Aggregation

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- What can we say about "aggregate demand" or "aggregate supply" based on these micro models of individual consumers and firms?

Aggregation of Production Technologies

- This question is relatively easy to answer for the production side.
- Suppose that there are J firms with production set $Y_j, j = 1, ..., J$.
- Let $Y = \sum_{j=1}^{J} Y_j$ be the **total production set** and consider a hypothetical representative firm, which maximizes $p \cdot y$ subject to $y \in Y$.
- Then the aggregate production $\sum_j y_j(p)$ coincides with this representative firm's optimal production $y(p)$.

Aggregation of Production Technologies

More precisely,

Theorem

- $\pi_j(p) < \infty$ and $y_j(p)$ is nonempty for every j if and only if $\pi(p) < \infty$ and $y(p)$ is nonempty.
- If $\pi(p) < \infty$ and $y(p)$ is nonempty, then $\pi(p) = \sum_j \pi_j(p)$ and $y(p) = \sum_j y_j(p)$.

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Comments.

- So if we are interested in the aggregate production, then we can pretend as if there is only one firm.
- We may impose an assumption directly on Y rather than on each Y_j, which is often a weaker requirement (ex. Y can be convex even when each Y_j is not).

Three Questions on Aggregation

The problem of aggregation is a bit more subtle for the demand side.

There is a different layer of issues.

- Can the aggregate demand $\sum_i x_i(p, w_i)$ be just a function of the price and the total wealth $(w = \sum_i w_i)$?
- Can the aggregate demand be regarded as the demand of some **representative consumer**?
- If there "exists" a representative consumer, then can we use the representative consumer's utility to measure social welfare?

The bottom line answer for all these questions is "yes, but under very restrictive assumptions".

Aggregate Demand Function

- Suppose that there are I consumers. Throughout we assume that consumer i's demand is given by Walrasian demand function $x_i(p, w_i)$.
- **Q:** Is there an aggregate demand function $x : \Re^{L+1}_{++} \to \Re^L_+$ to satisfy

$$x(p, w) = \sum_{i=1}^{I} x_i(p, w_i)$$

  where $w = \sum_{i=1}^{I} w_i$.
- We care about this because it is often the case that only aggregate data is available.

Aggregate Demand Function

- This requires that any transfer of wealth across consumers does not affect the total demand.
- It is necessary and sufficient for this that each $x_{i,\ell}(p, w_i)$ takes the following form:

$$x_{i,\ell}(p, w_i) = \alpha_{i,\ell}(p) + \beta_\ell(p) w_i.$$

Aggregate Demand Function

The necessary and sufficient condition for the aggregation in this sense is that each consumer's indirect utility function can take so-called **Gorman form**.

Gorman Form

The aggregate demand function depends on only prices and the aggregate wealth if and only if each consumer's indirect utility function can take the following form

$$v_i(p, w_i) = a_i(p) + b(p)w_i.$$

Aggregate Demand Function

Example 1.

- Suppose that $u_i(x, m) = v_i(x) + m$, where v_i is differentiable and strictly concave.
- Set the price of the numeraire to 1 and assume an interior solution. Then $x_i(p, w_i) = (Dv_i)^{-1}(p)$ and $m_i(p, w_i) = w_i - p \cdot (D_x v_i)^{-1}(p)$.
- So there exist such an aggregate demand function in this case:

$$x(p, w) = \sum_i (Dv_i)^{-1}(p)$$
$$m(p, w) = w - p \cdot x(p, w)$$

(but note that w_i is assumed to be large enough).

Aggregate Demand Function

Example 2.

- Consider Cobb-Douglas utility functions with the identical coefficients: $u_i(x_i) = x_{i,1}^{\alpha_1} ... x_{i,L}^{\alpha_L}, \alpha_\ell \geq 0, \sum_\ell \alpha_\ell = 1$.
- We know that $x_{i,\ell}(p, w_i) = \frac{\alpha_\ell}{p_\ell} w_i$. Hence the aggregate demand function is $x_\ell(p, w) = \frac{\alpha_\ell}{p_\ell} w$ for $\ell = 1, ..., L$, which is clearly rationalizable.
- In fact, $v_i(p, w_i) = \left(\frac{\alpha_1}{p_1}\right)^{\alpha_1} ... \left(\frac{\alpha_L}{p_L}\right)^{\alpha_L} w_i$.

Representative Consumer

- Let $(w_1(p,w)\dots,w_I(p,w))$ be a **wealth distribution rule**, which satisfies $w=\sum_i w_i(p,w)$ for every (p,w).
- Each such function represents a rule that relates the total wealth to individual wealth. Denote any set of wealth distribution rules by W.

