Producer Theory

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Production Set

Production Set

- Next we study the other side of markets: production.
- Producer's problem = maximizing profits subject to technological constraints.
- We first describe "production technology" formally.

Production Set

- We model a production process as a black box that transforms inputs into outputs.
- Each production technology is represented by a subset $Y \subset \Re^L$.
- Negative numbers correspond to inputs and positive numbers correspond to outputs. For example, $(-1, ..., -1, 2) \in Y$ means that 2 units of the Lth good can be produced by using one unit of good $1 ... L-1$.

Production Set

We always assume that Y is not empty and closed. In addition, Y may satisfy some of the following properties:

Various Properties of Production Set

- **No Free Lunch**: $Y \bigcap \Re^L_+ \subset \{0\}$.
- **Possibility of Inaction**: $0 \in Y$.
- **Free Disposal**: $Y - \Re^L_- \subset Y$.
- **Irreversibility**: $Y \bigcap (-Y) \subset \{0\}$
- **Additivity (Free Entry)**: $Y + Y \subset Y$.
- **Convexity**: Y is convex.

Production Set

Various Properties of Production Set

- **Returns to Scale:**
  - **Nonincreasing Returns to Scale**: $\alpha y \in Y$ for any $y \in Y$ and $\alpha \in [0, 1]$.
  - **Nondecreasing Returns to Scale**: $\alpha y \in Y$ for any $y \in Y$ and $\alpha \geq 1$.
  - **Constant Returns to Scale**: $\alpha y \in Y$ for any $y \in Y$ and $\alpha \geq 0$.
- **Convex Cone**: Y is a **convex cone** if Y is convex and satisfies constant returns to scale.
- **Boundedness**: Y is **bounded above** if $\exists \overline{y} \in \Re^L$ such that $y \leq \overline{y}$ for all $y \in Y$.

Production Set

- These properties are related to each other. For example
  - Possibility of inaction + convexity $\Rightarrow$ nonincreasing returns to scale.
  - Additivity + constant returns to scale $\Leftrightarrow$ convex cone.
  - ...
- The properties that are most useful for our analysis later are (1) convexity and (2) free disposal.

Transformation/Production Function

- There are many other ways to represent production technologies.
- It is possible to use a function (**transformation function**) to represent a production set.

$$Y = \left\{ y \in \Re^L : F(y) \leq 0 \right\}$$

- The properties of F can be translated into the corresponding properties of the production set Y.

Transformation/Production Function

- It is most common to use a specific version of transformation functions called **production function**. A production function specifies the maximum amount of a certain commodity that can be produced for a given set of inputs of other commodities. For example, $f:\Re_{+}^{L-1}\rightarrow\Re_{+}$ is a production function of some good that uses $L-1$ other goods as its inputs.
- Note that such production function corresponds to the following production set (with free disposal).

$$Y=\{(-z_1,...,-z_{L-1},q)|q\leq f(z_1,...,z_{L-1}),(z_1,...,z_{L-1})\geq 0\}$$

Is every technology constant returns to scale?

Remark. It is possible to regard any (convex) technology as a (convex) constant returns to scale technology by introducing some fixed, hidden input ("entrepreneurial factor"?, "management skill?). For this reason, CRS technology has some special place in the theory of production.

Example.

- Consider a production function $f(z_1, z_2) = z_1^{0.4} z_2^{0.4}$.
- This can be regarded as a technology $f(z_1, z_2, z_3) = z_1^{0.4} z_2^{0.4} z_3^{0.2}$ with $z_3 = 1$.

Profit Maximization

Profit Maximization

- The profit maximization problem given $p\Re^L_+$ is

Profit Maximization Problem

$\pi(p) = \sup_{y \in \Re^L} p \cdot y$ s.t. $y \in Y$ or $F(y) \leq 0$.

- This is similar to the cost minimization problem, but the domain may not be "compactifiable" (hence there may be no solution for this problem).

Profit Maximization

With production function f, this becomes

Profit Maximization with Production Function

$\pi(p) = \sup_{z \geq 0} rf(z) - w \cdot z.$

where $p = (r, w)$.

Profit Maximization

- Let $y(p)$ (**supply correspondence**) be the set of the solutions for the profit maximizing problem given $p \in \Re^L_+$.
    - $y(p)$ is nonempty for every $p \gg 0$ if Y is bounded above. But $y(p)$ may be empty in general.

Profit Maximization

- Let $P^* \subset \Re^L_{++}$ be the set of prices where $y(p)$ is nonempty (hence $\pi(p) < \infty$).
- It is a routine exercise to show the following properties of π in P^*.

Properties of $\pi(p)$

- $\pi(p)$ is homogeneous of degree 1.
- $\pi(p)$ is convex.
- $\pi(p)$ is continuous.

Properties of $y(p)$

The properties of $y(p)$ are parallel to those of Hicksian demand.

Properties of $y(p)$

- $y(p)$ is homogeneous of degree 0,
- $y(p)$ is convex (resp. single-valued) if Y is (resp. strictly) convex,
- **Hotelling's Lemma**: If $y(p)$ is a single point, then $\pi(p)$ is differentiable at p and $\nabla\pi(p) = y(p)$.
- If $y(p)$ is continuously differentiable, then $Dy(p) = D^2\pi(p)$ is symmetric and positive semi-definite with $Dy(p)p = 0$.

Profit Maximization

If F is differentiable, we can apply the programming technique to solve these problems.

- Kuhn-Tucker conditions are,
  - $p - \lambda \nabla F(y) = 0$
  - $F(y) = 0$
- With a production function,
  - $r \nabla f(z) - w = 0$

Factor Demand via Cost Minimization

- Let z^* be the solution for the profit maximization problem with a production function f given (r, w) and $q = f(z^*)$.
- Note that this z^* also solves the following problem.

Cost Minimization

$$\min_{z \geq 0} w \cdot z \text{ s.t. } f(z) \geq q$$

- Let $z(w, q)$ (**conditional factor demand**) be the set of solutions and $c(w, q)$ (**cost function**) be the minimized cost given (w, q).

Factor Demand

- This problem is parallel to the cost minimization problem for consumers.
- See MWG 5-C to find all the useful properties of $z(w, q)$ and $c(w, q)$.

Objectives of the Firm

Objectives of the Firm

We implicitly assumed that some "producer" maximizes her profits. But is this a reasonable assumption? **Who is maximizing profits?**

There are two issues.

Ownership and Management

- It is usually a manager/CEO who makes actual decisions. But her objective may not be profit maximization (ex. maximizing the size of the firm, maximizing her private benefits, etc.).
- This is an agency problem, which is a topic for more advanced courses.

Do shareholders agree?

- Suppose that there is no agency problem. The manager follows any suggestion by the shareholders.
- Do the shareholders maximize the profit of their firm?
- Let θ_i be the share of shareholder i of the firm. Since i is also a consumer, i solves the following problem given price p, his share of the profit $\theta_i p \cdot y$ and other income w_i:

$$\max_{x_i \in X_i} u_i(x_i) \text{ s.t. } p \cdot x_i \leq w_i + \theta_i p \cdot y$$

Do shareholders agree?

- Note that every shareholder would agree with the plan that maximizes the profit. So there is no conflict of interest among the shareholders.
- However, if the final profit is uncertain (say, the prices are stochastic), then there may not be a plan that is unanimously approved by all the shareholders. Some of them may be more risk averse and some may be more risk loving ("Decision under Risk and Uncertainty" will be covered later).