Introduction to Games

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Why Game?

- In price theory, consumers maximize their utility subject to their budget constraints and firms maximize their revenue given their technology and the market prices.
- In many interesting situations, each agent's optimal behavior depends on the behavior of the other agents' behavior directly. Examples of such situations abound in economics.
  - partnership problem
  - auctions
  - currency attack
- Game theory provides a language to describe and analyze such strategic situations.

Let's consider a few concrete examples.

Example 1: Bargaining

- Two individuals are discussing how to divide profits from some new joint project.
- There are three options for each individual: ask for 75 %, 50 %, or 25 % of the total profit.
- If their requests add up to 100 %, then the profits will be divided according to the requests. If the total request is less than 100 %, then the profits will be divided proportionally. If the total request exceeds 100%, then this project does not take off.

Here is a graphical representation of this game.

	25%	**50%**	**75%**
25%	(50, 50)	(33.3, 66.6)	(25, 75)
50%	(66.6, 33.3)	(50, 50)	(0, 0)
75%	(75, 25)	(0, 0)	(0, 0)

Another example.

Example 2: Competition by two rival firms (**Cournot duopoly model**)

- Two firms A and B are producing an identical product (say, cereal).
- Firm A can produce either 50 units or 100 units with per-unit cost c_A.

  Firm B can produce either 60 units or 120 units with per-unit cost c_B.

  Assume no fixed cost.
- The price of the product is $a(> 220)$ minus the total amount produced.

A graphical representation of this game is:

	60	120
50	(a- 110-c_A)50, (a- 110-c_B)60	(a- 170-c_A)50, (a- 170-c_B)120
100	(a- 160-c_A)100, (a- 160-c_B)60	(a- 220-c_A)100, (a- 220-c_B)120

Strategic Game

- These games are examples of **strategic game** or **strategic form game** (also called "normal form game").
- Here is the formal definition of strategic game.

Strategic Game

A **strategic game** $G = (N, (A_i), (u_i))$ consists of a finite set N, nonempty set A_i and a function $u_i : A \to \Re$ for each $i \in N$ $(A = \prod_{i \in N} A_i)$.

Strategic Game

- Interpretation:
  - N is a set of **players**.
  - A_i is a set of **actions** available to player i. Decisions are independent: actions are taken simultaneously or each player chooses an action without knowing the choice of the other players.
  - u_i is player i's **payoff (utility) function**.
- Graphical representation are useful to describe strategic games with two players and finite actions. But the formal definition of strategic game allows for many players and infinite actions.

Some Classical Examples of Strategic Game

Coordination Game

	B	C
B	1, 1	0, 0
C	0, 0	1, 1

- There are two players.
- Their payoffs are higher if and only if they succeed to coordinate.

Some Classical Examples of Strategic Game

Prisoner's Dilemma

	NC	C
NC	-1, -1	-5, 0
C	0, -5	-3, -3

- Two suspects are being questioned in different rooms. Each suspect can choose either "confess" or "do not confess".
- The number of years they will spend in a prison depend on how they respond to the questions.

Some Classical Examples of Strategic Game

Chicken Game (Hawk-Dove game)

	A	B
A	0, 0	4, 1
B	1, 4	3, 3

- Each player would like to induce the other player to her preferred outcome.

Some Classical Examples of Strategic Game

Matching Penny

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

- Player 1 likes to match her choice with player 2's choice.
- Player 2 likes to avoid the same choice as player 1.

