Perfectionism and Choice^{*}

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Abstract

Perfectionism can be mentally costly for people who deviate from selfimposed norms of behavior. I model perfectionism in Gul and Pesendorfer's (2001) framework where agents can be tempted by immediate consumption. Besides normative and temptation utilities, my model identifies a mental cost of perfectionism that is proportional to the difference between the decision maker's actual choices and her normative ideals. The coefficient in this proportion is uniquely derived from preference. My model accommodates the persistent desire 'to pay not to go to the gym' that has been observed empirically by DellaVigna and Malmendier (2006).

1 Introduction

Psychological research (e.g. Flett and Hewitt [15]) shows that many people are *perfectionists*: they set unrealistically high standards for their own behavior and suffer negative emotions—such as guilt, anger, or embarrassment—when these standards are not met. Antony and Swinson [3, p. 66], distinguish two general types of perfectionist behaviors. The first type is 'designed to help an individual meet his or her high standards,' such as healthy diets or regular exercise. The second type involves 'avoidance of situations that may require an individual to live up to his or her standards.'

This paper models perfectionist behaviors in Gul and Pesendorfer's [17] (henceforth GP) decision framework, where preferences are defined over *menus*—sets of consumption lotteries. Each menu A is interpreted as a physical action that, if chosen ex ante, makes the set A feasible ex post. When consumption becomes imminent ex post, the decision maker can be tempted to deviate from her long-term normative standards. Moreover, anticipating that it will be very hard for her to resist temptations in a menu A, she may choose to reject this menu ex ante even if A

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contains the best normative consumption across all feasible menus. Perfectionism can make such transgressions mentally costly and hence, affect choice behavior.

First, a perfectionist decision maker can desire flexility that she does not expect to use later. For example, she may have a strict preference $\{x, y\} \succ \{y\}$ even if she expects to choose the more tempting consumption y rather than the normatively better alternative x in the menu $\{x, y\}$ ex post. By keeping x feasible, she can pretend to follow her normative objectives (e.g. a diet, a New Year resolution, or a vow) and thus, feel better about her ex ante choice. Such behavior has some empirical support. DellaVigna and Malmendier [27] (henceforth DVM) observe that people frequently pay 'not to go to the gym'. In DVM's dataset, members of health clubs who chose a flat monthly rate paid about 70% more per visit than clients with a 10-visit pass. DVM's leading explanation for such behavior is overconfidence about future self-control. Perfectionism provides an alternative explanation that is consistent with rational expectations of self-control. (See a simple numerical example in Section 1.1 below.)

Second, perfectionism can motivate the decision maker to remove the best normative alternative from the feasible menu. For example, she can find x to be normatively better than y and rank $\{x\} \succ \{y\}$, but still have the strict preference $\{y\} \succ \{x, y\}$ because she expects that she will succumb to the temptation yanyway if the menu $\{x, y\}$ is available ex post, and the failure to choose x ex post will make her suffer from perfectionism.¹ Antony and Swinson [3, p. 80] give several anecdotal examples of such behavior. For instance, they describe a very bright and competent person who chose not to attend college because he believed that he could never be satisfied with his performance. Similarly, they argue that perfectionism can lead to *procrastination*: 'people who are constantly aiming for perfection may put off things for fear that they will never meet their targets or goals. By not starting things, perfectionist individuals do not need to confront the possibility of doing less than perfect job.'

Note that the above ranking $\{x\} \succ \{y\} \succ \{x, y\}$ violates GP's *Set-Betweenness* axiom, which requires that for all menus A and B,

$$A \succeq B \quad \Rightarrow \quad A \succeq A \cup B \succeq B.$$

This condition is also problematic if the perfectionist decision maker desires flexibility that she does not expect to use later. For example, she can plausibly rank $\{x, y, z\} \succ \{x, y\}$ and $\{x, y, z\} \succ \{y, z\}$ if z is her normatively best choice (e.g. intense exercise), y is her most tempting option (e.g. staying on the couch), and x is her expected choice (e.g. moderate exercise) in the menu $\{x, y, z\}$.

To accommodate perfectionism, I restrict Set-Betweenness to menus A and B that share the same best normative element z. It is assumed that perfectionism

¹A natural concern here is why the failure to choose x should not be costly ex ante. As I argue in Section 3.1, my model still applies if perfectionism is costly both ex ante and ex post, but the two costs are not uniquely identified by preferences over menus.

should not affect choice among such menus. Formally, it is required that

$$A \succeq B \quad \Rightarrow \quad \{z\} \succeq A \succeq A \cup B \succeq B$$

whenever $z \in A \cap B$ is such that $\{z\} \succeq \{x\}$ for all $x \in A \cup B$.

My main result (Theorem 2.1) shows that this condition—together with the standard Order, Continuity, and Independence—is necessary and sufficient for the preference \succeq to have a utility representation

$$U(A) = \max_{x \in A} [u(x) - \max_{y \in A} (v(y) - v(x))] + \kappa \max_{z \in A} u(z),$$
(1)

where $\kappa > -1$, and u and v are linear functions. This representation includes GP's model as a special case with $\kappa = 0$ and suggests similar interpretations for the functions u and v. The *commitment utility* u represents the ranking of singleton menus that is interpreted as the decision maker's long-term normative preference. The *temptation utility* v determines the negative component $\max_{y \in A}(v(y) - v(x))$, which reflects the anticipated cost of ex post self-control.

To interpret the functional form (1) for $\kappa \neq 0$, consider two cases. First, let $\kappa > 0$. Take any two menus A and B such that $\max_{z \in B} u(z) \ge \max_{z \in A} u(z)$. Then by (1), $A \succeq B$ if and only if

$$\max_{x \in A} [u(x) - \max_{y \in A} (v(y) - v(x))] - \max_{x \in B} [u(x) - \max_{y \in B} (v(y) - v(x))] \ge \kappa \left(\max_{z \in B} u(z) - \max_{z \in A} u(z) \right).$$

Here the non-negative component $\kappa (\max_{z \in B} u(z) - \max_{z \in A} u(z))$ can be interpreted as a mental cost of perfectionism that the decision maker incurs ex ante if she chooses the menu A when B is the only other feasible option. This cost is proportional to the difference in the commitment utility u between the best normative elements in the rejected menu B and the chosen menu A.

Second, let $-1 < \kappa < 0$. Then representation (1) is equivalent to

$$U(A) = \max_{x \in A} \left[u(x) - \theta \max_{y \in A} (v(y) - v(x)) - \lambda \max_{z \in A} (u(z) - u(x)) \right].$$

where $\theta = \frac{1}{1+\kappa} > 0$ and $\lambda = \frac{-\kappa}{1+\kappa} > 0$. The decision maker as portrayed by this utility function expects that both self-control and perfectionism should be mentally costly ex post. The two anticipated costs, $\theta \max_{y \in A}(v(y) - v(x))$ and $\lambda \max_{z \in A}(u(z) - u(x))$ respectively, are proportional to the ex post losses in temptation and commitment utilities that result from the choice of x in the menu A.

My second result (Theorem 2.2) establishes that under a mild regularity condition, the parameter κ in representation (1) is *unique*, and the pair of functions u and v is unique up to a positive linear transformation. Moreover, the sign of κ can be characterized in terms of preference: $\kappa > 0$ or $\kappa < 0$ must hold if and only if there are $A, B \in \mathcal{M}$ such that $A \cup B \succ A \succeq B$ or $A \succeq B \succ A \cup B$ respectively. The magnitude of the parameter κ can be interpreted in terms of a comparative desire for flexibility and commitment. (See Theorem 2.3 below.)

The decision maker as portrayed by (1) should plan her expost choice x_A in any menu A to strike an optimal compromise between her normative utility and expost emotional costs of self-control and perfectionism. This compromise is obtained by maximizing the function u + v. Theorem 2.4 shows that u + v is indeed a representation for the only expost choice rule that satisfies weak axiom of revealed preference, continuity, and a suitable consistency condition with the ex ante preference.

Representation (1) can be interpreted in terms of various emotions that are associated with perfectionism, most naturally *guilt*. In order to measure guilt empirically, Tangney and Dearing [26, p. 31–50] distinguish moral standards from *proneness to guilt*, that is, 'a tendency to experience guilt in response to one's failures or transgressions.' This distinction is captured by representation (1), in which the function u represents the decision maker's normative principles, while the parameter κ measures her tendency to suffer from violating these principles. My model shows that it may be possible to derive both normative standards and proneness to guilt from observable choice behavior rather than from purely verbal assessments that have been used in empirical research. Other possible interpretations of representation (1) are discussed in Section 3 below.

