First degree price discrimination

Introduction

- Annual subscriptions generally cost less in total than one-off purchases
- Buying in bulk usually offers a price discount
  - these are price discrimination reflecting quantity discounts
  - prices are *nonlinear*, with the unit price dependent upon the quantity bought
  - allows pricing nearer to willingness to pay
  - so should be more profitable than third-degree price discrimination
- How to design such pricing schemes?

Demand and quantity

- Individual inverse demand $p_i = D_i(q_i)$
- Interpretation: willingness to pay for the q_i^{th} unit. Also called reservation value.
- Examples:
  - Willingness to pay for each extra drink
  - Willingness to pay for the right of making an extra phone call
  - Willingness to pay for inviting an extra friend to a concert
- Decreasing with q_i

First-degree price discrimination 1

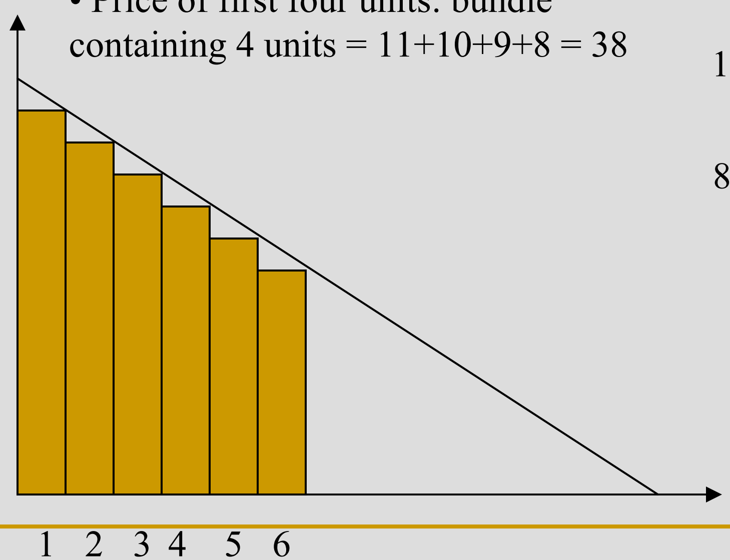
- Monopolist charges consumers their reservation value for each unit consumed.
- Extracts *all* consumer surplus
- Since profit is now total surplus, find that first-degree price discrimination is *efficient.*

First degree price discrimination - example

- One consumer type.
- Demand $p_i = 12\text{-}q_i$.
- Willingness to pay for first unit approximately 11
- Willingness to pay for 4^{th} unit: 8
- Charge consumer willingness to pay for each unit consumed.

• Charge 11 for the first unit, 10 for the second one…

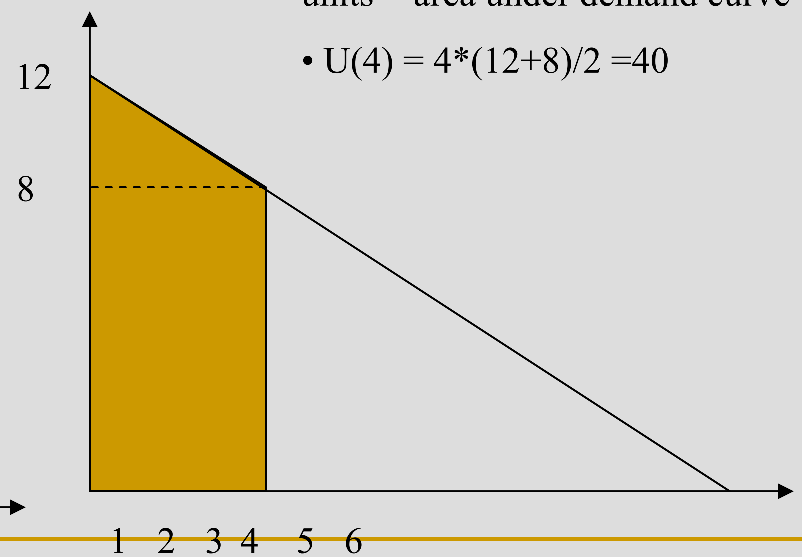
• Price of first four units: bundle containing 4 units = 11+10+9+8 = 38