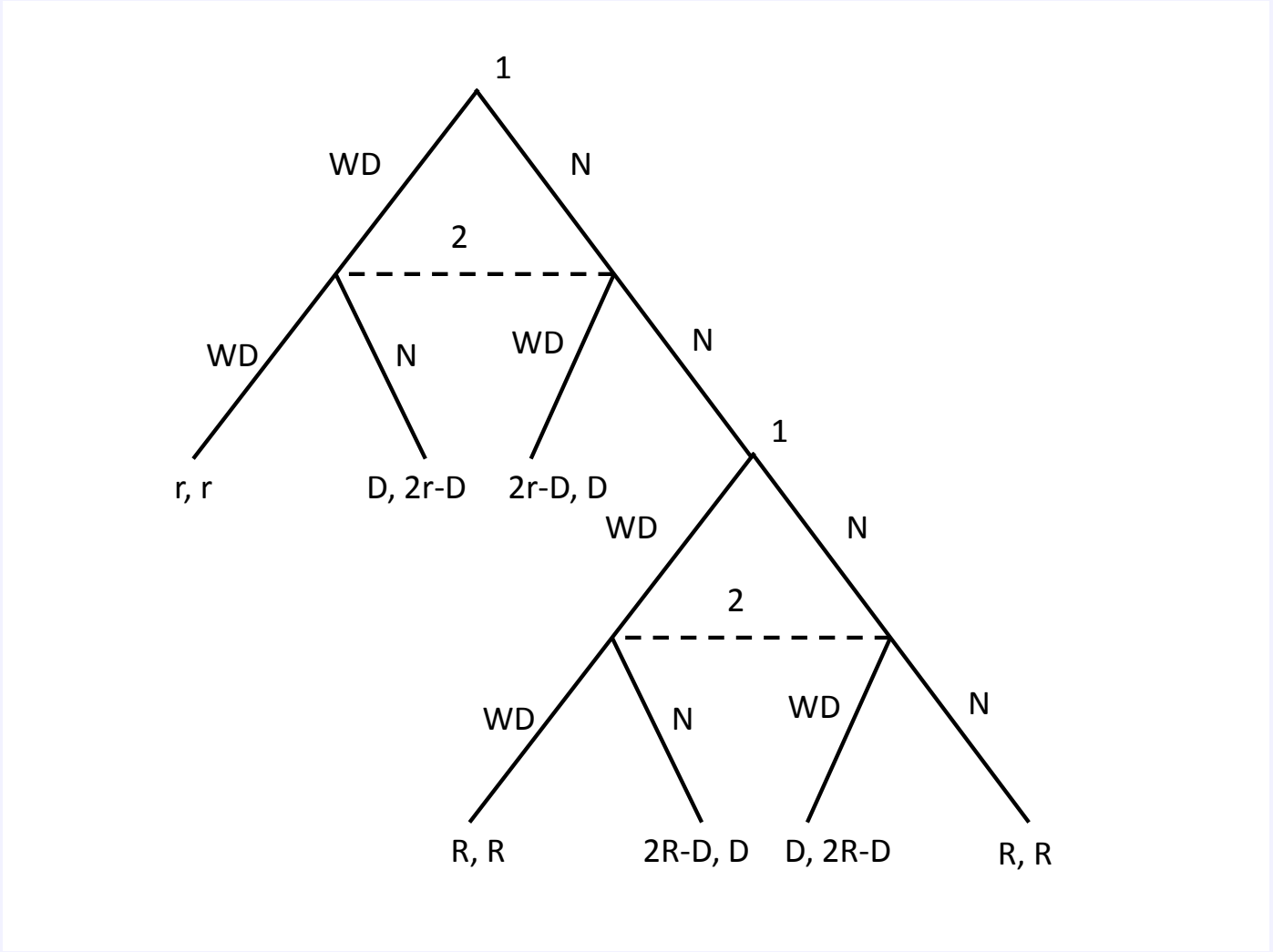
Extensive Game

- We are often interested in studying dynamic situations, where players make decisions sequentially.
- We can describe such situations by **extensive game** (**extensive form game**).
- Extensive games specify timing and information explicitly: when each player makes a decision and what each player knows when making a decision.

Example 1: Bank Runs

- Two investors with deposit $D > 0$ each at some bank, invested in some project.
- There are two periods. Each investor decides whether to withdraw her deposit in each period. If any investor withdraws her deposit in the first period, then the project is terminated and the game ends.
- The payoffs of the investors are as follows (assume $0.5D < r < D < R$):
  - each investor receives r if both investors withdraw in the 1st period.
  - if one investor withdraws and the other decides not to withdraw in the 1st period, then this investor secures D and the other investor receives the rest $(= 2r - D)$.
  - if both investors do not withdraw in the first period, then the game moves to the 2nd period and the project matures to generate $2R$.
  - each investor receives R if both investors withdraw or do not withdraw in the 2nd period.
  - if one investor withdraws and the other decides not to in the 2nd period, then this investor receives all the profits except for the other investor's original investment D, which is left for the other investor.