1.1 Example: Overpaying for Health Clubs

Consider a person who chooses between two health-club contracts A and B that are similar to those in DVM's dataset. The contract A has a flat monthly fee of \$70 with unlimited access, and the contract B offers the price of \$10 per visit without any additional fees. Suppose that the commitment and temptation utilities are

$$u(t,m) = -2.5(t-15)^2 - m$$

 $v(t) = -50t$

where t is the number of visits per month, and m is the monetary expense. This specification uses several simplifying assumptions. First, the normative utility uis quasi-linear and has a quadratic form with respect to the amount of exercise. Second, the temptation utility v is invariant of money and linear with respect to exercise. More precisely, the agent is assumed to incur a fifty-dollar self-control cost for each session at the gym regardless of the marginal monetary price of this session. The monetary effects of decisions are viewed as normative and are evaluated via the commitment utility u.

Let the person have utility (1). Then

$$U(A) = \max_{t \ge 0} [u(t, 70) + v(t)] - \max_{t \ge 0} v(t) + \kappa \max_{t \ge 0} u(t, 70) = -570 - 70\kappa.$$

The normatively best choice in this menu is $z_A = 15$, but the anticipated choice that maximizes u + v is $t_A = 5$. (The average monthly attendance in DVM's data is 4.3.) On the other hand,

$$U(B) = \max_{t \ge 0} [u(t, 10t) + v(t)] - \max_{t \ge 0} v(t) + \kappa \max_{t \ge 0} u(t, 10t) = -540 - 140\kappa.$$

Here the normatively best choice of exercise is $z_B = 13$, but the anticipated choice that maximizes u + v is $t_B = 3$.

If $\kappa > \frac{3}{7}$, then the contract A is strictly preferred to B even though the cost of the monthly membership (\$70) exceeds the pay-per-visit price (\$50) of the five work outs that the decision maker expects to make under the contract A. In this way, perfectionism can explain the overpaying for monthly contracts that is observed by DVM.

1.2 Related Literature

Representation (1) is a special case of *temptation-driven* preferences studied by Dekel, Lipman, and Rustichini [11] (henceforth, DLR). Their model suggests a different interpretation for the functional form (1) in terms of *random* and *cumulative* temptations for $\kappa > 0$ and $\kappa < 0$ respectively. DLR's interpretation suggests that if $\kappa > 0$, and if the decision maker correctly anticipates her temptations to be random, then her ex post choice should be random. By contrast, my model is deterministic. The characterization of the ex post choice rule in Theorem 2.4 provides a formal way to distinguish the two models.

On the other hand, if $\kappa < 0$, then cumulative temptation and perfectionism have the same behavioral implications both ex ante and ex post. Yet people who distinguish normative values from temptations should not be *tempted* to maximize normative utility.² Thus it seems more natural to interpret representation (1) for $-1 < \kappa < 0$ in terms of perfectionism rather than cumulative temptation.

Moreover, my model dispenses with DLR's two technical assumptions (Finiteness, Approximate Improvements are Chosen). To construct the utility representation (1), I use a technique that is different from GP's and DLR's and rely instead on the classification of finite subjective state spaces in Kopylov [21].

My model of perfectionism is also related to Sarver's [25] model of *regret*. In his framework, the decision maker has a strong desire for commitment $\{x\} \succeq \{y\} \succ \{x, y\}$ if she expects that she will regret her ex post choice x in the menu $\{x, y\}$ with a positive probability. To capture such behavior, Sarver uses a list of axioms and a utility representation that are both distinct from mine. Section 3.3 compares the two models in more detail.

Noor and Ren [24] formulate a three-period representation of guilt where the desire to avoid ex post guilt generates more guilt at the interim stage. In Section

 $^{^{2}}$ Some people do not distinguish *normative* and *tempting*. For instance, MGM Grand Casino has an advertising slogan: 'Resist the temptation to resist temptations.'

3.1, I show that representation (1) can be rewritten to accommodate perfectionism (or more specifically, guilt) that is costly at two time periods, ex ante and ex post. Yet the two costs cannot be identified uniquely in the two-period setting. This uniqueness is only obtained if one of the costs is assumed to be zero. A similar identification problem occurs in GP's model of self-control and in Sarver's model of regret.

Perfectionist emotions have been studied in some applied economic settings. In particular, *guilt* has been used to explain cooperation in firms and families (Casson [9], Kandel and Lazear [20], Becker [5]), and other contexts (Frank [16]). All of these models impose guilt as an ad hoc component of the utility function rather than derive it from preference.

2 Model

Let $X = \{x, y, z, ...\}$ be the set $\Delta(Z)$ of all Borel probability measures on a compact metric space Z of deterministic consumptions. Endow X with the weak convergence topology. This topology is metrizable.³

Let \mathcal{M} be the set of all *menus*—non-empty compact subsets $A \subset X$. Suppose that choices are made in two stages, *ex ante* and *ex post*. Interpret any menu $A \in \mathcal{M}$ as a course of action that, if taken ex ante, makes the set $A \subset X$ feasible ex post.

Endow \mathcal{M} with the Hausdorff metric topology. For any menus $A, B \in \mathcal{M}$ and $\alpha \in [0, 1]$, define a *mixture*

$$\alpha A + (1 - \alpha)B = \{ \alpha x + (1 - \alpha)y : x \in A, y \in B \}.$$

Let \succeq be the decision maker's ex ante preference over \mathcal{M} . Following GP and DLR, adapt the well-known conditions of the expected utility theory for the preference \succeq .

Axiom 1 (Order). \succeq is complete and transitive.

Axiom 2 (Continuity). For all $A \in \mathcal{M}$, the sets $\{B \in \mathcal{M} : B \succeq A\}$ and $\{B \in \mathcal{M} : B \preceq A\}$ are closed.

Axiom 3 (Independence). For all $\alpha \in [0,1]$ and menus $A, B, C \in \mathcal{M}$,

$$A \succ B \Rightarrow \alpha A + (1 - \alpha)C \succ \alpha B + (1 - \alpha)C.$$

Order and Continuity are standard. To motivate Independence, interpret any element $\alpha x + (1 - \alpha)y$ in the menu $\alpha A + (1 - \alpha)C$ as a lottery that delivers

³More generally, let X be the set of all Anscombe–Aumann acts f that map a finite state space Ω into $\Delta(Z)$, and endow X with a product metric.

consumptions $x \in A$ and $y \in C$ with probabilities α and $1 - \alpha$ respectively and is resolved after the ex post stage. If the time when this objective uncertainty is resolved is irrelevant for the decision maker's preferences, then she should be indifferent between the menu $\alpha A + (1 - \alpha)C$ and a hypothetical lottery $\alpha \circ A + (1 - \alpha) \circ C$ that yields the menus A or C with probabilities α and $1 - \alpha$ respectively, but is resolved before the ex post stage. (Here the preference \succeq is extended to lotteries over menus.) The standard separability argument suggests that

$$A \succ B \Rightarrow \alpha \circ A + (1 - \alpha) \circ C \succ \alpha \circ B + (1 - \alpha) \circ C$$

because the possibility of getting the menu C with probability $1 - \alpha$ should not affect the decision maker's comparison of A and B. Independence follows.

Given any menu $A \in \mathcal{M}$ and any element $z \in A$, say that z is *perfect* in A if $\{z\} \succeq \{y\}$ for all $y \in A$. For any $z \in X$, let

$$\mathcal{M}_z = \{ A \in \mathcal{M} : z \text{ is perfect in } A \}.$$

The perfect element $z \in A$ is interpreted as the decision maker's best *normative* consumption in the menu A. Psychological evidence (see Ainslie [1] and Loewenstein [22]) shows that people act less impulsively as their rewards become more distant temporally or spatially. It is therefore plausible that the expost consumption period can be sufficiently distant for the decision maker to have no impulsive cravings ex ante. Then her ex ante choice to commit to a consumption z rather than y should reveal her long-term normative preference between these two alternatives.

By contrast, the decision maker can succumb to spontaneous temptations when consumption becomes imminent ex post. Moreover, anticipating that it will be very hard for her to make the best normative choice in a menu A, she may strategically choose to reject this menu ex ante even if A contains the best normative consumption across all feasible menus. Perfectionism can make such transgressions costly and hence, affect both ex post and ex ante choices.

