A NOTE ON CONSUMPTION VERSUS INCOME TAXATION

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The tide of opinion among economists, especially those in the younger generation, has clearly shifted from income taxation to consumption taxation. Our laws, too, have been modified (through registered retirement plans, IRAs, Keogh plans, and the like) so as to partially convert our income tax system into a consumption tax. Given the relevance and importance of the topic, I would like to share with the profession what I think is the simplest, most straightforward treatment of the subject. Since I value simplicity and communicability very highly, I do not apologize for these aspects of the present note. Indeed, the essence of the story consists of a few "tricks" which render easily understandable what otherwise might be a quite complicated analytical structure. The analytical apparatus used is that of "standard" applied welfare economics or consumer surplus analysis. One of the standard assumptions of that literature, when applied to taxes, is that the proceeds of taxes are returned to the very taxpayers who paid them, but in a fashion (lump-sum transfers) which does not interfere with their choices at the margin. This enables one to isolate the pure efficiency effects of taxes that, like income and consumption taxes, do in fact distort the choices of economic agents. And it permits one to do so without entering into the morass of considering lots of alternative ways of how the government spends the money, or into the delicate and subtle question of how tax-induced changes in the distribution of income

might require modification of what otherwise would be "standard" analytical techniques or approaches.

The first simplification is to imbed saving in a two-period setting. Period 1 is when people work; period 2 is when they are retired. Consider it to be a constraint that people in period 2 will be consuming 8760 hours of leisure per year. Once this simplification is made, all the relevant choices are positioned in period 1. People can spend their hours working, or at leisure. From the hours that they work, part of the proceeds will be dedicated to current consumption, and part to saving for future consumption. We can thus set up the fundamental constraint as:

(1) $$C_1 + C_2 + L = 8760,$$

where C_1 is actual consumption in period 1, measured in wage units,

C_2 is the saving undertaken in period 1 for the purpose of consumption in period 2, and

L is leisure in period 1.

The second simplification comes from recognizing that a proportional consumption tax is, in these circumstances, "equivalent" to a subsidy to leisure in period 1. If we impose a tax at the rate t_c on both C_1 and C_2, it will generate a welfare cost equal to $-\frac{1}{2}(S_{11} + 2S_{12} + S_{22})t_c^2$.[2] But, by the adding-up property of substitution terms, we have $S_{11} = -S_{12} - S_{1L}$; $S_{22} = -S_{12} - S_{2L}$. Therefore $(S_{11} + 2S_{12} + S_{22}) = -S_{1L} - S_{2L}$. This, in turn is equal to the own-price substitution term for leisure, S_{LL}. Hence the welfare cost of a consumption tax at the rate t_c is equal to $-\frac{1}{2} S_{LL} t_c^2$. This permits an easy diagrammatic representation of the welfare costs of a consumption tax (see Figure 1).