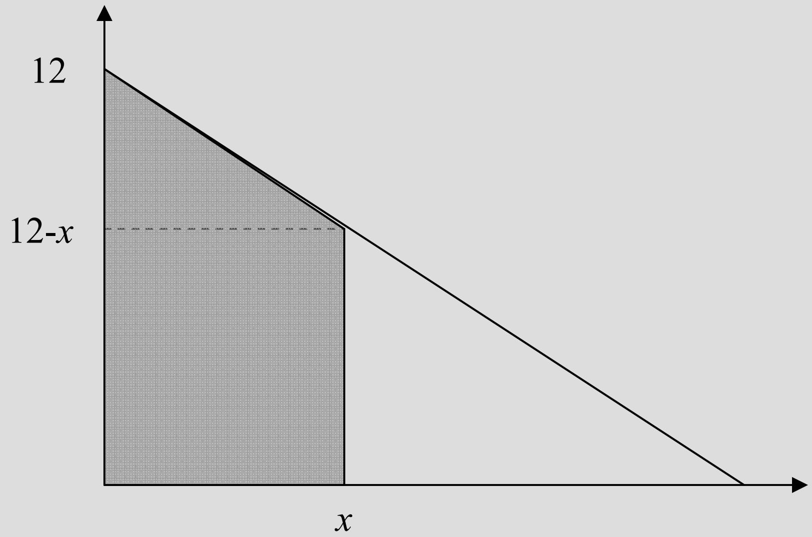
Continuous approximation

•Willingness to pay for the first 4 units = area under demand curve

• U(4) = 4*(12+8)/2 =40



More general case



- Willingnes to pay for first x units =

$x * (12+12-x)/2 = 12x - \frac{1}{2} x^2$

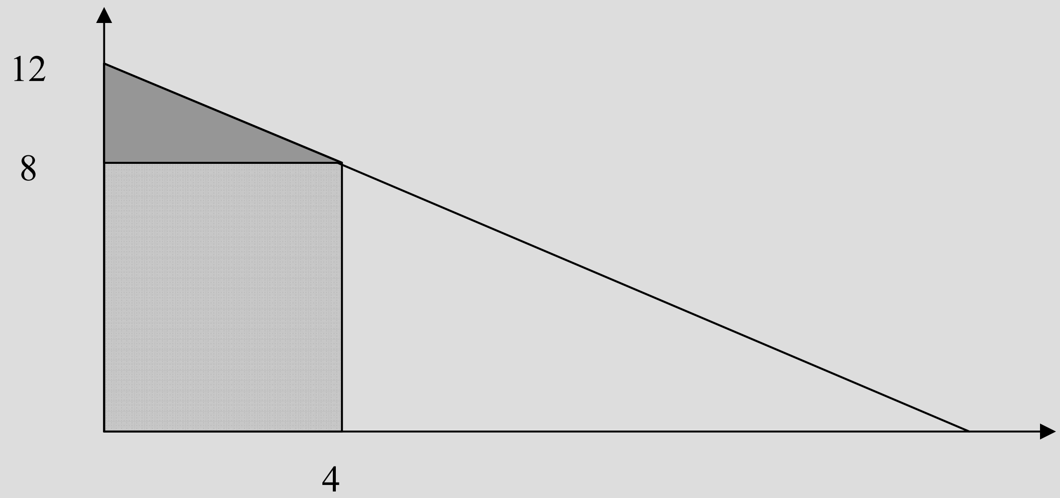
- More generally price for first x units:

$$\int_0^x P(q)dq$$

- Linear case $P(q) = A - Bq$

$$P(x) = Ax - \frac{1}{2} Bx^2$$

Implementation – two part tariffs



1. Charge different prices for each unit sold: *P(1)+P(2)+P(3)+P(4)*
2. Charge willingness to pay for the first 4 units (approximately 40).
3. Charge a price per 8 and a flat fee = to triangle = 4*4/2=8.
   - At a price of 8 per unit, consumer will buy 4 units.
   - Will pay 8*4=32 plus the flat fee (8), for total of 40.
   - This scheme is called a two-part tariff.
4. Charge flat fee of 40 with free consumption of 4 units and 8 for each extra unit consumed.

