# Econ 101: Problem Set I

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Due January 22, 2003 (in class).

1. For each (unconstrained) optimization problem, derive first order conditions, find critical points (points to solve FOC), and discuss whether the point you find is a (local)maximizer or (local)minimizer or neither of them.

   (a) $\max_{x} - (x - 7)^2$ , $x \in R$.

   (b) $\max_{x} x^3 - x$, $x \in R$

   (c) $\max_{x} - e^x - e^{-x}$, $x \in R$

2. For each constrained optimization problem, (1) define the Lagrangian function, (2) derive necessary conditions (including FOCs) for the optimal solution, and (3) find the optimal solution.

   (a) $\max_{x,y} 3x + 2y$ $s.t.$ $x, y \geq 0$, $x + y = 1$

   (b) $\max_{x} - (x - 1)^2$ $s.t. - 1 \leqq x \leqq 2$

   (c) $\max_{x} x^4$ $s.t. - 1 \leqq x \leqq 2$

3. Find all Nash equilibria of the following strategic form games.

   (a) :

|   | X | Y |
|---|---|---|
| H | 1, 2 | 0, 0 |
| L | 0, 0 | 2, 1 |

(b) :

|   | X | Y |
|---|---|---|
| H | 1, 1 | 0, 0 |
| L | 0, 0 | 0, 0 |

(c) :

|   | X | Y | Z |
|---|---|---|---|
| H | 1, 1 | 4, 0 | 0, - 1 |
| M | 0, - 1 | 3, 2 | 1, 3 |
| L | 2, - 1 | -1, - 3 | - 1, - 5 |

4. Consider the following (strategic form) game. Player $A$ announces $d_A$ $(0 \leqq d_A \leqq 100)$ and player $B$ announces $d_B$ $(0 \leqq d_B \leqq 100)$ at the same time. If $d_A + d_B > 100$, then both players get nothing. If $d_A + d_B \leq 100$, then player $A$ gets $d_A$ and player $B$ gets $d_B$.

   (a) If player $A$ announces $d_A = 30$, what is player $B$'s best response $d_B$?

   (b) One Nash equilibrium of this game is $(d_A^*, d_B^*) = (50, 50)$. Explain why this is a Nash equilibrium.

   (c) Is there any other Nash equilibrium? Show one Nash equilibrium if there is any.

5. (10.6 Nicholson) Consider the following dynamic game. Player B announces, "I have a bomb strapped to my body. If you (player A) do not give me \$1, I will set it off, killing each of us." Illustrate this game in extensive form and assess whether B's announced strategy for the game meets the criterion of subgame perfection.