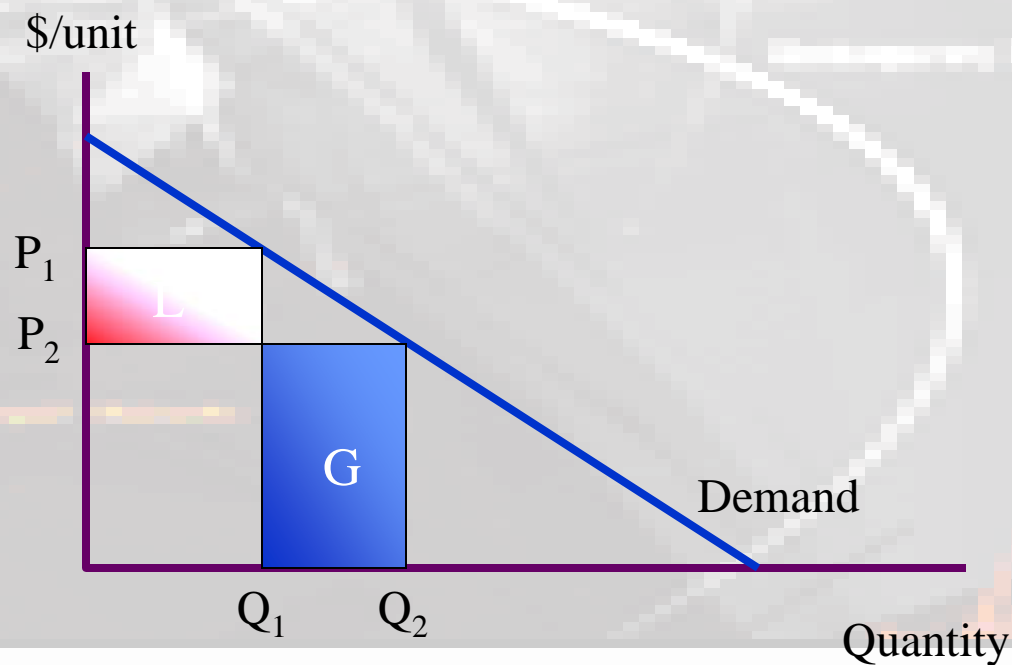




# Monopoly: Linear pricing

## Marginal Revenue

- The only firm in the market
  - market demand is the firm's demand
  - output decisions affect market clearing price