Quantity discount

- Monopolist will charge willingness to pay.
- With linear demand, total price paid is
  $Ax - \frac{1}{2} Bx^2$
- Called non-linear price
- Price per unit
  $A – \frac{1}{2} Bx$
- Decreasing in *x* – quantity discount

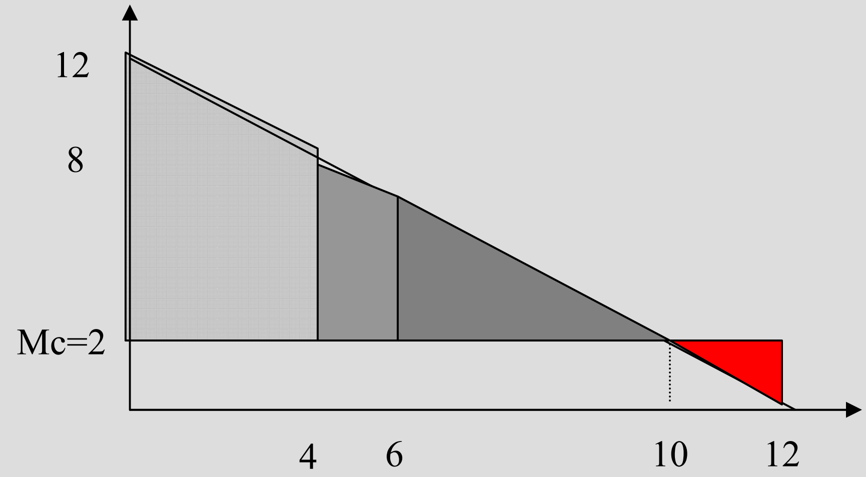
Take previous example ($A=12$, $B=1$)

x	Total price	Price /unit
4	40	10
6	54	9
8	64	8
10	70	7
12	72	6

Optimal quantity

- Take previous example with constant marginal cost $c = 2$

x	Total price	profits
4	40	32
6	54	42
8	64	48
10	70	50
12	72	48



Profits = $\pi(x) = \int_0^x P(q)dq - C(x)$

Maximum: $\frac{\partial \pi}{\partial x} = 0$ → $p(x)=MC(x)$

- Optimal rule: equate mg cost to reservation value
- Efficiency: no conflict between value creation and appropriation

