Introduction to Equilibrium Analysis

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Introduction

Motivation

Why do we study competitive (Walrasian) equilibrium?

- How are prices determined? Need a theory to pin down prices ("Theory of Value").

  $\rightarrow$ We derive the equilibrium prices & the equilibrium allocations from the primitives (preference and technology) of an economy.

- We like to analyze interactions between many markets (**general equilibrium analysis** vs **partial equilibrium analysis**).

Questions

Some theoretical questions we may ask:

- Is equilibrium "good"? (Efficiency)
- Is there any equilibrium? (Existence)
- Is equilibrium unique? (Uniqueness)

Economy and Equilibrium

Economy

- The basic data of an **economy** is summarized by

$$\mathcal{E} = \left(\{X_i, \succeq_i\}_{i \in I}, \{Y_j\}_{j \in J}, r \right).$$

  - $I = \{1, ..., I\}$ is the set of consumers and $J = \{1, ..., J\}$ is the set of producers.
  - $r \in \Re^L_+$ is the total resource for this economy.

Private Ownership Economy and Pure Exchange Economy

- A **private ownership economy** is

$$\mathcal{E}^{priv} = \left(\{X_i, \succeq_i, e_i\}_{i \in I}, \{Y_j\}_{j \in J}, \{\theta_{i,j}\}_{i \in I, j \in J} \right).$$

  - $e_i \in \Re_+^L$ is the initial endowment for consumer i ($\sum_i e_i = r$).
  - $\theta_{i,j}$ is consumer i's share of firm j (so $\sum_i \theta_{i,j} = 1$ for all $j \in J$).

- We often focus on **pure exchange economy**

$$\mathcal{E}^{pure} = \left(\{X_i, \succeq_i, e_i\}_{i \in I} \right).$$

, which can be interpreted as a special case of private ownership economy (more on this later).

Feasible Allocations

An allocation $(x, y) \in \Re^{2L}$ is feasible for $\mathcal{E}$ if (1) $\forall i, x_i \in X_i$, (2) $\forall j, y_j \in Y_j$, and (3) $\sum_i x_i = r + \sum_j y_j$.

Feasible Allocations

$$A = \{(x, y) \in \Re^{2L} | (x, y) \in X \times Y, \sum_{i \in I} x_i = r + \sum_{j \in J} y_j\}$$

Competitive Equilibrium

Now we introduce a competitive equilibrium. Let's start with a simple case: pure exchange economy.

Definition of CE in pure exchange economy

$(x^*, p^*) \in X \times \Re^L_+$ is a **competitive (Walrasian) equilibrium** for $\mathcal{E}^{pure}$ if

- x_i^* maximizes consumer i's utility.
  - $x_i^* \in B_i(p^*, p^* \cdot e_i)\,(= \{x_i \in X_i | p^* \cdot x_i \leq p^* \cdot e_i\})$
  - $x_i^* \succeq_i x_i$ for all $x_i \in B_i(p^*, p^* \cdot e_i)$ for all $i \in I$
- every market clears.
  - $\sum_i x_i^* \leq r$ and $\sum_i x_{i,\ell}^* = r_\ell$ if $p_\ell^* > 0$.

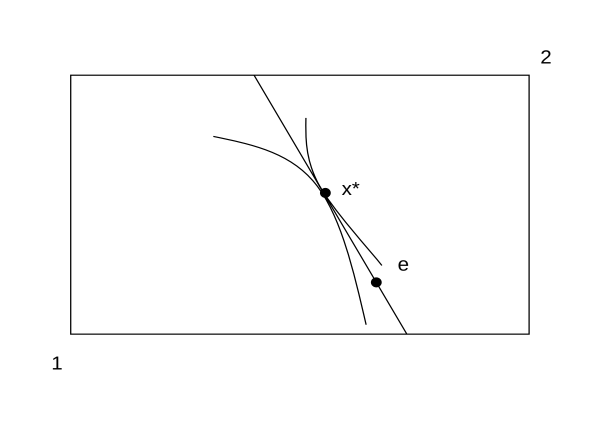
Remark

- $p^* > 0$ if the preferences are locally nonsatiated.
- If there is any consumer i with strongly monotone preference and $X_i = \Re^L_+$, then $p^* \gg 0$. Hence the second condition can be replaced by the market clearing condition $\sum_i x_i^* = r$.

Examples

Consider a pure exchange economy with two consumers and two goods.

Here is a typical CE in the **Edgeworth box**.



