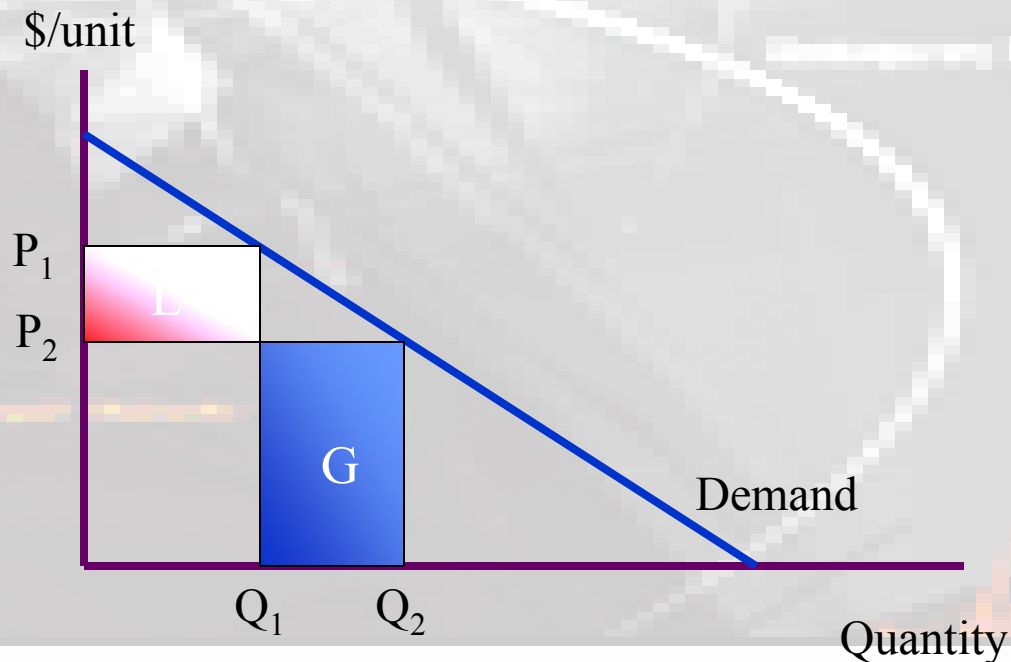




Monopoly: No discrimination

Marginal Revenue

- The only firm in the market
 - market demand is the firm's demand
 - output decisions affect market clearing price



Monopoly (cont.)