The third simplification comes from recognizing that an income tax,

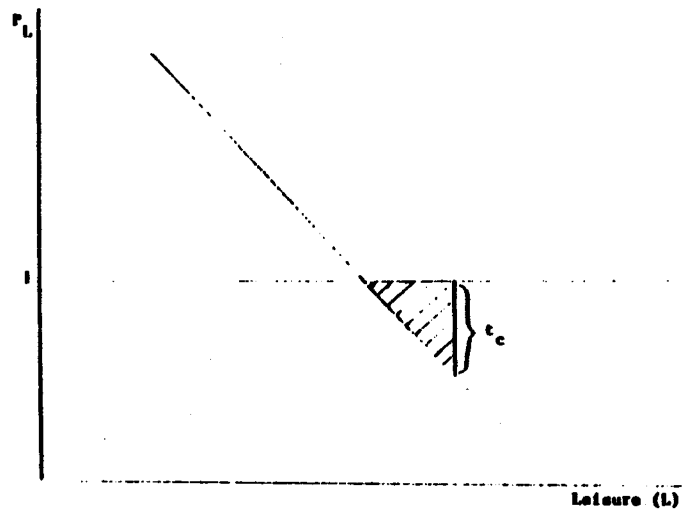

Figure 1

by its nature, double-taxes saving. All kinds of complications can enter here, if we let them. Dealing with a model that explicitly covers two periods, we would have to ask what is the interest rate, for how long is saving done in period 1 going to be held, on average, before it is consumed in period 2, etc. But with the "trick" of putting all relevant choices into time 1 we can "invent" the equivalent of an income tax, which, if at the stated rate of t_c would end up taxing C_1 at t_c and C_2 at $t_c + \delta$. Delta (δ) can be considered as the present value of the extra tax that would be paid on a dollar of period 1 saving, under an income tax as against a consumption tax at the rate of t_c. The interest rate lies in the background, changing it would change δ, and also change the pattern of choice between C_1, C_2 and L. One of the virtues of this simplified analysis is that neither the interest rate nor the time period need be introduced explicitly.

Applying the above simplification, we represent an income tax at the rate t_c as being the equivalent of a consumption tax at t_c plus an extra tax δ on C_2.[3] Diagrammatically, we have in Figure 2 a triangle identical to that of Figure 1, representing that a consumption tax is equivalent to a subsidy to leisure. In addition, we have a triangle whose height is δ, in the C_2 graph, plus a rectangle whose base is $S_{L2}\delta$, in the leisure graph. It is thus self-evident that, so long as period 1's leisure is a substitute for C_2, an income tax must be doubly inferior to a consumption tax at the same rate--doubly in the sense that both the triangle in the C_2 diagram and the rectangle in the L diagram constitute costs that are additional to those of a consumption tax. Q.E.D.

Figure 2 shows the superiority of the income tax in an unequivocal way (assuming $S_{L2} > 0$), but at the same time it may be regarded as answering a rather uninteresting question. To be sure, an income tax at the rate t_c

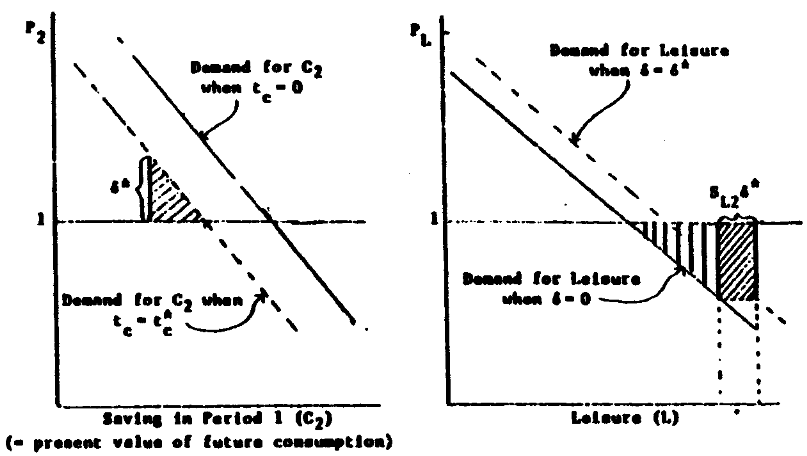

Figure 2

will have a higher welfare cost than a consumption tax at the same rate, but the income tax will yield more revenue. More interesting, it seems to me and to others, is the case in which the revenue yield of the two taxes is made to be the same. This case, built on the basis of our previous analysis, itself represents, in my view, a fourth simplification. Here the key is to set $t_yC_1 + (t_y + \delta)C_2 = t_c(C_1 + C_2)$. This leads to $t_c = t_y + [\delta C_2/(C_1 + C_2)]$.