## Monopoly (cont.)

- Derivation of the monopolist's marginal revenue

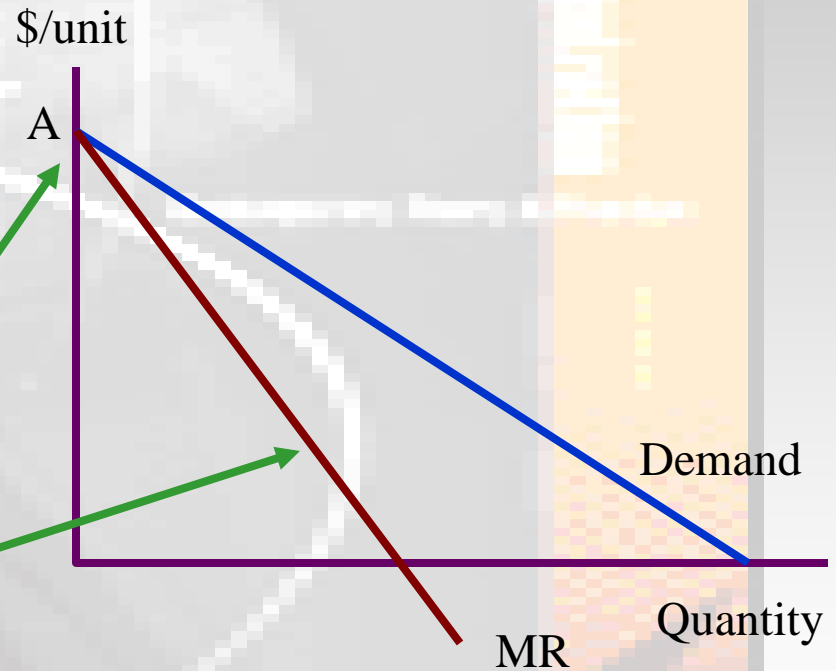
$$\text{Demand: } P = A - B \cdot Q$$

$$\text{Total Revenue: } TR = P \cdot Q = A \cdot Q - B \cdot Q^2$$

$$\text{Marginal Revenue: } MR = dTR/dQ$$

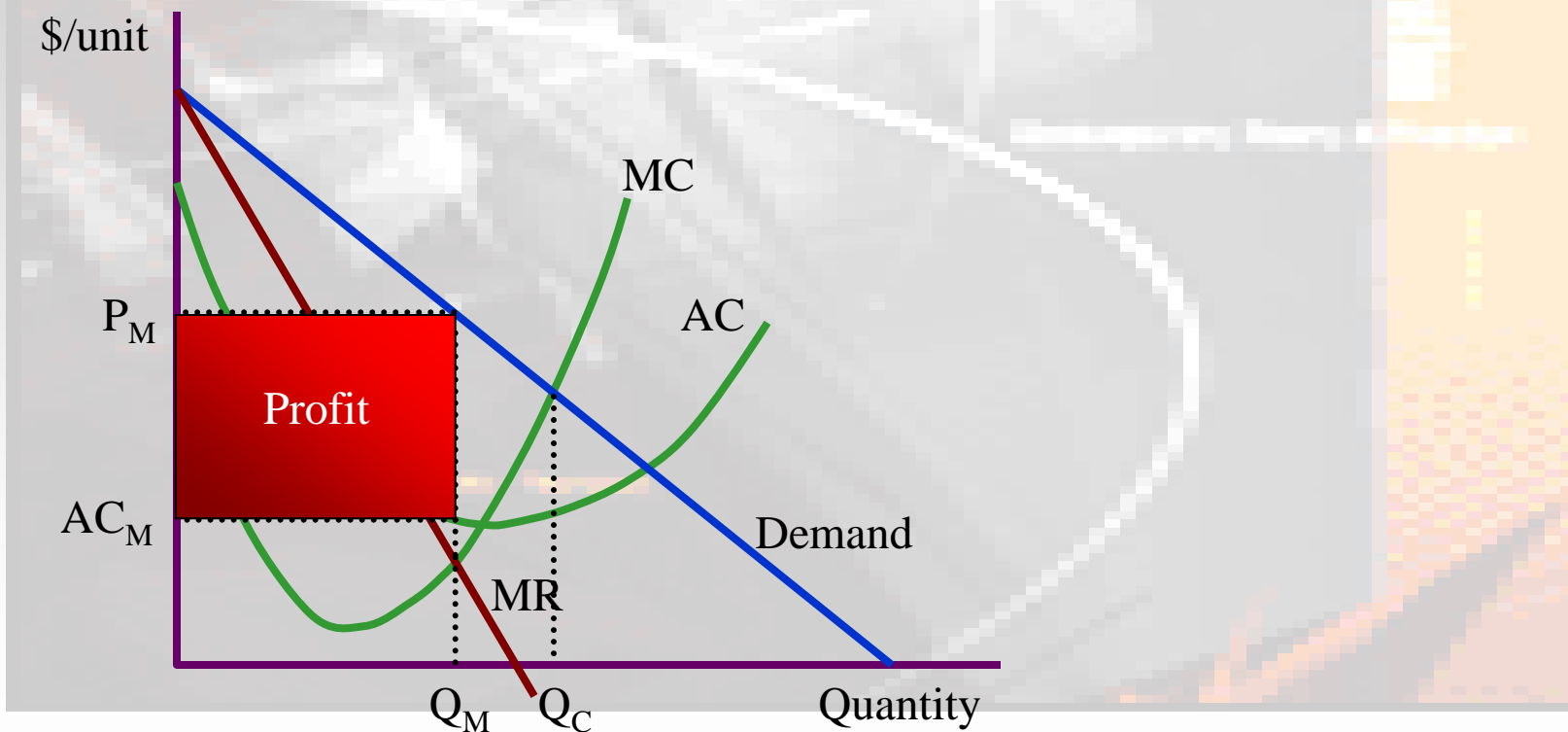
$$MR = A - 2B \cdot Q$$

With linear demand the marginal revenue curve is also linear with the same price intercept but twice the slope of the demand curve



