Extensive Game with Perfect Information

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Extensive Game with Perfect Information

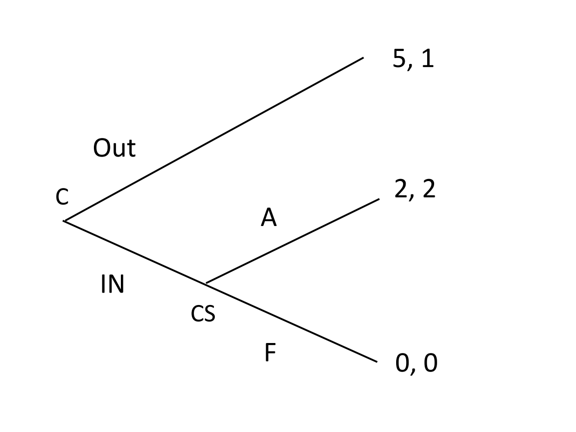
- We study dynamic games where players make a choice sequentially.
- We assume **perfect information**: each player can perfectly observe the past actions.

Example 1: Chain Store Game

- A chain store (CS) has a branch in a city
- There is one potential competitor (C) in the city.
- The game proceeds a follows:
  - C decides whether to enter the market or not.
  - Given C's choice, CS decides whether to accommodate or fight back.
- The profits are $(0, 0)$ (CS's profit, C's profit) if C enters and CS fights back, $(2, 2)$ if C enters and CS accommodates, and $(5, 1)$ if C does not enter.

Example 1: Chain Store Game

This game can be described as follows.

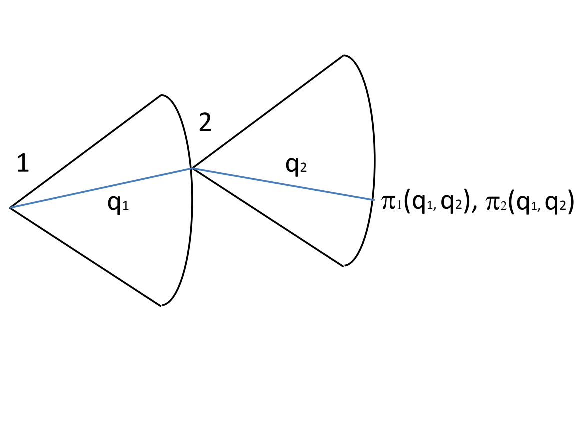


Example 2: Stackelberg Competion

- Consider the environment of the standard Cournot duopoly model.
- Suppose that the firms make decision sequentially.
  - Firm 1 (**leader**) first chooses how much to produce.
  - Then firm 2 (**follower**) decides how much to produce.

Example 2: Stackelberg Competion

This game looks like



Formal Model

Extensive Game with Perfect Information

- **Extensive game with Perfect Information** consists of
  - a finite set N
  - a set of sequences H such that
    - $\emptyset \in H$
    - $(a^1, ..., a^k) \in H \rightarrow (a^1, ..., a^\ell) \in H$ for any $\ell < k$
    - $(a^1, ...) \in H$ if $(a^1, ..., a^k) \in H$ for $k = 1, 2, ...$

    with $Z \subset H$ defined by $(a^1, ..., a^k) \in Z \Leftrightarrow \nexists a^{k+1}, (a^1, ..., a^{k+1}) \in H$.
  - a function $P : H/Z \rightarrow N$
  - a function $V_i : Z \rightarrow \Re$ for $i \in N$.

- N is the set of players.
- H is the set of **histories** with
  - Z as the set of **terminal histories**, and
  - $\emptyset$ as the **initial history**.
- P specifies who makes a choice at each history.
- $V_i(z)$ is player i's payoff at terminal history z.

- Let $H_i \subset H$ be the subset of histories such that $P(h) = i$. This is the set of histories where player i makes a choice.
- At history $h \in H/Z$, the set of actions that are available to player $P(h)$ is

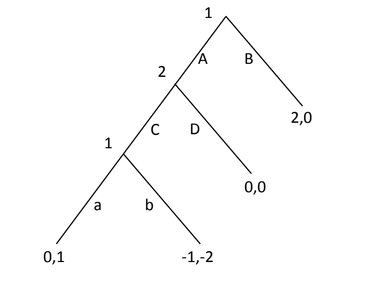
$$A(h) = \{a|(h, a) \in H\}$$

Strategy

- Player i's strategy for extensive game $(N, H, P, (V_i))$ is a mapping s_i that assigns an action in $A(h)$ at each $h \in H_i$. Let S_i be the set of player i's strategies.
- Every strategy profile $s = (s_1, ..., s_n)$ defines an **outcome** $O(s) = (a^1, ..., a^K) \in Z$ (K may be ∞) by
  - $s_{P(\emptyset)}(\emptyset) = a^1$
  - $s_{P(a^1)}(a^1) = a^2$
  - $s_{P(a^1,a^2)}(a^1, a^2) = a^3$....
- Thus player i's payoff is $V_i(O(s))$ given a strategy profile s.

Strategy

A strategy is not just a contingent plan of actions. It specifies an action at every history, even at histories that are never reached given the strategy(ex. strategy Ba for the game below).



Mixed Strategy and Behavior Strategy

- As in strategic games, we can define a mixed strategy for extensive games as a probability distribution over pure strategies ($\Delta(S_i)$).
- There is another way to express a mixed strategy. Player i's **behavioral strategy** σ_i is a mapping from H_i to a distribution on the set of available actions ($\sigma_i(h) \in \Delta(A_i(h))$ for each $h \in H$).
- They are different representations of the same thing. Every behavior strategy is clearly a mixed strategy. Every mixed strategy can be replicated by a behavior strategy.
- We will use behavior strategy representation most of the time.

Nash Equilibrium

- Note that an extensive game with perfect information $(N, H, P, (V_i))$ determines a strategic game $(N, (S_i), (V_i))$. So we can define Nash equilibrium for extensive game with perfect information.

Nash Equilibrium

For extensive game with perfect information $(N, H, P, (V_i))$, a profile of strategies s^* is a Nash equilibrium if

$$V_i(O(s^*)) \geq V_i(O(s_i', s_{-i}^*))$$

for any $s_i' \in S_i$ and any $i \in N$.

- Nash equilibrium is often too permissive.
- For the chain store game, there exists two NE: (In, A) and (Out, F). One may argue that (Out, F) is less reasonable, because F is not an optimal action once "In" is chosen.