Career Concerns, Project Choice, and Signaling*

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Abstract

This paper studies how career concerns affect an agent's choice between a risky and a safe project when the risky project's return depends on its quality and the agent's ability.

If the agent does not know her ability, career concerns lead to *underinvestment* in the risky project, which generalizes a result in Holmstrom (1999). In contrast, if the agent privately knows her ability, then project choice itself signals ability, resulting in *overinvestment*.

Moreover, if project quality is verifiable, first-best is attainable if the agent does not know her ability, but not attainable if she privately knows her ability.

Keywords: career concerns, underinvestment, overinvestment, signaling, verifiability

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1 Introduction

Consider the problem of a doctor choosing what treatment to give to her patient. One treatment, e.g., a drug regimen, is safe, in the sense that its expected benefits and side effects are known. The other treatment, e.g., surgery, is risky, because whether it succeeds or not depends on the doctor's skill as a surgeon and how appropriate it is given the patient's medical conditions. The doctor cares about the patient's well-being, but she also cares about how her ability will be perceived because her future labor market opportunities depends on her reputation. Similar situations arise in other principal-agent relationships whenever the agent's choices differ in their informativeness about her ability and the agent is (partly) motivated by her career concerns. Attorneys, on behalf of their clients, often choose between a settlement (with known reward or punishment) and a trial (whose outcome depends on the merit of the case and the talent of the attorney). Managers can continue investing in established and well-tested markets or start investment in new markets whose success or failure reflects their talent.

How will the agent's career concerns affect her decision? Will she act too conservatively because the safe project prevents unfavorable information being revealed about her ability? Or will she take too much risk, choosing the risky alternative against the principal's best interest, to show confidence in her ability?

These are the primary questions of this paper and the main finding is that an agent's career concerns can generate both kinds of distortions in her incentives. In particular, whether the agent acts too conservatively or takes too much risk (relative to the principal's best interest) crucially depends on whether the agent has private information on her ability. It also depends on whether the agent has the means to credibly reveal information about the characteristics of the risky project.

These results emerge in a simple model of project choice with career concerns, introduced in section 2. An agent, hired by a principal, chooses to invest in either a safe project or a risky project. The safe project's return is commonly known whereas the risky project's is unknown. The likelihood of the risky project being successful (if it is chosen) is determined by the agent's ability and the quality of the project, which is independent of the agent's ability. The agent privately observes project quality, but she may or may not privately know her ability, and she may or may not be able to verify project quality. The market knows neither the agent's ability nor the quality of the risky project, but it makes inferences about the agent's ability based on the observation of the agent's project choice, the outcome of the risky project if it is chosen and the message sent by the agent about project quality if she is able to reveal it. The agent's payoff depends on the market's perception of her ability at the end of the period as well as the chosen project's return.

Suppose the agent does not have better information on her talent than the market does. This is plausible, for example, in organizations where employees are monitored closely and evaluated frequently, resulting in the internal labor market having information on the agent's ability as precise as the agent's own. In this case, choosing the safe project reveals no information about the agent's talent and therefore her reputation neither goes up nor down if the safe project is chosen. Since the risky project's success rate is increasing in the agent's ability, however, the outcome of the risky project is partially informative of the agent's ability: she gains a higher reputation when the risky project succeeds than when it fails. In equilibrium the agent invests in the risky project if its quality is sufficiently high and invests in the safe project otherwise. At the threshold, the agent is indifferent between investing in either project.

When the market sees that a risky project is chosen, it infers that its quality must be relatively high (in the manager example, the mere fact that a new, unestablished market is chosen implies that it is promising). However, the market's belief is also coarse: without knowing the project quality exactly, the market's expectation of success rate is higher than it really is at the threshold. So at the threshold, the agent's expected reputation goes down if she chooses the risky project. Unless the agent is sufficiently risk loving (in which case she finds the uncertainty in her reputation attractive), her incentive to choose the risky project is diminished by the expected fall of her reputation at the threshold. So the agent may not choose the risky project even if it is in the principal's best interest to do so. This result, stated in Proposition 1 in section 3, generalizes the discussion given in Holmstrom's (1999, section 3) seminal paper. Holmstrom's examples illustrate that under symmetric information on managerial ability, a manager fails to invest in risky projects, and my paper shows a similar problem of *underinvestment* in a more general setting.

The problem of underinvestment arises when there is no asymmetric information on the agent's ability, but in many principal-agent relationships where the agent is kept at arm's length (Cremer, 1995), for example, when the agent is self-employed, it is likely that the agent knows more about her ability than the external labor market does. Although he does not provide any in-depth analysis, Holmstrom (1999) observes that the problem of underinvestment may not be so severe without the the assumption that the agent does not have private information on her ability. Indeed, I find that the agent's incentives are strikingly different when she has private information on her ability, resulting in *overinvestment* in the risky project, the opposite of what happens when she has no private information on her ability.

To understand why, note that if the agent privately knows her ability, then project choice itself may become an informative signal. The decision of which project to choose may depend on what the agent knows about her ability, and the market makes inferences accordingly. Since it is more likely for a talented agent to succeed with the risky project than an untalented one, it emerges in equilibrium that the choice of the safe project is a sign of weakness: the agent's reputation goes down if the safe project is chosen. On the other hand, the choice of the risky project shows the agent's confidence in the project's success, which in turn is a statstical indicator of her ability. Section 4 shows some sharp results when the success rate of the high-ability agent dominates the success rate of the low-ability agent in the likelihood ratio order. Under the likelihood ratio dominance, the agent's equilibrium reputation is always higher if she chooses the risky project than if she chooses the safe project, *even if the risky project fails*. It follows immediately that regardless of her true talent and risk attitude, the agent's reputation concern drives her to choose the risky project even if it is against the principal's interest.

The literature has given substantial attention to the problem of too little managerial risk taking and its implication for compensation design (see Milgrom and Roberts, 1992, chapter 13, for more examples and discussion). But as the results of this paper show, too much managerial risk taking may also arise from the same kind of career concerns. These results perhaps provide another explanation for the excessively risky behavior of the finanicial industry in the recent economic crisis: besides the distortions created by the explicit compensation structure, career concerns may have also fueled excessive risk taking because the labor market perceived bold behavior as a sign of strength.

The results in sections 3 and 4 are derived under the assumption that the agent cannot credibly convey the quality of the risky project, but this may not always be an appropriate assumption. For certain surgeries, the doctor may be able to explain how suitable it is for her patient by using test results, medical studies and related cases. Managers may have exploratory research of a new market that they can show the executives. These possibilities are modeled in section 6 where in addition to choosing a project, the agent sends a message to the principal about project quality. The message is verifiable in that the agent can be vague, but cannot lie.[1]

The verifiability of project quality has different implications for equilibrium project choice, depending on whether the agent has private information on her ability. For a risk-neutral agent with no private information on her ability, verifiability completely removes her incentive to distort project choice. Because the agent never wants the market to perceive the risky project as better than it really is, "unraveling" happens in which the

[1] See, for example, Grossman (1981) and Milgrom (1981).

agent completely reveals the risky project's quality if she chooses to invest in it. Given this, her expected reputation is the same as the prior no matter what project she chooses. In this case, career concern does not create any distortion and first best is attained in equilibrium.

In contrast, if the agent knows her ability, then no equilibrium exists in which first best is attained in equilibrium even if the agent can verify project quality. The reason is simple: for certain quality levels, first best requires that the talented agent invest in the risky project and the untalented agent invest in the safe project. But this separation implies that the market infers from investment in the risky project that the agent must be talented, giving the untalented agent an incentive to invest in the risky project as well. Hence, even if the market can perfectly observe project quality, the privately-informed agent will still overinvest in the risky project.

Related Literaure

Following Holmstrom's (1982, 1999) seminal work, my paper provides further investigation of the "implicit incentives" arising from agent's career concerns. Some findings confirm that the insight about underinvestment from Holmstrom's examples holds more generally. More importantly, other findings show that the incongruity in risk preference between the manager and the firm can go either way, depending on what private information the manager has.