- Derivation of the monopolist's marginal revenue

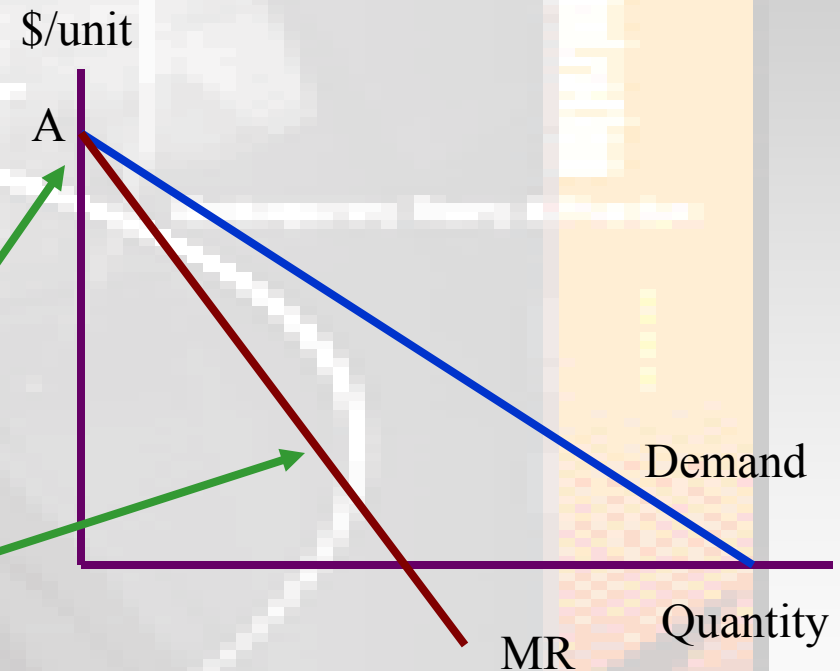
$$\text{Demand: } P = A - B \cdot Q$$

$$\text{Total Revenue: } TR = P \cdot Q = A \cdot Q - B \cdot Q^2$$

$$\text{Marginal Revenue: } MR = dTR/dQ$$

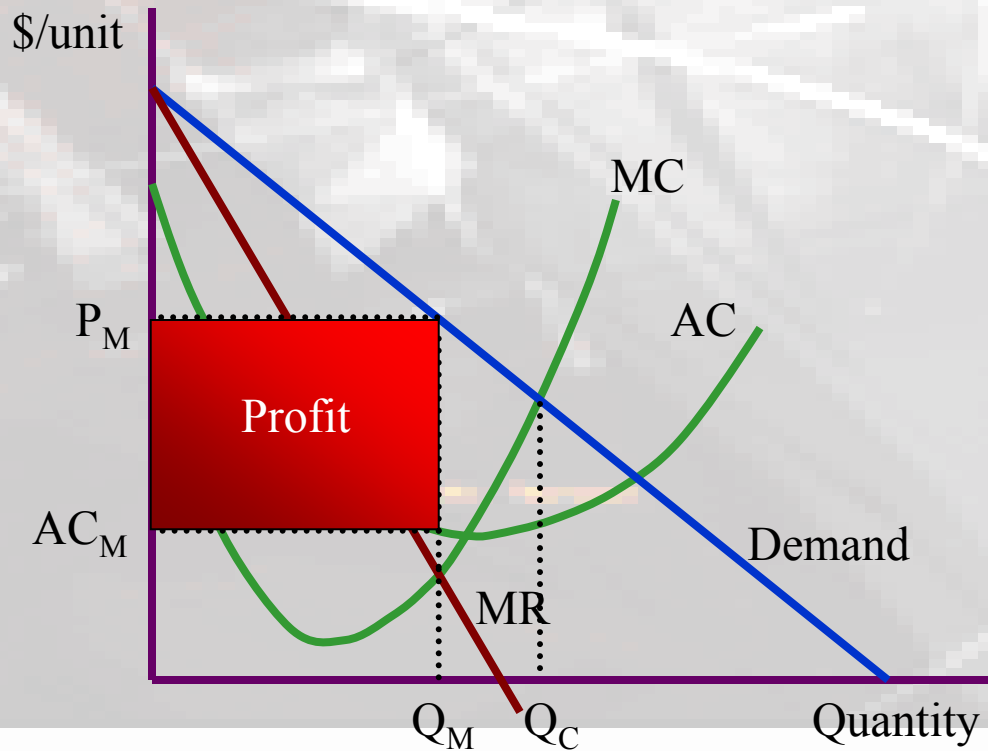
$$MR = A - 2B \cdot Q$$

With linear demand the marginal revenue curve is also linear with the same price intercept but twice the slope of the demand curve



Monopoly and Profit Maximization

- The monopolist maximizes profit by equating marginal revenue with marginal cost