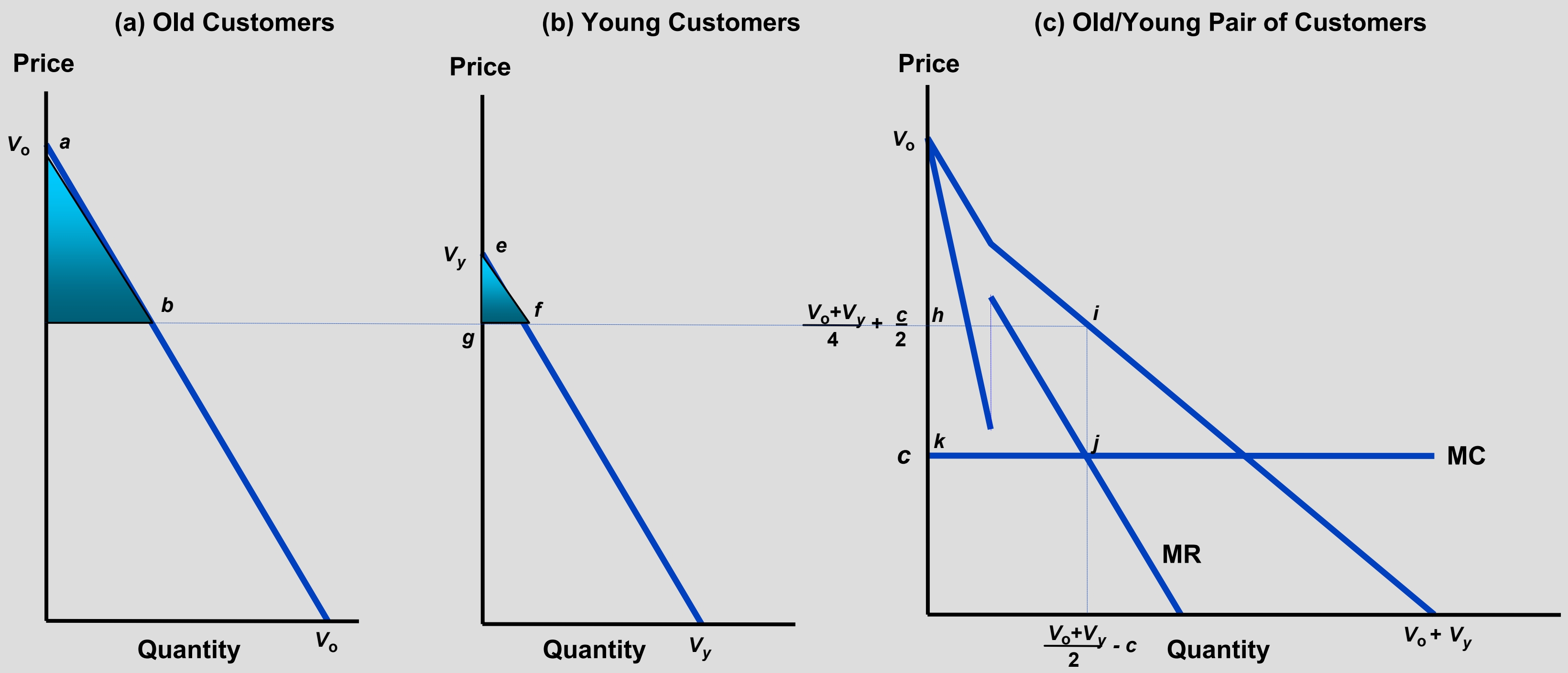
More customers: multiple nonlinear prices

- Jazz club serves two types of customer
  - Old: demand for entry plus Q_o drinks is $P = V_o - Q_o$
  - Young: demand for entry plus Q_y drinks is $P = V_y - Q_y$
  - Equal numbers of each type
  - Assume that $V_o > V_y$: Old are willing to pay more than Young
  - Cost of operating the jazz club $C(Q) = F + cQ$
- Demand and costs are all in daily units

Linear prices – no discrimination

- Suppose that the jazz club owner applies “traditional” linear pricing: free entry and a set price for drinks
  - aggregate demand is $Q = Q_o + Q_y = (V_o + V_y) - 2P$
  - invert to give: $P = (V_o + V_y)/2 - Q/2$
  - MR is then $MR = (V_o + V_y)/2 - Q$
  - equate MR and MC, where MC = c and solve for Q to give
  - $Q_U = (V_o + V_y)/2 - c$
  - substitute into aggregate demand to give the equilibrium price
  - $P_U = (V_o + V_y)/4 + c/2$
  - each Old consumer buys $Q_o = (3V_o - V_y)/4 - c/2$ drinks
  - each Young consumer buys $Q_y = (3V_y - V_o)/4 - c/2$ drinks
  - profit from each pair of Old and Young is $\pi_U = (V_o + V_y - 2c)^2$

This example can be illustrated as follows:



Linear pricing leaves each type of consumer with consumer surplus

Improvement 1: Add entry fee

- Jazz club owner can do better than this
- Consumer surplus at the uniform linear price is:
  - Old: $CS_o = (V_o - P_U).Q_o/2 = (Q_o)^2/2$
  - Young: $CS_y = (V_y - P_U).Q_y/2 = (Q_y)^2/2$
- So charge an entry fee (just less than):
  - $E_o = CS_o$ to each Old customer and $E_y = CS_y$ to each Young customer
    - check IDs to implement this policy
  - each type will still be willing to frequent the club and buy the equilibrium number of drinks
- So this increases profit by E_o for each Old and E_y for each Young customer