There is a substantial literature that study the distortions arising from career concerns in a number of different contexts. These include Holmstrom and Ricart I Costa (1986) on second-best contracts, Scharfstein and Stein (1990) on herding behavior, Milbourn, Shockley and Thakor (2001) on the manager's decision of information acquisition and Ottaviani and Sorensen (2006, 2006) on reputational cheap talk. Like Holmstrom (1982, 1999), these papers assume symmetric information and symmetric learning on managerial ability. As suggested by the results in this paper, conclusions may be different in these contexts if the agent has private information on her ability. A case in point is Avery and Chevalier (1999). In an environment similar to Scharfstein and Stein (1990), they show that if managers have private information on their ability (i.e., the precision of their private signals), then they may anti-herd, i.e., choose contrarian actions, to signal that they are the high type.[2]

My paper is also related to Prendergast and Stole (1996) and Chung and Eso (2008), both of which study dynamic signaling with career concerns and show interesting properties of the learning process. Chung and Eso's (2008) paper focuses on career concerns'

[2] Also, in a discussion of the robustness of their results, Ottaviani and Sorensen (2006) point out how some of their results will change if the expert privately knows her ability.

influence on an agent's choice to learn about her ability and Prendergast and Stole's (1996) paper highlights the difference in investment incentives in early and later periods. In Prendergast and Stole's model, initially the manager exaggerates her information to appear to be a fast learner but eventually becomes conservative to hide earlier errors. The key to the opposite effects in initial and later periods is that the manager has already made previous investments in later periods. This is different from the opposite effects identified in my paper, which arise from the difference in the agent's information on her own ability.

2 The Model

A principal (labor market) hires an agent to decide whether to invest in a risky project (A) or a safe project (B). Whether or not the risky project A will succeed depends the agent's ability, or his type, t, and another variable ω which measures the quality of the project. In particular, suppose project A's probability of success is $s(t, \omega)$ and $s(\cdot)$ is strictly increasing in both t and ω. So project A is more likely to succeed if the agent is more talented and if the project is better.

The agent has two possible types: t_H and t_L ($t_H > t_L$). The common prior is that $prob(t = t_H) = q \in (0, 1)$ and ω is drawn from a distribution F with continuous density function f on $\Omega = [\underline{\omega}, \bar{\omega}]$. Assume $f(\omega) > 0$, $\forall \omega \in [\underline{\omega}, \bar{\omega}]$ and that t and ω are independent.

The safe project B has a return of v and the risky project A has a return of x if it succeeds and 0 if it fails.[3] Assume $0 < v < x$. Let $y \in Y = \{A, B\}$ be the agent's project choice.

At the beginning of the game, the agent privately observes ω and decides which project to invest in. It is crucial what information the agent has on her ability. I will analyze both the case in which the agent does not know her ability and the case in which she privately knows her ability. The market observes neither ω nor t, but observes the agent's investment choice y; if A is chosen, the market also observes whether it succeeds or fails. So the market's posterior on the agent's type, denoted by β, depends on y and the outcome (success or failure) if $y = A$.

The principal cares about only the return of the chosen project. His payoff function is $U^p(y, s(t, \omega)) = v$ if $y = B$ and $U^p(y, s(t, \omega)) = s(t, \omega) \cdot x$ if $y = A$. The agent cares about both the return of the chosen project and how her investment decision and outcome

[3]I call project B safe because its return does not depend on the agent's type. The return of B can still be uncertain for reasons other than the agent's ability. One can think of v as the expected return of project B commonly known to both the principal and the agent.

reflects on her ability. For simplicity, assume that the agent's payoff function has two additively separable components: the return of the chosen project and the value of her reputation. That is, the agent's payoff is $U^a(y, s(t,\omega)) = \lambda v + (1-\lambda) g(\beta(B))$ if $y = B$ and $U^a(y, s(t,\omega)) = \lambda s(t,\omega) x + (1-\lambda)(s(t,\omega) g(\beta(A, \text{sucesss})) + (1 - s(t,\omega)) g(\beta(A, \text{failure})))$ if $y = A$. One implicit assumption, which is standard in the career concerns literature, is that the current period's payment to the agent does not depend on the project choice or the outcome of the project, i.e., contingent contracts are ruled out.

The parameter $\lambda \in [0,1]$ is the weight that the agent places on the return of the project relative to her reputational concern. If $\lambda = 1$, the agent cares solely about the return of the project. In this case, the principal's and the agent's interests coincide and the agent chooses the first best project. When $\lambda = 0$, the agent cares solely about her reputation, and this is the assumption made in Holmstrom (1999). Since in certain applications the agent does care about the project return as well as her reputation (for example, a doctor typically cares about her patient's well being) and the case of perfectly aligned interets ($\lambda = 1$) is uninteresting, I will assume that $\lambda \in [0,1)$.

The function $g(\cdot)$ is strictly increasing, indicating that the agent prefers to be perceived as talented. In most of the analysis below, I will follow Holmstrom (1999) and assume that $g(\cdot)$ *is linear in* β. This could happen, for example, if wage is linear in the agent's reputation and the agent is risk neutral.[4] I will also comment on what happens if $g(\cdot)$ is nonlinear.

3 Case I: The Agent Does Not Know Her Ability

Suppose the agent does not have private information on her ability. Let $\bar{s}(\omega)$ be the expected success rate of the risky project if it has quality ω, i.e., $\bar{s}(\omega) = qs(t_H, \omega) + (1-q) s(t_L, \omega)$. Note that $\bar{s}(\omega)$ is strictly increasing in ω. To avoid trivialities, assume $\bar{s}(\underline{\omega}) x < v < \bar{s}(\bar{\omega}) x$, which ensures that the agent's information on ω is valuable. Define ω^* by the equation $\bar{s}(\omega^*) x = v$. So the first-best investment rule is to invest in the risky project if $\omega \geq \omega^*$ and invest in the safe project if $\omega < \omega^*$.[5] Since $v < \bar{s}(\bar{\omega}) x$ and $\bar{s}(\omega)$ is strictly increasing in ω, it follows that $\omega^* < \bar{\omega}$.

Fix $\lambda \in [0,1)$. Let the agent's mixed strategy be $\alpha : \Omega \to [0,1]$, where $\alpha(\omega)$ is the probability that the agent chooses project A when she observes ω. If both projects A and B are chosen with positive probability ex ante, the market's posterior β can be found by

[4] Suppose the agent's value in the labor market is $v(t)$ and competition among either firms or divisions within a firm drives wage to be the expected value of labor. Then the agent's wage is $\beta v(t_H) + (1-\beta) v(t_L)$, which is linear in her reputation β.

[5] She is indifferent between A and B when $\omega = \omega^*$.

Bayes' rule. Given the agent's strategy $\alpha(\cdot)$, the market's posterior is

$$\begin{aligned}
\beta(B) &= p(t_H|y=B) = q, \\
\beta(A, success) &= p(t_H|y=A, success) = \frac{\int_\Omega q \cdot s(t_H,\omega) \cdot \alpha(\omega)\, dF(\omega)}{\int_\Omega \bar{s}(\omega) \cdot \alpha(\omega)\, dF(\omega)}, \text{ and} \\
\beta(A, failure) &= p(t_H|y=A, failure) = \frac{\int_\Omega q \cdot (1-s(t_H,\omega)) \cdot \alpha(\omega)\, dF(\omega)}{\int_\Omega (1-\bar{s}(\omega)) \cdot \alpha(\omega)\, dF(\omega)}.
\end{aligned}$$

Since $s(t_H,\omega) > s(t_L,\omega)\,, \forall\omega$, it follows that $\frac{q \cdot s(t_H,\omega)}{\bar{s}(\omega)} > q$ and $\frac{q\cdot(1-s(t_H,\omega))}{1-\bar{s}(\omega)} < q$, $\forall\omega$. Hence $\beta(A, success) > q$ and $\beta(A, failure) < q$. If either project A or B is chosen with zero probability, then Bayes' rule does not always apply. Following Kreps and Wilson's (1982) idea of sequential equilibrium,[6] I make the following *consistency* requirement that the players' beliefs are the limit of beliefs associated with totally mixed strategies. So the solution concept I use is Perfect Bayesian Equilibrium with the additional consistency requirement. Note that with totally mixed strategies, the market's posterior satisfies $\beta(B) = q$, $\beta(A, success) > q$ and $\beta(A, failure) < q$.[7] Hence, as the limit of these beliefs, the market's posterior (both on and off the equilibrium path) satisfies $\beta(B) = q$, $\beta(A, success) > q$ and $\beta(A, failure) < q$. That is, in equilibrium, if the agent chooses the safe project B, then no information will be revealed about her type and her reputation stays the same as the prior, but if she chooses the risky project A, then her reputation goes up if it succeeds and goes down if it fails.

