Revealed Preference

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Suppose that we obtained data of price and consumption pairs $D = \left\{(p^t, x^t) \in \Re^L_{++} \times \Re^L_{+}, t = 1, ..., T\right\}$ from a consumer.

Does this consumer maximize his or her utility?

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Here we take a very different approach to choice behavior. We start with some axioms on choice behavior, not on preference.

Consider the following properties.

- x^t is **revealed preferred** to x^s if $p^t \cdot x^s \leq p^t \cdot x^t$.
- x^t is **strictly revealed preferred** to x^s if $p^t \cdot x^s < p^t \cdot x^t$.

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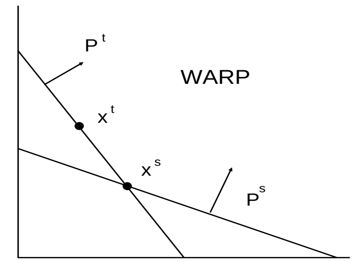
Here is one well-known axiom.

Weak Axiom of Revealed Preference (WARP)

For any $x^t, x^s \in D$, if x^t is revealed preferred to x^s and $x^t \neq x^s$, then x^s is not revealed preferred to x^t.



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Here is another important axiom.

Generalized Axiom of Revealed Preference (GARP)

For any $\left\{p^{t(n)}, x^{t(n)}, n = 1, ..., N\right\} \subset D$, if $x^{t(n)}$ is revealed preferred to $x^{t(n+1)}$ for $n = 1, ..., N-1$, then $x^{t(N)}$ is not strictly revealed preferred to $x^{t(1)}$.

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What is the use of axioms such as these?

- They may have a normative value. If you think that they are reasonable, you may apply their implications in your decision making.
- They may have a descriptive value. We may like to use it to...
  1. check if the data is consistent with some preference or a particular class of preferences,
  2. recover a preference from the data,
  3. make a prediction.

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- If a consumer's choice is based on his or her preference, then the following must be the case.
  - $u(x^t) \geq u(x^s)$ if x^t is revealed preferred to x^s, and
  - $u(x^t) > u(x^s)$ if x^t is strictly revealed preferred to x^s and u is locally nonsatiated.
  - $u(x^t) > u(x^s)$ if x^t is revealed preferred to x^s, $x^t \neq x^s$ and u is strictly quasi-concave.
- So GARP or WARP may be necessary for such consumers. Then (in principle) they can be used to check if the assumption of utility maximization is contradicted by the data.

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- Is the converse true? Can we regard a consumer as a utility maximizer when these axioms are satisfied?

Rationalizable Data

A finite data set D is **rationalized by a utility function** $u : \Re^L_+ \to \Re$ if $u(x^t) \geq u(x)$ for all $x \in B(p^t, p^t \cdot x^t)$ and $t = 1, ..., T$.

- More precisely, the question is: can we find a utility function that rationalizes the data when the data satisfies the above axioms?

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- The answer is NO for WARP.
  - **Example.** Consider three price-commodity bundle pairs given by

$$p^1 = (1, 1, 2), p^2 = (2, 1, 1), p^3 = (1, 2, 1 + e),$$
$$x^1 = (1, 0, 0), x^2 = (0, 1, 0), x^3 = (0, 0, 1).$$

This satisfies WARP, but violates GARP. Thus there is no (locally nonsatiated) utility function that rationalizes it.

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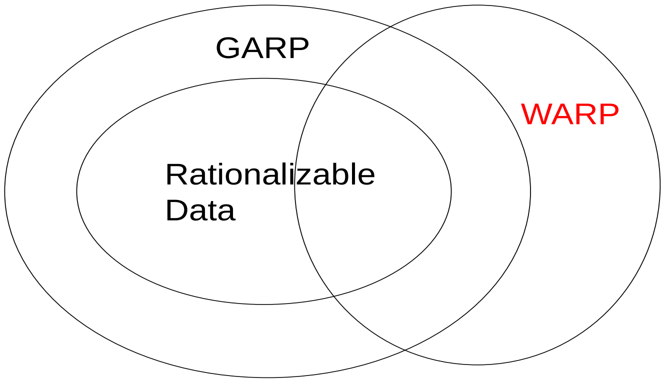
This example suggests that...



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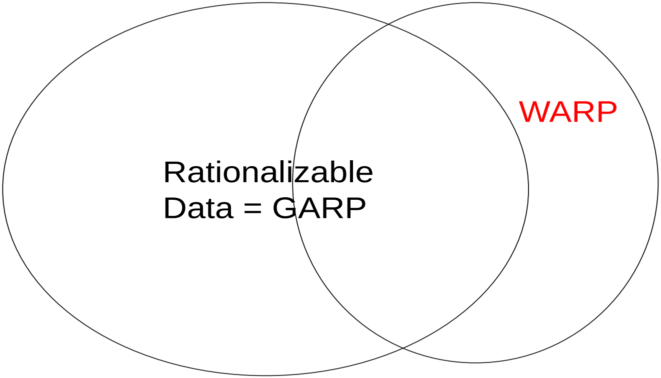
But it turns out that the right picture is the following.



