Nonlinear pricing

Second degree price discrimination
Self-selection

- What if the seller cannot distinguish between buyers?
  - perhaps they differ in income (unobservable)

- Then the type of price discrimination just discussed is impossible

- High-income buyer will pretend to be a low-income buyer
  - to avoid the high entry price
  - to pay the smaller total charge

- Take a specific example
  - \( P_h = 16 - Q_h \)
  - \( P_l = 12 - Q_l \)
  - \( MC = 4 \)
Second-degree price discrimination 2

- First-degree price discrimination requires:
  - High Income: entry fee $72 and $4 per drink or entry plus 12 drinks for a total charge of $120
  - Low Income: entry fee $32 and $4 per drink or entry plus 8 drinks for a total charge of $64

- This will not work
  - high income types get no consumer surplus from the package designed for them but get consumer surplus from the other package
  - so they will pretend to be low income even if this limits the number of drinks they can buy

- Need to design a “menu” of offerings targeted at the two types
Second-degree price discrimination

- The seller has to compromise

- Design a pricing scheme that makes buyers
  - reveal their true types
  - self-select the quantity/price package designed for them

- Essence of second-degree price discrimination

- It is “like” first-degree price discrimination
  - the seller knows that there are buyers of different types
  - but the seller is not able to identify the different types

- A two-part tariff is ineffective
  - allows deception by buyers

- Use quantity discounting
Simple example

- mc = 0

- If possible to separate:
  - Sell 2 units to each.
  - Charge consumer 1 a total of $8 and consumer 2 a total of $5.

- This does not work if unable to separate.

- Whatever alternative is offered to consumer 2, the other type could choose.

- Self-selection

- Instruments to discriminate: handicap one of the alternatives.

- How: making it smaller!
Simple example - continued

<table>
<thead>
<tr>
<th>Size of package</th>
<th>Value to consumer 1</th>
<th>Value to consumer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

- Looks like the self-selection problem of business and tourists. Discrimination:
  - Set price of package with one unit = 4
  - Price of package with two units = 7
  - Profits = $11
- Implementation:
  1. Sell two packages
  2. Offer a pair of two-part tariffs: \( F_1 = 4, p_1 = 3 \), \( F_2 = 0, p_2 = 4 \).
  3. Nonlinear price: unit 1 = $4, unit 2 = $3
Example 2 – discrimination not worthwhile

<table>
<thead>
<tr>
<th>Consumer 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumer 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

- Discrimination:
  - Set price of package with one unit = 3
  - Price of package with two units = 5
  - Profits = $8
  - Alternative 1: two units for both: $10
  - Alternative 2: exclude consumer 2: $6
Example 2 – Exclude consumer 2

<table>
<thead>
<tr>
<th>Consumer 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( q )</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumer 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( q )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
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<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

- Discrimination:
  - Set price of package with one unit = 2
  - Price of package with two units = 5
  - Profits = $7
  - Alternative 1: two units for both: $6
  - Alternative 2: exclude consumer 2: $8
Examples – conclusions

- Too discriminate, need to offer different alternatives.
- Consumers must self-select.
- This is done by “damaging” the alternative for low paying consumers: lower quantity
- Then offer a price for small package and price for large package.
- Cost: inefficient consumption of L type (deadweight loss)
- Gain: Extract more CS from high type
- When differences in willingness to pay are high, it pays to restrict quantity to L type.
- Leave no CS to L type:
  - Charge the highest possible price
  - This way get more from the L type and lower the CS of high type.
## Optimal exclusion of L type

<table>
<thead>
<tr>
<th>q</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>Gain from including L type (1)</th>
<th>Change in CS of high type (2)</th>
<th>Net: (1)-(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

### Example 2

<table>
<thead>
<tr>
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<th>$p_1$</th>
<th>$p_2$</th>
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<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

### Example 3

<table>
<thead>
<tr>
<th>q</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>Gain from including L type (1)</th>
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<tbody>
<tr>
<td>1</td>
<td>5</td>
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<td>2</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
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<td>1</td>
<td>1</td>
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<td>-1</td>
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</table>
With continuous demand functions

\[ P_h = 4 - Q_h \]
\[ P_L = 2 - Q_L \]
\[ MC = 0 \]

1. Sell 2 units to type L
2. Set price for this two units equal to total surplus = area under demand = \(2 \times 2 / 2 = 2\)
3. What can offer to H type?
   - Quantity = 4
   - Charge: ?
4. Profits = \(2 N_L + 4 N_h\)
5. e.g. if \(N_L = N_h = N\), profits = \(6N\)
Any better deal?

1. Sell 1 unit to type L
2. Set price for this unit equal to total surplus = area under demand = \((2+1)/2 \times 1 = 3/2\)
3. What can offer to H type?
   - Quantity = 4
   - \(CS_h\) under \(L\)-plan = 2
   - Maximum charge: 8-2=6
4. Profits = \(3/2 \times N_L + 6 \times N_h\)
5. e.g. \(N_L = N_h = N\), profit = 7 \(\frac{1}{2}\) \(N\)
Optimal quantity

Effect of a small decrease in $q_L$

- Loss from excluding L type: $N_L \times \Delta q_L \times p_L$

- $\Delta CS_h = N_h \times \Delta q_L \times (p_h - p_L)$

- Net gain $\Delta q_L \left[ N_h \times (p_h - p_L) - N_L \times p_L \right]$

If $N_h = N_L$ this is positive if $p_h - p_L > p_L$.