Next, I show that in equilibrium, the agent must follow a monotone strategy, that is, there exists a threshold $\tilde{\omega} \in [\underline{\omega}, \bar{\omega}]$ such that if $\omega < \tilde{\omega}$, the agent invests in the safe project and if $\omega > \tilde{\omega}$, the agent invests in the risky project.

For an agent with observation ω, her expected payoff if investing in A is $E_t(U^a(A, s(t,\omega)))$ $= \lambda\bar{s}(\omega)x + (1-\lambda)(\bar{s}(\omega)g(\beta(A, \text{success})) + (1-\bar{s}(\omega))g(\beta(A, \text{failure})))$, which is increasing in ω. Her expected payoff if investing in B is $E_t(U^a(B, s(t,\omega))) = \lambda v + (1-\lambda)g(\beta(B))$, which is independent of ω. So $E_t(U^a(A, s(t,\omega))) - E_t(U^a(B, s(t,\omega)))$ is strictly increasing in ω. Hence the agent must follow a monotone strategy in equilibrium and we can describe her equilibrium strategy by the threshold $\tilde{\omega} \in [\underline{\omega}, \bar{\omega}]$. Note that if $\tilde{\omega} = \underline{\omega}$, project A is always chosen; if $\tilde{\omega} = \bar{\omega}$, project B is always chosen; if $\tilde{\omega} \in (\underline{\omega}, \bar{\omega})$, then both A and B are chosen with positive probability ex ante and the agent must be indifferent between investing in A and investing B at the threshold $\tilde{\omega}$.

Since the agent is uninformed about her type, the belief over her type is a mar-

[6] Strictly speaking, Kreps and Wilson's sequential equilibrium does not apply here because they define it for finite games whereas here ω is continuous.

[7] Both $\beta(A, success)$ and $\beta(A, failure)$ are bounded away from q.

tingale (that is, the expectation of the belief is the same as the prior). In particular, since $\beta(B) = q$, it follows that $\beta(A) = q$, i.e., without conditioning on project outcome, the agent's expected reputation is the same as the prior if she chooses project A. Since the market does not observe ω but knows the agent's investment strategy, if project A is chosen, the expected success rate is $E(\bar{s}(\omega)|\omega \geq \tilde{\omega})$ and the expected failure rate is $(1 - E(\bar{s}(\omega)|\omega \geq \tilde{\omega}))$ from the market's point of view. It follows that $E(\bar{s}(\omega)|\omega \geq \tilde{\omega})\beta(A, success) + (1 - E(\bar{s}(\omega)|\omega \geq \tilde{\omega}))\beta(A, failure) = \beta(A) = q$. Note that unless $\tilde{\omega} = \bar{\omega}$, we have $\bar{s}(\tilde{\omega}) < E(\bar{s}(\omega)|\omega \geq \tilde{\omega})$ and therefore $\bar{s}(\tilde{\omega})\beta(A, success) + (1 - \bar{s}(\tilde{\omega}))\beta(A, success) < q$. This means that unless the agent always invests in the safe project (i.e., $\tilde{\omega} = \bar{\omega}$), at the threshold $\tilde{\omega}$, the agent expects her reputation to go down if she chooses the risky project A.

The following proposition compares the agent's equilibrium investment choice with the first best. It shows that the agent is too "conservative" in equilibrium in the sense that she underinvests in the risky project.

Proposition 1 *(Underinvestment in the risky project) Suppose the agent has no private information on her ability. Then in equilibrium, the threshold of the agent's strategy $\tilde{\omega}$ satisfies $\tilde{\omega} > \omega^*$.*

Proof. Suppose $\tilde{\omega} = \bar{\omega}$. Since $\omega^* < \bar{\omega}$, it follows that $\tilde{\omega} > \omega^*$.

Suppose $\tilde{\omega} < \bar{\omega}$. Then equilibrium condition requires that the agent with observation $\tilde{\omega}$ be indifferent between investing in project A and B. That is, $\lambda\bar{s}(\tilde{\omega})x + (1-\lambda)$ $(\bar{s}(\tilde{\omega})g(\beta(A, \text{sucesss})) + (1 - \bar{s}(\tilde{\omega}))g(\beta(A, \text{failure}))) = \lambda v + (1-\lambda)g(q)$. Since $\bar{s}(\tilde{\omega})$ $\beta(A, \text{sucesss}) + (1 - \bar{s}(\tilde{\omega}))\beta(A, \text{failure}) < q$ when $\tilde{\omega} < \bar{\omega}$ and $g(\cdot)$ is linear in β, it follows that $(\bar{s}(\tilde{\omega})g(\beta(A, \text{sucesss})) + (1 - \bar{s}(\tilde{\omega}))g(\beta(A, \text{failure}))) < g(q)$. Hence $\lambda\bar{s}(\tilde{\omega})x > \lambda v$. Since $\bar{s}(\omega^*)x = v$ and $\bar{s}(\cdot)$ is strictly increasing, it follows that $\tilde{\omega} < \omega^*$. ■

Proposition 1 says that in equilibrium, for some realization of ω (when $\omega \in (\tilde{\omega}, \omega^*)$), agent chooses to invest in the safe project, against the principal's best interest.

Why does the agent act too conservatively? Note that the agent's equilibrium strategy implies that the choice of project A conveys to the market that it has relatively high quality (i.e., $\omega \geq \tilde{\omega}$) and in equilibrium the market expects it to succeed with an average rate of $E(\bar{s}(\omega)|\omega \geq \tilde{\omega})$. But for an agent with the threshold observation $\tilde{\omega}$, she knows that project A's true quality is lower than her investment choice (of project A) conveys. Her expectation of the success rate is $\bar{s}(\tilde{\omega})$, lower than the market's. Since failure is bad news for the agent's reputation, the agent with the threshold observation expects that her reputation will go down if she chooses project A. For her to be willing to invest in A at the threshold $\tilde{\omega}$, the expected return from project A (the other component of her payoff function) must be higher than the return from project B, resulting in underinvestment in the risky project.

Remark 1 *The underinvestment result in Proposition 1 is derived under the assumption that the agent is risk neutral, i.e., $g(\cdot)$ is linear. More generally, if $g(\cdot)$ is concave, investing in the risky project is even less attractive and the underinvestment problem becomes worse.*[8] *But if $g(\cdot)$ is sufficiently convex (e.g., if the agent is highly risk loving), then overinvestment in the risky project can happen.*

Remark 2 *Proposition 1 generalizes the result in Holmstrom (1999, section 3.2). Holmstrom assumes that if the manager is untalented, the project succeeds with probability $\frac{1}{2}$ and if the manager is talented, the project succeeds with probability s.*[9] *From the firm's point view, the manager should invest if $s \geq \frac{1}{2}$, but Holmstrom shows that if the manager has no private information on her talent, but has private information on s, then the only equilibrium is the degenerate one in which no investment is ever made. The complete lack of investment arises from the assumption that the manager cares about only her reputation, which is equivalent to $\lambda = 0$ in my setting, but the reason for underinvestment is the same.*

4 Case II: The Agent Knows Her Ability

In the previous section, the agent is assumed to have no private information on her ability. In this section, I analyze the case in which the agent privately knows her type t.