The fifth and final simplification is the assumption that C_1 and C_2 are equally good substitutes for leisure. In symbols, $S_{1L} = -C_1S_{LL}/(C_1 + C_2)$ and $S_{2L} = -C_2S_{LL}/(C_1 + C_2)$. This brings true beauty to the picture, at the same time as it gives pause to dogmatists for the consumption tax. The beauty can be seen in Figure 3. From before, we know that an income tax carries three components of welfare cost--a triangle (A) in the leisure diagram, plus another (F) in the C_2 graph, plus a rectangle (B) in the leisure graph. The consumption tax entails only one triangle, but for equal revenue the consumption tax must be at a higher rate than the income tax. We have determined this extra rate to be $\delta^*C_2/(C_1 + C_2)$. The welfare cost of the consumption tax now becomes the larger triangle A + B + E. The neatness of the assumption that C_1 and C_2 are equally good substitutes for leisure lies in the fact that under this assumption the rectangle B is a component of <u>both</u> the welfare cost of the income tax <u>and</u> the welfare cost of the equal-revenue consumption tax. In analyzing the income tax, we think of imposing (on C_2) a tax of δ^* on top of a pre-existing consumption tax at the rate t^*_y (which by itself generated the initial triangle of welfare cost (A). When δ^* is set in place, the demand for leisure shifts, generating a rectangle of welfare cost whose base is $S_{L2}\delta^*$. When we increase the rate of consumption tax from t_y^* to $t_y^* + \delta^*C_2/(C_1 + C_2)$, we cause the base of

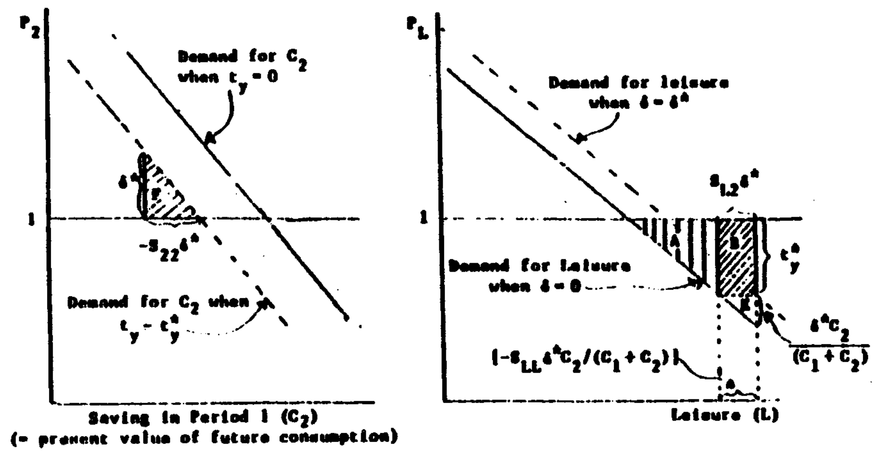

Figure 3

triangle A to increase by the amount $[-S_{LL}\delta^* C_2/(C_1 + C_2)]$. But under our assumption of equal substitutability, $S_{L2} = -S_{LL}C_2/(C_1 + C_2)$. Therefore the base of the leisure-graph triangle has to be expanded, in order to produce equal revenue with an income tax, by precisely the amount by which the income tax itself shifts the demand for leisure.

We now no longer have two extra reasons for preferring the consumption tax. All depends on the comparison between the size of triangle F (which is the part of the cost of the income tax not shared by the consumption tax) with the size of triangle E (which is a part of the cost of the consumption tax but not of the income tax). Purists may breathe a sigh of relief, however, for under our assumptions the consumption tax still wins. The height of triangle F is δ^*; that of triangle E is only a fraction of δ^*. Moreover, the base of triangle F is $-S_{22}\delta^*$, whereas that of triangle E is $S_{L2}\delta^*$. The adding-up property of substitution terms requires that $-S_{22} = S_{12} + S_{L2}$, so, assuming substitutability to prevail throughout, the base of triangle E would be less than that of F.

So it is still true, under our assumption, that a consumption tax is better than an income tax. But the issue turns, in the case we have examined, on the difference in size of two triangles. Subtract a bit of substitutability between C_2 and leisure, and the base of rectangle B ($S_{L2}\delta^*$) shrinks in size, at the same time as triangle F shrinks (because S_{L2} is a component of $-S_{22}$). Surely, then, a plausible picture could be built up in which an income tax would end up having lower welfare cost than a consumption tax.

This is done in Figure 4. Here the welfare cost of an income tax at the rate t_y^* is equal to A + B + F. That of an equal-revenue consumption tax is A + B + G + E. Clearly, as drawn, the area of G + E exceeds that of F.