Marginal Revenue and Demand Elasticity

Inverse demand: $P(q)$

Total revenue $R(q) = P(q)q$

Marginal revenue: $R'(q) = p + (\partial P / \partial q)q$

$$= p \left(1 + (\partial P / \partial q) \frac{q}{p} \right)$$

$$= p \left[1 - \frac{1}{\varepsilon_d} \right]$$

• *Max profits: $MR = MC$*

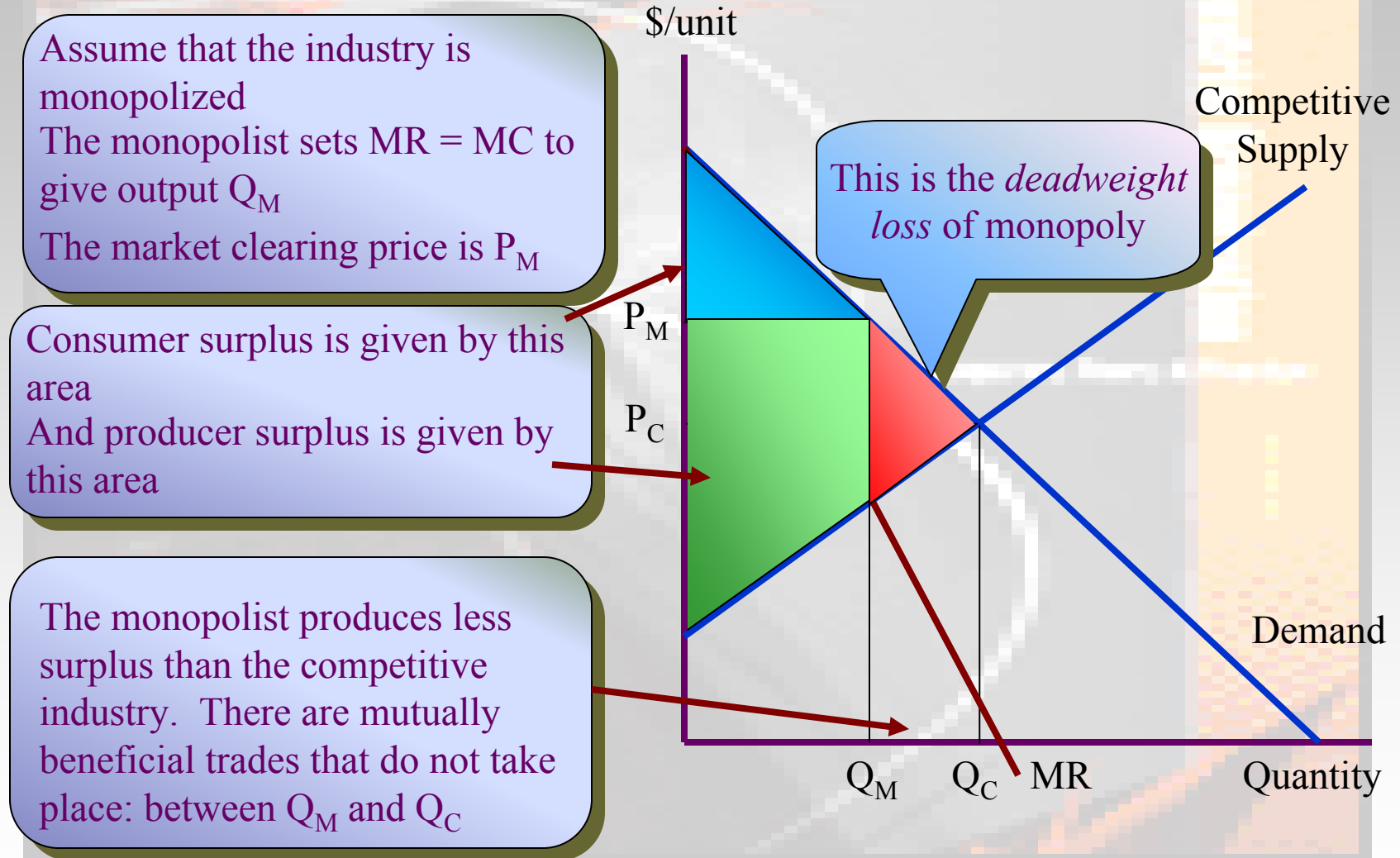
$$p \left(1 - \frac{1}{\varepsilon_d} \right) = MC$$

• *higher elasticity \rightarrow lower price*

Lerner Index:

$$L = \frac{p - MC}{p} = \frac{1}{\varepsilon_d}$$

Deadweight loss of Monopoly



Deadweight loss of Monopoly (cont.)

- Why can the monopolist not appropriate the deadweight loss?
 - Increasing output requires a reduction in price
 - *this assumes that the same price is charged to everyone.*
- The monopolist creates surplus
 - some goes to consumers
 - some appears as profit
- The monopolist bases her decisions purely on the surplus she gets, *not* on consumer surplus
- The monopolist undersupplies relative to the competitive outcome
- The primary problem: *the monopolist is large relative to the market*



Price Discrimination and Monopoly: Linear Pricing

Introduction

- Prescription drugs are cheaper in Canada than the United States
- Textbooks are generally cheaper in Britain than the United States
- Examples of *price discrimination*
 - presumably profitable
 - should affect market efficiency: not necessarily adversely
 - is price discrimination necessarily bad – even if not seen as “fair”?