Optimal quantity $q_L$: cut quantity until $N_h \times (p_h - p_L) = N_L \times p_L$

Note: if marginal cost is positive, until $N_h \times (p_h - p_L) = N_L \times (p_L - mc)$
Optimal quantity (continued)

- In previous example, assuming \( N_h = N_L \)
- \( p_h = 4-q, \quad p_L = 2-q \)
- Condition is: \( (4-q - (2-q)) = 2-q \rightarrow q=0! \)
- Best is to exclude completely type \( L \).
- Suppose \( N_L = 2 \times N_h \)
  - \( 2 = 2 \times (2-q) \)
  - \( q=1 \)
- Optimal quantity increases as \( N_L / N_h \) increases
- Intuition: losses from excluding \( L \) are proportional to \( N_L \) and gain from more surplus extraction from \( h \) are proportional to \( N_h \).
Optimal packages

In previous example with \( N_L = 2 \, N_h \)

- \( q_L = 1 \)
- Charge \( L \): $1.5 (their total surplus)
- \( q_h = 4 \) (always quantity is efficient for \( h \) type)
- Charge \( h \): Total surplus minus utility under \( L \) plan
  \[ = 8 - 2 = 6 \]
- Total profits: \( 1.5 \times N_L + 6 \times N_h \)
Optimal packages – general case

- Set $q_L$ so that $N_h \times (p_h-p_L) = N_L \times (p_L - mc)$
- Charge $L$-type their entire surplus (area under demand curve up to $q_L$) = \[ \int_0^{q_L} P_L(x) \, dx \]

- Set $q_h$ at the efficient level: $p(q_h) = mc$
- Charge $H$-type their entire surplus minus the utility they get under $L$-plan = \[ \int_0^{q_h} P_h(x) \, dx - \int_0^{q_L} [P_h(x) - P_L(x)] \, dx \]
Optimal packages – general principles

- Characteristics of second-degree price discrimination
  - extract all consumer surplus from the lowest-demand group
  - leave some consumer surplus for other groups
    - the self-selection (also called incentive-compatibility) constraint.
  - offer less than the socially efficient quantity to all groups other than the highest-demand group
  - offer quantity-discounting

- Second-degree price discrimination converts consumer surplus into profit less effectively than first-degree
- Some consumer surplus is left “on the table” in order to induce high-demand groups to buy large quantities.
- Fundamental tradeoff – restricting output to lower types to increase surplus extracted from higher types.
- The higher the cost of this distortion or lower the added surplus extracted, the higher $q_L$ will be.
- It may be optimal to exclude $L$ types.
Generalization to many consumer types

- Total price schedule is concave
- Derivative interpreted as marginal price, i.e., price for the last unit.
- Can approximate by offering these two plans:

\[ T_1(q) = F_1 + p_1 \times q \]
\[ T_2(q) = F_2 + p_2 \times q \]
Non-linear pricing and welfare

- Non-linear price discrimination raises profit
- Does it increase social welfare?
  - suppose that inverse demand of consumer group $i$ is $P = P_i(Q)$
  - marginal cost is constant at $MC - c$
  - suppose quantity offered to consumer group $i$ is $Q_i$
  - total surplus – consumer surplus plus profit –is the area between the inverse demand and marginal cost up to quantity $Q_i$
Pricing policy affects
- distribution of surplus
- output of the firm

First is welfare neutral

Second affects welfare

Does it increase social welfare?

Price discrimination increases social welfare of group \(i\) if it increases quantity supplied to group \(i\)
First-degree price discrimination always increases social welfare
- extracts all consumer surplus
- but generates socially optimal output
- output to group \( i \) is \( Q_i(c) \)
- this exceeds output with uniform (non-discriminatory) pricing
Menu pricing is less straightforward

- suppose that there are two markets
  - low demand
  - high demand

- Uniform price is $P_U$
- Menu pricing gives quantities $Q_1^s$, $Q_2^s$
- Welfare loss is greater than $L$
- Welfare gain is less than $G$
It follows that

\[ \Delta W \leq G - L \]

\[ = (P_U - MC)\Delta Q_1 + (P_U - MC)\Delta Q_2 \]

\[ = (P_U - MC)(\Delta Q_1 + \Delta Q_2) \]

A necessary condition for second-degree price discrimination to increase social welfare is that it increases total output.

“Like” third-degree price discrimination

But second-degree price discrimination is more likely to increase output.
The incentive compatibility constraint

- Any offer made to high demand consumers must offer them as much consumer surplus as they would get from an offer designed for low-demand consumers.

- This is a common phenomenon
  - performance bonuses must encourage effort
  - insurance policies need large deductibles to deter cheating
  - piece rates in factories have to be accompanied by strict quality inspection
  - encouragement to buy in bulk must offer a price discount