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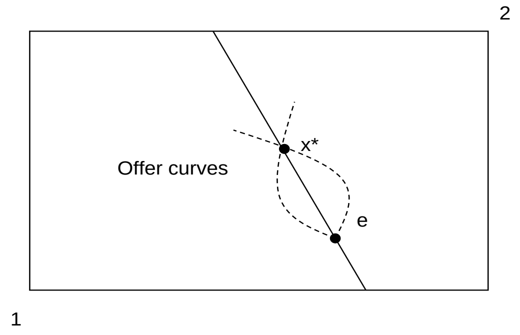
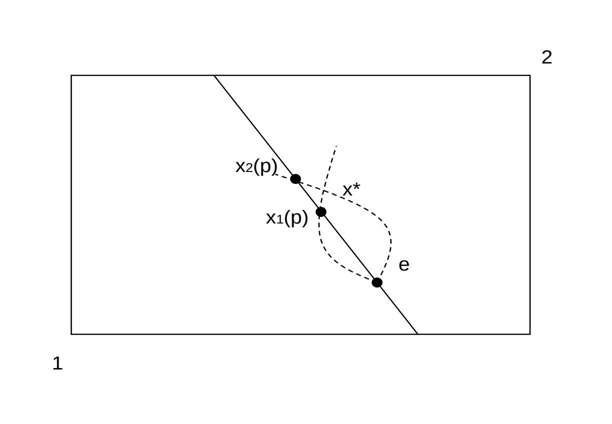


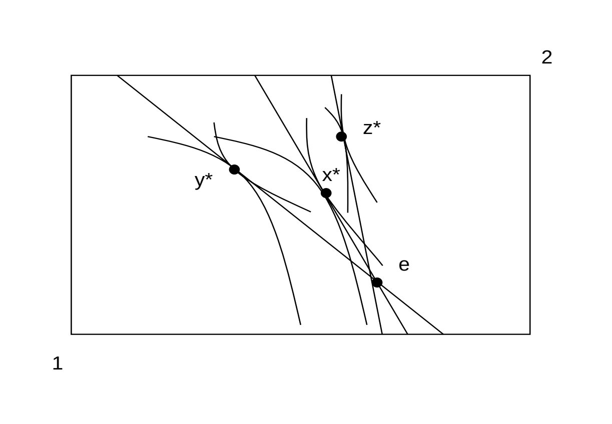
Examples

This is not a CE. Excess demand for good 1 and excess supply for good 2.



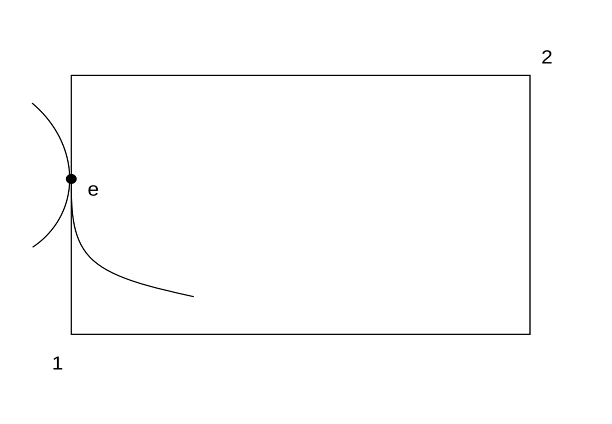
Examples

There may be multiple CE.



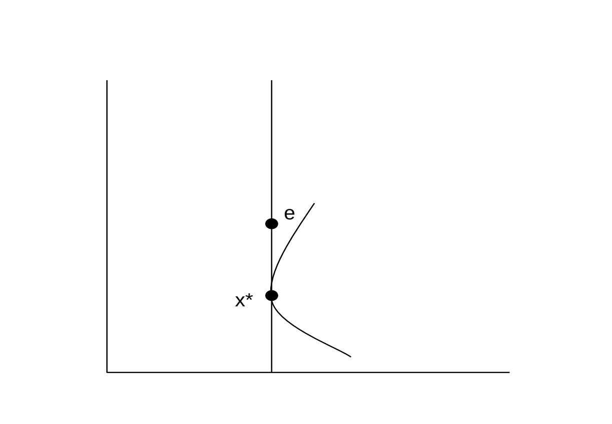
Examples

...or maybe none.



Examples

This is a CE with $p_2 = 0$ in a pure exchange economy with one consumer and two goods.



Cobb-Douglas Example

- Two consumers with Cobb-Douglas preference.
- $u_1(x) = x_{1,1}^{\alpha} x_{1,2}^{1-\alpha}$, $u_2(x) = x_{2,1}^{\beta} x_{2,2}^{1-\beta}$, $\alpha, \beta \in (0,1)$.
- $e_1 = (1,0), e_2 = (0,1)$.

Can you find a competitive equilibrium?