As illustrated by examples in the introduction, perfectionist behaviors can violate GP's Set-Betweenness. Consider the following weaker version of this axiom.

Axiom 4 (Perfectionist Set-Betweenness). For all $z \in X$ and $A, B \in \mathcal{M}_z$,

$$A \succeq B \quad \Rightarrow \quad \{z\} \succeq A \succeq A \cup B \succeq B.$$

To motivate this condition (PSB for short), take any menus $A, B \in \mathcal{M}_z$ that share the same perfect element z. The ranking $\{z\} \succeq A$ is intuitive because the singleton menu $\{z\}$ delivers the normatively best choice in A without any emotional distress. Note that the ranking $\{z\} \succeq A$ follows from GP's axioms.⁴ It

⁴Order and Set-Betweenness imply this ranking for finite menus A. By Continuity, it must hold for all menus A.

is also imposed as a separate axiom (Desire for Commitment) by DLR [11] and by Sarver [25].

Assume that the decision maker expects ex ante that if her ex post menu is A (or B) then she will choose $x_A \in A$ (or respectively, $x_B \in B$), but her strongest temptation will be $y_A \in A$ (or respectively $y_B \in B$). Then it is intuitive that

$$x_B \in A, \ y_A \in B \quad \Rightarrow \quad A \succeq B.$$
 (2)

Indeed, if $x_B \in A$ and $y_A \in B$, then the menu A contains the consumption x_B that the decision maker plans to choose in B. Moreover, if she makes the same choice in A, then it should be

- less costly for her to resist the strongest temptation y_A in A than the strongest y_B temptation in B because B contains y_A as well,
- equally costly to reject the same perfect alternative z in the menus A and B,
- costless to reject A in favor of B, or vice versa, because both A and B contain the same perfect alternative. (If a third menu C with better normative choices is feasible ex ante, then it should be equally costly to reject C in favor of either A or B.)

Therefore, if $x_B \in A$ and $y_A \in B$, then the menu A provides a weakly better combination of ex post choices and emotional costs than B does and hence, the ranking $A \succeq B$ is intuitive. Condition (2) implies PSB. To show this claim, suppose that $A \succeq B$. Let $C = A \cup B$. Then $x_A, x_B, y_A, y_B \in C$. By (2), if $x_C \in A$, then $A \succeq C$; if $x_C \in B$, then $B \succeq C$. In either case, $A \succeq C$. By (2), if $y_C \in A$, then $C \succeq A$; if $y_C \in B$, then $C \succeq B$. In either case, $C \succeq B$.

Note that PSB is problematic for some general types of self-control, perfectionism, and related emotions. For instance, PSB is violated in DLR's model of temptation-driven preferences, where the mental costs of self-control are cumulative or uncertain and hence, depend on several temptations in the feasible menu A. Similarly, PSB is problematic if perfectionism is driven by several different (and possibly contradictory) normative principles.

Say that a function $u: X \to \mathbb{R}$ is *linear* if for all $\alpha \in [0, 1]$ and $x, y \in X$,

$$u(\alpha x + (1 - \alpha)y) = \alpha u(x) + (1 - \alpha)u(y).$$

Let \mathcal{U} be the set of all continuous linear functions $u: X \to \mathbb{R}$.

The following is my main representation result.

Theorem 2.1. \succeq satisfies Axioms 1-4 if and only if \succeq can be represented by

$$U(A) = \max_{x \in A} \left[u(x) - \max_{y \in A} (v(y) - v(x)) \right] + \kappa \max_{z \in A} u(z)$$
(3)

for some $\kappa > -1$ and $u, v \in \mathcal{U}$.

Note that $U({x}) = (1 + \kappa)u(x)$ for all $x \in X$. As $\kappa > -1$, then u represents the restriction of \succeq to singleton menus. Accordingly, the function u is called *commitment utility*.

To interpret representation (3) for all menus A, consider several cases.

(i) $\kappa = 0$. Then the utility function (3) takes GP's form

$$U(A) = \max_{x \in A} \left[u(x) - \max_{y \in A} (v(y) - v(x)) \right],$$
(4)

and the negative component $\max_{y \in A}(v(y) - v(x))$ can be interpreted as the decision maker's anticipated cost of self-control. This cost equals the difference in the *temptation utility* v between the chosen consumption x and the most tempting alternative in the menu A.

(ii) $\kappa \ge 0$. Take any menus A and B such that $\max_{z\in B} u(z) \ge \max_{z\in A} u(z)$. By (3), $A \succeq B$ if and only if

$$\max_{x \in A} [u(x) - \max_{y \in A} (v(y) - v(x))] - \max_{x \in B} [u(x) - \max_{y \in B} (v(y) - v(x))] \ge \\ \kappa \left(\max_{z \in B} u(z) - \max_{z \in A} u(z) \right).$$
(5)

What differs (5) from the benchmark case (4) is the non-negative component $\kappa (\max_{z \in B} u(z) - \max_{z \in A} u(z))$. It can be interpreted as a mental cost of perfectionism that the decision maker incurs ex ante if she chooses the menu A when B is the only other feasible option. This cost is proportional to the difference in the commitment utility u between the perfect elements in the rejected menu B and the chosen menu A.

(iii) $-1 < \kappa < 0$. Then \succeq is represented by

$$U(A) = \max_{x \in A} \left[u(x) - \frac{1}{1+\kappa} \max_{y \in A} (v(y) - v(x)) + \frac{\kappa}{1+\kappa} \max_{z \in A} (u(z) - u(x)) \right].$$
 (6)

The decision maker as portrayed by (6) expects that both self-control and perfectionism will be mentally costly expost. The two anticipated costs, $\frac{1}{1+\kappa} \max_{y \in A}(v(y) - v(x))$ and $\frac{-\kappa}{1+\kappa} \max_{z \in A}(u(z) - u(x))$ respectively, are proportional to the losses in temptation and commitment utilities that occur if x is chosen expost in the menu A.

2.1 Uniqueness and Interpretation of κ

Under a mild regularity condition on the preference \succeq , the parameter κ is unique, and the pair of functions u and v in the utility representation (3) are unique up to a positive linear transformation.

Say that \succeq is represented by a triple (u, v, κ) if \succeq has the utility representation (3) with components $u, v \in \mathcal{U}$ and $\kappa > -1$.

Say that \succeq is *regular* if there exist $x, y, x', y' \in X$ such that $\{x\} \succ \{x, y\}$ and $\{x'\} \sim \{x', y'\} \succ \{y'\}$. These rankings are intuitive if the consumption x is normatively better, but less tempting than y, and if x' is both normatively better and more tempting than y'.

Theorem 2.2. Let \succeq be a regular preference represented by a triple (u, v, κ) . Then

- (i) \succeq is represented by another triple (u', v', κ') if and only if $\kappa' = \kappa$, $u' = \alpha u + \beta$, and $v' = \alpha v + \gamma$ for some $\alpha > 0$ and $\beta, \gamma \in \mathbb{R}$.
- (ii) $\kappa = 0$ if and only if \succeq satisfies Set-Betweenness.
- (iii) $\kappa > 0$ if and only if $A \cup B \succ A \succeq B$ for some $A, B \in \mathcal{M}$.
- (iv) $\kappa < 0$ if and only if $A \succeq B \succ A \cup B$ for some $A, B \in \mathcal{M}$.
- (v) $\kappa \geq 0$ if and only if for all $A, B \in \mathcal{M}, A \succeq B$ implies $A \cup B \succeq B$.
- (vi) $\kappa \leq 0$ if and only if for all $A, B \in \mathcal{M}, A \succeq B$ implies $A \succeq A \cup B$.

This result asserts that if a regular preference \succeq is represented by a triple (u, v, κ) , then the functions u and v are unique up to a positive linear transformation, the parameter κ is unique, and the sign of κ is positive (or negative) if and only if the decision maker reveals a strong desire for flexibility $A \cup B \succ A \succeq B$ (or respectively, a strong desire for commitment $A \succeq B \succ A \cup B$) for some menus A and B. Note that this classification does not hold if \succeq is not regular.⁵ For example, if $v = -\frac{1}{2}u$ and $\kappa = 1$, then \succeq satisfies Set-Betweenness even though $\kappa \neq 0$.