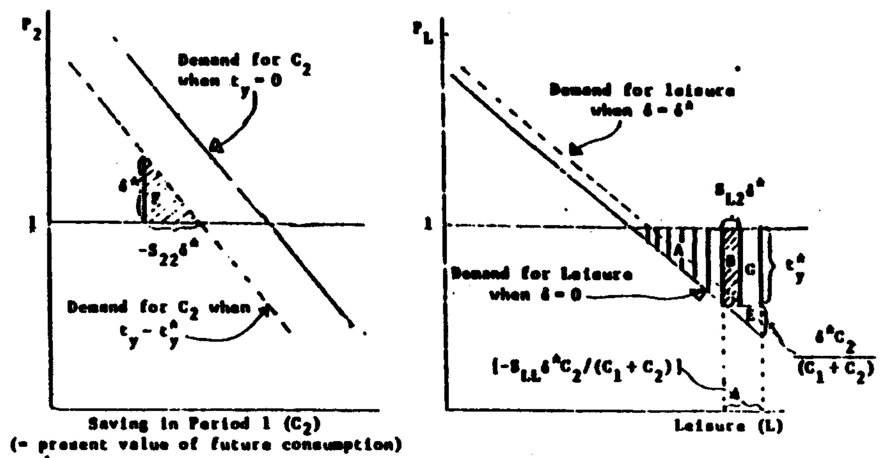

Figure 4

The precise condition that has to be met in order for the income tax to be superior is

$$(2) \qquad -\frac{S_{LL}\delta^{*2}}{2} + S_{L2}\delta^* t_y^* < -[S_{LL}\delta^* C_2/(C_1 + C_2)][t_y^* + C_2\delta^*/2(C_1 + C_2)]$$

$$(3) \qquad -\frac{S_{22}\delta^*}{2} + S_{L2}t_y^* < -[S_{LL}C_2/(C_1 + C_2)][t_y^* + C_2\delta^*/2(C_1 + C_2)]$$

$$(4) \qquad (S_{12} + S_{L2})\,\frac{\delta^*}{2} + S_{L2}t_y^* < [(S_{L1} + S_{L2})C_2/(C_1 + C_2)][t_y^* + C_2\delta^*/2(C_1 + C_2)]$$

$$(5) \qquad (S_{12} + S_{L2})\,\frac{\delta^*}{2} + S_{L2}t_y^*\left[\frac{C_1}{C_1 + C_2}\right] < \left(\frac{S_{L1}C_2}{C_1 + C_2}\right)t_y^* + (S_{L1} + S_{L2})\left(\frac{C_2}{C_1 + C_2}\right)^2\left(\frac{\delta^*}{2}\right).$$

Equation (2) is directly derived from the diagrammatic analysis of Figure 4. In equation (3) we simply divide each term by δ^*. In equation (4) we separate S_{22} and S_{LL} into their constituent parts, and in (5) we collect terms containing $S_{L2}t_y^*$, producing, on the left-hand side, $S_{L2}t_y^*[1 - C_2/(C_1 + C_2)] = S_{L2}t_y^* C_1/(C_1 + C_2)$.

Examining equation (5), it is useful to concentrate on the two terms in t_y^*. It is easy to see that the right hand term will be greater than the left hand term if $S_{L1}C_2 > S_{L2}C_1$. This is equivalent to saying that the cross-elasticity of demand (η_{1L}) for X_1 with respect to the price of leisure is greater than that of C_2 with respect to the same price. With initial prices of unity, we have that $\eta_{1L} = \frac{1}{C_1}\frac{\partial C_1}{\partial P_L} = S_{1L}/C_1$; $\eta_{2L} = \frac{1}{C_2}\frac{\partial C_2}{\partial P_L} = S_{2L}/C_2$. $\eta_{1L} > \eta_{2L}$ therefore implies $S_{1L}C_2 > S_{2L}C_1$. (Recall that, by the symmetry property $S_{1L} = S_{L1}$; $S_{2L} = S_{L2}$.) Thus, one can say that if the cross-elasticity of demand for present consumption, with respect to the price of present leisure, is <u>sufficiently</u> greater than that of future consumption, the income tax will turn out to have lower welfare cost than an equal-revenue consumption tax.