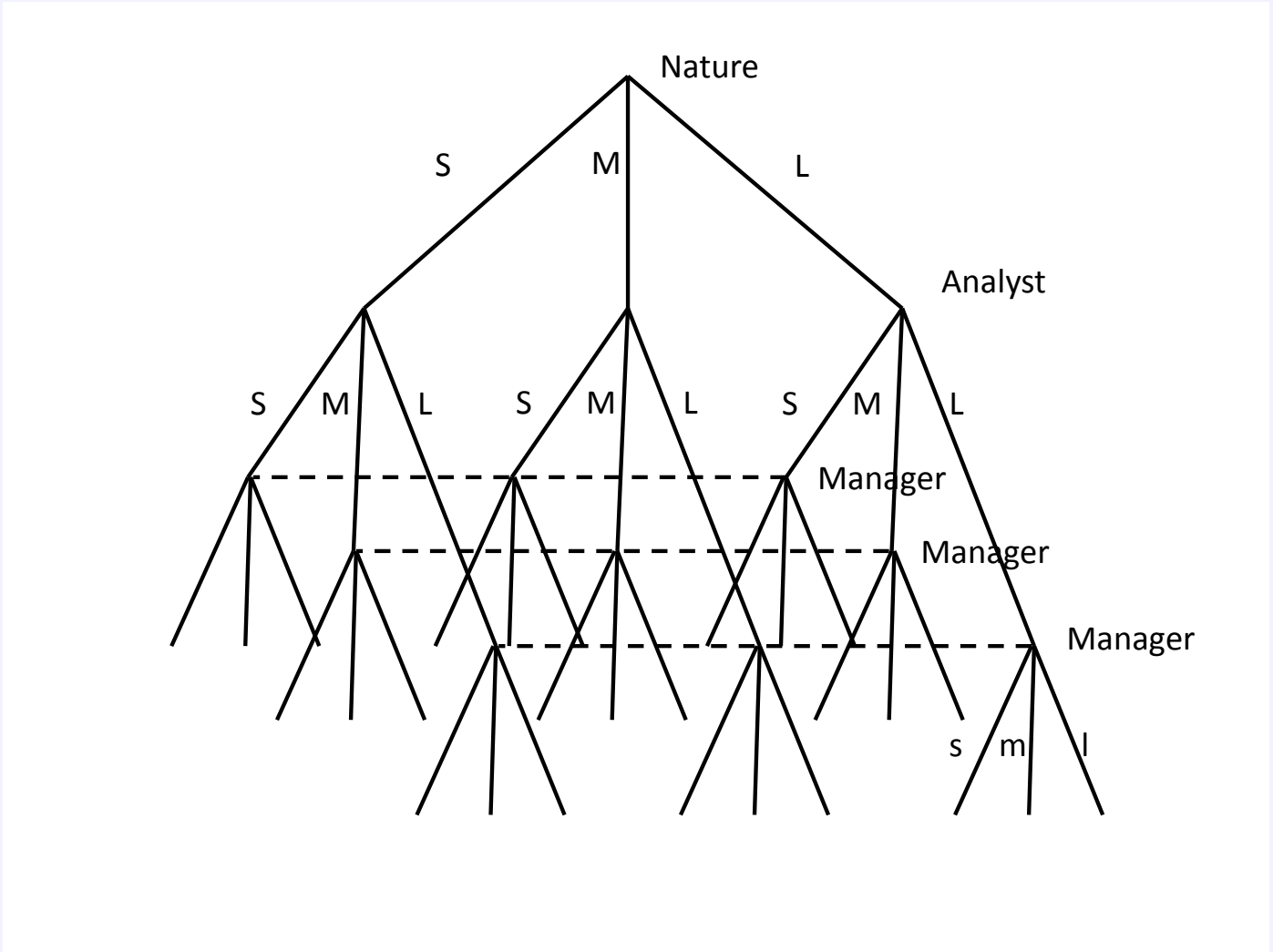
This game can be represented graphically as follows.



Example 2: Information Transmission

- There are two players: Manager and Analyst. Manager wants to figure out what is the right amount of investment to a new project. Analyst has an access to this information.
- Analyst learns the right amount of investment, which is either "small", "medium", "large". He conveys this information to the manager (but he can lie). Then the manager chooses the size of the investment.
- Analyst prefers a slightly excessive level of investment: he prefers "medium" when he learns "small", prefers "large" when he learns "medium" or "large".

This one looks like...



- The first example is an example of **extensive game with perfect information** (and with simultaneous moves). The second one is an example of **extensive game with imperfect information** (or extensive game of incomplete information).
- We postpone a formal description of these extensive games.

- Game theory provides a unified framework in which a variety of behavior in these strategic games and extensive games can be described and analyzed.