If the agent knows her type t in addition to observing ω, it is equivalent to observing project A's success rate $s(t,\omega)$. Let $s_L = s(t_L,\omega)$ and $s_H = s(t_H,\omega)$. Suppose s_L has distribution function $L(\cdot)$ and density function $l(\cdot)$ and s_H has distribution function $H(\cdot)$ and density function $h(\cdot)$. Suppose $0 < l(s) < \infty$ if and only if $s \in [\underline{s_L}, \bar{s}_L]$ and $0 < h(s) < \infty$ if and only if $s \in [\underline{s_H}, \bar{s}_H]$ where $\underline{s_L} \leq \underline{s_H}$ and $\bar{s}_L \leq \bar{s}_H$. Also assume that $\underline{s_H} < \bar{s}_L$, i.e., the support of s_L and the support of s_H overlap. Let S be $[\underline{s_L}, \bar{s}_L] \cup [\underline{s_H}, \bar{s}_H]$. The main results of this section are derived under the assumption that H dominates L

[8] Interestingly, Hermalin (1993) finds that a risk-averse manager may minimize his reputational risk by undertaking the most risky project available, *if the market can observe a project's risk.* This is because the greater known risk of a project, the more weight will be put on the prior when the market updates its assessment of managerial ability. So the manager is exposed to a lower reputational risk by choosing a riskier project.

[9] Although a quality parameter is not explicitly introduced in Holmstrom, the assumption is analogous: the success of the project is determined by both managerial talent and some other independent variable.

in the likelihood ratio order, that is, $l(s_1)h(s_2) \geq l(s_2)h(s_1)$ for any s_1, $s_2 \in S$ where $s_1 \leq s_2$ (equivalently, $\frac{h(s)}{l(s)}$ is weakly increasing in s if $l(s) \neq 0$).[10]

Let $s^* = \bar{s}(\omega^*)$. So the first-best investment rule is to invest in the risky project A if and only if $s \geq s^*$. Suppose $s^* \in (\underline{s}_L, \bar{s}_H)$, i.e., at first best, both project A and project B are chosen with positive probability ex ante.

Fix the market's belief $\beta(B)$, $\beta(A, success)$ and $\beta(A, failure)$. To simplify notation, let $g(\beta(A, success)) = g_1$ and $g(\beta(A, failure)) = g_0$. An agent's expected payoff if investing in project B is $U^a(B,s) = \lambda v + (1-\lambda) g(\beta(B))$ and her expected payoff if investing in project A is $U^a(A,s) = \lambda sx + (1-\lambda)(sg_1 + (1-s) g_0)$. Let $\Delta U^a(s) = U^a(A,s) - U^a(B,s)$. Then $\Delta U^a(s) = \lambda(sx - v) + (1-\lambda)(sg_1 + (1-s) g_0 - g(\beta(B)))$ and $\frac{d(\Delta U^a(s))}{ds} = \lambda x + (1-\lambda)(g_1 - g_0)$.

Since the agent's private information includes both t and ω, her investment choice can potentially depend on both t and ω. Note, however, that the agent's payoff depends only on s (a function of t and ω), with the market's posterior fixed. Below, I first focus on analyzing the case in which the agent's strategy depends only on s and then discuss what happens otherwise.

4.1 Project Choice Depends Only on the Success Rate

First, I show that if the agent's strategy depends only on s and project A is chosen with positive probability, then success is "good news" for the agent's reputation, that is, $\beta(A, success) \geq \beta(A, failure)$ in equilibrium.

If the type-t_L agent invests in project A with probability 0, then $\beta(A, success) = \beta(A, failure) = 1$. Now suppose both type t_H and type t_L invest in project A with positive probability. Let $\phi(s)$ be the probability that the agent with observation s invests in project A. Bayes' rule implies that

$$\begin{aligned}
\beta(A, success) &= \frac{q\int_S s \cdot h(s) \cdot \phi(s)\, ds}{q\int_S s \cdot h(s) \cdot \phi(s)\, ds + (1-q)\int_S s \cdot l(s) \cdot \phi(s)\, ds} \text{ and} \\
\beta(A, failure) &= \frac{q\int_S (1-s) \cdot h(s) \cdot \phi(s)\, ds}{q\int_S (1-s) \cdot h(s) \cdot \phi(s)\, ds + (1-q)\int_S (1-s) \cdot l(s) \cdot \phi(s)\, ds}.
\end{aligned}$$

So to show that $\beta(A, success) \geq \beta(A, failure)$, it is sufficient to show that $\frac{\int_S s \cdot h(s) \cdot \phi(s) ds}{\int_S h(s) \cdot \phi(s) ds} \geq \frac{\int_S s \cdot l(s) \cdot \phi(s) ds}{\int_S l(s) \cdot \phi(s) ds}$.

Let H^ϕ be a distribution with density $h^\phi = \frac{h(s) \cdot \phi(s)}{\int_{s \in S} h(s) \cdot \phi(s) ds}$ and L^ϕ be a distribution

[10] For a detailed discussion of the monotone likelihood ratio dominance, see Shaked and Shanthikumar (1994). See also Milgrom (1981).

with density $l^{\phi} = \frac{l(s)\cdot\phi(s)}{\int_{s\in S} l(s)\cdot\phi(s)ds}$. Since $\frac{h(s)}{l(s)}$ is increasing in s because H dominates L in the likelihood ratio order, it follows that $\frac{h^{\phi}(s)}{l^{\phi}(s)} \left(= \frac{h(s)\int_{s\in S} l(s)\cdot\phi(s)ds}{l(s)\int_{s\in S} h(s)\cdot\phi(s)ds}\right)$ is increasing in s, i.e., H^{ϕ} also dominates L^{ϕ} in the likelihood ratio order. As is well known, likelihood ratio dominance implies first order stochastic dominance.[11] Therefore $\frac{\int_S s\cdot h(s)\cdot\phi(s)ds}{\int_S h(s)\cdot\phi(s)ds} \geq \frac{\int_S s\cdot l(s)\cdot\phi(s)ds}{\int_S l(s)\cdot\phi(s)ds}$ and $\beta(A, success) \geq \beta(A, failure)$.

When success is good news for the agent's reputation, it becomes more attractive to invest in project A as s increases, since $\beta(A, success) \geq \beta(A, failure)$ implies that $\frac{d(\Delta U^a(s))}{ds} > 0$. So the agent follows a monotone strategy in equilibrium, and we can describe it by a threshold $\tilde{s} \in [\underline{s}_L, \bar{s}_H]$: both type t_L and type t_H invest in project A if and only if $s \geq \tilde{s}$.

4.1.1 Overinvestment in the Risky Project

There are four potential kinds of equilibria when the agent's strategy depends only on s and project A is chosen with positive probability ex ante. All of these equilibria involve overinvestment in the risky project and I discuss them in details below.

Interior Equilibrium

Definition 1 *An interior equilibrium is a Perfect Bayesian Equilibrium in which the agent invests in project A if $s \geq \tilde{s}$ and invests in project B if $s < \tilde{s}$ and $\tilde{s} \in (\underline{s}_H, \bar{s}_L)$.*

In an interior equilibrium, both types of the agent invest in both project A and project B with positive probability and the agent is indifferent between the two projects when $s = \tilde{s}$. Given the agent's strategy, the market's posterior must be

$$\begin{aligned}
\beta(B) &= \frac{qH(\tilde{s})}{qH(\tilde{s}) + (1-q)L(\tilde{s})}, \\
\beta(A, success) &= \frac{q\int_{\tilde{s}}^1 s dH(s)}{q\int_{\tilde{s}}^1 s dH(s) + (1-q)\int_{\tilde{s}}^1 s dL(s)}, \\
\beta(A, failure) &= \frac{q\int_{\tilde{s}}^1 (1-s) dH(s)}{q\int_{\tilde{s}}^1 (1-s) dH(s) + (1-q)\int_{\tilde{s}}^1 (1-s) dL(s)}.
\end{aligned}$$

The lemma below shows that under the likelihood ratio dominance, the posterior of the agent being type t_H is higher if she invests in A than if she invests in B, *even if project A fails.*

Lemma 1 *In an interior equilibrium, $\beta(A, failure) \geq \beta(B)$.*

[11] See, for example, Shaked and Shanthikumar (1994).