To interpret the magnitude of the parameter κ , consider a pair of preferences \succeq and \succeq^* . Following Dekel, Lipman, and Rustichini [10], say that \succeq desires more flexibility than \succeq^* if for all $A, B \in \mathcal{M}$,

$$A \cup B \succ^* A \quad \Rightarrow \quad A \cup B \succ A. \tag{7}$$

Similarly, say that \succeq desires more commitment than \succeq^* if for all $A, B \in \mathcal{M}$

$$A \succ^* A \cup B \quad \Rightarrow \quad A \succ A \cup B. \tag{8}$$

These conditions require that any flexibility (or respectively, any commitment) that has a positive benefit for \succeq^* should also have a positive benefit for \succeq .

⁵If \succeq is represented by a triple (u, v, κ) , then \succeq is regular if and only if the functions u, v, -u are non-constant and represent different rankings on X. See Lemma A.2 in the Appendix for more details.

Theorem 2.3. Let \succeq and \succeq^* be regular preferences that satisfy Axioms 1-4 on \mathcal{M} and coincide on X. Then the following statements are equivalent.

- (i) \succeq desires more flexibility than \succeq^* .
- (ii) \succeq^* desires more commitment than \succeq .
- (iii) \succeq and \succeq^* are represented by triples (u, v, κ) and (u, v, κ^*) respectively such that $\kappa \geq \kappa^*$.

This theorem establishes necessary and sufficient conditions for the pair of regular preferences \succeq and \succeq^* to have utility representations (3) with common commitment and temptation utilities u and v, but different parameters $\kappa \geq \kappa^*$. To interpret this inequality, consider three subcases.

- $\kappa \geq \kappa^* \geq 0$. Then both \succeq and \succeq^* have representations (5) with ex ante costs of perfectionism. The only difference between these representations is that perfectionism is costlier for \succeq than for \succeq^* .
- $\kappa \ge 0 \ge \kappa^*$. Then \succeq and \succeq^* have representations (5) and (6) where perfectionism is costly ex ante and ex post respectively. Besides the timing of perfectionism, the two representations differ also in the cost of self-control, which is higher for the preference \succeq^* .
- $0 \ge \kappa \ge \kappa^*$. Then \succeq and \succeq^* have representations (6) with expost costs of perfectionism. Here both perfectionism and self-control are costlier for \succeq^* than for \succeq .

Note that the comparative notions (7) and (8) have sharper interpretations in Theorem 2.3 than in the theory of subjective state spaces in DLR [10]. Their Theorem 2 asserts only that if \succeq desires more flexibility than \succeq^* (or if \succeq^* desires more commitment than \succeq) then \succeq should have an additive utility representation with a larger set of positive components (or respectively, with a smaller set of negative components) than \succeq^* .

2.2 Ex Post Choice

The decision maker as portrayed by representations (5) and (6) should expect her ex post choice x_A in any menu A to strike an optimal compromise between her normative utility and emotional costs of self-control and perfectionism. Consider two possible cases.

(i) $\kappa \geq 0$. Then x_A should maximize the function

$$w(x) = u(x) - \max_{y \in A} (v(y) - v(x)) = u(x) + v(x) - \max_{y \in A} v(y).$$

Here the cost of ex ante perfectionism is sunk at the ex post stage and hence, should not affect the ex post choice.

(ii) $-1 < \kappa < 0$. Then x_A should maximize

$$w(x) = u(x) - \frac{1}{1+\kappa} \max_{y \in A} (v(y) - v(x)) + \frac{\kappa}{1+\kappa} \max_{z \in A} (u(z) - u(x)) = \frac{1}{1+\kappa} [u(x) + v(x)] - \frac{1}{1+\kappa} \max_{y \in A} v(y) + \frac{\kappa}{1+\kappa} \max_{z \in A} u(z).$$

This function reflects the expost costs of both self-control and perfectionism.

Thus the anticipated choice x_A should maximize the function u + v in each of the above two cases. It should be emphasized though that actual ex post choices need not be determined by the ex ante preference \succeq or a fortiori, by any utility representation that is derived for this preference.

To model ex post choice, consider an additional primitive. For any menu $A \in \mathcal{M}$, let $C(A) \subset A$ be the non-empty set of all alternatives in A that the decision maker is willing to choose at the ex post stage. Consider two well-known conditions for the choice rule $C(\cdot)$.

Axiom 5 (Weak Axiom of Revealed Preference). For all $A, B \in \mathcal{M}$ and $x, y \in X$, if $x \in C(A)$, $y \in A$, $y \in C(B)$, and $x \in B$, then $x \in C(B)$.

Arrow [4] shows that this condition (WARP for short) is necessary and sufficient for the existence of a complete and transitive expost preference \succeq_1 that rationalizes the choice rule $C(\cdot)$. Arrow's result applies here because the space \mathcal{M} contains all finite menus.

Axiom 6 (Closed Graph). The set $\{(A, x) : A \in \mathcal{M}, x \in C(A)\}$ is closed in $\mathcal{M} \times X$.

As X is compact and Hausdorff, then Closed Graph is equivalent to the upper hemicontinuity of $C(\cdot)$ (see Aliprantis and Border [2, Theorem 16.12].)

Axiom 7 (Consistency). For all $z \in X$ and $A \in \mathcal{M}$,

$$A \succ A \setminus \{z\} \quad \Rightarrow \quad C(A) = \{z\} \quad or \quad A \in \mathcal{M}_z.$$
 (9)

This condition requires that the decision maker can refuse to remove an element z from a menu A only if she plans to choose z in A, or if z is her normatively best choice in this menu.⁶ In the latter case, she can desire flexibility which she does no plan to use ex post, but which allows her to feel better about her ex ante choice.

Theorem 2.4. Let \succeq be a regular preference represented by a triple (u, v, κ) . Then a choice rule $C(\cdot)$ satisfies WARP, Closed Graph, and Consistency if and only if for all $A \in \mathcal{M}$,

$$C(A) = \arg\max_{x \in A} [u(x) + v(x)].$$

$$(10)$$

⁶If $A = \{z\}$, then by convention, let $A \succ A \setminus \{z\}$ be a false statement.

This result establishes necessary and sufficient conditions for the expost choice rule $C(\cdot)$ to comply with the decision maker's anticipation of perfectionism and self-control as revealed by her ex ante utility (3). Note that neither the parameter κ , nor the functions u and v can be derived from expost consumption choices. To determine the triple (u, v, κ) , one needs to observe the ex ante preference \succeq over menus.

2.3 Sketch of Proof

The necessity of the axioms in Theorem 2.1 is straightforward.

Turn to sufficiency. Suppose that \succeq satisfies Axioms 1–4. Construct the utility representation (3) as follows. First, use PSB to show that for any three menus $A_1, A_2, A_3 \in \mathcal{M}$,

$$A_1 \cup A_2 \succeq A_1 \cup A_3 \succeq A_2 \cup A_3 \quad \Rightarrow \quad A_1 \cup A_2 \succeq A_1 \cup A_2 \cup A_3 \succeq A_2 \cup A_3.$$

Invoke Kopylov's [21] Theorem 2.1 to conclude that \succeq is represented by

$$U(A) = \max_{x \in A} u_1(x) + \max_{x' \in A} u_2(x') - \max_{y \in A} u_3(y) - \max_{y' \in A} u_4(y')$$

for some $u_1, u_2, u_3, u_4 \in \mathcal{U}$.

Let $u_0 = u_1 + u_2 - u_3 - u_4$. Use PSB again to show that at least one of the functions u_1 and u_2 can be replaced by αu_0 for some $\alpha > 0$, and at least one of the functions u_3 and u_4 by βu_0 for some $\beta > 0$. Without loss in generality, rewrite the utility function U as

$$U(A) = \max_{x \in A} u_1(x) - \max_{y \in A} u_3(y) + \gamma \max_{z \in A} u_0(z)$$

where $\gamma = \alpha - \beta$ and $u_1 - u_3 = (1 - \gamma)u_0$. Consider the regular case where $\gamma < 1$. Then U has the required form (3) with $u = u_1 - u_3$, $v = u_3$, and $\kappa = \frac{\gamma}{1 - \gamma} > -1$.

All formal proofs are relegated to the appendix.

3 Discussion

3.1 Ex Ante vs Ex Post Perfectionism

The decision maker as portrayed by representations (5) or (6) is perfectionist about her choices only at one time period, either ex ante or ex post. This sharp dichotomy is plausible in some settings. For example, a person may have a mental commitment (e.g. a New Year resolution or a new diet) to comply with normative values ex ante, but anticipate that she will no longer have this commitment ex post. For this person, perfectionism should be costly ex ante, but not ex post. On the other hand, there is psychological evidence (e.g. Besser, Flett, and Hewitt [8]) that people are most negatively affected by perfectionism when they receive feedback on their previous performance that shows their lack of ability or willpower. There is also evidence (see Higgins, Snyder and Berglas [19, p. 192]) that people tend to focus their negative emotions on most recent choices and are less likely to challenge their previous self-handicapping decisions. Therefore, it is also plausible that some subjects in the two-period menu framework can be perfectionist mainly about their ex post choices that are temporally close to the consumption period when their lack of willpower and self-control becomes evident.

