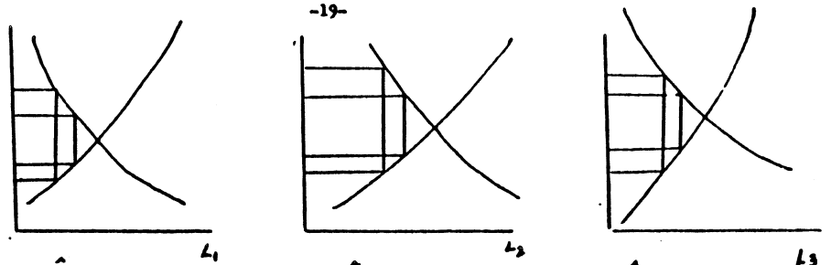
We are always counterbalancing a distributional gain with an efficiency loss.

Income Taxes



Does this generate a progressibe income tax? Not really. If labor supply is completely inelastic we are dealing with pure rents—no efficiency costs. We end up with confiscatory taxation.
If there are administrative costs, there would be a small band around $\bar{y}$ ($y - \delta_1$ to $y + \delta_1$).

Suppose we have a set of noncompeting groups: $\phi_1 < 1$

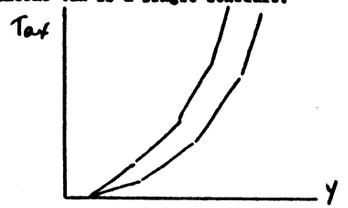


$\hat{t}_1 = \frac{(1 - \phi_1)}{1}$ $\hat{t}_2 = \frac{(1 - \phi_2)}{2}$ $\hat{t}_3 = \frac{(1 - \phi_3)}{3}$

$\phi_1 > \phi_2 > \phi_3$

If $\epsilon_1 = \epsilon_2 = \epsilon_3$ the tax will be progressive. $\hat{t}$ is a proportionate rate of tax. When we raise the tax we take in, we increase the percentage of entire income earned.

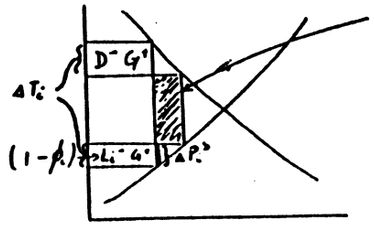
But the real income tax is a single schedule:



A higher tax on the poor raises tax for all other groups. There is an externality effect: A distributional externality from changing the tax at the low end of the scale.

Lecture 5

Difference between raising a proportional tax on non-competing group versus a tax affecting that group within a context of global income tax structure:
Assume the weight of demanders is 1--neutrality



Loss to the government = $T_1 \Delta L_1$

Gain = $(1 - \beta_1)\Delta L_1 |\Delta P^s|$

Optimum tax = $\hat{t}_1 = \hat{T}_1$

$$= (1 - \beta_1) \left(\frac{L_1 |\Delta P^s|}{P^s \Delta L_1}\right)$$

$$\hat{t}_1 = \frac{(1 - \beta_1)}{\epsilon_1}$$

The social value of the tax will depend on which group it will affect
$\sum_j (1 - \beta_j)\Delta T L_j$
So the total effect of the tax is the prior effect on L_1 plus later effects (externalities):

$$T_1 \Delta L_1 = (1 - \beta_1) L_1 |\Delta P^s| + \sum_j (1 - \beta_j)\Delta T_1 L_j$$

L_j does not change because we are dealing with the inframarginal groups

Dividing both sides by $P_1 \Delta L_1$:

$$\frac{T_1}{P_1} = \frac{(1 - \beta_1) L_1 |\Delta P^s|}{P_1 \Delta L_1} + \sum \frac{L_j}{L_1} \frac{(1 - \beta_j)\Delta T_1 / \Delta P_1}{P_1 \Delta L_1 / \Delta P_1 L_1}$$

$$= \frac{(1 - \beta_1) L_1 |\Delta P^s|}{P_1 \Delta L_1} + \sum \frac{L_j}{L_1} \frac{(1 - \beta_j) \Delta T_j}{\epsilon_1 \Delta P_1^s}$$

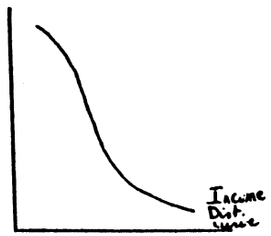
$$\frac{\Delta T_j}{\Delta P_1^s} > 1 \Rightarrow \frac{1/\epsilon + 1/|\eta|}{1/\epsilon}$$

If $\frac{(1 - \beta_j)}{1} = .1$ Group L_1 has $\beta_1 = .9$ $\epsilon_1 = 1$

Quantifying the externality: Assume $\frac{\Delta T_j}{\Delta P_1^s} = 2$

$$\hat{t} = \frac{T_1}{P_1} = .1 + 2 \sum \frac{L_j}{L_1} (1 - \beta_j)$$

We think of an income tax affecting only the upper tail of the distribution where we have a Pareto law applying. So a very simpple kind of representation of this sort of distribution would be an exponential or geometric progression.



Group	L_j/L_1	β_j	$(1 - \beta_j)$
2	1/2	.81	.19
3	1/4	.72	.28
4	1/8	.64	.36
5	1/16	.57	.43
etc...			

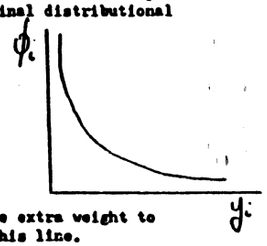
Pop. in each successive group is half of the previous group so L_j/L_1 are changing

For group 1 the eff. tax would be: $\hat{t} = .1 + 2\ (.24) = 58\%$

For group 2 the eff. tax would be: $\hat{t} = .19 + 2\ (.31) = 81\%$ because the L_j/L_1 changed so that group 3's ratio becomes 1/2, group 4's ratio becomes 1/4 etc...

For the last group, group 5, the eff. tax would be: $\hat{t} = 43\%$
It becomes a regressive tax.
This shows how distributional weights lead to nonsense because we are pushing the tax to where marginal efficiency cost is equal to the external benefits. It is "like throwing away the baby with the bath water".
If you want to use distributional weights, you must put a limit to the price you are willing to pay for any implicit transfer.
For example; the Mc Namara function: He suggested, in Nairobi 1973, that countries with same rate of growth in income should get the same weight. If the recipient group has a β of 10 times that of another group, you can transfer up to the point where .9 of the transfer is lost in the transfer and still justify your transfer. "It's like carrying ice cream across the desert until 90% melts."
The difference in marginal weights is the point to which we push efficiency losses to counterbalance the marginal distributional gains.
The problem with the McNamara function is that the factors can go to infinity--it invites incredible marginal inefficiencies in generating transfers. It is a member of a general class of functions:

$\log \beta_i = 1 - \log Y_i$
$\log \beta_i = \gamma - \beta \log Y_i$



An alternative: Draw a poverty line and give extra weight to transfers which would benefit people below this line.
The distribution function should be of the sort:

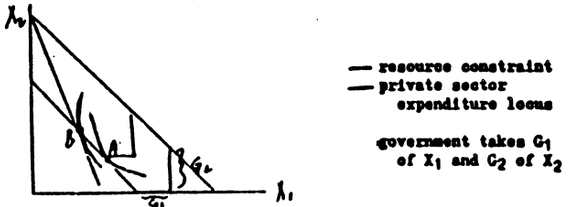
f(poverty level, % inefficiency cost we are willing to bear to effect the transfer)

Economists should not get involved in political opinions but should be professional economists--and stick to the conventions: the three postulates----is N' opinion.

Lecture 6

Demand and Supply curves

What we are working with are substitution effects—compensated demand curves. However it does not have to be so; we can modify the analysis to include income effects.



— resource constraint
— private sector expenditure locus

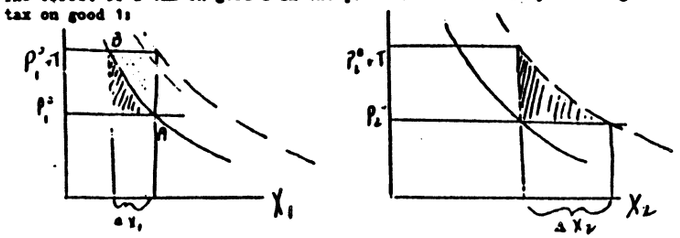
government takes G_1 of X_1 and G_2 of X_2

So private sector is constrained by the expenditure locus.
If X_1 and X_2 are the only relevant goods—an equal rate excise tax on both goods would look like the expenditure locus whereas an excise tax on X_1 would have the property of generating a change in the slope of the constraint. One can use both and raise the y intercept by income subsidies and change slopes by excise tax on one good.
Assume you start at an equilibrium like point A— $\sum P_1^0 X_1 = Y^0$ is the constraint. The true compensating variation in income is bounded by using the two measures. At point B we give people income to enable them to get the old bundle of goods at the new prices. It cuts through the first budget constraint and the consumer is on a higher indifference curve. This is an overstatement.
$\sum P_1^1 \Delta X_1$ overstates the true compensation in income from going from A to B.
If we consider $\sum P_1^0 X_1 = Y^0$, this measure results in an understatement of the true compensation ; it is $\sum P_1^1 \Delta X_1$.
So we take the average:

$$\frac{\sum P_1^0 \Delta X_1 + \sum P_1^1 \Delta X_1}{2}$$

$$= \frac{\sum P_1^0 \Delta X_1 + \sum (P_1^0 + T)\Delta X_1}{2} = \frac{\sum T_1 \Delta X_1}{2} = \frac{1}{2} \sum T_1 \Delta X_1 = \frac{1}{2} T_1 \Delta X_1$$

If we were to put an equal percentage tax on X_2 then you would go back to A from B.
The effect of a tax on good 2 in the presence of an already existing tax on good 1:

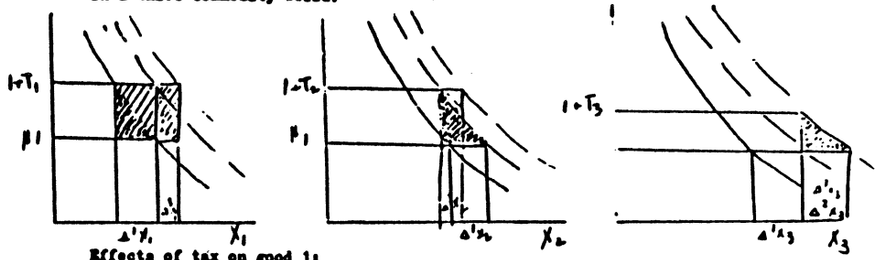


There is no welfare connotation to a shifting demand curve for X_2.
You put a tax on X_1— Welfare change: $\frac{1}{2} t \Delta X_1 < 0$

You put a tax on X_2, given a tax on X_1—Welfare change:
$\frac{1}{2} t \Delta X_2 < 0$ plus $t \Delta X_1 > 0$
Sum of the welfare changes is 0.

There is a welfare connotation to shifting the demand curve for X_1 (because it already has a tax). Going back to A from B involves a subsidy or giving back money to the people.

