Revealed Preference

Ichiro Obara

UCLA

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Suppose that we obtained data of price and consumption pairs

$D = \left\{(p^t, x^t) \in \Re^L_{++} \times \Re^L_{+}, t = 1, ..., T\right\}$ from a consumer.

Does this consumer maximize his or her utility?

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Here we take a very different approach to choice behavior. We start with some axioms on choice behavior, not on preference.

Consider the following properties.

- x^t is **revealed preferred** to x^s if $p^t \cdot x^s \leq p^t \cdot x^t$.
- x^t is **strictly revealed preferred** to x^s if $p^t \cdot x^s < p^t \cdot x^t$.

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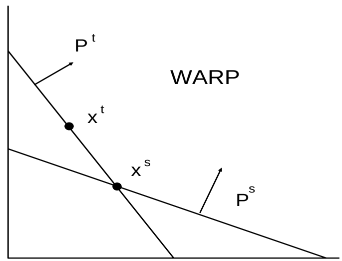
Here is one well-known axiom.

Weak Axiom of Revealed Preference (WARP)

For any $x^t, x^s \in D$, if x^t is revealed preferred to x^s and $x^t \neq x^s$, then x^s is not revealed preferred to x^t.



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Here is another important axiom.

Generalized Axiom of Revealed Preference (GARP)

For any $\left\{p^{t(n)}, x^{t(n)}, n = 1, ..., N\right\} \subset D$, if $x^{t(n)}$ is revealed preferred to $x^{t(n+1)}$ for $n = 1, ..., N-1$, then $x^{t(N)}$ is not strictly revealed preferred to $x^{t(1)}$.

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- If a consumer's choice is based on his or her preference, then the following must be the case.
  - $u(x^t) \geq u(x^s)$ if x^t is revealed preferred to x^s, and
  - $u(x^t) > u(x^s)$ if x^t is strictly revealed preferred to x^s and u is locally nonsatiated.
  - $u(x^t) > u(x^s)$ if x^t is revealed preferred to x^s, $x^t \neq x^s$ and u is strictly quasi-concave.
- So GARP is necessary for consumers with locally nonsatiated preference and WARP is necessary for consumers with strictly convex preference. Hence (in principle) we can check the assumption of utility maximization by checking these properties in data.

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- Is the converse true? Can we regard a consumer as a utility maximizer if these axioms are satisfied?

Rationalizable Data

A finite data set D is **rationalized by a utility function** $u : \Re^L_+ \to \Re$ if $u(x^t) \geq u(x)$ for all $x \in B(p^t, p^t \cdot x^t)$ and $t = 1, ..., T$.

- More precisely, the question is: can we find a utility function that rationalizes the data when the data satisfies the above axioms?

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- The answer is NO for WARP.
  - **Example.** Consider three price-commodity pairs given by

$$p^1 = (1,1,2), p^2 = (2,1,1), p^3 = (1,2,1+\epsilon),$$
$$x^1 = (1,0,0), x^2 = (0,1,0), x^3 = (0,0,1).$$

This satisfies WARP, but violates GARP. Thus there is no (locally nonsatiated) utility function that rationalizes it.

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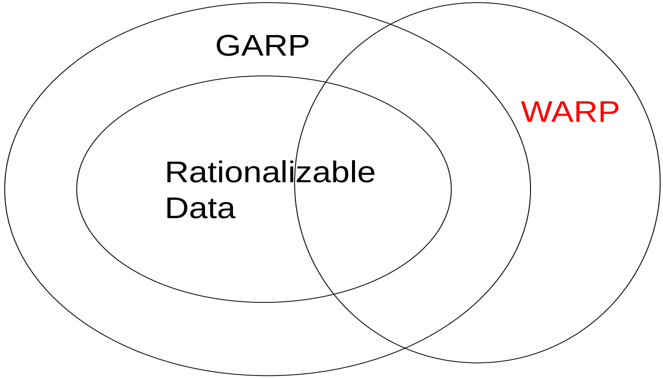
This example suggests that...