Yet in general, perfectionism and related emotions, such as guilt, anger, or embarrassment, are experienced both ex ante and ex post. For instance, Elster [13, p. 65] argues that a person who feels guilty if she chooses not to return a book to a library ex post, should also feel guilty if she commits to do so ex ante. In Elster's words, 'to want to be immoral is to be immoral.'

To model perfectionism that is costly both ex ante and ex post, one can adopt the same Axioms 1–4 (and PSB in particular) and rewrite representation (3) as follows. For all $A, B \in \mathcal{M}$,

$$A \succeq B \quad \Leftrightarrow \quad U_0(A) - U_0(B) \ge \kappa_0 \left(\max_{z \in B} u(z) - \max_{z \in A} u(z) \right)$$

where

$$U_0(A) = \max_{x \in A} \left[u(x) - (1 + \kappa_1) \max_{y \in A} (v(y) - v(x)) - \kappa_1 \max_{z \in A} (u(z) - u(x)) \right],$$

and $\kappa_0, \kappa_1 \ge 0$ are such that $\kappa = \frac{\kappa_0 - \kappa_1}{1 + \kappa_1}$. Providing that $\max_{z \in B} u(z) \ge \max_{z \in A} u(z)$, the components $\kappa_0 (\max_{z \in B} u(z) - \max_{z \in A} u(z))$ and $\kappa_1 \max_{z \in A} (u(z) - u(x))$ can be interpreted as the ex ante and ex post costs of perfectionism.

Note that the equality $\kappa = \frac{\kappa_0 - \kappa_1}{1 + \kappa_1}$ does not determine the parameters $\kappa_0, \kappa_1 \ge 0$ uniquely. The uniqueness is only obtained if either κ_0 or κ_1 is taken to be zero. Uniqueness can be also obtained if a *discount factor* $\delta > 0$ is given such that $\kappa_1 = \delta \kappa_0$.

3.2 Self-Oriented vs Socially Prescribed Perfectionism

Flett and Hewitt [14] distinguish *self-oriented perfectionism* (SOP) that is based on self-imposed standards of behavior and 'unrealistic self-expectations in the face of failure' from *socially prescribed perfectionism* (SPP) that reflects 'a strong concern over obtaining and maintaining the approval and care of other people and a sense of belonging that could be attained if it were possible to be perfect in the eyes of others.'

My model seems problematic for SPP because socially prescribed values may differ from personal normative principles. To illustrate, consider a decision maker who plans to go to a charity event where her normatively best choice would be a donation of \$100. She expects that she will face a social pressure to donate more than this amount, but will be tempted to donate less. Then she should prefer to commit to give at most \$100 (e.g. bring only that much in cash) or to commit to give at least \$100 (e.g. make a pledge beforehand) rather than to make no commitments at all. Accordingly, her preference over menus of possible donations should be

$$A = \{\$100, \$1000\} \succ \{\$100, \$1000, \$0\} = A \cup B$$
$$B = \{\$100, \$0\} \succ \{\$100, \$1000, \$0\} = A \cup B.$$

This preference violates PSB because both menus A and B share the same perfect alternative (\$100). Moreover, if the decision maker expects that she will yield to the social pressure and donate \$1000 when the menu $A \cup B$ is feasible, then she may choose to stay at home rather than to attend the charity event:

$$\{\$0\} \succ \{\$100, \$1000, \$0\}.$$

This behavior is similar to a century-old observation due to Newcomb [23]: 'The fact that the benevolent gentleman may wish there were no beggars, and may be very sorry to see them, does not change the economic effect of his readiness to give them money.' In a similar vein, Becker [6] writes: 'People do not want to encounter beggars, even though they may contribute handsomely after an encounter.' The discrepancy between ex ante commitment ranking and socially prescribed behavior motivates Sadowski and Dillenberger's [12] model of *shame*.

Note that in the absence of temptations and self-control costs, my model can accommodate both SPP and SOP. In this case, the function v can be reinterpreted as a *socially prescribed* ranking, and the negative component $\max_{y \in A}(v(y) - v(x))$ in the utility function (3) as the anticipated cost of resisting the social pressure. The negative sign of this component suggests that the social pressure should be costly ex post rather than ex ante.

The ex post timing of SPP can be intuitive if the social values are instilled on the decision maker *between* the ex ante and the ex post periods. For instance, this may happen in firms and organizations, such as the military, which promote loyalty and team spirit (see Kandeal and Lazear [20]).

The ex post timing of SPP can be also motivated by the framing of ex ante and ex post choices. For example, a person who crosses a street to avoid meeting a beggar may feel little or no SPP because she makes this decision without seeing the beggar's deliberate exhibit of his bad fortune, and without openly revealing her lack of charity. It is indeed common for people to feel more guilt or shame about choices that are observed by others than about similar choices that are hidden from the public scrutiny. For instance, more than 90% of all guilt experiences reported by subjects in the empirical study of TD [26, p.14–16] had occurred in public. Therefore, it is plausible that SPP should occur only ex post if menus are chosen privately, but choices in menus are publicly observable. To adopt this interpretation, the modeler may have to observe ex ante choices without making subjects suffer from SPP. This can be achieved if (i) subjects are not aware that they are being observed, or if (ii) they perceive the modeler as a neutral figure who is not affected by their actions and would not reprimand them for asocial behavior, or if (iii) observations involve a large group of subjects (such as potential Army recruits) but do not focus on any particular individual.

3.3 Perfectionism vs Temptation and Regret

My model is a special case of a broad class of *temptation-driven* preferences studied by DLR [11]. However, their interpretations of the utility representations (5) and (6) are different from mine.

First, they interpret both negative components in (6) as cumulative costs of self-control, and the functions v and u as two distinct *temptation utilities*. Yet it is counterintuitive that the decision maker should be 'tempted' to maximize the commitment utility u because temptations should be distinct from normative preferences. Therefore, it seems more natural to attribute the negative component $\frac{-\kappa}{1+\kappa} \max_{z \in A}(u(z) - u(x))$ to perfectionism.

Second, DLR interpret representation (5) with positive $\kappa > 0$ in terms of random temptation. Indeed, the utility function (5) can be rewritten as

$$U(A) = \pi \max_{x \in A} \left[u(x) - \max_{y \in A} (v(y) - v(x)) \right] + (1 - \pi) \max_{z \in A} u(z),$$
(11)

where $\pi = \frac{1}{1+\kappa} \in (0,1]$. The decision maker as portrayed by this representation believes that with probability π , she will incur the cost of self-control $\max_{y \in A}(v(y) - v(x))$ in order to resist ex post temptations, but with probability $1 - \pi$, she will maximize her normative utility u without being tempted ex post. For example, she may perceive her ex post temptation for an addictive substance to be contingent on the uncertain event that she encounters a cue that triggers her craving for this substance. (Such cue-triggered temptations are studied by Bernheim and Rangel [7].)

Even though perfectionism and random temptation have equivalent utility representations (5) and (11), they have different implications for expost choice behavior. Random temptation suggests that at the expost stage, the decision maker should maximize u + v if the temptation strikes and maximize u otherwise. This random choice violates both WARP and Consistency.

My model of perfectionism is also related to Sarver's [25] model of regret. A simple case of Sarver's representation is

$$U(A) = \max_{x \in A} [u(x) + v(x) - K \max_{y \in A} (v(y) - v(x)) - K \max_{z \in A} (u(z) - u(x))], \quad (12)$$

where $K \ge 0$ and $u, v \in \mathcal{U}$. The decision maker as portrayed by (12) learns whether her true consumption utility is u or v after making her ex post choice x_A in a menu A and feels regret if x_A does not maximize this utility. Then the two negative components in representation (12) can be interpreted as two expected costs of regret. (Alternatively, one can still interpret these components as cumulative costs of self-control.)