In a three commodity world:



Effects of tax on good 1:
$\sum P_1^0 X_1 = K$ $\sum P_1^0 \Delta X_1 = 0$ $P_1^0 = 1$

$\Delta^1 X_1 + \Delta^1 X_2 + \Delta^1 X_3 = 0$

the own effect: (-) $\frac{1}{2} S_{11} T_1^2$

Effects of a tax on good 2 given the tax on good 1:
$\Delta^2 X_1 + \Delta^2 X_2 + \Delta^2 X_3 = 0$
effect on good one: (+) $S_{12} T_2 T_1$
own effect: (-) $\frac{1}{2} S_{22} T_2^2$

Effects of a tax on good 3 given taxes on goods 1 and 2:
$\Delta^3 X_1 + \Delta^3 X_2 + \Delta^3 X_3 = 0$
effect on good 1: (+) $S_{13} T_3 T_1$
effect on good 2: (+) $S_{23} T_3 T_2$
own effect: (-) $\frac{1}{2} S_{33} T_3^2$

Own effects in total:
$\frac{1}{2}(S_{11} T_1^2 + S_{22} T_2^2 + S_{33} T_3^2)$

Effects on other goods
$S_{12} T_1 T_2 + S_{13} T_1 T_3 + S_{23} T_3 T_2$

$S_{12} = S_{21}$ ~~Symmetry property~~ Slutsky's substitution property

So, $S_{12}T_1T_2 = \frac{1}{2} S_{12}T_2T_1 + \frac{1}{2} S_{21}T_1T_2$

$S_{13}T_1T_3 = \frac{1}{2} S_{13}T_1T_3 + \frac{1}{2} S_{31}T_1T_3$

$S_{23}T_2T_3 = \frac{1}{2} S_{23}T_2T_3 + \frac{1}{2} S_{32}T_2T_3$

Can also be written as, $\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} S_{ij}T_iT_j$ or $\frac{1}{2}\sum\sum S_{ij}P_i^0P_j^0t_it_j$

Generally: $L = U(x_1, x_2, x_3) - \lambda(P_1X_1 + P_2X_2 + P_3X_3 - T^0)$

$\Delta U = \sum U_i \Delta X_i + \frac{1}{2}\sum\sum U_{ij}\Delta X_i \Delta X_j$

First order conditions from differentiating the Lagrangian:

$U_i = \lambda P_i$

$\sum U_i^0 \Delta X_i = \lambda \sum P_i^0 \Delta X_i = 0$

$U_i = U_i(x_1, x_2, \ldots x_n)$

$\Delta U_i = \sum U_{ij}\Delta X_j$

$\Delta U_i = P_i \Delta\lambda + \lambda \Delta P_i$

$\Delta U_i = \frac{1}{2}\sum \Delta U_i \Delta X_i$

$\Delta U = \frac{1}{2}\Delta\lambda \sum P_i^0 \Delta X_i + \frac{1}{2}\lambda \sum \Delta P_i \Delta X_i$

$\frac{\Delta U}{\Delta \lambda} = \frac{1}{2}\sum \Delta P_i \Delta X_i = \frac{1}{2}\sum T_i \Delta X_i = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} S_{ij}T_iT_j$

$\Delta X_i = \sum s_j T_j$

Restrictions: $\sum P_i^0 S_{ij} = 0 \quad \sum a_i \eta_{ij}^0 = 0 \quad \eta_{ij}^s = 0$

Hicks- Slutsky substitution terms:

$S_{ij} = S_{ji}$ $\qquad a_i\eta_{ij}^0 = a_j\eta_{ji}^0$

$S_{ii} \leq 0$ $\qquad \eta_{ii}^s \leq 0$

$S_{ij} = \frac{\partial X_i^0}{\partial P_j}$ $\qquad \eta_{ij}^0 = \frac{\partial X_i^0}{\partial P_j} \cdot \frac{P_j}{X_i}$ $\qquad a_i = \frac{X_i P_i}{\sum X_i P_i}$

So the change in welfare is

$\frac{\Delta U}{\Delta \lambda} = \frac{1}{2}\sum T_i \Delta X_i$ Dividing by λ converts utility into money by subjective judgment on the marg. utility of income

this is the formula for the generalized triangle which when approximated by a Taylor's series can be restated as:

$\Delta V = \int_{t^0}^{t^1} \sum D_i(Z) \frac{\partial X_i}{\partial Z} dZ$ where Z is the distortion

(a change in X_i will only have a cost or a benefit if " i " is a distorted activity)

Lecture 7

Derivatives of the welfare cost formula

If you have a tax on all goods, then the welfare cost O:

$W = \frac{1}{2}\sum\sum S_{ij}T_iT_j$

The own effects are negative, cross effects are positive but own effects dominate— proof: semidefiniteness of bordered hessian.

I $\Delta V = \frac{1}{2}\sum T_i \Delta X_i$

From $\Delta W = \frac{1}{2}\sum\sum S_{ij}T_iT_j$ we derive: $\Delta W = (-)\frac{1}{2}\sum_{i,j} P_i^0 P_j^0 S_{ij}(t_1 - t_2)^2$

using the adding up property: $P_1^0 S_{11} + P_2^0 S_{21} = 0$

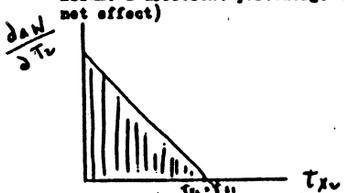
(The number of independent substitution terms $\frac{n(n-1)}{2}$ = the number of off-diagonal elements divided by two)

If $t_1 = t_1$ then the welfare cost = 0

Implications: Only the relative distortions that count.
Equivalent subsidies and taxes have the same effect.

These three ways of measuring welfare cost are equivalent but may be more convenient depending on how the question is posed.

Applications: A tax on good 2 in the presence of a tax on good 1
Assume a different percentage tax: (if they are equal there is no net effect)

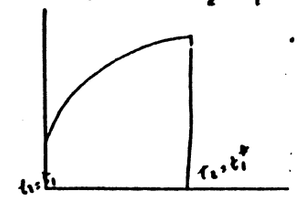


$\Delta W = -\frac{1}{2} S_{12}(t_1 - t_2)^2$

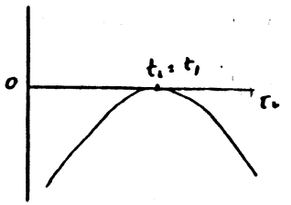
$\frac{\partial \Delta W}{\partial T_2} = S_{12}(t_1 - t_2)$

This is an instantaneous effect on welfare due to T_2.

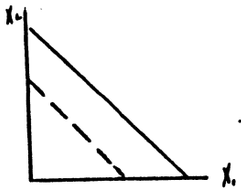
Integrating the previous graph you get the cummulative effect on welfare compared with $t_2 = t_1$



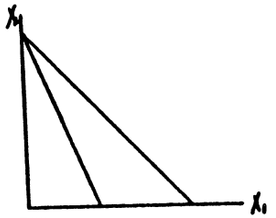
The effect on welfare compared to optimum:



Excise Tax vs. Income Tax

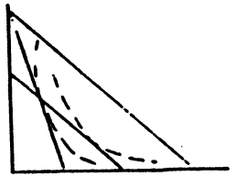


Effect of an income tax



Effect of an excise tax

Why an income tax is preferable to an excise tax:



When you have only two goods the income tax looks preferable to the excise tax. But if you include leisure you can no longer say an income tax is better than the excise tax without looking at the details of the case:

Excise tax on good 1 distorts the choice between 1 and 2, 1 and 3 but not 2 and 3
Excise tax on good 2 distorts the choice between 2 and 1, 2 and 3 but not between 1 and 3
Excise tax on good 3 distorts the choice between 3 and 1 , 3 and 2 but not between 1 and 2.

Once you have the labor-leisure choice you cannot make a choice between income tax and excise tax without working through the specifics of the case. The excise tax would be better than an income tax if it could catch what the income tax does not. For example the tax on a commodity which is a complement of leisure (or the less substitutable commodity for leisure). As long as the two goods are equally substitutable then the excise tax is equivalent to the income tax.

Lecture 8

Conditions under which an income tax will be better than an excise tax—

The income tax is a suboptimal way of raising revenue, a second best solution. The income tax does not tax leisure.
The excise tax on goods 1 and 2 is equivalent to a subsidy on leisure and a subsidy on goods 1and 2 is equivalentto a tax on leisure, given linear demand and supply curves.

Welfare cost of a tax on good 1: $WC_1 = -\frac{1}{2}(S_{11}T_1^2)$

Welfare cost of a ~~tax on good two given a tax on good one~~ INCOME TAX: $WC_2 = -\frac{1}{2}(S_{11}T_1^2 + 2S_{12}T_1T_2 + S_{22}T_2^2)$

If $T_1^2 = t^2$ $T_1T_2 = t^2$ $T_2^2 = t^2$ because initial prices = 1

Then $WC_2 = -\frac{1}{2} S_{33} t^2$

$X_1T_1 = (X_1 + X_2)\, t$

$\frac{X_1}{X_1 + X_2} T_1 = t$

$a_1 T_1 = t$ where a_1 is the fraction of good one in National Income

$WC_1 - WC_2 = -\frac{1}{2} S_{11}T_1^2 \left(1 - a_1^2 \frac{S_{33}}{S_{11}}\right)$

$\frac{-S_{33}}{X_1 + X_2} = \epsilon$ The elasticity of the supply of labor

S_{33} is the slope of leisure demand function or the labor supply function

$\frac{P_3 \partial L}{L \partial P_3} = \epsilon$ $P_3 = 1$ $L = X_1 + X_2$ $\frac{S_{11}}{X_1} = \eta_{11}$

Therefore,

$$WC_1 - WC_2 = -\tfrac{1}{2} S_{11}T_1^2 \left(1 - a_1 \frac{\epsilon}{|\eta_{11}|}\right)$$

If labor were completely inelastic $-\frac{1}{2}S_{11}T_1^2$ would exactly measure the WC of the excise tax and the WC of the income tax = 0.

Example: Tobacco

$\eta_{11} = -.6$ $\epsilon = .2$ $a_1 = .02$

$WC_1 - WC_2 = \text{Triangle}\left(1 - \frac{.02\,(.02)}{.6}\right) = \text{Triangle}\,(.99)$

Value added tax

Consider $\eta_{11} = -.1$ $a_1 = .8$

For a aggregate good: $a_1 \eta_{12} = a_2 \eta_{21}$ $\qquad \eta^s_{22} = -\eta^s_{21} - \eta^s_{23}$

So $WC_1 - WC_2$ = Triangle (-.6)
Triangel analysis does not do any good anymore . What lies behind this is that good one is a complement to leisure, or less substitutable for leisure than good 2 so the excise tax is better than an income tax. $0 < \eta^s_{13} < \eta^s_{23}$

The optimum tax on goods 1 and 2: Given that a certain revenue is raised.

$$\text{Min}_{T_1, T_2} \quad WC = -\tfrac{1}{2}\left(S_{11}T_1^2 + 2S_{12}T_1T_2 + S_{22}T_2^2 \right) - \lambda(X_1T_1 + X_2T_2 - R)$$

$$\frac{\partial WC}{\partial T_1} = -S_{11}T_1 - S_{12}T_2 - \lambda X_1 - \lambda(S_{11}T_1 + S_{21}T_2) = 0$$

$$\frac{\partial WC}{\partial T_2} = -S_{12}T_1 - S_{22}T_2 \lambda X_2 - \lambda(S_{12}T_1 + S_{22}T_2) - \lambda X_2 = 0$$

$$\lambda X_1 = -(1+\lambda)(S_{11}T_1 + S_{21}T_2)$$

$$\lambda X_2 = -(1+\lambda)(S_{12}T_1 + S_{22}T_2)$$

$$\Lambda X_1 = S_{11}T_1 + S_{21}T_2 \qquad \text{where } \Lambda = \frac{\lambda}{1+\lambda}$$

$$\Lambda X_2 = S_{12}T_1 + S_{22}T_2$$

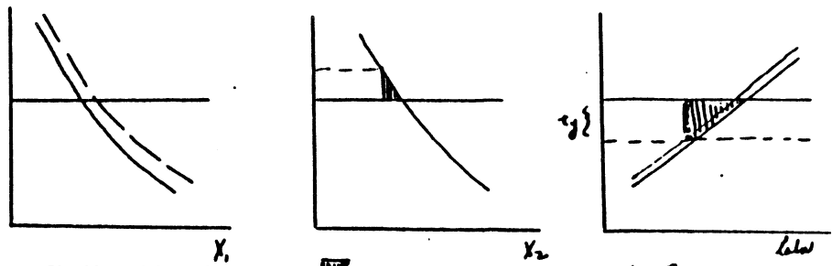
$$\hat{T}_1 = \frac{\begin{vmatrix} \Lambda X_1 & S_{21} \\ \Lambda X_2 & S_{22} \end{vmatrix}}{\Delta} = \frac{\Lambda[X_1S_{22} - X_2S_{21}]}{\Delta} = \frac{\Lambda[-X_1S_{12} - X_1S_{13} - X_2S_{21}]}{\Delta}$$

$$\hat{T}_2 = \frac{\begin{vmatrix} S_{11} & \Lambda X_1 \\ S_{12} & \Lambda X_2 \end{vmatrix}}{\Delta} = \frac{\Lambda[-X_1S_{12} + X_2S_{11}]}{\Delta} = \frac{\Lambda[-X_2S_{12} - X_2S_{13} - X_1S_{12}]}{\Delta}$$

Optimum tax turns out to be greater on the good which is a complement to leisure or less than average substitute to leisure.