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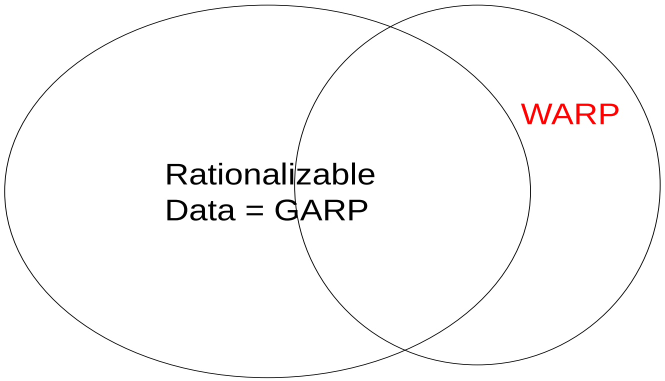
But it turns out that the right picture is the following.



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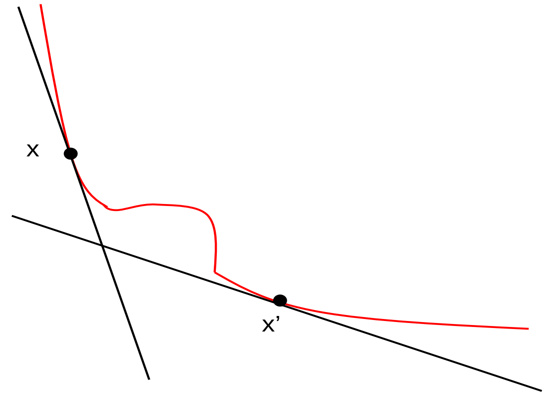
Theorem (Afriat)

The following statements are equivalent.

1. A finite data set D can be rationalized by a locally nonsatiated utility function.
2. A finite data set D satisfies GARP.
3. A finite data set D can be rationalized by a continuous, concave, and strongly monotone utility function.

The only difficult step is $2 \to 3$.

Some intuition: Whether a consumer's indifferent curve is a red curve or a blue curve cannot be detected by two data points.



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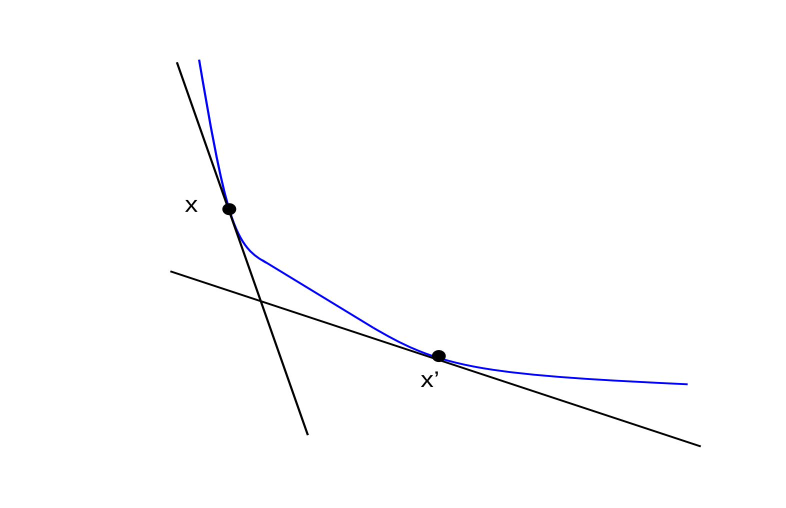

Proof of $2 \to 3$.

- It can be shown that $\exists U^t \in \Re, \lambda^t > 0, t = 1, ..., T$ such that for all i, j,

$$U^j \leq U^i + \lambda^i p^i \cdot (x^j - x^i).$$

- Define u by

$$u(x) := \min_t \left\{ U^t + \lambda^t p^t \cdot (x - x^t) \right\}.$$

- Verify that $u : \Re^L_+ \to \Re$ is continuous, concave, and strongly monotone.
- Show $p^t \cdot x \leq p^t \cdot x^t \to u(x^t) \geq u(x)$.

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Remark.

- There is a natural interpretation of the inequality that appears in the beginning of the proof. Suppose that a consumer maximizes a concave utility function u. Then for any $x^i, x^j \in D$,

$$u(x^j) \leq u(x^i) + Du(x^i)\left(x^j - x^i\right)$$

The inequality is obtained if $Du(x^i)^{\perp} = \lambda^i p^i$.

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Comment

- One way to interpret Afriat's theorem is that, if GARP is satisfied, then we can treat the consumer as if he or she is maximizing a (well-behaved) utility function. So we may expect that all the implications of utility maximization apply to this consumer's demand behavior.