Besides the differences in the motivation and the utility functional forms, the models of perfectionism and regret can be distinguished by choice behavior. First, the case $\kappa > 0$ of ex ante perfectionism requires a desire for flexibility $A \cup B \succ A \succ B$ that is inconsistent with Sarver's representation that has only one positive component. Second, the case $\kappa < 0$ of ex post perfectionism violates Sarver's model as well. To illustrate, let K = 1 and

Then the regret representation (12) implies that $U(\{x, y, z\}) = 2 < 4 = U(\{x, z\}) = U(\{y, z\})$, and

$$\{x, z\} \sim \{y, z\} \succ \{x, y, z\},\$$

even though $\{x, z\}, \{y, z\} \in \mathcal{M}_z$. Thus Sarver's model violates PSB.

On the other hand, if $\kappa = \frac{1}{5}$, then representation (3) implies that $\{x\} \succeq \{z\}$, but $U(\{x, y, z\}) = 0 > -1 = U(\{x, y\})$. This comparison violates Sarver's Dominance axiom, which requires that if $\{x\} \succeq \{z\}$ and $x \in A$, then $A \succeq A \cup \{z\}$.

A APPENDIX: PROOFS

The proofs of Theorems 2.1–2.4 require some preliminaries.

Given any $u \in \mathcal{U}$ and $A \in \mathcal{M}$, write

$$u(A) = \max_{x \in A} u(x).$$

Then representation (3) can be written as

$$U(A) = (u+v)(A) - v(A) + \kappa u(A).$$
(13)

Given any function $u \in \mathcal{U}$, let

$$\mathcal{T}(u) = \{ \alpha u + \beta : \alpha \ge 0, \beta \in \mathbb{R} \}$$

be the set of all non-negative transformations of u. By Herstein–Milnor's [18] Theorem, $v \in \mathcal{T}(u)$ if and only if u and v represent the same ranking on X, or v is constant. It follows that $v \notin \mathcal{T}(u)$ if and only if v(y) > v(x) and $u(x) \ge u(y)$ for some $x, y \in X$. The following result extends this observation to any number of functions.

Lemma A.1. Let S be a finite index set, and let $u_i \in \mathcal{U}$ for all $i \in S$. Then there exist $x_i \in X$ such that for all $i, j \in S$, $u_i(x_i) \ge u_i(x_j)$, and

$$u_i \notin \mathcal{T}(u_j) \quad \Leftrightarrow \quad u_i(x_i) > u_i(x_j).$$
 (14)

Proof. Take any $k, l \in S$. If $u_k \notin \mathcal{T}(u_l)$, then take $x_{kl}, y_{kl} \in X$ such that $u_k(x_{kl}) > u_k(y_{kl})$ and $u_l(y_{kl}) \ge u_l(x_{kl})$. If $u_k \in \mathcal{T}(u_l)$, then take $x_{kl}, y_{kl} \in X$ such that $u_k(x_{kl}) \ge u_k(y_{kl})$.

For all $i \in S$, let $x_{kl}^i = x_{kl}$ if $u_i(x_{kl}) > u_i(y_{kl})$ and $x_{kl}^i = y_{kl}$ otherwise. Let

$$x_i = \sum_{k,l \in S} \frac{1}{|S|^2} x_{kl}^i$$

Take any $i, j \in S$. Then $u_i(x_{kl}^i) \ge u_i(x_{kl}^j)$ for all $k, l \in S$, and hence,

$$u_i(x_i) = \sum_{k,l \in S} \frac{1}{|S|^2} u_i(x_{kl}^i) \ge \sum_{k,l \in S} \frac{1}{|S|^2} u_i(x_{kl}^j) = u_i(x_j).$$

Moreover, if $u_i \notin \mathcal{T}(u_j)$, then

$$u_i(x_{ij}^i) = u_i(x_{ij}) > u_i(y_{ij}) = u_i(x_{ij}^j),$$

and hence, $u_i(x_i) > u_i(x_j)$. Conversely, the inequalities $u_i(x_i) > u_i(x_j)$ and $u_j(x_j) \ge u_j(x_i)$ imply that $u_i \notin \mathcal{T}(u_j)$.

Say that u_1, \ldots, u_n are redundant if $u_i \in \mathcal{T}(u_j)$ for some $i \neq j$.

Lemma A.2. Suppose that \succeq is represented by a triple (u, v, κ) . Then the following statements are equivalent.

(i) \succeq is regular.

(ii) if $\alpha u + \beta v + \gamma = 0$ for $\alpha, \beta, \gamma \in \mathbb{R}$, then $\alpha = \beta = \gamma = 0$.

- (iii) u, v, and u + v are not redundant.
- (iv) u, -u, and v are not redundant.

Proof. (i) \Rightarrow (ii). Suppose that \succeq is regular and $\alpha u + \beta v + \gamma = 0$ for $\alpha, \beta, \gamma \in \mathbb{R}$. If u is constant, then $\{x'\} \sim \{y'\}$ for all $x', y' \in X$, which contradicts regularity. Assume that u is not constant. Then $\beta \neq 0$, and $v = -\frac{\alpha}{\beta}u - \frac{\gamma}{\beta}$. Let $u_{-} = -u$. Consider three possible cases.

(1) $\frac{\alpha}{\beta} \leq 0$. Then $U(A) = (1 + \kappa)u(A)$. Thus $\{x, y\} \succeq \{x\}$ for all $x, y \in X$, which contradicts regularity.

- (2) $1 \ge \frac{\alpha}{\beta} > 0$. Then $U(A) = (1 \frac{\alpha}{\beta} + \kappa)u(A) \frac{\alpha}{\beta}u_{-}(A)$. Thus for all $x', y' \in X$ such that $\{x'\} \succ \{y'\}, u_{-}(y') > u_{-}(x')$ and hence, $\{x'\} \succ \{x', y'\}$. This contradicts regularity.
- (3) $\frac{\alpha}{\beta} > 1$. Then $U(A) = -u_{-}(A) + \kappa u(A)$. Thus for all $x', y' \in X$ such that $\{x'\} \succ \{y'\}, u_{-}(y') > u_{-}(x')$ and hence, $\{x'\} \succ \{x', y'\}$. This contradicts regularity.
- It follows that $\alpha = \beta = 0$. Then $\gamma = 0$ as well.
- (ii) \Rightarrow (iii). If u, v, and u + v are redundant, then $\alpha u + \beta v + \gamma = 0$ for some $\alpha, \beta, \gamma \in \mathbb{R}$ such that $\alpha \neq 0$ or $\beta \neq 0$.
 - (iii) \Rightarrow (iv). Suppose that u, -u, and v are redundant. Consider three cases.
- (1) u or v is constant. Then u, v, and u + v are redundant as well.
- (2) $v = \alpha u + \beta$ for some $\alpha \ge -1$ and $\beta \in \mathbb{R}$. Then $u + v = (1 + \alpha)u + \beta$. Thus u, v, and u + v are redundant.
- (3) $v = \alpha u + \beta$ for some $\alpha < -1$ and $\beta \in \mathbb{R}$. Then $u + v = \frac{1+\alpha}{\alpha}v \frac{\beta}{\alpha}$. Thus u, v, and u + v are redundant.

By contradiction, if u, v, and u + v are not redundant, then u, -u, and v are not redundant either.

(iv) \Rightarrow (i). Suppose that the functions u, v, -v are not redundant. By (A.1), there are $x, y, x', y' \in X$ such that u(x) > u(y) and v(y) > v(x), and u(x') > u(y') and -v(y') > -v(x'). By (13) $\{x\} \succ \{x, y\}$ and $\{x'\} \sim \{x', y'\} \succ \{y'\}$, that is, \succeq is regular.

A.1 Proofs of Theorems 2.1 and 2.2

Suppose that \succeq is represented by a triple (u, v, κ) . Order, Continuity, and Independence are straightforward. Show that \succeq satisfies PSB. Take any $z \in X$ and $A, B \in \mathcal{M}_z$ such that $U(A) \ge U(B)$. As $u(A) = u(B) = u(A \cup B) = u(z)$, then

$$U(\{z\}) = (1+\kappa)u(z) = u(A) + \kappa u(A) \ge (u+v)(A) - v(A) + \kappa u(A) = U(A).$$

When restricted to the menus $A, B, A \cup B$, the preference \succeq is represented by $(u+v)(\cdot) - v(\cdot)$ and hence, satisfies $A \succeq A \cup B \succeq B$.

Turn to sufficiency. Suppose that \succeq satisfies Axioms 1–4. Show that \succeq can be represented by (13).