Representative Consumer

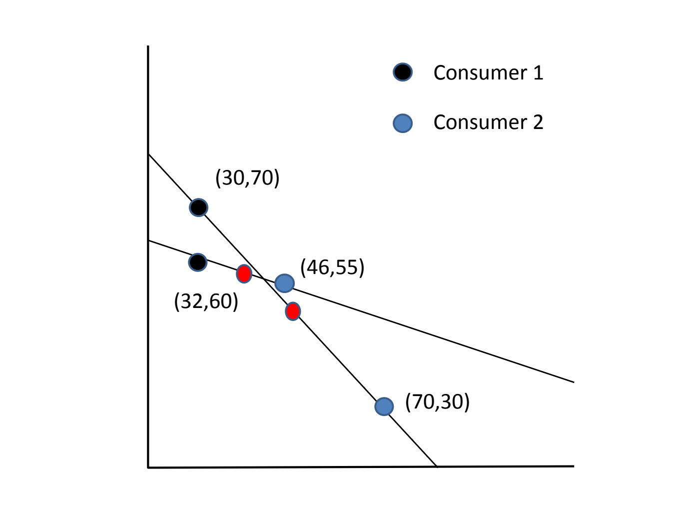
Let's make the notion of representative consumer a bit more precise.

Representative Consumer

A **representative consumer** exists with respect to W if there exists an aggregate demand function $x(p, w)$ that coincides with $\sum_i x_i(p, w_i(p, w))$ given any wealth distribution rule $(w_1, ..., w_I)$ in W and there exists a rational preference from which $x(p, w)$ is derived as the unique optimal choice given any $(p, w) \gg 0$.

Representative Consumer

- In general, it is difficult to find a representative consumer.
- **Example**
  - Two consumers with \$1000 each. Suppose there are two goods and prices are $p^1 = (10, 10)$ or $p^2 = (5, 14)$.
  - Consumer 1 buys $(30, 70)$ given p^1 and buys $(32, 60)$ given p^2. Consumer 2 buys $(70, 30)$ given p^1 and buys $(46, 55)$ given p^2. This satisfies GARP, hence rationaliable.
  - But the aggregate demand is $(100, 100)$ given p^1 and $(92, 110)$ given p^2, which violates GARP.


Consumer 1
Consumer 2
(30,70)
(46,55)
(32,60)
(70,30)

Representative Consumer

- Then when can we find a representative consumer?
- Observe that there exists a representative consumer independent of wealth distribution rules in some special cases we have seen: (1) quasi-linear utility function and (2) Cobb-Douglas utility function with the identical coefficients across consumers.
- More generally, Gorman form of indirect utility functions (with common $b(p)$) is (necessary and) sufficient.

Representative Consumer

- We can get a more positive result if wealth distribution rules are not arbitrary.
- Fix one wealth distribution rule $(w_1, ..., w_I)$. In this case, clearly we can get the aggregate demand function for free without any assumption: $(x(p, w) = \sum_i x_i(p, w_i(p, w))$. Now preference?
- Let $W : \Re^I \to \Re$ be a **social welfare function** that assigns some value to any vector of utilities of the consumers. Assume that it is increasing and concave.

Representative Consumer

Positive Representative Consumer

Suppose that $(w_1(p,w),...,w_I(p,w))$ solves the following problem for every (p,w)

$$\max_{(w_1,...,w_I)} W\left(v_1(p,w_1),...,v_I(p,w_I)\right) \ s.t. \ \sum_i w_i \leq w.$$

Then there exists a representative consumer whose indirect utility function is the maximized value of the above problem and whose demand $x(p,w)$ coincides with the aggregate demand $(\sum_i x_i(p,w_i(p,w)))$ for any (p,w).

Representative Consumer

Remark.

- It is not difficult to verify that this theorem is consistent with our observation regarding quasi-linear utility and Cobb-Douglas utility.
- Maybe we don't really need a representative consumer as long as we can find enough restriction on the aggregate demand. MWG 4.C looks for a sufficient condition for the aggregate demand to satisfy WARP.

Representative Consumer

- In the above theorem, the representative consumer's demand maximizes some social welfare function. So the representative consumer's preference has some welfare meaning in this sense.
- In general, this may not be the case. For example, it can be the case that there exists a (positive) representative consumer, but there does not exist any social welfare function that rationalizes the representative consumer's demand. See MWG 4.D.

Is aggregation always a problem?

- Sometimes aggregation makes our life easier.
- This is due to the **regularizing effect** in **large economy**.
  - Discontinuous demand functions due to indivisibility can be turned into a continuous aggregate demand function.
  - (Many identical) nonconvex demand functions due to nonconvex preference can be turned into a convex aggregate demand correspondence.