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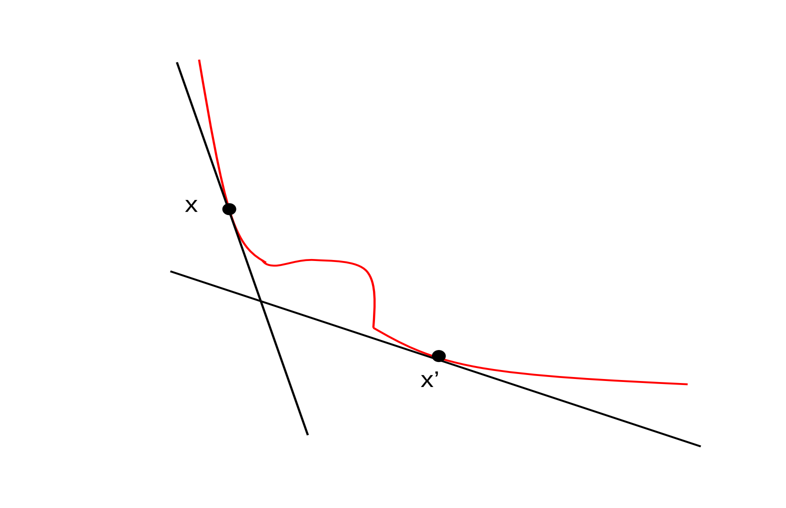
Theorem (Afriat)

The following statements are equivalent.

1. A finite data set D can be rationalized by a locally nonsatiated utility function.
2. A finite data set D satisfies GARP.
3. A finite data set D can be rationalized by a continuous, concave, and strongly monotone utility function.

The only difficult step is $2 \to 3$.

Some intuition: Whether a consumer's indifferent curve is a red curve or a blue curve cannot be detected by two data points.



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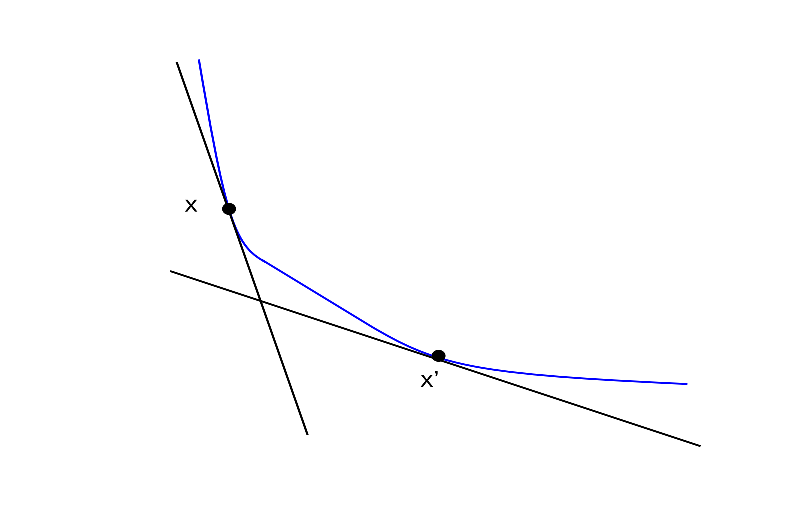

Proof of $2 \to 3$.

- It can be shown that $\exists U^t \in \Re, \lambda^t > 0, t = 1, ..., T$ such that for all i, j,

$$U^j \leq U^i + \lambda^i p^i \cdot (x^j - x^i).$$

- Define u by

$$u(x) := \min_t \left\{ U^t + \lambda^t p^t \cdot (x - x^t) \right\}.$$

- Verify that $u : \Re^L_+ \to \Re$ is continuous, concave, and strongly monotone.
- Show $p^t \cdot x \leq p^t \cdot x^t \to u(x^t) \geq u(x)$.

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Remark.

- There is a natural interpretation of the inequality that appears in the beginning of the proof. Suppose that a consumer maximizes a concave utility function u. Then for any $x^i, x^j \in D$,

$$u(x^j) \leq u(x^i) + Du(x^i)\left(x^j - x^i\right)$$

The inequality is obtained if $Du(x^i)^\perp = \lambda^i p^i$ (we will see this formula soon).

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Remark

- One way to interpret Afriat's theorem is that, if GARP is satisfied, then we can treat the consumer as if he or she is maximizing a (well-behaved) utility function. So we may expect that all the implications of utility maximization apply to this consumer's demand behavior.