## Monopoly and Profit Maximization

- The monopolist maximizes profit by equating marginal revenue with marginal cost



## Marginal Revenue and Demand Elasticity

Inverse demand:  $P(q)$

Total revenue  $R(q) = P(q)q$

Marginal revenue:  $R'(q) = p + (\partial P / \partial q)q$

$$= p \left( 1 + (\partial P / \partial q) \frac{q}{p} \right)$$

$$= p \left[ 1 - \frac{1}{\varepsilon_d} \right]$$

- *Max profits:  $MR = MC$*

$$p \left( 1 - \frac{1}{\varepsilon_d} \right) = MC$$

- *higher elasticity  $\rightarrow$  lower price*

Lerner Index:

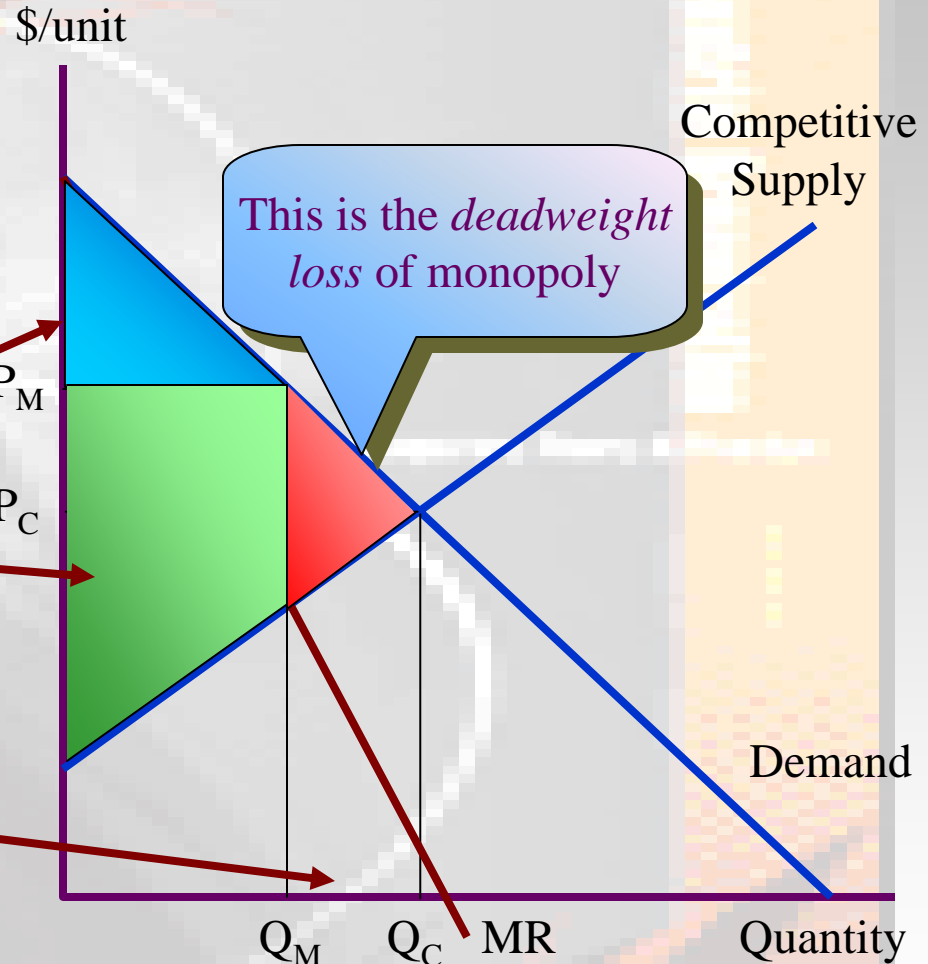
$$L = \frac{p - MC}{p} = \frac{1}{\varepsilon_d}$$