Feasibility of price discrimination

- Two problems confront a firm wishing to price discriminate
 - *identification*: the firm is able to identify demands of different types of consumer or in separate markets
 - easier in some markets than others: e.g tax consultants, doctors
 - *arbitrage*: prevent consumers who are charged a low price from reselling to consumers who are charged a high price
 - prevent re-importation of prescription drugs to the United States
- The firm then must choose the *type* of price discrimination
 - first-degree or personalized pricing
 - second-degree or menu pricing
 - third-degree or group pricing

Third-degree price discrimination

- Consumers differ by some observable characteristic(s)
- A uniform price is charged to all consumers in a particular group – linear price
- Different uniform prices are charged to different groups
 - “kids are free”
 - subscriptions to professional journals e.g. *American Economic Review*
 - airlines
 - early-bird specials; first-runs of movies

Third-degree price discrimination (cont.)

- The pricing rule is very simple:
 - consumers with low elasticity of demand should be charged a high price
 - consumers with high elasticity of demand should be charged a low price

Third degree price discrimination: example

- Harry Potter volume sold in the United States and Europe
- Demand:
 - United States: $P_U = 36 - 4Q_U$
 - Europe: $P_E = 24 - 4Q_E$
- Marginal cost constant in each market
 - $MC = \$4$

The example: no price discrimination

- Suppose that the same price is charged in both markets
- Use the following procedure:
 - calculate aggregate demand in the two markets
 - identify marginal revenue for that aggregate demand
 - equate marginal revenue with marginal cost to identify the profit maximizing quantity
 - identify the market clearing price from the aggregate demand
 - calculate demands in the individual markets from the individual market demand curves and the equilibrium price

The example (npd cont.)

United States: $P_U = 36 - 4Q_U$ Invert this:

$$Q_U = 9 - P/4 \text{ for } P \leq \$36$$

Europe: $P_E = 24 - 4Q_E$ Invert

$$Q_E = 6 - P/4 \text{ for } P \leq \$24$$

Aggregate these demands

$$Q = Q_U + Q_E = 9 - P/4 \text{ for } \$36 \geq P \geq \$24$$

$$Q = Q_U + Q_E = 15 - P/2 \text{ for } P < \$24$$

**At these prices
only the US
market is active**

**Now both
markets are
active**

The example (npd cont.)