Improvement 2: optimal prices

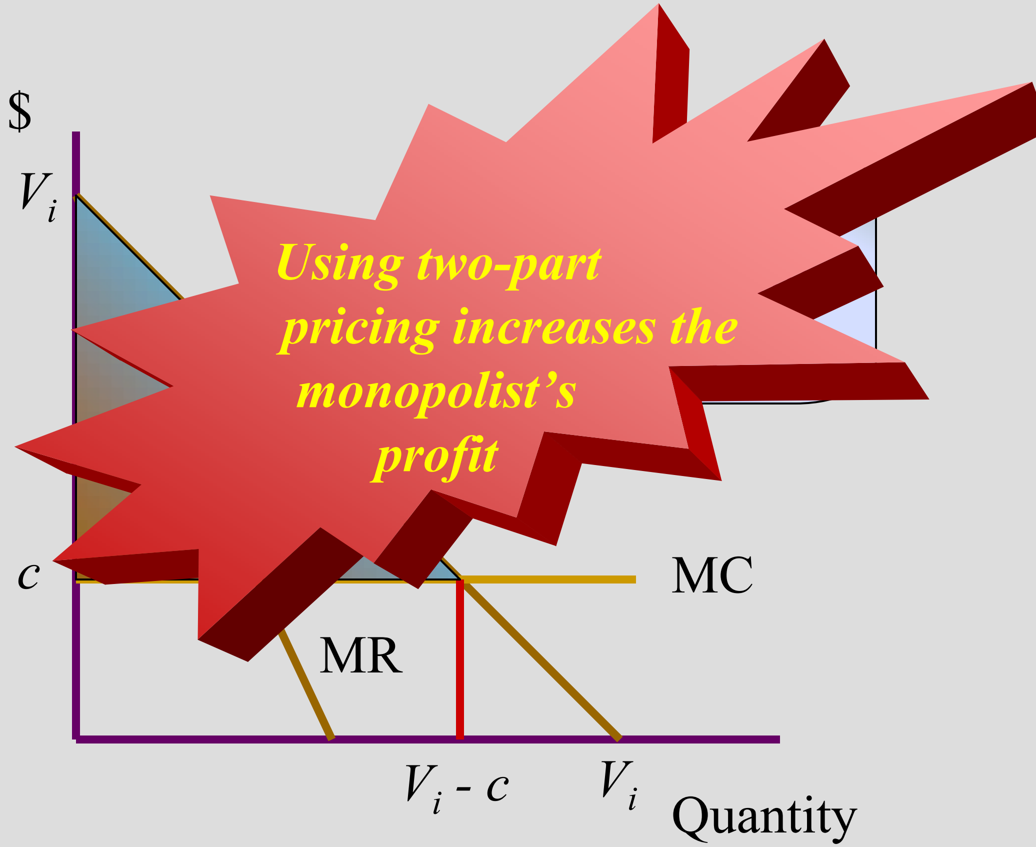
- The jazz club can do even better
  - reduce the price per drink
  - this increases consumer surplus
  - but the additional consumer surplus can be extracted through a higher entry fee
- Consider the best that the jazz club owner can do with respect to each type of consumer