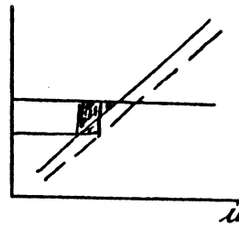
$\hat{T}_1 > \hat{T}_2$ when $\frac{S_{23}}{X_2} > \frac{S_{13}}{X_1}$ or when $\eta^s_{23} > \eta^s_{13}$

$\hat{T}_2 > \hat{T}_1$ when $X_2S_{13} > X_1S_{23}$ or when $\eta^s_{23} < \eta^s_{13}$

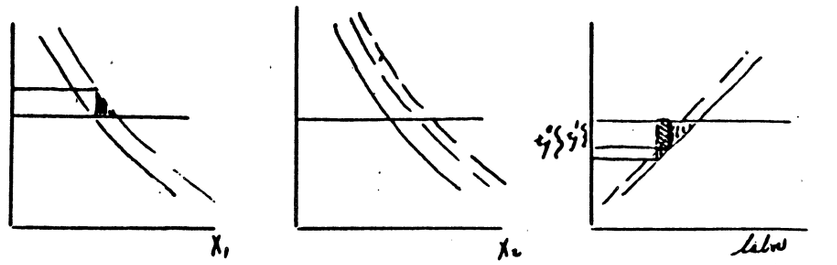


Starting with an income tax then we put a tax on X_2

If labor were a complement, then we would have a gain:



Keeping revenue constant, as we increase the tax on X_2, we decrease the income tax: We start with an income tax , then we reduce the rate of the income tax assuming both X_1 and X_2 are substitutes for leisure. Now we put a tax on good 1. This increases the cost because good 1 is less than average substitute for leisure.

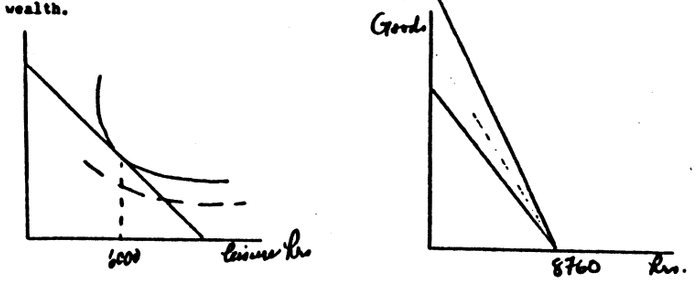


What goods are complements to leisure? sporting goods, alcohol, housing etc; everything is a substitute for leisure. This is the wrong way to measure the substitution effect.

Let's do an experiment:
Suppose a law is enacted which changes the work week—everyone works two hours less. There is no income effect—there is a loss in income but there is more leisure. People would not behave differently than if 2 hours worth of income were taxed away. The only difference is that there are more free hours.

When you measure the income elasticity of leisure: (Time series estimate) (Income elasticities are just cross elaticities with respect to leisure because most income is earned at the expense of leisure).

Income effect = elasticity of substitution with respect to the price of leisure + elasticity with respect to the true inherited wealth.



Lecture 9

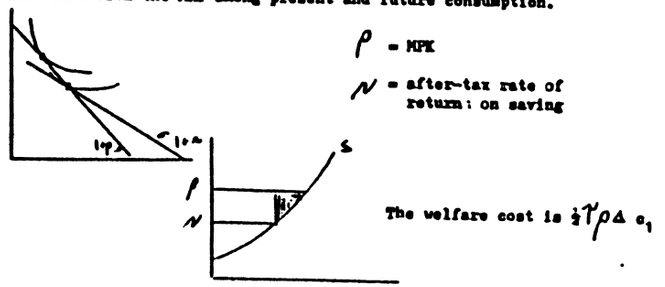
Labor-leisure and Saving decision

Complements to leisure: If people react the same way when income decreases by 25% due to a tax or when income decreases by 25% due to a reduction in working hours then

$$\frac{\Delta w}{w} = k \frac{\Delta y}{y}$$

then the test candidates for complements to leisure are inferior goods. And they should be taxed more heavily.

With enough assumptions, you can show that an income tax is better than an excise tax. We introduce complications when we introduce leisure. We also introduce more complications when we introduce the savings decision.
The income tax doubles the taxation of savings—income is taxed before it is saved and the income gotten from the savings is also taxed. In the presence of the tax, the MRS becomes different from the MRT among present and future consumption.



The welfare cost is $\frac{1}{2}\tau\rho\Delta c_1$

When we have a world with a labor-leisure choice (a life cycle model with a retirement period) C_1 C_2 and L. In a way the model allocates 8760 hours per year in period one. Consider the neutral tax

$$T = T_1 = T_2 \quad \text{and} \quad T_1 = -T_2$$

It creates a distortion between C_1 and L equal to that between C_2 and L but no distortion between C_1 and C_2.

Treating both C_1 and C_2 as substitutes for leisure, the income tax is worse than the consumption tax on two grounds:

- It introduces a distortion between C_2 and L greater than before
- To the extent that L is a substitute for C_2 the expansion of L causes an excess of social cost over social benefit.

When C_2 and L are substitutes the consumption tax is preferable. The income tax yielding the same revenue as a consumption tax will be at a lower rate, including, as it does, savings.

(This lecture was taken from someone else's notes since I was absent)

Lecture 10

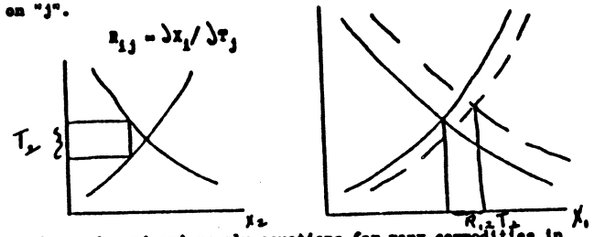
Measurement of Waste

In the Three Postulates, $(P_i^1 - P_i^0) = D(Z)$ is the distortion due to activity Z. It can be rewritten as a line integral:

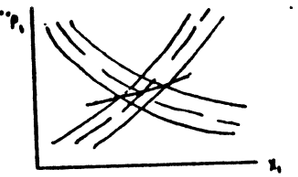
$$\int_0^{Z^0} \sum_i D_i(Z)\, \partial X_i/\partial Z \; dZ \qquad \text{where } \partial X_i/\partial Z = f(S_{ij})$$

Up to now we have been assuming constant costs. When costs are not constant, the S_{ij} is altered and we use R_{ij}. We want a reaction coefficient which shows how the level of activity "i" changes with the tax on "j".

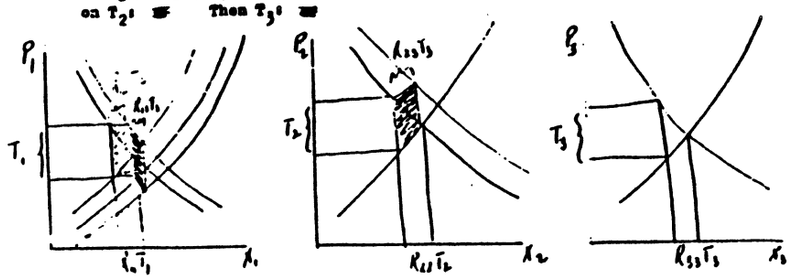
$$R_{ij} = \partial X_i/\partial T_j$$



When solving demand and supply equations for many commodities in order to get tax effects—you get a jamble. The formula of the Three Postulates $\int \sum D_i(Z)\, \partial X/\partial Z \; dZ$ simplifies it all. Using R's we have the linear approximation (linear reaction models). For small changes "the world is quadratic" (in the neighborhood of an equilibrium position)..



Quadratic approximations are useful and practical to approximate changes. Below, the curves are defined with respect to T_1, we put on T_2: ≡ Then T_3: ≡



Here we are assuming that the demand for X_1 does not change with the tax on X_3 and the Supply of X_2 does not change due to the tax on X_3; the $R_{ij}T_j$ are lateral distances.
All of this stuff sort of flows out, a graphical representation of the Three Postulates leading to the same quadratic form as before. The thing that is sort of neat about this kind of formulation is that the slutsky-Hicks properties of substitution terms can be extended to these R's as well as to the S's.

Considering the adding-up property:

An equiproportional tax on all goods (including leisure) will not alter the equilibium quantities. It is like giving money back to the people through the back door somehow.

$$\Delta X_i = \sum_j P_j^0 R_j^i \; T_j/P_j^0 = \sum_j P_j^0 R_{ij} t_j = 0 \text{ when } t_j = t^* \text{ for all } j$$

This will work like a neutral tax if we exclude the problems of depreciation, different economic lives, and different amounts of capital inputs.

$$\sum_j P_j^0 R_{ij} = 0 \quad \text{so} \quad X_i\left(\sum_j P_j^0 R_{ij}/X_i\right) = 0$$

$$\text{which becomes } \sum_j \eta_{ij} = 0 \text{ since } X_i \neq 0$$

Proving symmetry: Consider the expressions for welfare cost when T_1 is imposed first and when T_2 is imposed first:

T_1 first action $= -\frac{1}{2}R_{11}\,T_1^2$

T_2 second action $= \frac{1}{2}R_{22}T_2^2 + R_{12}T_1T_2$

and

T_2 first action $= -\frac{1}{2}\,R_{22}\,T_2^2$

T_1 second action $= -\frac{1}{2}\,R_{11}T_1^2 + R_{21}\,T_1T_2$

The two are equivalent if $R_{12} = R_{21}$

The general quadratic form for the change in welfare is...

$$\tfrac{1}{2}\sum\sum R_{ij}T_iT_j \leq 0$$

It follows that $\frac{1}{2}\,R_{ii}\,T_i^2 \leq 0$ if we only have one tax and $R_{ii} \leq 0$

These reaction coefficients have all the properties which follow from the quadratic form—negative semidefiniteness; each principal minor is ≥ 0.

We should go a step further than the B's because sometimes the distortions are in the factor markets. We set up a framework capable of handling them which is just an extension of the same thing: we have a demand and supply for the factor and distortions create discrepancy among supply and demand prices.

We have: B_i = tax on capital in activity i
E_j = tax on labor in activity j

We define four reaction coefficients:

$$G_{ij} = \partial K_i / \partial B_j \qquad M_{ij} = \partial L_i / \partial E_j$$

$$H_{ij} = \partial K_i / \partial E_j \qquad N_{ij} = \partial L_i / \partial B_i$$

Testing for symmetry:
When the tax on capital comes first you have a welfare cost of...

$$T_K = \frac{1}{2} \sum_i \sum_j G_{ij} B_i B_j$$

$$T_L = \frac{1}{2} \sum_i \sum_j M_{ij} E_i E_j + \sum_i \sum_j H_{ij} B_i E_j$$

When the tax on labor comes first:

$$T_L = \frac{1}{2} \sum_i \sum_j M_{ij} E_i E_j$$

$$T_K = \frac{1}{2} \sum_i \sum_j G_{ij} B_i B_j + \sum_j \sum_i N_{ji} E_j B_i$$

If there is to be symmetry, then : $\partial K_i / \partial E_j = \partial L_j / \partial B_i$

i.e. $N_{ji} = H_{ij}$

$G_{ji} = G_{ij}$ and $M_{ij} = M_{ji}$ but $H_{ij} \neq H_{ji}$ and $N_{ij} \neq N_{ji}$

because the last two relations are effects of a tax on one factor affecting the other factor in the other industry.

Problems in building up some tax equivalences in terms of B_i and E_j:

Take an excise tax on product: t^o_X is equivalent to t_{Lx}, t_{Kx}, t_{Mx}

where Lx are labor costs, Kx are capital costs including depreciation and Mx are materials costs
If materials enter in fixed proportions then t^o_X is equivalent to:

$$t_{Lx} = \left(X / Lx + Kx \right) t^o_X$$

$$t_{Kx} = \left(X / Lx + Kx \right) t^o_X$$

Example: If material is 40% of total cost then a 10% tax on X is a 16.7% tax on gross value added (i.e. $t_{Lx} = t_{Kx} = 16.7\%$) if you can't substitute away from capital and labor.