Proof. Since H dominates L in the likelihood ratio order, it follows that H dominates L in the reverse hazard rate order, i.e., $\frac{h(v)}{H(u)} \geq \frac{l(v)}{L(u)}$ for any $v \geq u > \underline{s_H}$. Hence $\frac{\int_{\tilde{s}}^{1}(1-s)h(s)ds}{H(\tilde{s})} \geq \frac{\int_{\tilde{s}}^{1}(1-s)l(s)ds}{L(\tilde{s})}$, which implies that $\frac{L(\tilde{s})}{H(\tilde{s})} \geq \frac{\int_{\tilde{s}}^{1}(1-s)l(s)dt}{\int_{\tilde{s}}^{1}(1-s)h(s)dt}$. Since $\beta(B) = \frac{q}{q+(1-q)\frac{L(\tilde{s})}{H(\tilde{s})}}$ and $\beta(A, failure) = \frac{q}{q+(1-q)\frac{\int_{\tilde{s}}^{1}(1-s)dL(s)}{\int_{\tilde{s}}^{1}(1-s)dH(s)}}$, it follows that $\beta(A, failure) \geq \beta(B)$.
■

The agent's indifference at $\tilde{s}$ requires that $\lambda(\tilde{s}x - v) + (1-\lambda)(\tilde{s}g_1 + (1-\tilde{s})g_0 - g(\beta(B))) = 0$. Since $\beta(A, success) \geq \beta(A, failure) \geq \beta(B)$ and $g(\cdot)$ is increasing, it follows that $g_1 \geq g_0 > g(\beta(B))$. Hence $\tilde{s}x - v < 0$. Since $s^*x = v$, it follows that $\tilde{s} < s^*$.

Proposition 2 *(Overinvestment in the risky project) In an interior equilibrium, the threshold of the agent's strategy $\tilde{s}$ satisfies $\tilde{s} < s^*$.*

Proposition 2 says that when the agent privately knows her type, then in an interior equilibrium, the agent sometimes invests in the risky project A even if it is against the principal's best interest (i.e., when $s \in (\tilde{s}, s^*)$). Note that if $\lambda = 0$ (i.e., the agent cares about only her reputation), as assumed in Holmstrom (1999), then the agent always invests in the risky project when she privately knows her type.

In both case I and case II, the incongruity in preference beween the principal and the agent arises from the agent's career concerns, but the effect on project choice is exactly the opposite, as shown in Propositions 1 and 2. What drives the difference? When the agent privately knows her type, signaling through project choice becomes possible. Since it is more likely for the high ability agent to make project A successful, the high ability agent has a lower cost of choosing A than the low ability agent, for any ω. Hence in equilibrium the lower ability agent chooses the safe project B more often and the choice of the safe project is a sign of weakness whereas the high ability agent chooses the risky project A more often and the choice of the risky project signals confidence.

So there are two channels through which the market makes inferences about the agent's ability: project choice and project outcome. And it is useful to think of the market's updating as having two stages. In stage one, the market updates its belief based on the project choice (A or B); if A is chosen, there is stage-two updating based on project outcome (success or failure). Similar to what happens in case I, stage-2 updating is unfavorable to the agent with the threshold observation $\tilde{s}$. That is, $\tilde{s}\beta(A, success) + (1-\tilde{s})\beta(A, failure) \leq \beta(A)$ because the market expects project A to succeed with a higher probability than $\tilde{s}$.[12] So stage-two updating makes choosing the risky project

[12] More precisely, when project A is chosen, the market expects it to succeed with probability $\int_{\tilde{s}}^{1}(qh(s) + (1-q)l(s))ds > \tilde{s}$.

less attractive. However, when the agent privately knows her ability, the additional signaling effect in stage-one updating makes choosing the risky project more attractive. In particular, under the likelihood ratio dominance, although failure of the risky project is still bad news for the agent's reputation, the signaling effect of project choice is so strong that even if project A turns out to fail, the agent's reputation is still higher than if she had chosen project B. Hence, to gain in reputational payoff the agent sometimes chooses the risky project even when the safe project has a higher return.

It is worth noting that this overinvestment result does not depend on the shape of the function $g(\cdot)$. As long as the agent prefers higher reputation for being talented (i.e., $g(\cdot)$ is strictly increasing), overinvestment arises.

Corner Equilibrium Unlike an interior equilibrium, a corner equilibrium is one in which at least one type of the agent chooses one project with probability one. There are three possible kinds of corner equilibrium.

(i) Consider a strategy in which the agent of type t_H always invests in project A and the agent of type t_L invests in both projects A and B with positive probability, i.e., $\tilde{s} \in (\underline{s}_L, \underline{s}_H]$. Given this strategy, $\beta(B) = 0$ and $\beta(A, success) \geq \beta(A, failure) > 0$. Type t_L's indifference implies that $\tilde{s} < s^*$, i.e., the agent overinvests in project A.

(ii) Consider a strategy in which the agent of type t_L always invests in project B and the agent of type t_H invests in both projects A and B with positive probability, i.e., $\tilde{s} \in [\bar{s}_L, \bar{s}_H)$. Given this strategy, $\beta(A, success) = \beta(A, failure) = 1 > \beta(B)$. Again, investing in project A generates a higher reputation payoff than investing in B and this leads to the agent to overinvest in A.

(iii) Consider a strategy in which both types t_L and t_H always invest in project A, i.e., $\tilde{s} = \underline{s}_L$. Clearly, the agent overinvests in the risky project by following this strategy. Given this strategy, $\beta(A, success) = \frac{q\int_{s\in S} sdH}{q\int_{s\in S} sdH + (1-q)\int_{s\in S} sdL} > q$ and $\beta(A, failure) = \frac{q\int_{s\in S} sdH}{q\int_{s\in S} sdH + (1-q)\int_{s\in S} sdL} < q$.[13] Since project B is chosen with zero probability, Bayes' rule does not apply to the posterior $\beta(B)$. When can this be an equilibrium? The worst belief that the market can have is $\beta(B) = 0$. So for it to be an equilibrium that both types always invest in project A, a necessary (also sufficient if we assume $\beta(B) = 0$) condition is that the agent with observation $\underline{s}_L$ prefers to invest in project A, i.e., $\lambda \underline{s}_L x + (1-\lambda)\left(\underline{s}_L g(\beta(A, success)) + \left(1 - \underline{s}_L\right) g(\beta(A, failure))\right) \geq \lambda v + (1-\lambda) g(0)$.

[13] Since H dominates L in the likelihood ratio order (and hence the weaker first order stochastic dominance order), we have $\int sdH > \int sdL$.

4.1.2 A Refinement to Rule Out the Risky Project Never Being Chosen

The discussion in the last section shows that if project A is chosen with positive probability, the agent overinvests in the risky project in equilibrium. Can it happen in equilibrium that both type t_L and t_H always invest in project B? (Clearly, this would be a case of underinvestment.) If both types of the agent always invests in project B, then $\beta(B) = q$. But since project A is chosen with 0 probability, Bayes' rule does not apply if investment in A is observed and one needs to specify the market's belief off the equilibrium path. To have it as an equilibrium that the agent always invests in project B, the market's posterior when observing project A being chosen must be sufficiently low. One cannot directly apply standard equilibrium refinements to the game considered here because it is not a standard signaling game.[14] In particular, unlike the standard signaling game in which only the costly action is observed by the receiver, here the receiver also observes the outcome of the risky project if it is chosen and the outcome is also informative about the sender's type. Below, I show that with restrictions on the market's belief in the same spirit as well-known refinements such as divinity (Banks and Sobel, 1987) and D1 (Cho and Kreps, 1987), one can rule out this as an equilibrium.

Consider a putative equilibrium in which the agent always invests in project B. Let $p(s|A)$ be the posterior on s conditional on A being chosen (unlike β, which is the posterior on the agent's ability, $p(s|A)$ is the posterior on the success rate). For notational simplicity, let $\beta_1 = \beta(A, success)$ and $\beta_0 = \beta(A, failure)$.

Condition (i): $\beta_1 \geq \beta_0$. The justification of this condition comes from the arguments on page 12, which show that $\beta_1 \geq \beta_0$ if the probability of the agent's deviating and choosing project A depends only on s and not on t directly. Since the agent's expected payoff of investing in A is $\lambda sx + (1-\lambda)(sg(\beta_1) + (1-s)g(\beta_0))$, which depends only on s, it is plausible that the probability of deviating to choosing project A depends only on s.