Two-Part Pricing

Set the unit price equal to marginal cost

This gives consumer surplus of $(V_i - c)^2/2$

Set the entry charge to $(V_i - c)^2/2$

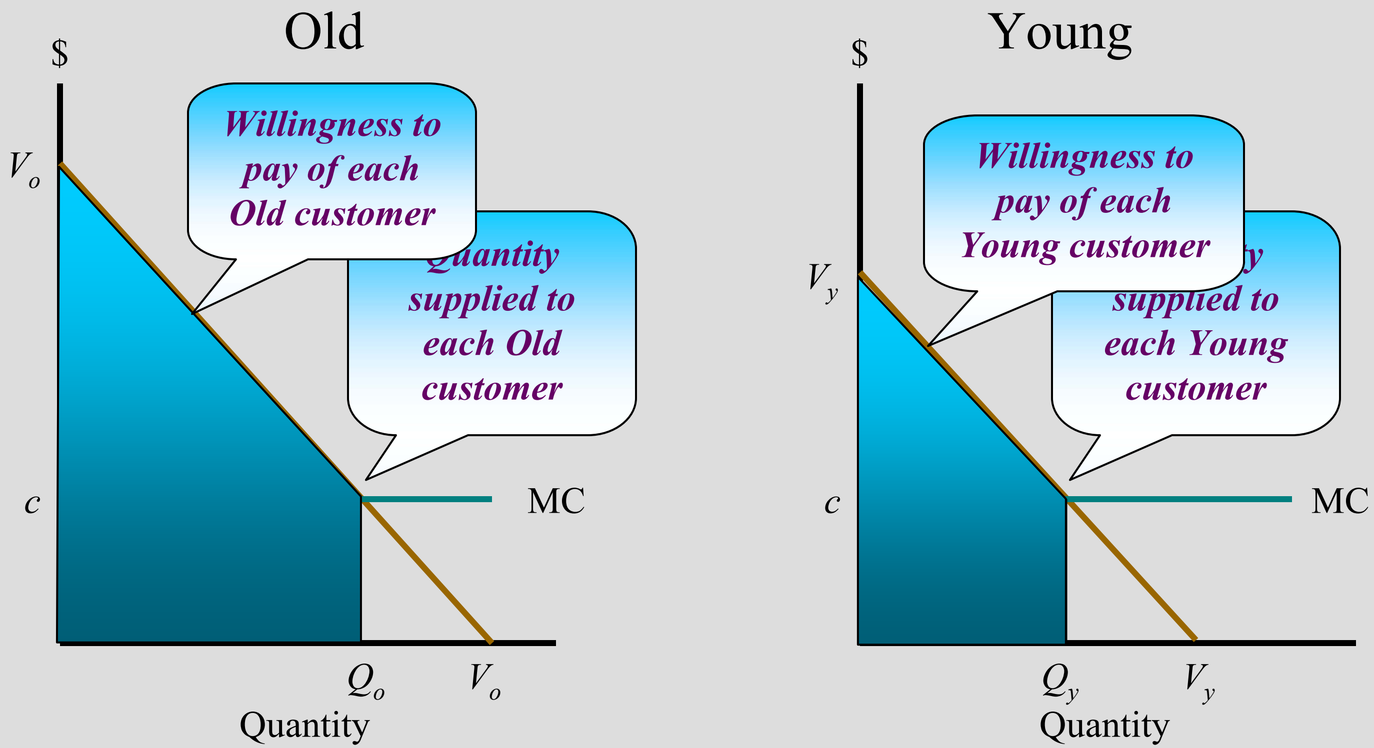


Profit from each pair of Old and Young is now $\pi_d = [(V_o - c)^2 + (V_y - c)^2]/2$

Block pricing

- Offer a *package* of “Entry plus X drinks for $Y”
- To maximize profit apply two rules
  - set the quantity offered to each consumer type equal to the amount that type would buy at price equal to marginal cost
  - set the total charge for each consumer type to the total willingness to pay for the relevant quantity
- Return to the example:

Block pricing 2



$$WTP_o = (V_o - c)^2/2 + (V_o - c)c = (V_o^2 - c^2)/2$$

$$WTP_y = (V_y - c)^2/2 + (V_y - c)c = (V_y^2 - c^2)/2$$

Block pricing 3

- How to implement this policy?
  - card at the door
  - give customers the requisite number of tokens that are exchanged for drinks

Summary

- First degree price discrimination (charging different prices for additional units) allow monopolist to extract more surplus.
- Optimal quantity = efficient, where reservation value = mc
- Can be implemented with two-part tariff: p=mc and F=CS
- Can also be implemented with block pricing: Charge a flat fee in exchange for total “package” . Size of package where reservation value=mc (same as before), fee=area under demand curve.
- Average price decreases with quantity (non-linear price)