Invert the direct demands

$$P = 36 - 4Q \text{ for } Q \leq 3$$

$$P = 30 - 2Q \text{ for } Q > 3$$

Marginal revenue is

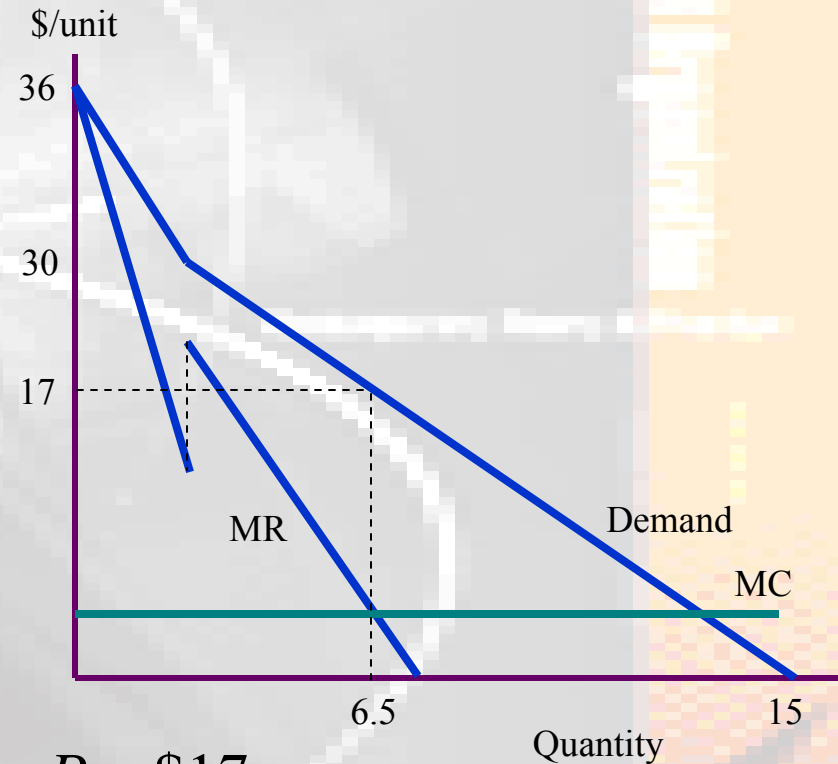
$$MR = 36 - 8Q \text{ for } Q \leq 3$$

$$MR = 30 - 4Q \text{ for } Q > 3$$

Set $MR = MC$

$$Q = 6.5$$

Price from the demand curve $P = \$17$



The example (npd cont.)

Substitute price into the individual market demand curves:

$$Q_U = 9 - P/4 = 9 - 17/4 = 4.75 \text{ million}$$

$$Q_E = 6 - P/4 = 6 - 17/4 = 1.75 \text{ million}$$

$$\text{Aggregate profit} = (17 - 4) \times 6.5 = \$84.5 \text{ million}$$

The example: price discrimination

- The firm can improve on this outcome
- Check that MR is not equal to MC in both markets
 - $MR > MC$ in Europe
 - $MR < MC$ in the US
 - the firms should transfer some books from the US to Europe
- This requires that different prices be charged in the two markets
- Procedure:
 - take each market separately
 - identify equilibrium quantity in each market by equating MR and MC
 - identify the price in each market from market demand

The example: (pd cont.)

Demand in the US:

$$P_U = 36 - 4Q_U$$

Marginal revenue:

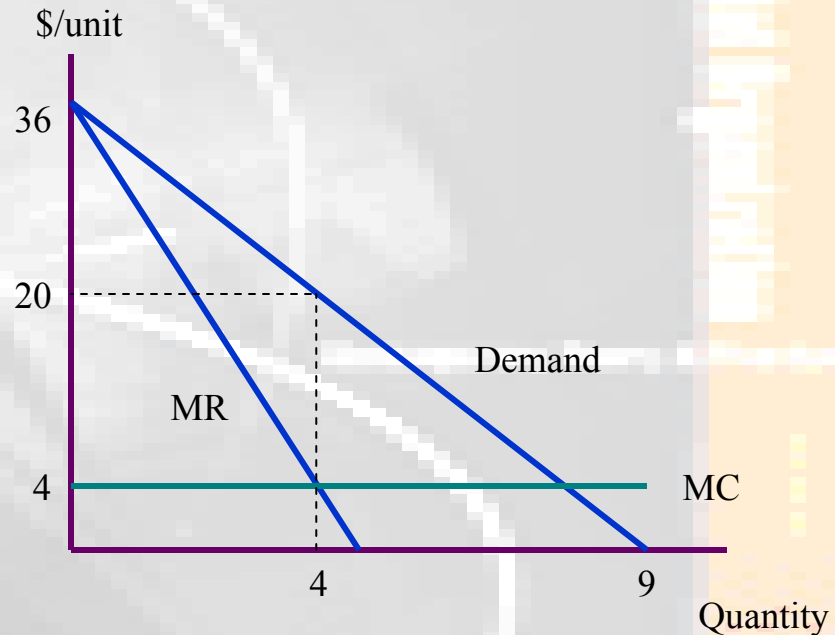
$$MR = 36 - 8Q_U$$

$$MC = 4$$

Equate MR and MC

$$Q_U = 4$$

Price from the demand curve $P_U = \$20$



The example: (pd cont.)

Demand in the Europe:

$$P_E = 24 - 4Q_E$$

Marginal revenue:

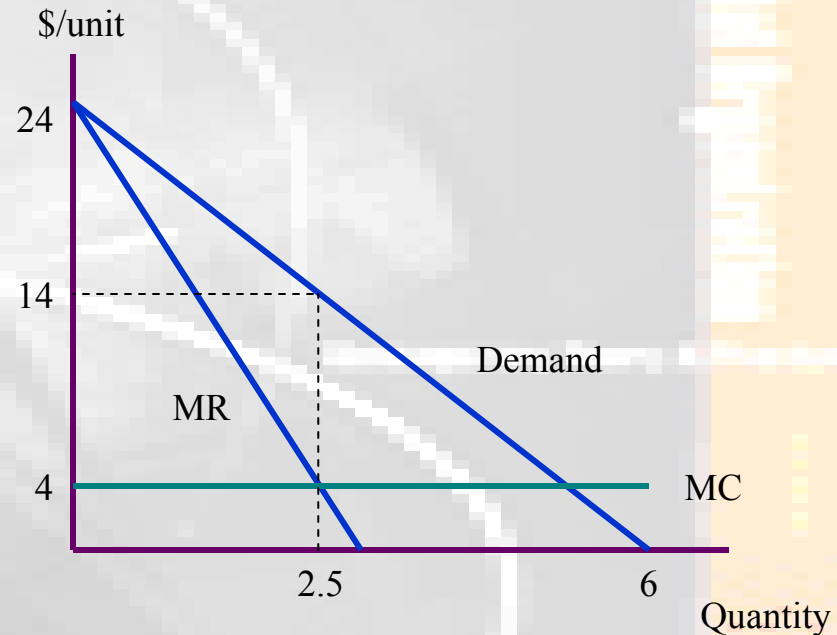
$$MR = 24 - 8Q_E$$

$$MC = 4$$

Equate MR and MC

$$Q_E = 2.5$$

Price from the demand curve $P_E = \$14$



The example (pd cont.)

- Aggregate sales are 6.5 million books
 - the same as without price discrimination
- Aggregate profit is $(20 - 4) \times 4 + (14 - 4) \times 2.5 =$ \$89 million
 - \$4.5 million greater than without price discrimination

No price discrimination: non-constant cost

- The example assumes constant marginal cost
- How is this affected if MC is non-constant?
 - Suppose MC is increasing

An example with increasing MC

$$MC(q) = 2 \cdot (q-1)$$

D market 1

P	q
7	1
5	2

D market 2

P	q
4	1
3	2

No discrimination

p	q	TR	MR	MC	TC
7	1				
5	2				
4	3				
3	4				

An example with increasing MC

D market 1

P	q
7	1
5	2

Previous solution: $p=5$, $q=2$, $TC=2$, $\pi=8$

Anything better?

Consider selling one unit in each market:

$p_1=7$, $p_2=4$ $TR=11$ and $\pi=9$

D market 2

P	q
4	1
3	2

Where is the difference coming from?

$$MC(q) = 2 \cdot (q-1)$$

Example (continued)

market 1

p	q	TR	MR
7	1	7	7
5	2	10	3

market 2

p	q	TR	MR
4	1	4	4
3	2	6	2

Key idea: order consumers by MR

q	MR	MC
1	7	0
2	4	2
3	3	4
4	2	8

The optimum is to include only the first two consumers:

$$p_1=7, p_2=4.$$

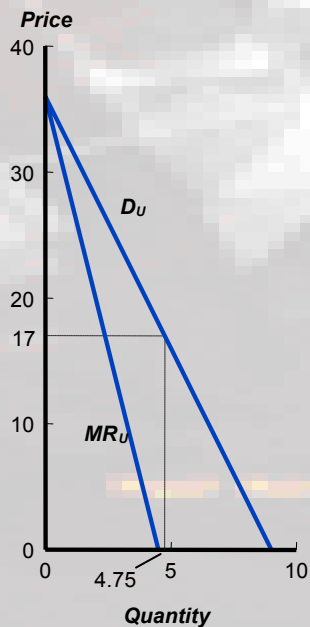
No price discrimination: non-constant cost

- More general linear demand case
- No price discrimination procedure
 - Calculate aggregate demand
 - Calculate the associated MR
 - Equate MR with MC to give aggregate output
 - Identify price from aggregate demand
 - Identify market demands from individual demand curves

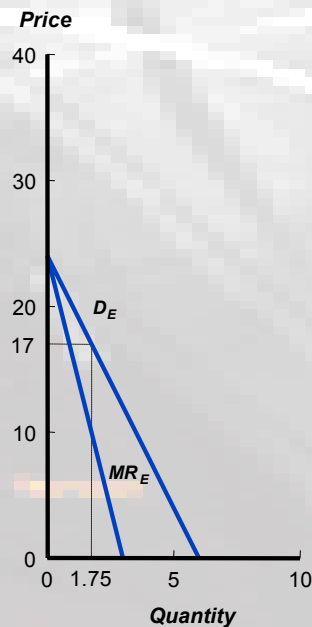
The example again

Applying this procedure assuming that $MC = 0.75 + Q/2$ gives: $0.75 + Q/2 = 30 - 4Q \rightarrow Q = 6.5$