Take any three menus $A_1, A_2, A_3 \in \mathcal{M}$ and let $A = A_1 \cup A_2 \cup A_3$. Suppose that $A = \bigcup_{z \in A} \{x \in X : \{z\} \succ \{x\}\}$. As A is compact, then there is a finite subcover $A = \bigcup_{i=1}^n \{x \in X : \{z_i\} \succ \{x\}\}$ for some $z_1, \ldots, z_n \in A$. Yet by Order, there exists z_j such that $\{z_j\} \succeq \{z_i\}$ for all i. This contradiction shows that there is $z \in A$ such that $A \in \mathcal{M}_z$.

For concreteness, let $z \in A_1$ and $A_1 \cup A_3 \succeq A_1 \cup A_2$. These conditions can be always satisfied by renumbering A_i 's. By PSB,

$$A_1 \cup A_3 \succeq A \succeq A_1 \cup A_2.$$

Thus an arbitrary triple $A_1, A_2, A_3 \in \mathcal{M}$ has at most two *positive* and at most two *negative* menus as defined by Kopylov [21]. His Theorem 2.1 implies that \succeq can be represented by

$$U(A) = u_1(A) + u_2(A) - u_3(A) - u_4(A)$$
(15)

for some $u_1, u_2, u_3, u_4 \in \mathcal{U}$.

Fix any $x_* \in X$. Let all the functions u_i satisfy

$$u_1(x_*) = u_2(x_*) = u_3(x_*) = u_4(x_*) = 0.$$
 (16)

Take any $j \neq i$ such that $u_i \neq 0$ and $u_i \in \mathcal{T}(u_j)$. By (16), $u_i = \alpha u_j$ for some $\alpha > 0$. If $\alpha \leq 1$, replace u_i and u_j by 0 and $(1-\alpha)u_j$ respectively. If $\alpha > 1$, replace u_i and u_j by $(\alpha - 1)u_j$ and 0 respectively. Thus it is without loss in generality to assume that for all $i \neq j$,

$$u_i \in \mathcal{T}(u_j) \quad \Rightarrow \quad u_i = 0.$$
 (17)

Let $u_0 = u_1 + u_2 - u_3 - u_4$. The function $u_0 \in \mathcal{U}$ represents the preference \succeq restricted to singleton menus. Take $x_0, x_1, x_2, x_3, x_4 \in X$ that satisfy conditions of Lemma A.1. Suppose that $u_1 \notin \mathcal{T}(u_0)$ and $u_2 \notin \mathcal{T}(u_0)$. Let $B_1 = \{x_0, x_2, x_3, x_4\}$ and $B_2 = \{x_0, x_1, x_3, x_4\}$. As $u_1 \neq 0$ and $u_2 \neq 0$, then $u_1 \notin \mathcal{T}(u_j)$ for all $j \neq 1$ and $u_2 \notin \mathcal{T}(u_j)$ for all $j \neq 2$. By (14), $u_1(x_1) > u_1(B_1), u_2(x_2) > u_2(B_2)$, and $u_j(B_1) = u_j(B_2) = u_j(x_j)$ for j = 0, 3, 4. Representation (15) implies that $B_1 \cup B_2 \succ B_1$ and $B_1 \cup B_2 \succ B_2$, which contradicts PSB because B_1 and B_2 share the same perfect element x_0 . Thus $u_1 \in \mathcal{T}(u_0)$ or $u_2 \in \mathcal{T}(u_0)$. For concreteness, assume that $u_2 \in \mathcal{T}(u_0)$. By (16), $u_2 = \alpha_2 u_0$ for some $\alpha_2 \geq 0$.

Suppose next that $u_3 \notin \mathcal{T}(u_0)$ and $u_4 \notin \mathcal{T}(u_0)$. Let $B_3 = \{x_0, x_1, x_2, x_4\}$ and $B_2 = \{x_0, x_1, x_2, x_3\}$. Analogously to the previous case, representation (15) implies that $B_1 \succ B_1 \cup B_2$ and $B_2 \succ B_1 \cup B_2$, which contradicts PSB as well. Thus $u_3 \in \mathcal{T}(u_0)$ or $u_4 \in \mathcal{T}(u_0)$. For concreteness, assume that $u_4 \in \mathcal{T}(u_0)$. By (16), $u_4 = \alpha_4 u_0$ for some $\alpha_4 \ge 0$.

Thus representation (15) can be written as

$$U(A) = u_1(A) - u_3(A) + \gamma u_0(A)$$
(18)

where $\gamma = \alpha_2 - \alpha_4 \in \mathbb{R}$ and $u_1 - u_3 + \gamma u_0 = u_0$. Consider two cases.

Case 1. The functions u_0, u_1, u_3 are not redundant. Consider several subcases.

(i)
$$\gamma > 1$$
. By (14), $u_1(x_1) > u_1(x_3)$ and $u_0(x_0) > u_0(x_3)$. Then

$$U(\{x_0, x_1, x_3\}) = u_1(x_1) - u_3(x_3) + \gamma u_0(x_0) > (u_1 - u_3)(x_3) + \gamma u_0(x_0) = \gamma u_0(x_0) - (\gamma - 1)u_0(x_3) > \gamma u_0(x_0) - (\gamma - 1)u_0(x_0) = u_0(x_0) = U(\{x_0\}).$$

However, the ranking $\{x_0, x_1, x_3\} \succ \{x_0\}$ contradicts PSB because x_0 is the perfect element in the menu $\{x_0, x_1, x_3\}$.

- (ii) $\gamma = 1$. Then $u_1 u_3 = 0$. By (17), $u_1 = u_3 = 0$. Then U is constant, which is impossible.
- (iii) $\gamma < 1$. Then U has the form (13) for $u = u_1 u_3$, $v = u_3$, and $\kappa = \frac{\gamma}{1 \gamma} > -1$.

Case 2. The functions u_0 , u_1 , u_3 are redundant. Then (17) and $u_1 - u_3 + \gamma u_0 = u_0$ imply that $u_1 = \alpha_1 u_0$ and $u_3 = \alpha_3 u_0$ for some $\alpha_1, \alpha_3 \in \mathbb{R}$ (not necessarily positive). Then representation (18) can be written as

$$U(A) = \beta u_0(A) + (\beta - 1)u_-(A)$$

where $u_{-} = -u_0$ and $\beta \in \mathbb{R}$. Consider several subcases.

- (i) $\beta > 1$. Take any $x, y \in X$ such that $u_0(x) > u_0(y)$. Then $\{x, y\} \succ \{x\} \succ \{y\}$, which contradicts PSB. Thus u_0 is constant. By (16), $u_0 = 0$, $u_1 = u_3$ and hence, U = 0 has the form (13) for u = v = 0. Here \succeq is not regular.
- (ii) $\beta = 1$. Then U has the form (13) for $u = u_0$, v = 0, and $\kappa = 0$. Here the ranking $\{x\} \succ \{x, y\}$ is impossible, and the preference \succeq is not regular.
- (iii) $\beta < 1$. Then U has the form (13) for $u = (1 \beta)u_0$, $v = (\beta 1)u_0$, and $\kappa = \frac{\beta}{1-\beta} > -1$. Here the ranking $\{x\} \sim \{x, y\} \succ \{y\}$ is impossible, and the preference \succeq is not regular.

To show the uniqueness statement (i) in Theorem 2.2, suppose that the preference \succeq is regular, and is represented by a triple (u', v', κ') . As \succeq is regular, then by Lemma A.2, u' + v', u', v' are not redundant. Also the functions u_1, u_3 , and u_0 in representation (18) are not redundant because Case 2 cannot hold. Kopylov [21, Theorem 2.1] shows that the triples $(u' + v', v', \kappa'u')$ and $(u_1, u_3, \gamma u_0)$ are equal up to a positive linear transformation with a common factor $\alpha > 0$. It follows that $\kappa' = \frac{\gamma}{1-\gamma} = \kappa$, and the couple (u', v') equals $(u, v) = (u_1 - u_3, u_3)$ up to a positive linear transformation.

To show the other statements of Theorem 2.2, consider several cases.

(1) \succeq satisfies Set-Betweenness. Then the uniqueness of κ together with GP's Theorem 1 imply that $\kappa = 0$.

- (2) There are $A, B \in \mathcal{M}$ such that $A \cup B \succ A$ and $A \cup B \succ B$. Let w = u + v. Without loss in generality, assume that $w(A) \ge w(B)$. By (13), if $\kappa \le 0$, then $A \succeq A \cup B$. Thus $\kappa > 0$.
- (3) There are $A, B \in \mathcal{M}$ such that $A \succ A \cup B$ and $B \succ A \cup B$. Without loss in generality, assume that $v(A) \ge v(B)$. By (13), if $\kappa \ge 0$, then $A \cup B \succeq A$. Thus $\kappa < 0$.