Condition (ii): Let $U_a^*(s)$ be the agent's equilibrium payoff when the observation is s. Then $U_a^*(s) = \lambda v + (1-\lambda) g(q)$. Let $D(A,s) = \{\beta_1, \beta_0 : \beta_1 \geq \beta_0 \text{ and } \lambda sx + (1-\lambda)(sg(\beta_1) + (1-s)g(\beta_0)) \geq U_a^*(s)\}$. That is, $D(A,s)$ is the set of the posteriors (β_1, β_0) that satisfy condition (i) and lead to an expected payoff (if A is chosen) to the agent with observation s at least as high as her equilibrium payoff. In the same spirit as divinity (Banks and Sobel, 1987), define condition (ii) as follows: if $D(A, s_1)$ is strictly contained in $D(A, s_2)$, then $\frac{p(s_2|A)}{p(s_1|A)} \geq \frac{qh(s_2)+(1-q)l(s_2)}{qh(s_1)+(1-q)l(s_1)}$. That is, the relative likelihood of the types more likely to deviate increases when A is chosen.[15]

[14] See Sobel (2009) for a survey of signaling games and equilibrium selections.

[15] If $g(\cdot)$ is nonlinear, then a stronger condition is needed to rule out project A never being chosen.

Lemma 2 *Under conditions* (i) *and* (ii) *on the market's belief off the equilibrium path, the strategy that both types* t_L *and* t_H *always invest in the safe project* B *cannot be supported as an equilibrium.*

The proof is in the appendix.

4.2 Project Choice Depends on the Agent's Type Directly: A Discussion

So far we have seen that if the agent's strategy depends on s only, then overinvestment in the risky project arises in equilibrium. Since the agent's type t is payoff-relevant only through s, it is natural to make this case the focus of analysis. To further investigate the robustness of the overinvestment result, this subsection provides a discussion of what happens if the project choice depends on t directly.

One important variable for equilibrium characterization is $\Delta U^a(s)$ and the analysis in section 4.1 already shows that if $\frac{d(\Delta U^a(s))}{ds} > 0$, then overinvestment happens. Next, let's look at the other two cases, $\frac{d(\Delta U^a(s))}{ds} < 0$ and $\frac{d(\Delta U^a(s))}{ds} = 0$, which can potentially arise when the agent's strategy depends on t directly.

Suppose $\frac{d(\Delta U^a(s))}{ds} < 0$. Then the agent must follow a monotone strategy in equilibrium, but in the opposite direction of what happens when $\frac{d(\Delta U^a(s))}{ds} > 0$. That is, there exists an $\tilde{s} \in [\underline{s}_L, \bar{s}_H]$ such that the agent with observation $s < \tilde{s}$ invests in project A and the agent with observation $s > \tilde{s}$ invests in project B. Note that this is a strategy that depends only on s, but as shown in section 4.1, when the agent's strategy depends only on s, then $\beta(A, success) \geq \beta(A, failure)$ and $\frac{d(\Delta U^a(s))}{ds} > 0$. Hence it cannot happen in equilibrium that $\frac{d(\Delta U^a(s))}{ds} < 0$.

Suppose $\frac{d(\Delta U^a(s))}{ds} = 0$. If $\Delta U^a(s) > 0$, then for any s, investing in A is strictly better and in equilibrium the agent always invests in project A. But then $\beta(A, success) \geq \beta(A, failure)$, which contradicts $\frac{d(\Delta U^a(s))}{ds} = 0$. If $\Delta U^a(s) < 0$, then for any s, investing in B is strictly better and in equilibrium the agent always invests in B. As shown in section 4.1, this is ruled out in equilibrium by imposing conditions (i) and (ii). Finally, suppose $\Delta U^a(s) = 0$. Then, for any s, the agent is indifferent between the two projects. One can carefully construct a mixed strategy equilibrium to support $\Delta U^a(s) = 0$, but such an equilibrium has an unappealing property that the success of project A is "bad

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Consider the following condition, which has the same spirit as D1 (Cho and Kreps, 1987): if the market observes A being chosen, then the market's belief is supported on those s's for which $D(A, s)$ is maximal (that is, $D(A, s)$ is not a proper subset of any $D(A, s')$ where $s' \in S$.) Call this condition (ii'). It is straightforward to show that under conditions (i) and (ii'), the strategy that the agent always invests in project B cannot be supported as an equilibrium, as long as $g(\cdot)$ is increasing.

news" for the agent's reputation, i.e., $\beta(A, success) < \beta(A, failure)$. This can only be consistent with a (not very plausible) strategy that type t_H chooses the risky project with a high probability than the type t_L does when the risky project is more likely to fail.

5 An Example

To illustrate equilibrium underinvestment in case I and equilibrium overinvestment in case II, this section provides a simple numerical example.

Example 1 *Suppose ω is uniformly distributed on $[0,1]$ and the prior probability that the agent is the high ability type is $q = \frac{1}{2}$. Also, suppose $s(t_H, \omega) = \omega$, $s(t_L, \omega) = \frac{4}{5}\omega$, $v = 1$, $x = 2$ and $\lambda = \frac{1}{2}$.*

Case I: Suppose the agent does not know her type. Since the expected probability of success is $\bar{s}(\omega) = \frac{1}{2}\omega + \frac{1}{2}\left(\frac{4}{5}\omega\right) = \frac{9}{10}\omega$ and $\bar{s}(\omega^*)x = v$, it follows that $\omega^* = \frac{5}{9}$. So the first best is to invest in the risky project if and only if $\omega \geq \omega^* = \frac{5}{9}$.

Suppose the agent's strategy is to invest in project A if and only if $\omega \geq \tilde{\omega}$ where $\tilde{\omega} \in (0,1)$. Then the posterior satisfies $\beta(B) = \frac{1}{2}$, $\beta(A, success) = \frac{\int_{\tilde{\omega}}^1 \omega d\omega}{\int_{\tilde{\omega}}^1 \omega d\omega + \int_{\tilde{\omega}}^1 \left(\frac{4}{5}\omega\right) d\omega} = \frac{5}{9}$ and $\beta(A, failure) = \frac{\int_{\tilde{\omega}}^1 (1-\omega) d\omega}{\int_{\tilde{\omega}}^1 (1-\omega) d\omega + \int_{\tilde{\omega}}^1 \left(1 - \frac{4}{5}\omega\right) d\omega} = \frac{5\tilde{\omega}-5}{9\tilde{\omega}-11}$. Equilibrium condition requires that $\lambda v + (1-\lambda) g(\beta(B)) = \lambda \bar{s}(\tilde{\omega}) x + (1-\lambda)(\bar{s}(\tilde{\omega}) g(\beta(A, success)) + (1 - \bar{s}(\tilde{\omega})) g(\beta(A, failure)))$. If $g(\cdot)$ is linear, then $\tilde{\omega} \approx 0.576 > \omega^*$. So when the agent does not know her type, her career concerns lead to underinvestment in the risky project.

Case II: Suppose the agent privately knows her type. Under the parametric assumptions, s_H is uniformly distributed on $[0,1]$ and s_L is uniformly distributed on $[0, \frac{4}{5}]$. So $H(y) = y$, $h(y) = 1$ for $y \in [0,1]$ and $L(y) = \frac{5}{4}y$, $l(y) = \frac{5}{4}$ for $y \in [0, \frac{4}{5}]$. The first-best rule is to invest in the risky project if and only if $s \geq s^*$ where $s^* = \bar{s}(\omega^*) = \frac{1}{2}$.