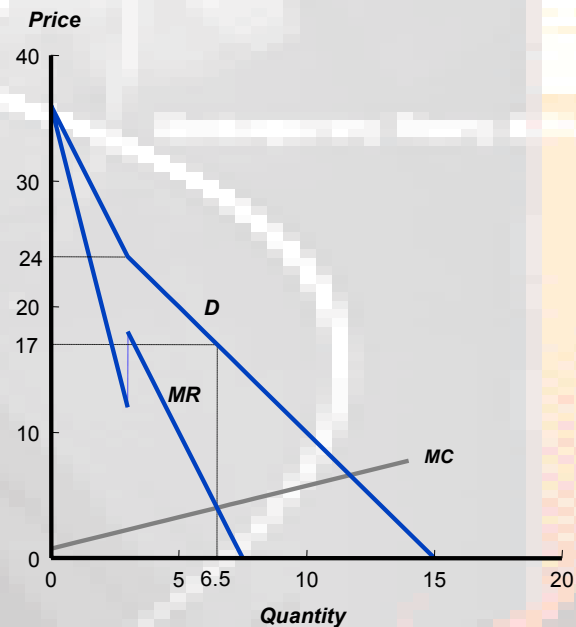
(a) United States



(b) Europe



(c) Aggregate

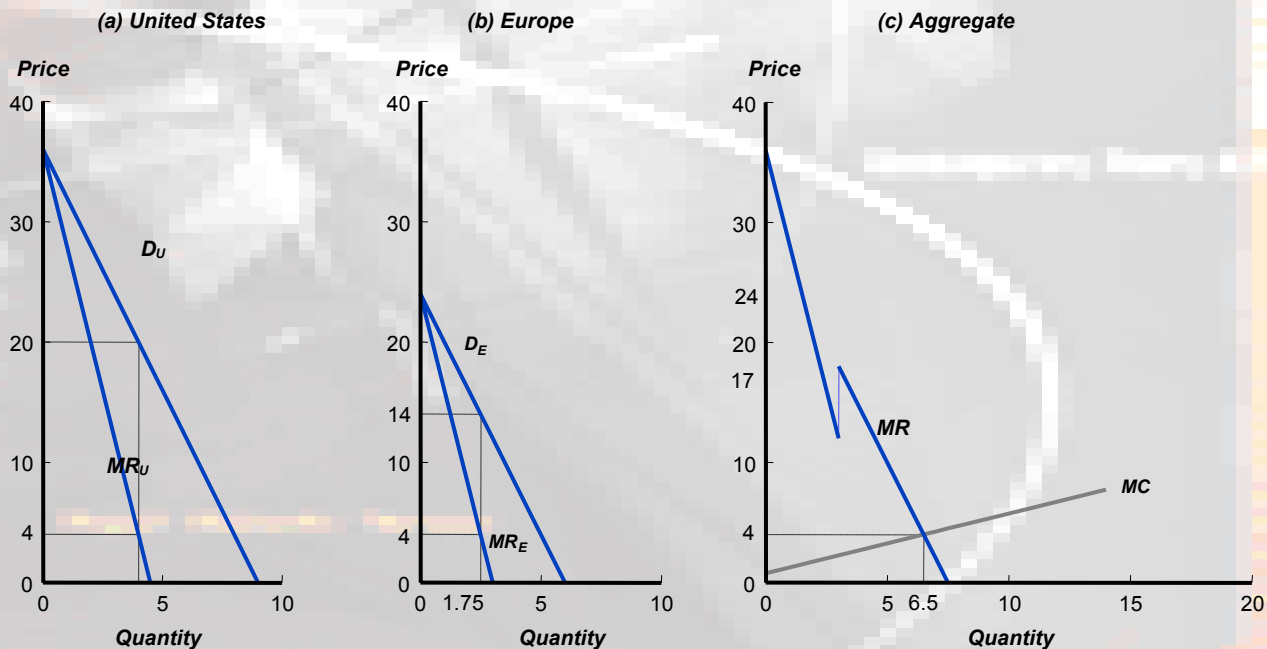


Price discrimination: non-constant cost

- With price discrimination the procedure is
 - Identify marginal revenue in each market
 - Aggregate these marginal revenues to give aggregate marginal revenue
 - Equate this MR with MC to give aggregate output
 - Identify equilibrium MR from the aggregate MR curve
 - Equate this MR with MC in each market to give individual market quantities
 - Identify equilibrium prices from individual market demands

The example again

Applying this procedure assuming that $MC = 0.75 + Q/2$ gives:



Necessary conditions for optimal prices

Above procedure:

$$Q_U = 36/8 - MR/8$$

1. Invert MR functions

$$Q_E = 24/8 - MR/8$$

2. Add them up

$$Q = 60/8 - 2MR/8$$

3. Replace MR by MC

$$= 60/8 - 2/8(0.75 + Q/2)$$

$$Q = 6.5, MC = 4, Q_U = 4, Q_E = 2.5$$

General necessary conditions (for continuous demands)

Equate marginal revenues in both markets

Equate those marginal revenues to marginal cost

$$MR_U = 36 - 8Q_U = 24 - 8Q_E = MR_E$$

$$MC = 0.75 + (Q_U + Q_E) / 2 = 24 - 8Q_E \quad (\text{could have used } MR_U \text{ instead})$$

Some additional comments

- With linear demands:
 - price discrimination results in the same aggregate output as no price discrimination
 - price discrimination always increases profit
- For any demand specifications two rules apply
 - marginal revenue must be equalized in each market
 - marginal revenue must equal aggregate marginal cost

Price discrimination and elasticity

- Suppose that there are two markets with the same MC
- MR in market i is given by $MR_i = P_i(1 - 1/\eta_i)$
 - where η_i is (absolute value of) elasticity of demand
- From rule 1 (above)
 - $MR_1 = MR_2$
 - so $P_1(1 - 1/\eta_1) = P_2(1 - 1/\eta_2)$ which gives