Lecture 11

Tax equivalences (cont.)

We can find a true equivalence with $t_X = 10\%$ only if materials enter in fixed proportions.

t_{Kx} is tax on gross earnings of capital

t_X is a tax on the net earnings of capital

$t_{Lx} = (1 / 1 - \alpha_{Mx})\ 10\%$

$t_X = (1 / 1 - \alpha_{Mx})\ 10\% \left(\Pi_X + D_{Kx} \right) / \Pi_X$

Π_X is the net return on capital

D_{Kx} is depreciation or user cost of capital

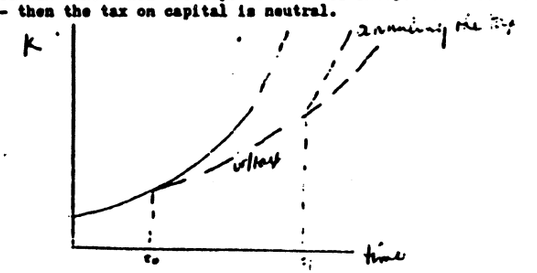
$\frac{\Pi_X + D_{Kx}}{\Pi_X}$ is the ratio of gross earnings of capital to net earnings of caital

When is a tax on net earnings the same in effect as a tax on gross earnings? When something blocks the normal response to the tax. The normal response is to shift the lives of the assets towards short end. (when there is a shift from gross to net earnings tax) due to depreciation which is hugher for short than for long term assets. If a production relationship were to prevent this kind of shift (example: trucking activity--the design of trucks will hot be changed) then equivalence between tax on gross earnings and net earnings.
there is
Equivalence depends on D_{Kx} / Π_X remaining constant.

Tax Neutrality

A flat rate tax on labor earnings is neutral if S_L is zero elastic. A flat rate tax on labor earnings imputing the same hourly rate on leisure time will be neutral no matter which supply elasticity labor has.

With respect to capital it is only sensible to talk of t_K when we talk of a neutral tax. When the capital stock is independent of the rate of return-- then the tax on capital is neutral.



How much efficiency would be reduced if we put capital earnings tax? It is hard to measure by figuring the dynamics of capital stock growth. Our method assumes a given capital stock so that calculation becomes less hairy than simulating growth. If equal across all activities, t_{Lx} and t_{Kx} might be neutral.

An equal rate on products cannot be neutral unless there are special asumptions: first we must distinguish between final products or inputs—we must define the base.

The value added tax—the base of the tax is sales minus material purchases That tax has potential for suffering from depreciation business (depreciation at different rates for different assets). Net return on capital will be taxed more heavily when it has longer lived liabilities. To get around this you should have a VAT of the consumption type.

Three types of VAT:

Product type— the base is sales minus materials purchased

Income type— the base is sales minus materials purchased minus depreciation

Consumption type—the base is sales minus materials purchased minus investment

The product type will be discriminatory against activities with shorter asset lives. The other types solve this problem—the difference between the two others is with their incentives for saving

The income type , when aggregated, it is like the income tax.
The product type, when aggregated, it is like the GNP type tax.
The consumption type, when aggregated, it is like a tax on consumption expenditure.

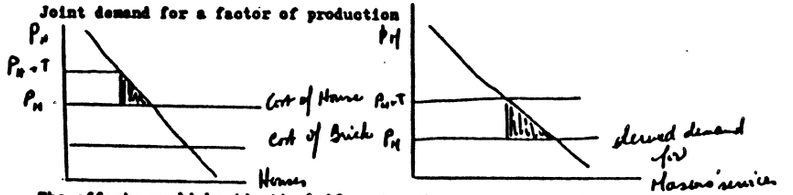
The income type entails a double taxation of savings—it is nonneutral.
The consumption type is neutral except for the labor-leisure choice.
The labor-leisure choice is important only to a certain sector which has an alternative to work... example: students, housewives— high income elasticity types. The consumption type is therefore not too bad compared to other taxes. It comes as close as possible to neutrality.

The base of the income type tax is bigger than the base of the consumption type VAT.

The corporation income tax

Analyzing the tax on factors of production in terms of the product market: The extent to which the welfare cost of a tax on the factor can be transferred to the market for the product...

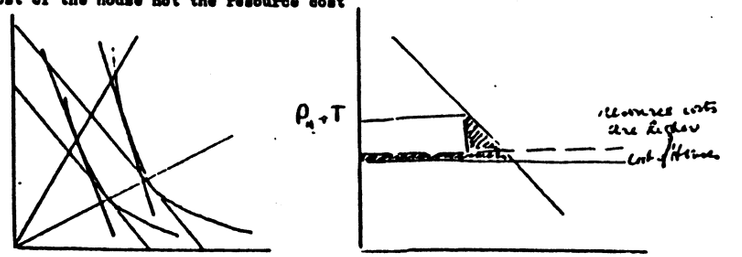
Joint demand for a factor of production



The effects would be identical if we tax the masons or the brick or the house if there are fixed proportions. But you must look at the tax only in one market otherwise you are double counting.
There is an equivalent tax on the product which will have the same effect as a tax on a factor.

What happens when you have variable proportions?

The welfare cost of a tax on factors will be greater than that of a corresponding tax on the product: (assuming variable proportions)
When you change relative prices the tax on the house minimizes monetary cost of the house not the resource cost



Unit resource costs are higher.

Elasticity of demand for a factor: example: masons

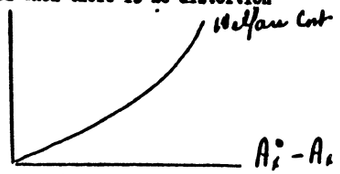
$$\eta_{mm} = \alpha_{hh} \times \eta_{hh} + (1 - \alpha_{mh})\,\sigma_{hm} \qquad \sigma_{hm} > 0$$

Lecture 12

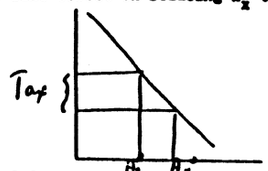
Factor tax vs. Product tax yeilding the same revenue

The impact of the tax includes the extent of use of the factor, the product and the amount of the revenue.

Factor A is used in the production of product X. A^0 is the amount of factor A that is used when there is no distortion

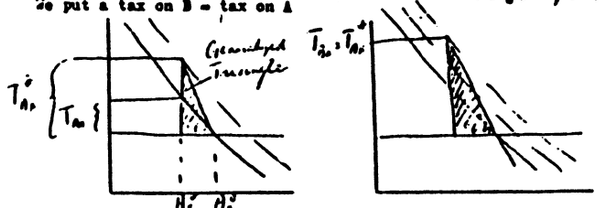


Bringing about the same reduction in the use of factor A by putting a tax on X, what happens to welfare cost ? We want a $t_x = t_{bx} = t_{ax}$ that will have the same effect on reducing A_x^0 to A_x^*



Since we have 2 factors, they are substitutes $\partial A/\partial t_b > 0$; $\partial b/\partial t_a > 0$

We put a tax on B = tax on A



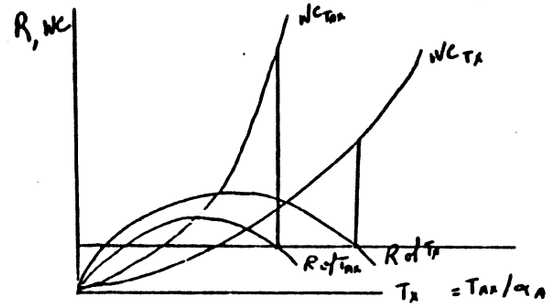
$t_{ax} = t_{bx}$ will lead to a substitution effect which dominates the scale effect. The welfare cost will be greater than the one from the sole tax on A. If $\sigma_{ab} = 0$ then the welfare costs are the same for the factor and product tax.

$$d \log A = (\alpha_a \eta_x + (1 - \alpha_a)\sigma_{ab}) d \log P_a + (\alpha_b \eta_x - (1 - \alpha_b)\sigma_{ab}) d \log P_b$$

1) When the $(\alpha_b \eta_x - (1 - \alpha_b)\sigma_{ab}) > 0$ then the welfare cost of the product tax lies above the factor tax.

2) When the $(.....) < 0$ then the welfare cost of the product tax is equal the welfare cost of the factor tax. $\sigma_{ab} = 0$

The lesson to be gotten is that a tax on a factor is the most efficient way to restrict the use of the factor.
A tax on output is the most efficient way to restrict utput.
To raise revenue, the product tax is the most efficient way:



$(WC - R)_{t_x} < (WC - R)_{t_{ax}}$ t_x is more desirable

Revenues are equal in the case of zero substitutability between A and B inputs.
One should not make the mistake of picking the rising portion of the revenue curve; it may give the false conclusions.

The corporation income tax

The most important lesson to be drawn from H's work in this field is that "capital bears the full burden of the corporation income tax" is not an extreme statement.

Mythology in the subject: the traditional trichotomy

$$0 \leq \frac{\text{Burden borne by capital}}{\text{Burden of the corporate income tax}} \leq 1$$

The two sector analysis method used by H comes up with:

$$\left(\begin{matrix}\text{capital's share} \\ \text{in nat'l income}\end{matrix}\right) \alpha_K \leq \frac{\text{Burden borne by capital}}{\text{Burden of the corporate income tax}} \leq \frac{K_X + K_Y}{K_X} \left(\frac{\text{Total K stock}}{\text{K in corp. sector}}\right)$$

The 2 sector model is a closed economy model. Since capital is fluid in movement across national boundaries, this assumption is unrealistic and can lead to erroneous conclusions. However, the corporate income tax is a general tax across countries. The model can be applicable when you can talk of capital in developed countries as a whole—the effect of these taxes reduces the rate of return on capital across countries. In the case where one country reduces its rate of tax, the analysis is a bit different and the results of the

model are erroneous.

The traditional trichotomy

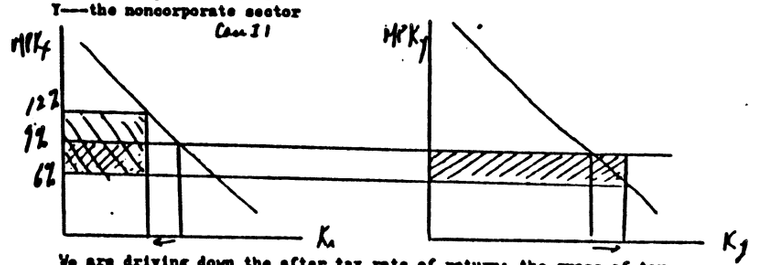
The burden of the tax is divided between shareholders, labor and consumers. If this is the way it is then the burden borne by capital cannot be less than of equal to one because everyone who owns capital in this market must share the burden borne by capital.

Shareholders are thought of in the short run as residual recipients but the argument does not take into account resource allocation due to restoration of equilibrium.

Consumers, we can consider them as the other face of workers and shareholders. When we speak of the corporation income tax we are speaking of the tax on capital in the corporate sector.
We do not allow for debt issues because the firm cannot finance itself with 100% debt. A tax of 50% on equity will be approximately a 50% tax however if the firm finances with debt it can lower this tax rate by lowering the amount of equity. Since firms have not been successful in avoiding the tax in this way to any large extent, we will neglect it.

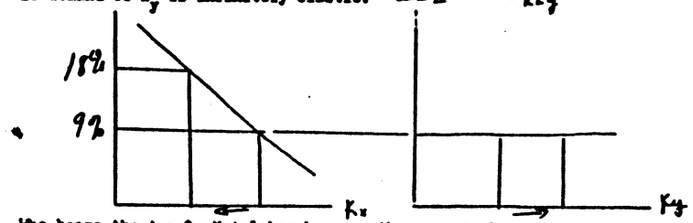
The traditional trichotomy

The third mistake—whatever be the categories we are separating out, the fractions of the tax burden that they bear does not have to be positive.
X—the corporate sector
Y—the noncorporate sector



We are driving down the after tax rate of return; the gross of tax rate of return rises
What is the burden of the tax ? The difference between the gross and net returns on taxed activity times the amount of initial capital in that activity.
Burden borne by capital is the reduction in yield times $\bar{K}$ so K bears more burden than the burden of the tax. Therefore labor must be bearing a negative burden.