Suppose the agent's strategy is to invest in project A if and only if $s \geq \tilde{s}$ where $\tilde{s} \in \left(0, \frac{4}{5}\right)$. Then, the posterior satisfies $\beta(B) = \frac{\tilde{s}}{\frac{5}{4}\tilde{s} + \tilde{s}} = \frac{4}{9}$, $\beta(A, success) = \frac{\int_{\tilde{s}}^1 s ds}{\int_{\tilde{s}}^1 s ds + \int_{\tilde{s}}^{\frac{4}{5}} \left(\frac{5}{4}s\right) ds} = \frac{20\tilde{s}^2 - 20}{45\tilde{s}^2 - 36}$ and $\beta(A, failure) = \frac{\int_{\tilde{s}}^1 (1-s) ds}{\int_{\tilde{s}}^1 (1-s) ds + \int_{\tilde{s}}^{\frac{4}{5}} \left(1 - \frac{5}{4}s\right) ds} = \frac{20\tilde{s}^2 - 40\tilde{s} + 20}{45\tilde{s}^2 - 90\tilde{s} + 44}$. Equilibrium condition requires that $\lambda v + (1-\lambda) g(\beta(B)) = \lambda \tilde{s} x + (1-\lambda)(\tilde{s} g(\beta(A, success)) + (1-\tilde{s}) g(\beta(A, failure)))$. If $g(\cdot)$ is linear, then $\tilde{s} \approx 0.456 < s^*$. So when the agent privately knows her type, her career concerns lead to overinvestment in the risky project.

6 Verifiability of Project Quality

One important assumption in the previous analysis is that the quality of project A, ω, cannot be verified by the agent. In this section, I relax this assumption and study the agent's equilibrium project choice when ω is verifiable. Formally, in addition to choosing a project, the agent also sends a message m. The market updates its belief over the agent's type based on the project choice, the outcome (if project A is chosen) and the message sent by the agent.

Verifiability of ω is defined as follows. (See Grossman (1981) and Milgrom (1981) for earlier work on games with verifiable information.) Let $M(\omega)$ be the collection of all subsets of Ω such that ω is an element of the subset. Verifiability of ω means that if the agent observes ω, then the set of messages available to her is $M(\omega)$. That is, she can choose to be imprecise in her revelation, but she cannot lie about ω. Note that in particular, $\{\omega'\} \in M(\omega)$ if and only if $\omega' = \omega$. So verifiability enables the the agent to completely reveal ω if she chooses to.

The agent's strategy has two components, what message to send and what project to invest in. Let $m(\cdot)$ be the agent's (pure) message strategy and $y(\cdot)$ be the agent's (pure) investment strategy. If the agent does not know her type, then $m(\cdot)$ is a mapping from Ω to $M(\omega)$ and $y(\cdot)$ is a mapping from Ω to Y. If the agent knows her type, then $m(\cdot)$ is a mapping from $\Omega \times T$ to $M(\omega)$ and $y(\cdot)$ is a mapping from $\Omega \times T$ to Y. Let $\beta(m, B)$ be the market's posterior when the message is m and project choice is B and let $\beta(m, A, success)$ $(\beta(m, A, failure))$ be the market's posterior when the message is m and the project choice is A and it succeeds (fails). Again, whether or not the agent knows her own ability has strikingly different implications for her equilibrium project choice.

6.1 Case I': The Agent Does Not Know Her Ability

Proposition 3 below shows that if ω is verifiable, then in equilibrium the agent chooses the first-best project and she also reveals ω completely when she chooses project A.

Proposition 3 *(First best in equilibrium) Suppose the agent does not know her type and ω is verifiable. Then in equilibrium, the agent chooses project A if and only if $\omega \geq \omega^*$ and she reveals ω completely if $\omega \geq \omega^*$.*

The proof is in the appendix.

The intuition for this result is straightforward. The problem of underinvestment when ω is not verifiable arises precisely because the agent's choice of project A does not fully convey the quality of the project. When ω is verifiable, however, the agent can convey precisely the quality of the project. Since the agent would not want the market to believe that ω is higher than it really is, an "unraveling" result, in which the agent reveals ω completely when she chooses project A, immediately follows. When ω is revealed to the market, her expected reputation is the same as the prior q, no matter what project she chooses. So for a risk-neutral agent, the distortion in incentives disappears and she chooses the first-best project in equilibrium.

Remark 3 *The result that the agent reveals ω completely when she chooses project A does not depend on the linearity of $g(\cdot)$, but the result that the agent chooses the first-best project does. If $g(\cdot)$ is concave, then the expectation of $g(\cdot)$ is lower if the agent chooses project A than if the agent chooses B. Hence the agent still underinvests in the risky project, although the verifiability of ω makes the underinvestment problem less severe. If $g(\cdot)$ is convex, then verifiability of ω compounds the problem of overinvestment.*

Remark 4 *The discussion in section* 3.1 *of Holmstrom (1999) is a special case of the setting discussed here. Holmstrom shows that when the project's quality (equivalently, expected return) is observable to the principal, then a risk-neutral manager is indifferent between investing and not investing, but a risk-averse agent prefers not investing. Like my Proposition 3, Holmstrom shows that first best is attainable when the agent is risk neutral. But Proposition 3 does not assume public observability of project quality. Instead, it shows that project quality will be revealed endogenously in equilibrium by the agent under verifiability.*

6.2 Case II': The Agent Privately Knows Her Ability

This is a case of multi-dimensional signaling through multi-dimensional actions, since the agent has private information on both her ability and the quality of project A and she makes project choice and can (partially) reveal project quality. In constrast to Proposition 3, Proposition 4 below shows that with some restrictions on beliefs off the equilibrium path, there exists no equilibrium in which the agent makes the first-best project choice. The restriction of belief is the following. Suppose at ω, the project choices of the type$-t_L$ and the type$-t_H$ agent are different (i.e., $y(t_L, \omega) \neq y(t_H, \omega)$). If the market observes project choice $y = y(t_H, \omega)$ and receives $m = \{\omega\}$, then its posterior is that with probability 1, the agent's type is t_H; if the market observes project choice $y = y(t_L, \omega)$ and receives $m = \{\omega\}$, then its posterior is that with probability 1, the agent's type if t_L.

Call this restriction $(*)$.[16] For notational convenience, let ω_H and ω_L be defined by $s_H(\omega_H) = s^*$ and $s_L(\omega_L) = s^*$. First best requires that type t_H invests in project A if and only if $\omega \geq \omega_H$ and type t_L invests in project A if and only if $\omega \geq \omega_L$.

Proposition 4 *(No first best in equilibrium) Suppose the agent privately know her type and ω is verifiable. Then there exists no equilibrium that satisfies $(*)$ in which the agent chooses the first-best project.*

It is easy to see why first best fails when the agent has private information on her type. First best requires that for a certain range of ω (i.e., when $\omega \in (\omega_H, \omega_L)$), type t_H invests in the risky project and type t_L invests in the safe project, but such separation implies that the agent's reputation would jump from 0 to 1 if she chooses the risky project instead of the safe project for ω in this range, giving her an incentive to invest in the risky project, irrespective of her ability.

Not surprisingly, there are many equilibira in this game of multi-dimensional signaling. One focal point is perhaps the set of equilibria in which ω is completely revealed.[17] (The result also applies if ω is publicly observable.) It is straightforward to show that in this set of equilibria, both types of the agent's signaling incentives still lead them to overinvest in the risky project (i.e., there exists $\omega' < \omega_H$ and $\omega'' < \omega_L$ such that the type-t_H agent invests in project A if $\omega > \omega'$ and type t_L invests in project A if $\omega > \omega''$).[18]

7 Conclusion

In this paper, we revisit the important problem raised in Holmstrom (1982, 1999) about the implication for project choice when an agent is motivated (partly) by career concerns. The main insight is that the direction of distortion (in the setting of this paper, whether the agent overinvests or underinvests in the risky project) depends crucially on the information environment. This is illustrated by comparison of equilibrium project choice under different assumptions on what information the agent possesses and what means are available to convey her private information. Although a full analysis is beyond the scope of this paper, the findings also suggest that second-best contractual remedies in the presence of career concerns are likely to be sensitive to informational assumptions.

[16] If the agent's strategy is to reveal ω completely, then restriction $(*)$ is simply the Bayes' rule. It has bite when precise revelation of ω is unexpected.

[17] This can be supported in equilibrium by the belief that if ω is not completely revealed, then the market believes that with probability 1, the agent is type t_L. This belief does not violate any (variations) of the standard refinements.