$$\frac{P_1}{P_2} = \frac{(1 - 1/\eta_2)}{(1 - 1/\eta_1)} = \frac{\eta_1\eta_2 - \eta_1}{\eta_1\eta_2 - \eta_2}$$

Price is lower in the market with the higher demand elasticity

Third-degree price discrimination (cont.)

- Often arises when firms sell *differentiated products*
 - hard-back versus paper back books
 - first-class versus economy airfare
- Price discrimination exists in these cases when:
 - “two varieties of a commodity are sold by the same seller to two buyers at different *net* prices, the net price being the price paid by the buyer corrected for the cost associated with the product differentiation.” (Phlips)
- The seller needs an easily observable characteristic that signals willingness to pay
- The seller must be able to *prevent arbitrage*
 - e.g. require a Saturday night stay for a cheap flight

Product differentiation and price discrimination

Suppose there are two types of travellers:

Business (B)

Tourists (T)

Additional cost for first class = 100

(1) Both first class:

$$P=250, \text{ profit}=150*N$$

(2) Both Coach:

$$P=200, \text{ profit} = 200*N$$

(3) Separate:

$$P_C = 200$$

$$P_B=?$$

For example: $N_B = 50$, $N_T = 200$

(1) $150*250=37,500$

(2) $200*250=50,000$

(2) $200*200+400*50=60,000$

Utilities:

	B	T
Coach	500	200
First Class	800	250

If $P_B - P_C > 300$, B will choose coach.

Possibility of arbitrage puts limits on P_B .

U_{BC} : utility B flying coach

U_{BF} : utility B flying first

$$p_F - p_C < U_{BF} - U_{BC}$$

Known as *self-selection or no-arbitrage constraint*

Other mechanisms for price discrimination

- Impose restrictions on use to control arbitrage
 - Saturday night stay
 - no changes/alterations
 - personal use only (academic journals)
 - time of purchase (movies, restaurants)
- “Crimp” the product to make lower quality products
 - *Mathematica*®
- Discrimination by location