If demand of K_y is infinitely elastic: Case II $\sigma_{K_x K_y} = \infty$

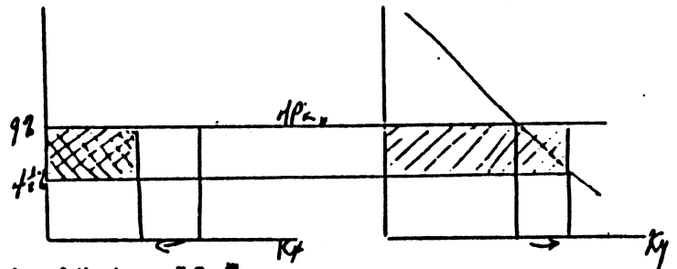


Who bears the tax ? Not labor because the wage to labor is taken as numeraire on the vertical axis. The price of corporate products rises relative to noncorporate price—consumers bear the tax.

Example:	Y_L	Y_K	Gov't revenue	Price
No tax	60	40	0	1
Case II	60	40	40	1.40

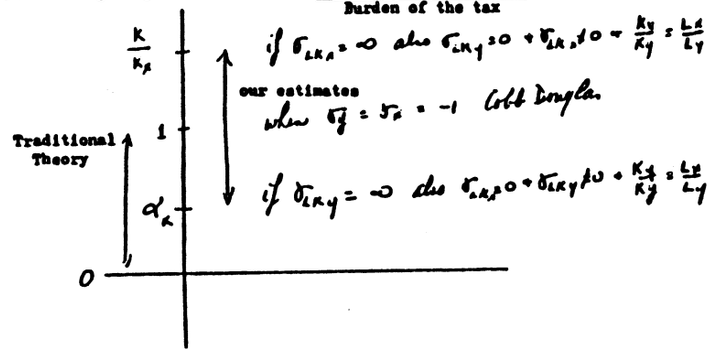
Real income of labor and capital has fallen by their share of total income incase II.

Case III $\sigma_{KLx} = \infty$.

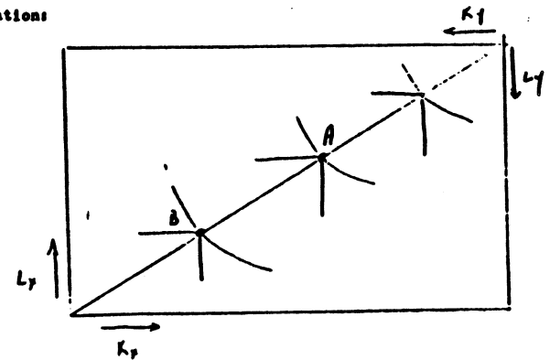


Burden of the tax = $K_f T$ ≷
What capital bears = $\bar{K}T$ ≷

The interesting range of the ratio: $\frac{\text{Burden borne by capital}}{\text{Burden of the tax}}$



Explanations:

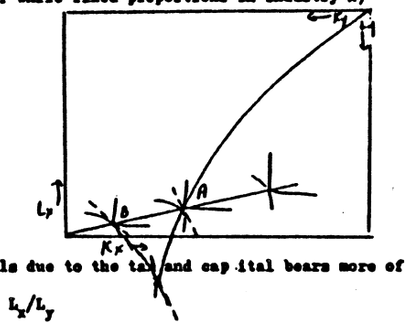


There is a straight line contract curve due to the fact that factor proportions are the same in the two activities. Both industries have constant returns to scale so there are parallel tangencies. We start at point A and as a consequence of the tax, we go to point B, where K receives the same return gross of tax. Industry X will not be able to eject more capital and labor. The price of capital will rise by T so at point B the MRS is $\frac{P_K + T}{P_L}$. At point A it is $\frac{P_K}{P_L}$.

$P_K / P_L = (P_K + T) / P_L$

When $\sigma_{LK_f} = 0$ The case where consumers bear the tax

If when $\sigma_{LK_f} = 0$ the X industry is capital intensive, (Variable Proportions in industry Y while fixed proportions in industry X)

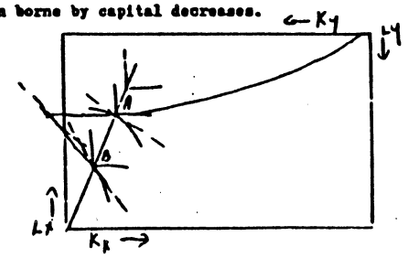


P_K / P_L falls due to the tax and capital bears more of the burden.

$K_x/K_y > L_x/L_y$

If when $\sigma_{LK_f} = 0$ industry X is labor intensive, $K_x/K_y < L_x/L_y$

the burden borne by capital decreases.



The Cobb Douglas Case

$U = X^{\beta} Y^{(1-\beta)}$

Prod. fn. $X = L_x^{\gamma} K_x^{(1-\gamma)}$

$Y = L_y^{\delta} K_y^{(1-\delta)}$

$X P_x = \beta Z$

$Y P_y = (1-\beta) Z$

$L_x P_L = \gamma X P_x = \gamma\beta Z$

$L_y P_L = \delta Y P_y = (1-\beta)\delta Z$

$K_x P_K = (1-\gamma) X P_x$

$K_y P_K = (1-\delta) Y P_y = (1-\beta)(1-\delta) Z$

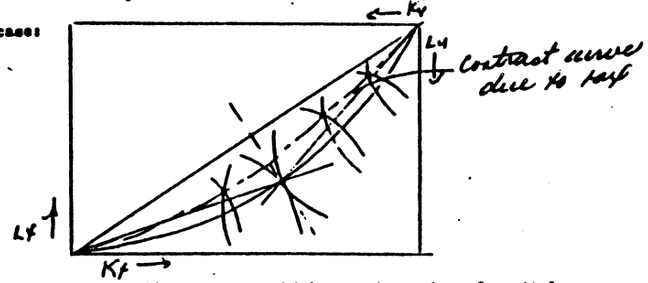
	L_y	K_y	L_x	K_x	
No tax	.10	.40	.20	.30	
Tax on X of 50%	.10	.40	.20	.15 (K_x)	.15 (Gov't)

Capital loses exactly what the gov't is getting—that is what we mean by burden being borne completely by capital.

$\sigma_{LK_X} = \sigma_{LK_Y} = -1$ Cobb-Douglas Case

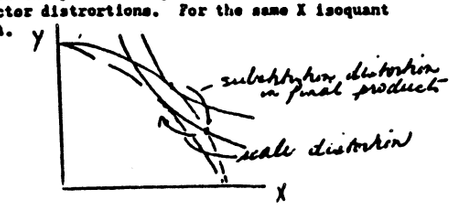
So that is not an extreme position but a middle of the range effect.

The general case:



In the taxed activity you will be paying a higher price price of capital You will be on a point of equilibrium in which output is less.

The contract curve determines the production possiblities frontier which gets pulled in due to the factor distrortions. For the same X isoquant you ~~are on a lower~~ have less Y isoquant.



Corporation income tax (cont.)

Partial factor taxes in a two factor two product world

1. Demand for the product $D_x = D_x(P_x/P_y)$
2. Supply for the product $Q_x = Q_x(K_x, L_x)$
3. Equilibium in the market $Q_x = D_x$
4. Factor substitution in X
5. Factor substitution in Y
6. Price formation in X
7. Price formation in Y
8. Labor market equilibrium
9. capital market equilibrium

1. $dX/X = E_x (dP_x/P_x - dP_y/P_y)$
2. $dX/X = f_L dL_x/L_x + f_K dK_x/K_x$ homogeneous of first degree prod. fn.
3. $dX^A/X = dX^B/X \qquad dP_L/P_L = 0$ (choice of numeraire is arbitrary)
4. $\sigma_{LK_x}(dP_{Kx}/P_{Kx} - dP_{Lx}/P_{Lx}) = dK_x/K_x - dL_x/L_x$
5. $\sigma_{LK_y}(dP_{Ky}/P_{Ky} - dP_{Ly}/P_{Ly}) = dK_y/K_y = dL_y/L_y$
6. $dP_x/P_x = f_k dP_{Kx}/P_{Kx} + f_L dP_{Lx}/P_{Lx}$ Since $P_x, P_y, P_k = 1$

   $dP_x = f_k dP_k + T$

   T is a % of the initial price of capital

7. $dP_y/P_y = s_K dP_{Ky}/P_{Ky} + s_L dP_{Ly}/P_{Ly}$ so $dP_y = s_k dP_k$
8. $L_x + L_y = \bar{L} \qquad \frac{L_x}{L_y}\frac{dL_x}{L_x} + \frac{dL_y}{L_y} = 0$

9. $K_x + K_y = K$ $\qquad \frac{K_x}{K_y}\frac{dK_x}{K_x} + \frac{dK_y}{K_y} = 0$

There are nine unknown variables: dK_x/K_x, dK_y/K_y, dL_x/L_x, dL_y/L_y, dX/X
dP_x/P_x, dP_y/P_y, dP_k/P_k, dP_L/P_L

Assume no distortion in the market for labor

$$dP_{Lx}/P_{Lx} = dP_{Ly}/P_{Ly}$$

And in the capital market:

$$dP_{kx}/P_{kx} = dP_{ky}/P_{ky} + T/P_{ky} \qquad \text{and } dP_{ky}/P_{ky} = dP_k/P_k$$

General equilibrium problem: When looking at efficiency costs we assume the government gives money back in the form of neutral tax or transfer—a nice way to handle the problem on a theoretical level.
However, when we talk of incidence--who bears the direct burden? Then how do they suffer if they are getting back the money? So to handle the incidence of taxation--we assume the government impounds the tax (we don't look at how it is spent). So we look at how people do without this money.i.e. assume the government buys the goods and services and throws them away. We don't contemplate any benefits from government's use of those goods and services . So we assume government spends the money as the citizens would have(we impute to the government. the same tastes asthe citizens) for the neatest solution.

Now working through the model:

From equations 1,2,6,7

$$E_x(f_x dP_x + f_x T - \epsilon_x dP_k) = f_L dL_x/L_x + f_K dK_x/K_x$$

$$E_x f_x T + E_x(f_x - \epsilon_x)\, dP_K$$

From equations 4,5,8,9

$$S_x T + S_x dP_K = -dL_x/L_x + dK_x/K_x$$

$$S_y dP_K = dK_y/K_y - dL_y/L_y = \frac{L_x}{L_y}\frac{dL_x}{L_x} - \frac{K_x}{K_y}\frac{dK_x}{K_x}$$

Now the three equations are:

$$E_x f_x T = E_x(\epsilon_x - f_x)dP_K + f_L dL_x/L_x + f_K dK_x/K_x$$

$$S_x T = -S_x dP_K - dL_x/L_x + dK_x/K_x$$

$$0 = -S_y dP_k + \frac{L_x}{L_y}\frac{dL_x}{L_x} - \frac{K_x}{K_y}\frac{dK_x}{K_x}$$

$$dP_k = -T$$

$$dP_k = -T\,\frac{K_x}{K_x + K_y}$$

$$dP_k = 0$$

What is dP_k? When $dP_k = 0$ capital bears α_k of the burden because P_1 is the numeraire.

When $dP_k = -T$ capital bears K/K_x of the burden

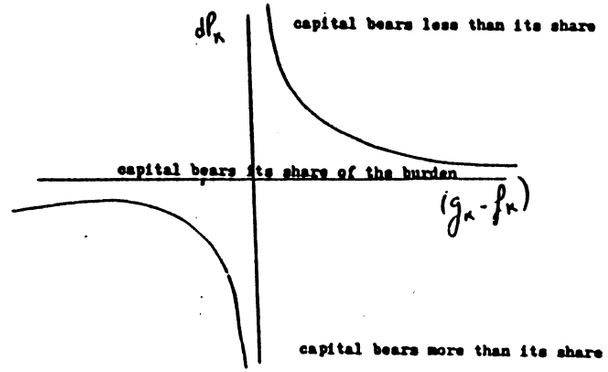
To determine the incidence of the tax, dP_k is critical.