[18] Details of the characterization (not intended for publication) are in the appendix B.

Appendix A

Proof of Lemma 2. Since $D(A, s_1) \subset D(A, s_2)$ if and only if $s_1 < s_2$, condition (ii) implies that $\frac{p(s_2|A)}{p(s_1|A)} \geq \frac{qh(s_2)+(1-q)l(s_2)}{qh(s_1)+(1-q)l(s_1)}$ if $s_1 < s_2$.

Since $\frac{p(s_2|A)}{p(s_1|A)} = \frac{\beta(A)h(s_2)+(1-\beta(A))l(s_2)}{\beta(A)h(s_1)+(1-\beta(A))l(s_1)}$ and $\frac{h(s_2)}{l(s_2)} > \frac{h(s_1)}{l(s_1)}$ because H dominates L in the likelihood ratio order, it follows that $\beta(A) \geq q$. Since $g(\cdot)$ is linear, there exists an $s' \in (s^*, \bar{s}_H]$ such that for any $s > s'$, $sg(\beta_1) + (1-s)g(\beta_0) \geq g(\beta(A)) \geq g(q)$ and therefore s has a strictly higher payoff by investing in A than by investing in B. Hence, conditions (i) and (ii) rule out as an equilibrium strategy that both types t_L and t_H always invest in project B. ■

Proof of Proposition 3. First, I show that if the agent chooses project A in equilibrium, then she reveals ω completely. Suppose not. Then there exists an ω' such that the agent chooses project A if $\omega = \omega'$, sends the message m' and (because the agent does not reveal ω' completely,) $\omega' < E(\omega|m')$ where $E(\omega|m')$ is the market's conditional expecation of ω when observing project A and m'. That is, there exists an ω' such that the expected quality of project A when the message is m' is higher than ω'. So the agent can do better by revealing that $\omega = \omega'$, a contradiction.

Suppose the agent with observation ω chooses project A in equilibrium. As shown above, she reveals ω to the market. So the market's posterior is $\beta(\{\omega\}, A, success) = \frac{q \cdot s_H(\omega)}{\bar{s}(\omega)}$ and $\beta(\{\omega\}, A, failure) = \frac{q \cdot (1-s_H(\omega))}{1-\bar{s}(\omega)}$. Since $g(\cdot)$ is linear, her expected payoff if choosing A is $\lambda\bar{s}(\omega)x + (1-\lambda)(\bar{s}(\omega)g(\beta(\{\omega\}, A, success)) + (1-\bar{s}(\omega))g(\beta(\{\omega\}, A, failure))$ $= \lambda\bar{s}(\omega)x + (1-\lambda)g\left(\bar{s}(\omega) \cdot \frac{q \cdot s_H(\omega)}{\bar{s}(\omega)} + (1-\bar{s}(\omega))\frac{q \cdot (1-s_H(\omega))}{1-\bar{s}(\omega)}\right) = \lambda\bar{s}(\omega)x + (1-\lambda)g(q)$. If the agent with observation ω chooses project B, then her expected payoff is $\lambda v + (1-\lambda)g(q)$. Hence the agent chooses A if and only if $\omega \geq \omega^*$. ■

Proof of Proposition 4. By contradiction. Note that first-best requires that when $\omega \in (\omega_H, \omega_L)$, only type t_H invests in project A.

Suppose the market observes that project A is chosen and $m = \{\omega_L - \varepsilon\}$, where $\varepsilon > 0$.

If this is on the equilibrium path (i.e., $m(t_H, \omega_L - \varepsilon) = \{\omega_L - \varepsilon\}$), then the market's posterior must be that the agent is type t_H with probability 1. But then, for ε sufficiently small, the type-t_L agent with observation $\omega = \omega_L - \varepsilon$ would want to deviate and invest in project A and gain a reputation of 1, a contradiction.

If this is not on the equilibrium path (i.e., $m(t_H, \omega_L - \varepsilon) \neq \{\omega_L - \varepsilon\}$), then restriction $(*)$ impliest that the market's posterior must put probability 1 on the agent being type t_H, again giving type-t_L agent with observation $\omega = \omega_L - \varepsilon$ an incentive to deviate, a contradiction. ■

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Appendix B (not intended for publication)

Characterization of equilibrium in case $II'$when ω is completely revealed (Overinvestment):

Fix ω. Let $\phi_H(\omega)$ $(\phi_L(\omega))$ be the probability that type t_H (t_L) invests in project A when the quality parameter is ω. If $\phi_H(\omega) = 0, \phi_L(\omega) > 0$, then $\beta(\omega, A, success) = \beta(\omega, A, failure) = 0$. If $\phi_L(\omega) = 0, \phi_H(\omega) > 0$, then $\beta(\omega, A, success) = \beta(\omega, A, failure) = 1$. If $\phi_H(\omega) > 0, \phi_L(\omega) > 0$, then $\beta(\omega, A, success) = \frac{q\phi_H(\omega)s_H(\omega)}{q\phi_H(\omega)s_H(\omega)+(1-q)\phi_L(\omega)s_L(\omega)}$ and $\beta(\omega, A, failure) = \frac{q\phi_H(\omega)(1-s_H(\omega))}{q\phi_H(\omega)(1-s_H(\omega))+(1-q)\phi_L(\omega)(1-s_L(\omega))}$. Since $s_H(\omega) > s_L(\omega)$, it follows that $\beta(\omega, A, success) > \beta(\omega, A, failulre)$. So if project A is chosen with positive probability at ω, then $\beta(\omega, A, success) \geq \beta(\omega, A, failure)$. Type t_L's expected payoff if investing in project A is $\lambda s_L(\omega)x + (1-\lambda)(s_L(\omega))g(\beta(\omega, A, success)) + (1 - s_L(\omega))$ $g(\beta(\omega, A, failure))$ and type t_H's expected payoff if investing in project A is $\lambda s_H(\omega)x +$ $(1-\lambda)$
$(s_H(\omega))g(\beta(\omega, A, success)) + (1 - s_H(\omega))g(\beta(\omega, A, failure))$. If investing in project B, either type's expected payoff is $\lambda v + (1-\lambda)g(\beta(\omega, B))$. Since $s_H(\omega) > s_L(\omega)$, this implies that in equilibrium, if $\phi_L(\omega) > 0$, then $\phi_H(\omega) = 1$. This also implies that if project A is chosen with probability 0 at ω, then divinity requires that the probability that the deviation comes from type t_H is higher than the probability that the deviation comes from type t_L.

Next, I show that in equilibrium, if $\omega \geq \omega_H$, then $\phi_H(\omega) = 1$. Suppose not, then $\phi_H(\omega) = \phi_L(\omega) = 0$. Divinity implies that $s_H(\omega)g(\beta(\omega, A, success)) + (1 - s_H(\omega))$ $g(\beta(\omega, A, failure)) > g(q) = g(\beta(\omega, B))$. Since $s_H(\omega_H)x = v$, it follows type t_H has a strict incentive to deviate and invest in project A, a contradiction. Note that as long as $\lambda s_H(\omega)x + (1-\lambda)(s_H(\omega))g(\beta(\omega, A, success)) + (1 - s_H(\omega))g(\beta(\omega, A, failure)) >$ $\lambda v + (1-\lambda)g(q)$, type t_H has a strict incentive to deviate. Hence there must exist an $\omega' < \omega_H$ such that if $\omega > \omega'$, then $\phi_H(\omega) = 1$.

Suppose at $\omega \geq \omega_H$, $\phi_L(\omega) < 1$. Since $\phi_H(\omega) = 1$, it follows that $\beta(\omega, B) = 0$ and $\beta(\omega, A, success) > \beta(\omega, A, failure) > 0$. If $\omega \geq \omega_L$, then $s_L(\omega)x \geq v$ and type t_L has a strictly higher payoff by deviating and investing in project A with probability 1. Hence it cannot happen in equilibrium that $\phi_L(\omega) < 1$ if $\omega \geq \omega_L$. Since the reputational payoff from investing B is strictly lower than the reputational payoff from investing in A, there must an $\omega'' < \omega_L$ such that $\phi_L(\omega) = 1$ if $\omega > \omega''$.