## Deadweight loss of Monopoly

Assume that the industry is monopolized  
The monopolist sets  $MR = MC$  to give output  $Q_M$   
The market clearing price is  $P_M$

Consumer surplus is given by this area  
And producer surplus is given by this area

The monopolist produces less surplus than the competitive industry. There are mutually beneficial trades that do not take place: between  $Q_M$  and  $Q_C$



## Deadweight loss of Monopoly (cont.)

- Why can the monopolist not appropriate the deadweight loss?
  - Increasing output requires a reduction in price
  - *this assumes that the same price is charged to everyone.*
- The monopolist creates surplus
  - some goes to consumers
  - some appears as profit
- The monopolist bases her decisions purely on the surplus she gets, *not* on consumer surplus
- The monopolist undersupplies relative to the competitive outcome
- The primary problem: *the monopolist is large relative to the market*



# Price Discrimination and Monopoly: Linear Pricing



# Introduction

- Prescription drugs are cheaper in Canada than the United States
- Textbooks are generally cheaper in Britain than the United States
- Examples of *price discrimination*
  - presumably profitable
  - should affect market efficiency: not necessarily adversely
  - is price discrimination necessarily bad – even if not seen as “fair”?

## Feasibility of price discrimination

- Two problems confront a firm wishing to price discriminate
  - *identification*: the firm is able to identify demands of different types of consumer or in separate markets
    - easier in some markets than others: e.g tax consultants, doctors
  - *arbitrage*: prevent consumers who are charged a low price from reselling to consumers who are charged a high price
    - prevent re-importation of prescription drugs to the United States
- The firm then must choose the *type* of price discrimination
  - first-degree or personalized pricing
  - second-degree or menu pricing
  - third-degree or group pricing

## Third-degree price discrimination

- Consumers differ by some observable characteristic(s)
- A uniform price is charged to all consumers in a particular group – linear price
- Different uniform prices are charged to different groups
  - “kids are free”
  - subscriptions to professional journals e.g. *American Economic Review*
  - airlines
  - early-bird specials; first-runs of movies

## Third-degree price discrimination (cont.)

- The pricing rule is very simple:
  - consumers with low elasticity of demand should be charged a high price
  - consumers with high elasticity of demand should be charged a low price

## Third degree price discrimination: example

- Harry Potter volume sold in the United States and Europe
- Demand:
  - United States:  $P_U = 36 - 4Q_U$
  - Europe:  $P_E = 24 - 4Q_E$
- Marginal cost constant in each market
  - $MC = \$4$

## The example: no price discrimination

- Suppose that the same price is charged in both markets
- Use the following procedure:
  - calculate aggregate demand in the two markets
  - identify marginal revenue for that aggregate demand
  - equate marginal revenue with marginal cost to identify the profit maximizing quantity
  - identify the market clearing price from the aggregate demand
  - calculate demands in the individual markets from the individual market demand curves and the equilibrium price

## The example (npd cont.)

United States:  $P_U = 36 - 4Q_U$  Invert this:

$$Q_U = 9 - P/4 \text{ for } P \leq \$36$$

Europe:  $P_E = 24 - 4Q_E$  Invert

$$Q_E = 6 - P/4 \text{ for } P \leq \$24$$

Aggregate these demands

$$Q = Q_U + Q_E = 9 - P/4 \text{ for } \$36 \geq P \geq$$

$$Q = Q_U + Q_E = 15 - P/2 \text{ for } P < \$24$$

**At these prices  
only the US  
market is active**

**Now both  
markets are  
active**

## The example (npd cont.)

Invert the direct demands

$$P = 36 - 4Q \text{ for } Q \leq 3$$

$$P = 30 - 2Q \text{ for } Q > 3$$

Marginal revenue is

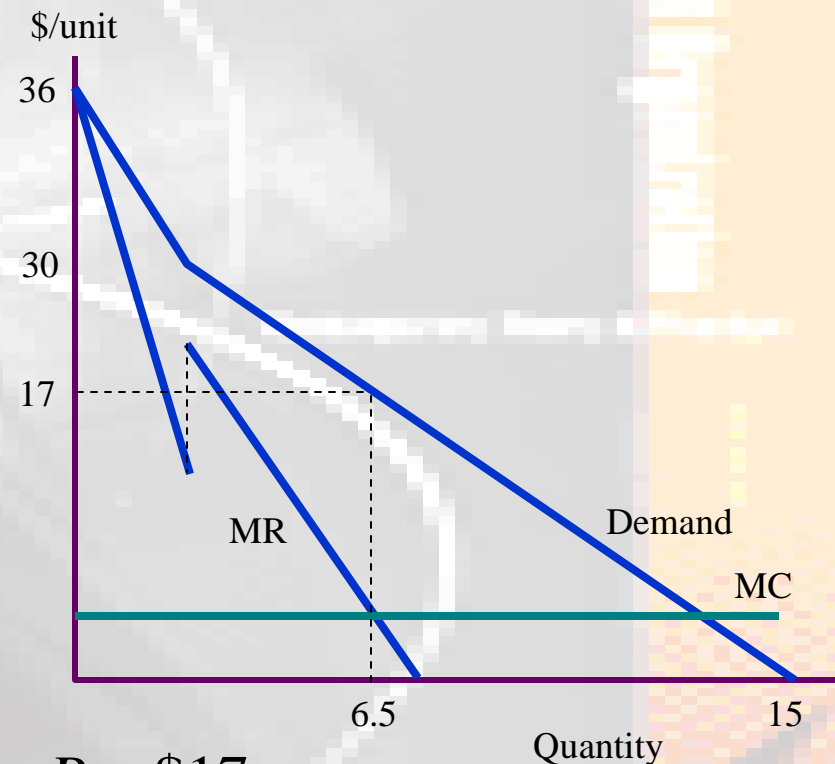
$$MR = 36 - 8Q \text{ for } Q \leq 3$$

$$MR = 30 - 4Q \text{ for } Q > 3$$

Set  $MR = MC$

$$Q = 6.5$$

Price from the demand curve  $P = \$17$





## The example (npd cont.)

Substitute price into the individual market demand curves:

$$Q_U = 9 - P/4 = 9 - 17/4 = 4.75 \text{ million}$$

$$Q_E = 6 - P/4 = 6 - 17/4 = 1.75 \text{ million}$$

Total output = 6.5 million

Aggregate profit =  $(17 - 4) \times 6.5 = \$84.5 \text{ million}$

## The example: price discrimination

- The firm can improve on this outcome
- Check that MR is not equal to MC in both markets
  - $MR > MC$  in Europe
  - $MR < MC$  in the US
  - the firms should transfer some books from the US to Europe
- This requires that different prices be charged in the two markets
- Procedure:
  - take each market separately
  - identify equilibrium quantity in each market by equating MR and MC
  - identify the price in each market from market demand

## The example: (pd cont.)

Demand in the US:

$$P_U = 36 - 4Q_U$$

Marginal revenue:

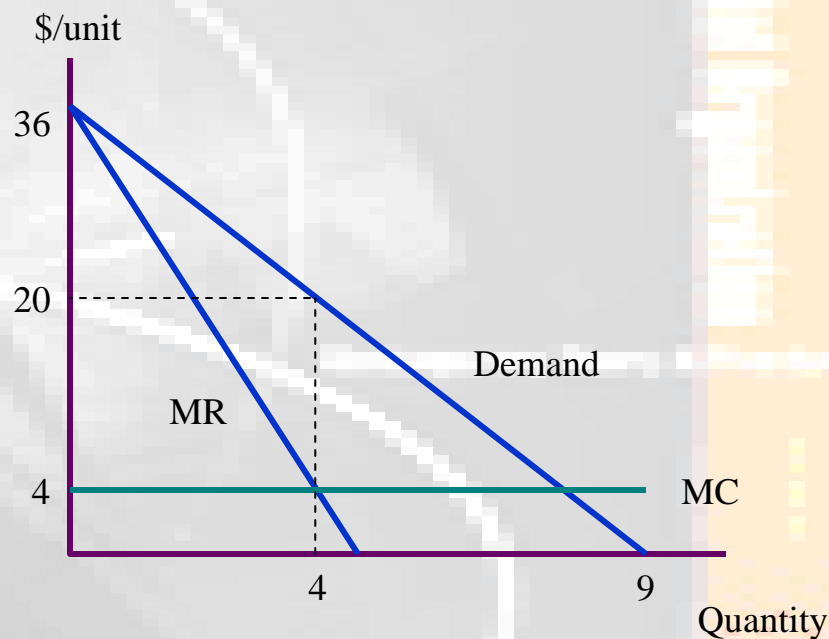
$$MR = 36 - 8Q_U$$

$$MC = 4$$

Equate MR and MC

$$Q_U = 4$$

Price from the demand curve  $P_U = \$20$



## The example: (pd cont.)

Demand in the Europe:

$$P_E = 24 - 4Q_E$$

Marginal revenue:

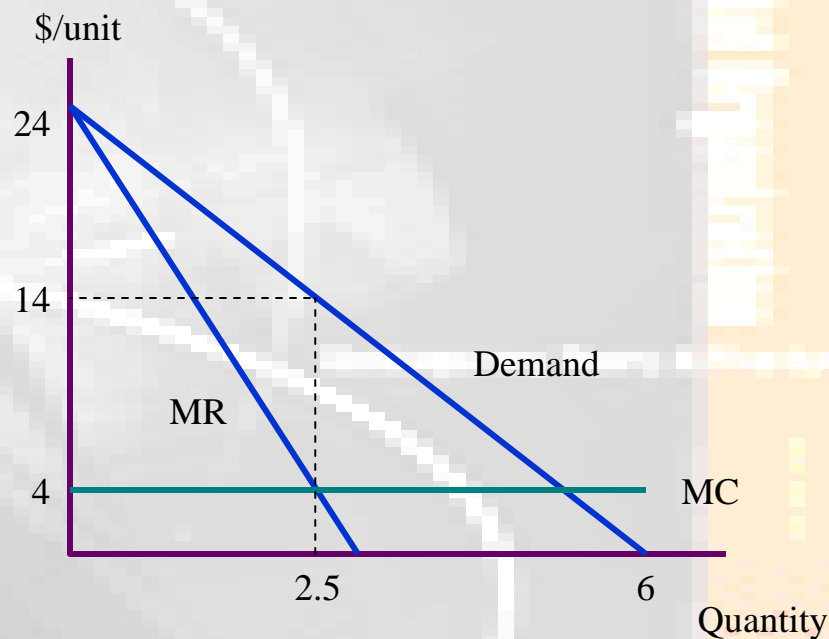
$$MR = 24 - 8Q_E$$

$$MC = 4$$

Equate MR and MC

$$Q_E = 2.5$$

Price from the demand curve  $P_E = \$14$



## The example (pd cont.)

- Aggregate sales are 6.5 million books
  - the same as without price discrimination
  - This property holds always with linear demands
- Aggregate profit is  $(20 - 4) \times 4 + (14 - 4) \times 2.5 =$   
\$89 million
  - \$4.5 million greater than without price discrimination

## Additional considerations

- The example assumes constant marginal cost
- Rule obtained is to equate MR to MC in each market – independent decisions.
- Possible connections between markets:
  - Arbitrage: limits price differences
  - Capacity constraints or non-constant Marginal cost
- Tradeoff: to what market should you sell extra unit?