Cobb-Douglas Example

Consumers maximize their utility,

$$
\begin{aligned}
D_\ell u_i(x_i) &= \lambda_i p_\ell \text{ for } \ell = 1,2 \text{ and } i = 1,2 \\
p \cdot x_i &= p \cdot e_i \text{ for } i = 1,2
\end{aligned}
$$

and the markets clear:

$$
x_{1,\ell} + x_{2,\ell} = e_{1,\ell} + e_{2,\ell} \text{ for } \ell = 1,2.
$$

Cobb-Douglas Example

- We know that:
  - $x_{1,1}(p) = \alpha \frac{p \cdot e_1}{p_1} = \alpha$.
  - $x_{1,2}(p) = (1-\alpha)\frac{p \cdot e_1}{p_2} = (1-\alpha)\frac{p_1}{p_2}$.
  - $x_{2,1}(p) = \beta \frac{p \cdot e_2}{p_1} = \beta \frac{p_2}{p_1}$.
  - $x_{2,2}(p) = (1-\beta)\frac{p \cdot e_2}{p_2} = (1-\beta)$.

  (**Offer curves** are vertical).
- Combined with the market clearing conditions, we can derive $x_1^*(p) = (\alpha, \beta), x_2^*(p) = (1-\alpha, 1-\beta)$, $p_2^*/p_1^* = \frac{1-\alpha}{\beta}$.
- Note that only the price ratio can be determined (if p^* is a CE price, then αp^* is also a CE price for any $\alpha > 0$).

Here is the definition of CE for private ownership economies.

Definition of CE in private ownership economy

$(x^*, y^*, p^*) \in X \times Y \times \Re^L$ is a **competitive (Walrasian) equilibrium** for $\mathcal{E}^{priv}$ if

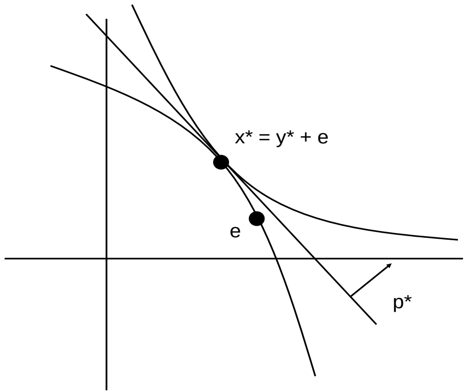
- x_i^* maximizes consumer i's utility.
  - $x_i^* \in B_i(p^*, p^* \cdot e_i + \sum_j \theta_{i,j} p^* \cdot y_j^*)$ and
  - $x_i^* \succeq_i x_i$ for all $x_i \in B_i(p^*, p^* \cdot e_i + \sum_j \theta_{i,j} p^* \cdot y_j^*)$ for all $i \in I$.
- y_j^* maximizes firm j's ptofit.
  - $p \cdot y_j^* \geq p \cdot y_j$ for all $y_j \in Y_j$ for all $j \in J$,
- Every market clears.
  - $\sum_i x_i^* = r + \sum_j y_j^*$.

Remark.

- CE in pure exchange economy can be regarded as CE in the private ownership economy with the free disposal technology $Y = \Re^L_-$. Pay attention to:
  - the space of prices
  - irrelevance of the ownership structure ($\pi(p) = 0$).
  - the market clearing condition (equality and inequality)

Example

This figure illustrates a CE in a economy with one consumer and one producer.



Walras' Law

Walras' Law

Note that one market clearing condition was redundant in the Cobb-Douglas example. This is true in general.

"Walras' Law"

Suppose that, for all $i \in I$, $\succeq_i$ is locally nonsatiated and a consumption vector $x_i' \in X_i$ satisfies $x_i' \succeq x_i$ for every $x_i \in B_i(p, p \cdot e_i + \sum_{j \in J} \theta_{i,j} p \cdot y_j)$ given (p, y). If $L - 1$ markets clear and $p \gg 0$, then the last market must clear as well.

Proof.

- **Step 1** Since the preferences are locally nonsatiated, $p \cdot x_i = p \cdot e_i + \sum_j \theta_{i,j} p \cdot y_j$ for all $i \in I$. Sum up these equations with respect to i to obtain $p \cdot \sum_i x_i = p \cdot r + p \cdot \sum_j y_j$.
- **Step 2** Suppose that the markers clear ($\sum_i x_{i,\ell} = r_\ell + \sum_j y_{j,\ell}$) for (say) $\ell = 1, ..., L-1$. Then $p_L \sum_i x_{i,L} \;=\; p_L r_L + p_L \sum_j y_{j,L}$. Since $p_L > 0$, $\sum_i x_{i,L} \;=\; r_L + \sum_j y_{j,L}$.