A numerical example might be helpful to some readers. Let us assume that the cross-elasticity of demand for C_1 with respect to the price of leisure is 0.2, while that of C_2 with respect to the same price is 0.1. If C_1 is initially 2000 and C_2 is 500 this implies that $S_{1L} = 400$, and $S_{2L} = 50$. By the adding-up property, S_{LL} is -450. Assume, too, that the cross-elasticity $\eta_{12} = .1$. This means that $S_{12} = 200$, implying $S_{11} = -600$; $S_{22} = -250$.

Now assume $t_y^* = .2$ and $\delta^* = .05$, and make the necessary substitutions in (3):

$$(3) \qquad -\frac{S_{22}\delta^*}{2} + S_{L2}t_y^* < [-S_{LL}C_2/(C_1+C_2)][t_y^* + C_2\delta^*/(C_1+C_2)]$$

$$\frac{250(.05)}{2} + 50(.2) < 450\ (.2)\ [.2 + \frac{.2(.05)}{2}]$$

$$16.25 < 90(.205) = 18.45.$$

The results are quite sensitive to movements of δ^* relative to t^*. In this particular case the inequality becomes a virtual equality when $\delta^* = .07$. I certainly do not want to argue for the empirical veracity of the numerical assumptions involved in the above example. I would only assert that the assumptions are not patently absurd. Other assumptions can be made about the basic parameters, and it still will be possible, by manipulating the ratio of η_{1L} to η_{2L} to produce a similar result. I really believe, however, that working with "most plausible" assumptions, a consumption tax will prove to be more conducive to economic efficiency than an income tax, but, as was shown in Figure 3, by a surprisingly narrow margin.

The lesson of all of this, it seems to me, is that it is too close a horse race, between consumption and income taxation on the purely theoretical

level, for us in the economics profession to go to the barricades for one or the other. We probably need to worry much more about issues of tax design. What may seem initially to be tiny loopholes, we have learned by sad experience can turn out to have huge welfare costs. So my recommendation is, let the partisans of the income tax design the best comprehensive income tax system that modern economic science plus our accumulated administrative experience and wisdom can produce. Also, let the partisans of a consumption tax as a substitute for the income tax do the same. Then we can all relax and flip a coin.

FOOTNOTES

[1]This note is extracted from classroom presentations of the last several years in my course in public finance at the University of Chicago. When, through casual conversations, I presented its basic outlines to a number of members of the public finance fraternity--specifically Glenn P. Jenkins, Attiat F. Ott, and James Roumasset--they urged me to make it available in printed form. The most direct lineage into which this paper fits would start with Corlett and Hague's classic article (1954) and go through my two papers (1964a and 1964b) on the subject of the applied welfare economics of taxation in a general equilibrium setting. The specific underlying methodology is expounded in (1964a).

[2]$S_{11} = \partial C_1/\partial P_1$, $S_{12} = \partial C_1/\partial P_2$, etc. The prices of all "commodities" can be considered to be 1 plus the associated tax or minus the associated subsidy. The properties of the S_{ij} are: adding-up, $S_{11} + S_{12} + S_{1L} = 0$; $S_{21} + S_{22} + S_{2L} = 0$; $S_{L1} + S_{L2} + S_{LL} = 0$; and also symmetry $S_{12} = S_{21}$; $S_{1L} = S_{L1}$; $S_{2L} = S_{L2}$. The general formula from which the expression in the text is derived expresses the welfare cost of a set of taxes t_i by a quadratic form: $W.C. = -\frac{1}{2}\sum_i \sum_j S_{ij} t_i t_j$ (see Harberger 1964a and 1964b). Similar expressions appear in H. Hotelling (1938) and J. R. Hicks (1946, p. 331).

[3]If one contemplates varying the rate of income tax then δ should be considered to be a monotonically increasing function of $t_y - t_c$ over the relevant range.

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