Using Cramer's Rule we can drive dP_k from the three equations(above) in terms of S_y and S_x:

1. When S_y is very large ($\to \infty$), then $dP_k = 0$
2. when S_x is very large ($\to \infty$) then $dP_k = -1$
   when we have equal factor proportions
3. when $S_y = 0$ then $dP_k = -T$
4. when $S_x = 0$ then $dP_k = 0$

When we don't have equal factor proportions, we have capital bearing more burden if we are dealing with the capital intensive industry and capital bearing less of the burden if we are dealing with the labor intensive industry.

When $dP_k > 0$ and we put a tax on the labor intensive industry, capital will bear less of its share of the burden of the tax.



Our model looks at point A---full employment and only one set of relative prices P_x/P_y which prevails. $dP_x - dP_y = 0$

Example:

T = 50% of price of capital

$\theta_k = .6$ $\quad f_k = .4$

$dP_k = 1$ $\quad P_L = 1$

$P_k = 2$

$P_y = .4(1) + .6\,(2) = 1.6$

$P_x = .4(2) + .6(1) + .2 = 1.6$

if $\theta_k = .8$ $\quad f_k = .2$

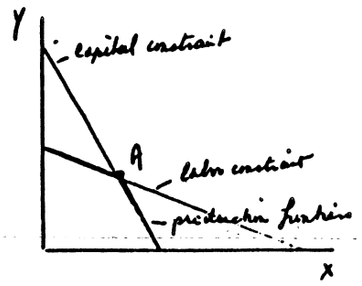
then

$dP_k = 1/6$ $\quad P_L = 1$

$P_k = 1.167$

$P_y = .8\,(1.167) + .2\,(1) = 1.13$

$P_x = .2\,(1.167) + .8\,(1) + 1.132 = 1.13$



Differences in factor proportions act as a shock absorber for effects of T. If factor proportions are close it is hard.

Lecture 15

The corporation income tax (cont.)

The model explores the condition of full employment (i.e. point A of the previous graph)

That factor which will be more hurt is that which is more intensive in the taxed industry. If factor proportions are very different, then th e burden borne by the intensive factor is less than if the factor proportions were relatively close together.

Reason:

$$P_x^o = f_L P_L^o + f_k P_k^o = f_L P_L' + f_k P_k' + f_k T$$

$$P_y^o = \theta_L P_L^o + \theta_k P_k^o = \theta_L P_L' + \theta_k P_k'$$

$$P_k' = \frac{f_L P_y^o - \theta_L P_x^o + \theta_L f_x T}{f_L \theta_k - f_k \theta_L}$$

The closer together are factor proportions, the closer to zero will be the numerator.

When $\sigma_{xy} = \sigma_{LK} = \sigma_{LK} = -1$ $\quad dP_k = \frac{-TK_x}{K_x + K_y}$ capital bears the whole burden of the tax: $(K_x + K_y)\, dP_k = -TK_x$

The case where the capital market is equilibrated across national boundaries:

$$\mathcal{E}_{xPx}\,(dP_x - dP_y) = f_k\, dK_x/K_x + f_L\, dL_x/L_x$$

$$dK_x/K_x - dL_x/L_x = S_x\,(dP_k + T - dP_L) \qquad \text{where } S_x = \sigma_{LK_x}$$

$$S_y = \sigma_{LK_y}$$

$$dK_y/K_y - dL_y/L_y = S_y\,(dP_k - dP_L)$$

If $dP_k = 0$ then $dL_x + dL_y = 0$

$$dP_x = f_k T$$

$$dP_y = 0$$

$$f_k\, dK_x/K_x = -f_L\, dL_x/L_x + \mathcal{E}_{xPx}\, f_k T$$

$$dK_x/K_x - dL_x/L_x = S_x T$$

$$dK_y/K_y = dL_y/L_y$$

$$\frac{L_x}{L_y}\frac{dL_x}{L_x} + \frac{dL_y}{L_y} = 0 \quad \text{(the constraint on labor)}$$

Incidence when the elasticity of demand for the corporate sector's product is large relative to other items:

$$dP_k = \frac{\mathcal{E}_{xPx}\, f_k\,(K_x/K_y - L_x/L_y) + S_x(f_k L_x/\;\; + f_L K_x/K_y)}{\mathcal{E}_{xPx}\,(\;\; - f_k)(\;\;\ldots\;\;) - S_x(\ldots) - S_y} \cdot T$$

If $S_x = 0$ and $S_y = 0$ you have fixed proportions
You will have a smaller corporate sector if $\mathcal{E}_{xPx} > \sigma_{kLx}$

That means we will be ejecting labor as well as capital from the corporate sector as if it were a fixed proportions sector--in this case labor and capital are pushed out in exactly the same proportions but its first approximation is the fixed proportions case. Capital will bear more than its share if the corporate sector is more capital intensive.

It is probably true that the corporate sector is more labor intensive--in a geographical setting. Therefore the burden will depend on the nature of the particular corporate sector in the region. To the extent that there are effects on relative prices of labor and capital, there will be effects throughout the entire economy--the essence of the two sector approach. This might not be the right way to handle the problem of the small geographical area. As we go to small geographical regions, the models we have looked at don't make sense. A partial equilibrium model makes more sense. A disaggregative approach for example: The effect of the corporate income tax will be felt by some immobile factor, in part by consumers in the area (regionally specific demand will be inelastic and a rise in price will work however if demand can be elastic it is amovement down the demand curve of a monopoly). Besides that, there is an outflow in activities with high demand elasticities.

If the world were really Cobb-Douglas, the aggregative approach will account a small decrease of the rate of return to all the capital in the world.

Franco Modigliani and Merton Miller, The Cost of Capital, Corporation Finance and the Theory of Investment AER June 1958, p.292

"Misinterpretation of the scope of Proposition III can be avoided by remembering that this Proposition tells us only that the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worth while. This does not mean that the owners (or the managers) have no grounds whatever for preferring one financing plan to another; or that there are no other policy or technical issues in finance at the level of the firm.

That grounds for preferring one type of financial structure to another will still exist within the framework of our model can readily be seen for the case of common-stock financing. etc... "

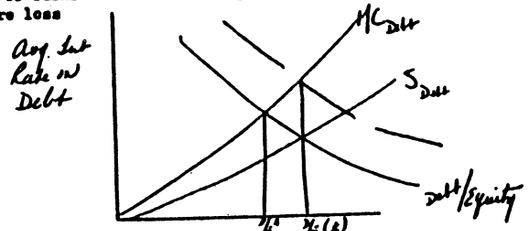
Lecture 16

The Financing Effect

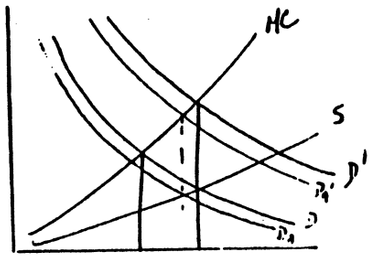
The corporation income tax is a tax on return denominated to equity capital plus the residual share (rents, monopoly profits, return to the owned factor).

The question of financing—debt capital is considered an expense while equity is taxed at 50% marginal tax rate. Why not finance 100% by debt?

1) there must be some residual claimant
2) there is a given optimum debt/equity ratio for a given class of firms—taxes will change that optimum and cause a welfare loss



Marginal cost of debt increases as you move to more debt financing and there arises a discrepancy between the supply of debt and the MC. There is also a rising supply price of equity due to higher leverage factor: as D/E ratio rises you cause a negative externality in the equity market.
D_d is a hypothetical demand curve for debt, net of this externality



The Miller-Modigliani theory is correct but the homogeneous perceptions assumption of theirs is a subtle oversimplification. but there are preferences for bonds, for stocks etc...In a capital market people have different attitudes apart from institutions (mutual funds, and insurance companies which prefer bonds). They may want to hedge against inflation, therefore they would not want stable stocks (ITT). There is a division in the market on the individual level: bondholders are disjoint from stockholders. H feels this contradicts MM theory.

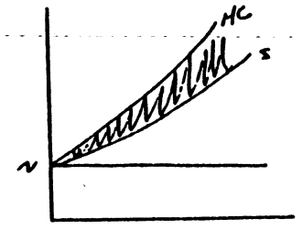
The rising supply curve is due to different individuals having different appreciation of the future earnings of particular companies.

Reason for $(MC_{debt} - S_{debt}) > 0$:

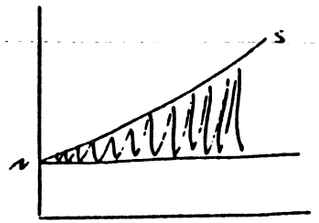
Larry Fischer explains risk premium on bonds from cross-sectional data on American Companies: in 1947 and 1954
risk premium= f(D/E, last default, size of issues outstanding, variance in earnings) gets $R^2 = .75$
He found the same coefficients for depression years and boom years
R^2 for pooled regression was .73

Tambini's thesis—brought Fischer's work up to date. The same regression fits the data from the 1960's.

Recent problems from the developing countries'angle:
We usually associate rising supply curve as behavior of a monopsonist. A firm can be behaving like a monopsonist although it has no effect on world capital because of default risk.
The marginal paticipant inthe market perceives default risk and as you move out to more debt more and more individuals which perceive higher risks enter into the market. If the firm perceives its own default risk, it will not look at the risk free rate as the cost of capital because it doesn't believe that the probability of default rises but its suppliers have a different idea so the firm looks like a monopsonist. i.e. acts such that MC_d is higher than S_d rather than at the risk free rate. If the company believes that the probability of its default rises as D/E rises as well as the suppliers do, then it will take the risk free rate as its cost of capital.



Case I

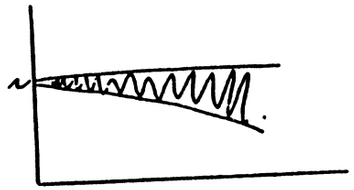


Case II

The range of disagreement between the market and the firm.
Case I is where the firm feels the suppliers of funds are unfair and takes its cost of capital as the MC_d.

Case II is where the firm feels that the suppliers of funds are fair and takes its cost of capital at the risk free rate.

Case III
Fleecing the public---Doesn't happen often. If people (the market) thi nk the probability of default is less than the firm thinks.



Lecture 17

Stimulation of investment

Tax stimuli to investment---a feature of legislation common in all countries

The simple income tax already entails the double taxation of savings in thesense that it is nonneutral; the third time by property taxation and the fourth time by corporation income tax. Then we say we want to stimulate investment---special treatment for capital gains, special tax stimuli to invest etc... It cannot be a rational process out of which this sort of pot-pourri emerges.

We can go into the political arena and say"We think that the earnings of labor are hard earned and that the earnings of capital are fruits of idleness, represent no true effort on the part of the people. Shouldn't we tax capital more heavily?...Oh yes, yes, yes...But we need investment for our economic growth, so shouldn't we have some special incentives...yes, yes,yes...You see it's like that! Economics is a wonderful playground for a born charlatan...you can con people up and down the avenue and they don't know it. Really it's true. You can sell ideas under some labels and not under other labels although they're exactly the same. For example, in Latin America, if you say 'I'm a free trader' there's no better way to have yourself never taken seriously again but if you say here we are under import tariffs on a lot of different industries, protecting these activities. Now if we calculate the effective protection we find it varies among activities wouldn't it be sensible to make that more equal so that it could make it easier to handle? Oh yes... However we are neglecting exports. Oh yes, we never thoughtabout that... But if we put a subsidy to exports equal inmagnitude to the tax that we are putting on imports so that we equalize effective protection, then we are giving the same treatment to imports and exports...Oh yes that's a great idea...Well that free trade. The same is true with taxation on capital."

There are subtleties in the political process but certainly what we see is not rational economics.

The tax incentives for investments are various kinds: there are tax holidays, accelerated depreciation, direct expensing of investment costs, and there are interest tax credits etc...Each has a special role.

We can also anlyze them in more than one plane. If we have a corporation income tax that already discriminates between corporate and noncorporate uses of capital: Example: Accelerated depreciation to certain industries.