# An example with increasing MC

$$MC(q) = 2 \cdot (q-1)$$

D market 1

P	q
7	1
5	2

D market 2

P	q
4	1
3	2

No discrimination

p	q	TR	MR	MC	TC
7	1				
5	2				
4	3				
3	4				

# An example with increasing MC

D market 1

P	q
7	1
5	2

D market 2

P	q
4	1
3	2

$$MC(q) = 2 \cdot (q-1)$$

Previous solution:  $p=5$ ,  $q=2$ ,  $TC=2$ ,  $\pi=8$

Anything better?

Consider selling one unit in each market:

$$p_1 = 7, p_2 = 4 \quad TR = 11 \quad \text{and} \quad \pi = 9$$

Where is the difference coming from?



## Example (continued)

market 1

p	q	TR	MR
7	1	7	7
5	2	10	3

market 2

p	q	TR	MR
4	1	4	4
3	2	6	2

Key idea: order consumers by MR

q	MR	MC
1	7	0
2	4	2
3	3	4
4	2	6

The optimum is to include only the first two consumers:

$$p_1=7, p_2=4.$$

## Non-constant costs: general principle

- Key principle: think *Marginal Revenue*:
  - Sell next unit to market with highest marginal revenue
- With continuous demands:
  - Equate marginal revenue in all active markets
  - Equate this marginal revenue to marginal cost
  - If in some market  $MR(Q)$  less than this marginal cost, do not serve.

## Price discrimination and elasticity

- Suppose that there are two markets with the same MC
- MR in market  $i$  is given by  $MR_i = P_i(1 - 1/\eta_i)$ 
  - where  $\eta_i$  is (absolute value of) elasticity of demand
- From previous rule
  - $MR_1 = MR_2$
  - so  $P_1(1 - 1/\eta_1) = P_2(1 - 1/\eta_2)$  which gives

$$\frac{P_1}{P_2} = \frac{(1 - 1/\eta_2)}{(1 - 1/\eta_1)} = \frac{\eta_1\eta_2 - \eta_1}{\eta_1\eta_2 - \eta_2}$$

Price is lower in the market with the higher demand elasticity

## Price discrimination and product differentiation

- Often arises when firms sell *differentiated products*
  - hard-back versus paper back books
  - first-class versus economy airfare
- Price discrimination exists in these cases when:
  - “two varieties of a commodity are sold by the same seller to two buyers at different *net* prices, the net price being the price paid by the buyer corrected for the cost associated with the product differentiation.” (Phlips)
- The seller needs an easily observable characteristic that signals willingness to pay
- The seller must be able to *prevent arbitrage*
  - e.g. require a Saturday night stay for a cheap flight

# Product differentiation and price discrimination

Suppose there are two types of travellers:

Business (B)

Tourists (T)

Additional cost for first class = 100

(1) Both first class:

$$P=250, \text{ profit}=150*N$$

(2) Both Coach:

$$P=200, \text{ profit} = 200*N$$

(3) Separate:

$$P_C = 200$$

$$P_B=?$$

For example:  $N_B = 50$  ,  $N_T = 200$

(1)  $150*250=37,500$

(2)  $200*250=50,000$

(2)  $200*200+400*50=60,000$

Utilities:

	B	T
Coach	500	200
First Class	800	250

If  $P_B - P_C > 300$ , B will choose coach.

Possibility of arbitrage puts limits on  $P_B$ .

$U_{BC}$  : utility B flying coach

$U_{BF}$  : utility B flying first

$$p_F - p_C < U_{BF} - U_{BC}$$

Known as *self-selection or no-arbitrage constraint*

## Other mechanisms for price discrimination

- Impose restrictions on use to control arbitrage
  - Saturday night stay
  - no changes/alterations
  - personal use only (academic journals)
  - time of purchase (movies, restaurants)
- “Crimp” the product to make lower quality products
  - *Mathematica*®
- Discrimination by location