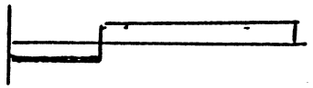
We might use the global picture between the corporate and noncorporate sectors as a basis of comparison. So effectively we are reducing the degree of didtortion imposed by the corporation income tax. In any way that we give tax stimulus to investment in the corporate sector it will have that effect. It is, any excuse under which the corporate tax , where the excuse affects the marginal activity, will have the effect of reducing the overall disincentive in the corporate sector.

Our area of interest however lies in more specific properties of investment incentives: if you are stimulating, what particular ways can be recommended? What are their attributes? Can we attachadvantages and disadvantages?

We begin with two propositions:

Expensing—A total expensing of investment costs against the corporation income tax is neutral in all relvant respects—what that means is that the government is getting a certain piece of the action.

If you have an investment with this profile:



and the tax rate is 30%, the government pays [illegible] and gets [illegible]. i.e. when there is a loss government puts up 30% and when there is a benefit, the government takes 30% of the net benefit. It is equivalent to the government being a 30% partner—an automatic partner.

Investing 70% rather than 100% does not change the investment decisions concerning risk or life of the investment. If investment from now on were expensed, there would be a net tax revenue. In our modern growing economy the income from capital would be greater than gross investment. (income from capital gross of tax and depreciation that is)

In a stable economy, gross investment equals depreciation; in a growing economy, the difference is profits. Investment = profit in the Golden rule world—which is optimal if capital in the initial position is used. However, since [illegible] nature is not in the havit of giving us the intial capital stock we chose, we are usually short of being in that world. So you would make money--it isn't exactly equivalent to annulling the corporation income tax but it is similar in all effects.(resource allcation effects are essentially zero).

There is another kind of neutrality which is possible in the corporation taxation. Full expensing is equivalent to government partnership—the other treatment that has a certain neutrality to it is income taxation of true economic income—i.e. we tax income after true economic depreciation is calculated. There is no bias in favor of or against long lived or short lived assets. Many ways of seeing this: the elegant way is as a series of negative elements in the vector of outputs in the firms.True economic depreciation viewed as wastage to be charged to costs of operation. Then what you end up with is a concept of income such that a proportional tax on that income will not bias the choice of investment. Consider assets of 3, 6, 20 yrs—Associate with each asset some kind of typical profile of an investment. There are several different kinds of investment interventions that starting with a bunch of these investment prospects which are exactly at the margin of the acceptance decision—due to the tax they will no longer be at the margin of indifference. The proportional taxation of income with full correction for true economic depreciation will not have that effect.

Suppose a stationary state, a bunch of assets of 3 years and

investment of each year replaces one third of your investments. Every year we replace investment and we are earning 20% after cost of replacement (which is exactly the true depreciation). Now we have a tax of 40% on income from capital ne to depreciation. We now earn 12%. That will apply all across the board. If you were indifferent between assets of types ABC you will again be indifferent except now the rate of return will be lower. We have a wedge going between the gross rate of return and the net rate of return on capital which is equivalent to all of the activities in the corporate sector. The introduction of such a wedge is going to maintain whatever relationships might exist between the gross of tax rate of return of projects and the net of tax rate of return. Therefore the projects that survive the corporation income tax will be projects that yield 6% net; 12% gross. The ranking of those projects which aremarginal at the pre-tax rate of return is the same when considering their post-tax rates of return.

The main case where there is a bias towards short lived assets is the investment tax credit which is the one which has been most recently used in the United States. How it has become so popular device? Obviously a mistaken policy measure. The government pays a certain fraction of investment to the investor and allows you not to deduct but allows you to take say 25%. It is not like being able to write off because you can now write off the full amount of the investment. It favors short lived assets.

(This is a partial transcription from a tape which was used due to my absence)

Lecture 19

Accelerated depreciation makes long-lived projects look artificially better.
Tax credits have artificial incentives for short-lived projects.

Why investment tax credits create artificial incentives for short-lived projects?
Let ρ be the yield and δ be the rate of depreciation and P_K the price of equipment.
$(\rho+\delta)P_K$ is the rental value of the asset in equilibrium.

You could think of a tax credit whichpays 10% of the price of the asset when you acquire it as being equivalent to paying 10% deductions on depreciation as it depreciates.
Depreciation is smaller for long-lived assets than for short-lived assets so you get a bigger subsidy when using short lived assets.

If you produce the same economic rent with long lived assets as you do with short lived ones, shouldn't we be comparing the generation of the same economic rent. What does it mean to have equalized economic rent ? $N_o$thing

Subsidy	Econ. Rent			
20	120	= (.10 +.02)	1000	long-lived asset
80	120	= (.10 +.20)	400	short-lived asset

It is the rate or return that should be equalized-- $\frac{\rho+\delta}{\rho} = 1+\frac{\delta}{\rho}$

Investment in short-lived assets will go until the marginal return is lower than investment in long-lived assets in the presence of an investment tax credit.
One way to get a 10% return:

2 yr asset	100	100	100	100
Before tax return		20	20	20
After tax return		10	10	10
Steady state replacement depreciation		50	50	50

For a 5 yr. asset the before-tax return is 20, the after tax return is 10 and depreciation is 20.
For a twenty yr asset the before tax return is 20 the after tax return is 10 and depreciation is 5.

Due to the tax credit:	2 yr. asset	5yr asset	20 yr asset
	100	100	100
Before tax return	20	20	20
After tax return	15	12	10.5
Depreciation	50	20	5
Tax credit	10% of 50=5	10% of 20 =2	10% of 5 = .5

This is not an equilibrium situation. This will lead to allowing short lived assets to be carried to equal after tax returns: It is not to be assumed that quantities of investments will be very different . It depends on the elasticities of the rate of return schedule ρ.
The equilibrium is:

	2yr asset	5 yr asset	20 yr asset
	100	100	100
Before tax return	10	16	19
After tax return	10	10	10
Depreciation	50	20	5

So we can see that short lived investments are favored by the tax credit. It enhances demand for short-lived assets. It bids up the price of funds (savings). It reduces the tax rate on capital. It reduced a preexisting distortion but it introduces other distortions.

If you want to stimulate investment in a sector subject to income tax, one way to do it is to reduce the income tax rate. But its not popular. because not enough"bang for the buck" since it reduces tax on old investment. A way to reduce tax on new investment--allow expensing of new investment 30% and allow depreciation of 70%. This is the same as allowing a reduction of income tax from 50% to 35% on new investment.

Tax Holidays

Tax holidays are exemption from income tax for an investment for a specified amount of time. (usually 4-5 yrs.)
A five-year holiday will make incentives for investments which have positive flows during the period of the holiday. If it is to take place after a certain date--it gives incentive to have losses until then and then have tremendous profits for the specified period.
Tax holidays have very different effects on different firms which are at different stages of their profitability schedule; which have different ages of assets etc... It introduces tremendous distortions.

Percentage Depletion and Related Matters (Good transition to Pers. Inc. Tax)

Percentage depletion is only part of the tax treatment of the mineral industry. Capital gains tratment to mineral industry activities takes place with respect to an individual.

M's work on % depletion is now 23 yrs. old. Assumes capital makes a normal profit and saw what was inefficiency introduced by the % depletion allowance. %depletion laws have less force now that oil prices have risen so much.

% depletion amounts to the same thing as a tax credit--the problem is that you still have to tax the whole depletion. No expensing allowance. Percentage depletion allows overdepreciation of taxed good in question.
History: Depreciation of costs of wells (new wells) will be allowed; however it will not apply to cost of already existing wells but their fair market value. New well explorers complained that they were getting unfair treatment. So the discovery depletion was created. True economic value which is depreciated over time of the cost of discovery , whichever is greater, can be written off.for each well. This was a type of double

incentive because you write off all bad wells and the value of the good wells. Problems arose in determining the value. There was a legal hassle to the percentage depletion was instituted. 27½%--The fraction of the value of oil which the well yielded which was permitted to be written off. The intention was an administrative simplification of discovery depletion. Percentage depletion amounted to double deduction--more was written off interms of costs than costs incurred.

How do you analyze percentage depletion?

You make a comparison between what conditions would be if normal taxation had been instituted instead of percentage depletion.

The value of the oil well subject to % edpletion is not different from the value of the oil well subject to cost depreciation. What is the price that an investor would pay for an operation?

Y = present value of future gross income stream
(before taxes, depreciation and depletion)
d--depreciation (discounts R)
R--price you pay for an asset(undiscounted)

Taxable income: Y-dR Taxes: t(Y - dR)

After tax income:(PV) Y - t(Y - dR)
What you are willing to pay for the asset:(Assuming you depreciate it with cost depletion)

$$R_1 = \frac{Y(1 - t)}{1 - dt}$$

What you are willing to pay for the asset assuming you depreciate it with % depletion:

$$R_2 = Y - t(Y - pY) = Y(1 - t + tp)$$

p = 35% which corresponds to 27½% of gross income(which includes asset expenses etc...)

d depends of the time shape of the asset. Assuming a 10 yr. life and 10% rate of discount then d = 65%

Now comparing R_1 with R_2:

$R_1 = .5\ Y/\ 1 - (.65)(.5) = .74Y$

$R_2 = Y(1 - .5 + .5(.35)) = .675\ Y$

From the data it looks like people chose cost depletion when they have bought the well and when they have discovered the well they prefer percentage depletion.

If they purchase a well: they consider an alternative project--cost of a machine for example-- $C_1 = .7Y$

If they discover the well: they consider the total cost--cost of bad wells, looking for wells as well as that of the good well. The cost associated with the well is greater than the cost of a bad well.

$C_2 - t(.8C_2) = .67Y$ $C_2 = 1.1Y$

Combined with the percentage depletion, you have the deduction of 80% of costs against income tax which makes the % depletion alternative more profitable for those who discover wells.

Lecture 19

The Choice Between Percentage Depletion and Cost Depletion (cont.)

Theoretically percentage depletion is less favorable for firms than cost depletion except when firms discover wells. With percentage depletion you attach depletion expenses which are far greater than the cost of the successful well because you write off the cost of dry holes. So firms who discover wells prefer percentage depletion.

Capital gains is a special case which is connected with the same phenomenon. Possible to write off dry holes expenses from personnal income and pay capital gains tax on the net profit from the successful well. For example:

Expenses	1000	Sales price minus capital gains tax =
Successful well	200	900 - .25 (900-200) = 725
Dry hole	800	this is the tax treatment on the
You receive from sales	900	successful well

Tax treatment on the dry hole:

Rate	loss written off
50%	400
70%	560
90%	720

V- vlue of oil

t_g-capital gains tax

t_y-income tax rate

C_t total costs

$$C_t = V - t_g(V - .2C_t) + t_y(.8C_t)$$

($.2C_t$: cost of successful well; $.8C_t$: cost of dry holes)

C/7 is approximately 1.45

H's suggestion for reform:
Instantaneous write off of all costs is equivalent to a subsidy to costs which can be written off instantaneously--it gets around the problem.

Nobody can deny that you are given special treatment(happens now for research expenditures) How to analyze instantaneous expensing? We would still have the problem of capital gains. There are two possibilities:

when assets that have special treatment , they are not eligible for the capital gains tax (which is less than the income tax) so that V/C = 1 (V/C =(1 - .2 t_g - .8 t_y)/ 1 - t_y). The problem is that the two sides of the balance sheet are affected with different tax rates that is why you end up with different after tax incomes.

Different Capital gains treatments

Tricky to pin down the nature of the special treatment--It is critical whether you net out all costs first before calculating income or whether you can get treatment if you have no income.(The latter is exemplified in the fattening cattle example)

Example of expensing costs before figuring out income:

Costs of cattle	100	Tax flow:
Feeding cost	60	.5 x 60 = 30 from expensing
Sales revenue	150	.25 x (150 - 100) = -12.5
Cash flow	- 10	from capital gains
		total= + 17.5
		net flow = 7.5

If the cattle was sold for 160, the net flow would have been 5.

Lecture 20.

Capital Gains tax (cont.)

We don't have data on the accrual of capital gains but we have data on realized capital gains. The richer will prefer longer term capital gains.

Assume a portfolio of $1000.

	Bought at	Worth +	10% return		capital gain
A	100	150	165	sell 66	26
B	200	250	275		
C	400	400	440		
D	200	150	165		
E	100	50	55	sell 33	-27
	1000		1100	sell 99	- 1

66 - (66/1.65)= 26 = 26%
33 - (33/-.55)= -27 = -27%
99 - (99/1.1) = 9 = .9% ≈ 1%

You can chose judiciously which assets to liquidate in order to offset capital gains with capital losses.
If the portfolio is growing (at 10%/yr.) you may run out of losses.

This postponement effect reduces the taxes--the gain is realized long after it is accrued and discounted the tax is much less.
A reform which will never be implemented is taxing capital gains as they accrue. It would eliminate the locked-in problem without eliminating the revenue.

Capital gains and inflation

The purist way--build an indexation in the system to tax only the real value however the world is messy!
There are problems--money holdings would lose, bond holdings reflect expectations of inflation so paying tax on interest payment is wrong because it biases towards the bondholder; why should recipient of capital gains get special treatment--it is not clear. The more judicious statement to make is to index the entire tax system--for large rates of inflation.
Unindexed tax treatment has built in stabilizing properties against small rates of inflation.

Lecture 21

Tax Treatment of Housing; Negative Income Tax; Averaging

Housing
Tax exemptions on income from owner-occupied housing is one of the most important subsidies both in the United States and the rest of the world. It seems that there is a kind of insurmountable political barrier associated with this particular provision. No economist has made a good case for it. The government is in effect paying a regressive share of the rent: $f(Y) > 0$
Why? "Home ownership is a good thing" Poor people look forward to it while rich people enjoy it. It is a deduction which goes up with income rather than a tax.
From the rent which you figure out , you deduct real estate taxes, maintenance and depreciation charges. If the owner which occupies the house does not declare a rent he cannot deduct maintenance and depreciation; he can deduct real estate taxes and mortgage interest. Hence a tax offset. Industry income from owner-occupied housing is more blatant regression, with reform being within the range of feasibility. It is truly difficult to redistribute income by taxation. If we took all income over $50,000 and spread it over the poor, they wouldn't be very much less poor. It is unlikely that there is a system which actually redistributes. If it does, most countries don't want to emulate it for example the U.K. There are allsorts of problems which come up such as braindrain, etc...
The tax system is not strong enough to accomplish income redistribution.

Negative income tax

Experiment in New Jersey--but people knew they were a special program divorced from the sea around them. Windfall vs. permanent effect. Evidence leans in the direction that income maintenance payments were treated as windfalls. Looks nice, uncomplicated but it doesn't do what people want it to do.
Does it make sense to impose higher marginal rates on our poorer than our richer people? In the main, there is no happy solution to the problem.
Perhaps this explains the unkindness of much of our tax expenditure.

Averaging under personal income tax

There is a discrimination between transitory and permanent income. This is a problems especially for athletes. In the U.S. it is not automatic, not everyone must or can average their income.
The simple concept behind it:
A moving average 1/n of income this year gets innov and 1-1/n in the future. This is equity for people with variations around the mean which rises.

Cumulative income averaging

The best of the varieties. It imputes a lower marginal rate for windfall gains and a higher rate for losses.

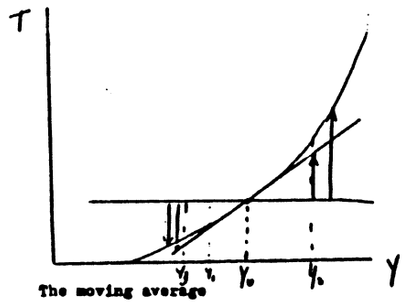
(This lecture was copied from someone else's notes due to my absence)

Lecture 22

Income averaging; Property taxation; Consumption taxes

Moving average vs. contemporaneous taxation

Contemporaneous taxation is better from built in stabilising view



The moving average

$$\frac{Y_t - \bar{Y}_{t-1}}{t-b} = \bar{Y}_t - \bar{Y}_{t-1}$$

You pay less extra tax when income rises than with contemporaneous tax.
You get a bigger reduction in tax by going from Y_0 to Y_1 twice than by going from Y_0 to Y_3 once.

Cumulative taxation with interest

b is the beginning date

$$\frac{(t-1-b)\,\bar{Y}_{t-1} + Y_t}{t-b} = \frac{\text{all the income earned until time } t-1}{\#\text{ of periods over which he has earned}} = \bar{Y}_t$$

Average tax due associated with average income:

$$\bar{T}_t = \tau(\bar{Y}_t)$$

Total tax payments minus what is already paid = what one owes

$$(t-b)\,\bar{T}_t \quad - \quad (t-1-b)\,\bar{T}_{t-1} \quad = \quad T_t$$

We should discriminate on timing of relevant payments of the tax. We should have a certain present value of taxes paid and income earned so that any two people at same time and if they earned the same amount and present value of income should pay the same present value of tax.

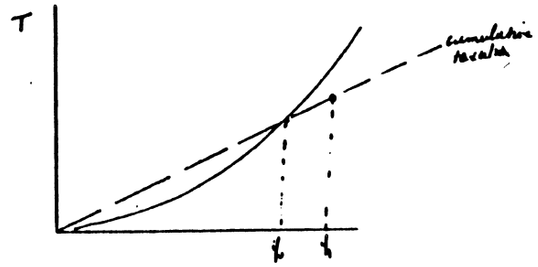
Present value of all income to date $= \bar{Y}_t(t-b) = (t-1-b)\bar{Y}_{t-1}(1+r) + Y_t$

$\bar{Y}_{t-1}$ = present value of all income up to Y_{t-1} /t-b-1

The tax bill :

$$(t-b)\,\bar{T}_t - (t-1-b)\,\bar{T}_{t-1}(1+r) = T_t$$

Equalising the tax payments in terms of present value in terms of the diagram:



Property tax
Valuation

It is hard to value things—self assessment is notrealistic
Variants of self assessment—government assesses your property with your help
In Latin America for example, the government asks owner to assess his land and then taxes on that assessment

Beyond valuation, what is to be said for prperty tax?

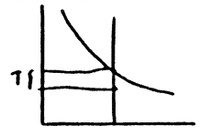
If you have a fixed stock of capital in an economy, and you put on a prperty which takes away a certain fraction of income generated by that capital:

r is gross of tax rent $\quad r = iK$
Property tax is set at $\quad T = .02\,K$

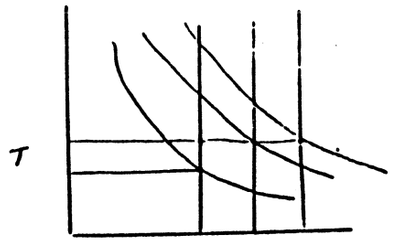
r (gross of tax not of dep.) $= (i_0 - .02)\,K$

We don't change scarcity or distribution of capital when we put on a property tax. What happens is that the interest rate falls (closed economy assumption) by just enough to reflect the tax.

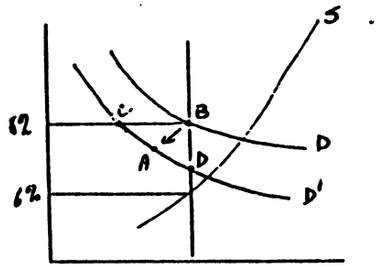
If there is an inelastic supply, the tax will come out of the rent. The way it happens is that the after tax yield decreases. And the capital value of existing assets falls and of reproducible assets falls.



As the economy grows the tax raises house rents and user cost of capital equipment.



In a closed economy, a dynamic analysis:



S is the supply curve of funds or the flow supply of savings or the long term supply of stock of capital

D is the flow demand for increments of capital stock or the MPK stock

D' is the demand affected by the tax

B is the before tax equilibrium
The tax is imposed and if the equilibrium moves to point C it means that the interest rate remained constant and the value falls by the full amount of the capitalized value of all future taxes.

Rent on land stays the same all along curve D' because it is a fixed component.

If the interest rate falls, the value stays the same for example pt. D.

At point A we have an intermediate position when there is a response from capital value and interest rate.

Consumption taxes

Kaldor-An Expenditure Tax

Earlier, Irving Fischer proposed a consumption-expenditure tax. Milton Friedman proposed one during the war and now Martin Feldstein is proposing it.
If you value savings because it is useful in promoting growth in the econoy, then you are in favor of a consumtpion tax.

The consumption tax taxes what people take out of the economy, the income tax taxes what people put into the economy.
The consumption tax gets rid of one distortion, the labor-leisure choice distortion, of a tax.

With equal revenue, the welfare cost will be less.

How administrable is the expenditure tax? In order to administer the consumption-expenditure tax you need more data than for an income tax.

Consider a household— you need to know:
Receipts from labor services + new debt + profits in interest, rents + sales of eligible assets - current costs of doing business - old debt repaid - interest paid on debt - taxespaid - purchases of eligible assets = consumption.
Consumer durables are hard to deal with. And housing is it considered as an investment and you impute a rent on housing.
If the house is held as an asset—you receive rent from the house.

Treatment of depreciation—you can maintain a sinking fund and put it into some bank account the amount that would be depreciation.

Depreciation would not appear in taxation.
You would have a charge on owner-occupied housing and real estate.

Would you want to have an inheritance tax along with the consumption tax? With the consumption tax, Kaldor would like to have wealth taxes.

Difference between an inheritance tax and a wealth tax:
An inheritance tax—the person did not himself generate income—he should be taxed
A wealth tax—wealth carries economic power, therefore has more taxable capacity. Wealth is net worth—what would accrue to an estate if the person were to die , does not cover human wealth.

For the consumtion tax you need to know:
Where the money comes from
What assets were acquired and sold

It can't just be feasibility of administration which blocks the consumtion tax: The problem is the force with which administration would have to come down with the tax; the enforcement of facing down people's portfolios is difficult; the information requirement is not greater but you have to keep a running account of assets faithfully.

Lecture 23

Value added tax and Border tax adjustments

No group is the beneficiary of the tax
The French first instituted the Value-added tax in 1954
Each level pays the tax on what it sells and gets an offset for all that it buys which has already been taxed. Modifications: buying from outside the system you get no deduction.
Farmers are out of the VAT system—they are the exempt sector because they are the early part in the production chain. Users of farm products pay the tax for them. Actually farmers pay more because they don't deduct taxes on fertilizer etc...(inputs in farming) Reason farmers are left out—administrative reason.

Retailers at final sales are sometimes left out—not burden of the tax goes down for them and this stimulates the activity. Many people try to hide activities under this retailing label to reduce their taxes.

All types of VAT tax sales but differ in what you can deduct from sales: A tax of the product type —taxes gross value added i.e. they deduct current material inputs.

A tax of the income type—it covers whole economy, the net national product by allowing sales minus current material inputs minus depreciation.

A tax of the consumption type—added up over all activities, it would end up taxing consumption in nationalincome accounting language. It is sales minus current material inputs minus capital goods acquisition.

All existing VAT are of the consumption type.
The most neutral and easiest to administer is the income type: it is problematic because deducting depreciation creates ambiguity.

The product type is hard to administer because it is hard to distinguish between capital and current inputs.

Border tax adjustments in the VAT

Border tax adjustments is a mechanism which levies indirect taxes in addition to tariffs on goods which are imported and which rebates taxes when they are exported.
With a fixed exchange rate— VAT makes internal prices higher but when goods go in and out they are adjusted. Border tax adjustments do not give an advantage to a country in international trade;they permit a tax to be accomodated without causing a change in the exchange rate or internal prices. They are a trick to avoid a reaction to the tax by factor prices.

	Initial Situation	20% VAT	VAT & BTA	VAT & No Border tax adjustments	
Exchange Rate	1	1	1	1.2	1
Internal factor Prices	1	1	1	1	.833
Price of imports	100	120	100	100	100
Domestic Prices	100	120	120	120	100
Price of import Substitutes	100	120	120	120	100
Price of exports	100	120	100	100	100

Another way we can look at border tax adjustments— Suppose we decide to have an excise tax: where should we collect it? BTA convert tax on production to tax on consumption. It makes it possible to tax consumption while we tax producers.

If Value added fell predominantly on national versus international goods, there will be distortions and reverberations in other markets however no direct interference with trade patterns. If all sectors of production are affected similarly—there is no distortion.