As κ is unique, then these three cases are mutually exclusive. The statements (ii)-(vi) of Theorem 2.2 follow.

Proof of Theorem 2.3

Let \succeq and \succeq^* be regular preferences that are represented by triples (u, v, κ) and (u, v, κ^*) such that $\kappa \geq \kappa^*$. Take any two menus $A, B \in \mathcal{M}$. By (13),

$$[U(A \cup B) - U(A)] - [U^*(A \cup B) - U^*(A)] = (\kappa - \kappa^*)(u(A \cup B) - u(A)) \ge 0.$$

Therefore, $U^*(A \cup B) - U^*(A) > 0$ implies $U(A \cup B) - U(A) > 0$, and $U(A \cup B) - U(A) < 0$ implies $U^*(A \cup B) - U^*(A) < 0$. By definition \succeq desires more flexibility and less commitment than \succeq^* .

Conversely, suppose that \succeq and \succeq^* are regular preferences that share the same commitment rankings and satisfy Axioms 1–4. By Theorem 2.1, these preferences can be represented by some triples (u, v, κ) and (u^*, v^*, κ^*) . Without loss in generality, $u = u^*$ because u and u^* represent the same ranking on X.

Let $u_0 = u$, $u_1 = u + v$, $u_2 = v$, $u_3 = u + v^*$, and $u_4 = v^*$. By Lemma A.2, u_0, u_1, u_2 are not redundant, and u_0, u_3, u_4 are not redundant either.

Take $x_0, x_1, x_2, x_3, x_4 \in X$ that satisfy Lemma A.1. Take $\varepsilon > 0$ such that for all i, j,

$$u_i(x_i) > u_i(x_j) \quad \Rightarrow \quad u_i(x_i) > u_i(x_j) + \varepsilon.$$

As all the functions u_i are continuous and X is compact, then there exists $\delta > 0$ such that $|u_i(\delta x + (1 - \delta)y) - u_i(y)| < \varepsilon/2$ for all i and $x, y \in X$. Let $y_1 = \delta x_0 + (1 - \delta)x_1$ and $y_4 = \delta x_0 + (1 - \delta)x_4$.

Let $A = \{x_0, y_1, y_4\}$ and $B = \{x_2, x_3\}$.

(i) v and v^* are not redundant. Then $u_2(x_2) > u_2(x_i) + \varepsilon$ for i = 0, 1, 4,

$$u_1(A) \ge u_1(y_1) > u_1(x_1) - \varepsilon/2 \ge u_1(A \cup B) - \varepsilon/2$$

$$u_2(A) \le u_2(x_2) - \varepsilon = u_2(A \cup B) - \varepsilon,$$

and $u_0(A \cup B) = u_0(A) = u_0(x_0)$. Thus

$$U(A \cup B) = u_1(A \cup B) - u_2(A \cup B) + \kappa u_0(A \cup B) \le u_1(A) + \varepsilon/2 - u_2(A) - \varepsilon + \kappa u_0(A) < U(A).$$

On the other hand, $u_4(x_4) > u_4(x_i) + \varepsilon$ for i = 0, 2, 3 implies that $u_4(y_4) > u_4(B)$. Thus $u_4(A \cup B) = u_4(x_4) = u_4(A)$,

$$u_3(A) = \delta u_3(x_0) + (1 - \delta)u_3(\{x_0, x_1, x_4\}) < u_3(x_3) = u_3(A \cup B)$$

and hence,

$$U^*(A \cup B) = u_3(A \cup B) - u_4(A \cup B) + \kappa u_0(A \cup B) > u_3(A) - u_4(A) + \kappa u_0(A) = U(A).$$

Thus $A \cup B \succ^* A$ and $A \succ A \cup B$.

(ii) u + v and $u + v^*$ are not redundant. Then $u_1(x_1) > u_1(x_i) + \varepsilon$ for i = 0, 2, 3 implies that $u_1(y_1) > u_1(B)$. Thus $u_1(A \cup B) = u_1(x_1) = u_1(A)$

$$u_2(A \cup B) = u_2(x_2) > \delta u_2(x_0) + (1 - \delta)u_2(\{x_0, x_1, x_4\}) = u_2(A)$$

and hence,

$$U(A \cup B) = u_1(A \cup B) - u_2(A \cup B) + \kappa u_0(A \cup B) < u_1(A) - u_2(A) + \kappa u_0(A) = U(A).$$

On the other hand, $u_3(x_3) > u_3(x_i) + \varepsilon$ for i = 0, 1, 4 implies that $u_3(A \cup B) = u_3(x_3) > u_3(A) + \varepsilon$. Moreover,

$$u_4(A) \ge u_4(y_4) \ge u_4(x_4) - \varepsilon/2 \ge u_4(A \cup B) - \varepsilon/2$$

and hence,

$$U^*(A \cup B) = u_3(A \cup B) - u_4(A \cup B) + \kappa u_0(A \cup B) > u_3(A) + \varepsilon - u_4(A) - \varepsilon/2 + \kappa u_0(A) = U(A) + \varepsilon/2.$$

Thus $A \cup B \succ^* A$ and $A \succ A \cup B$.

(iii) v and v^* are redundant, and u + v and $u + v^*$ are redundant. Then there are $\alpha > 0, \beta > 0$, and $\gamma, \gamma' \in \mathbb{R}$ such that $u + v^* = \alpha(u + v) + \gamma$ and $v^* = \beta v + \gamma'$. Then $(1 - \alpha)u + (\beta - \alpha)v + (\gamma - \gamma') = 0$. By Lemma A.2, $\alpha = \beta = 1, \gamma = \gamma'$ and hence, $v = v^*$.

Assume that \succeq desires more flexibility than \succeq^* , or alternatively, that \succeq^* desires more commitment than \succeq . Then the rankings $A \cup B \succ^* A$ and $A \succ A \cup B$ are inconsistent. Thus case (iii) must hold, and hence, $v = v^*$.

I claim that $\kappa \geq \kappa^*$. Let $Y_0 = \delta X + (1 - \delta) \{x_0\}$, $Y_1 = \delta X + (1 - \delta) \{x_1\}$ and $Y_2 = \delta X + (1 - \delta) \{x_2\}$. Consider three cases.

(i) $0 \ge \kappa^* > \kappa$. Since neither u nor $u_1 = u + v$ is constant, then there exist $y_0, z_0 \in Y_0$ and $y_1, z_1 \in Y_1$ such that $u(z_0) > u(y_0), u_1(z_1) > u_1(y_1)$, and

$$0 \ge \kappa^*(u(z_0) - u(y_0)) > u_1(y_1) - u_1(z_1) > \kappa(u(z_0) - u(y_0)).$$

Let $A = \{y_0, y_1, x_2\}$ and $B = \{z_0, z_1\}$. The definition of δ implies that $u_0(A \cup B) = u_0(z_0) > u_0(y_0) = u_0(A), u_1(A \cup B) = u_1(z_1) > u_1(y_1) = u_1(A),$ and $u_2(A \cup B) = u_2(A) = u_2(x_2)$. Thus

$$U(A \cup B) = u_1(z_1) - u_2(x_2) + \kappa u_0(z_0) < u_1(y_1) - u_2(x_2) + \kappa u_0(y_0) = U(A)$$
$$U^*(A \cup B) = u_1(z_1) - u_2(x_2) + \kappa^* u_0(z_0) > u_1(y_1) - u_2(x_2) + \kappa^* u_0(y_0) = U^*(A)$$

and hence, $A \cup B \succ^* A$ and $A \succ A \cup B$.

(ii) $\kappa^* > 0$ and $\kappa^* > \kappa$. Since neither u nor $u_2 = v$ is constant, then there exist $y_0, z_0 \in Y_0$ and $y_2, z_2 \in Y_2$ such that $u(z_0) > u(y_0), u_2(z_2) > u_2(y_2)$, and

$$\kappa^*(u(z_0) - u(y_0)) > u_2(z_2) - u_2(y_2) > \kappa(u(z_0) - u(y_0)).$$

Let $A = \{y_0, x_1, y_2\}$ and $B = \{z_0, z_2\}$. Then the definition of δ implies that $u_0(A \cup B) = u_0(z_0) > u_0(y_0) = u_0(A), u_2(A \cup B) = u_2(z_2) > u_1(y_2) = u_2(A),$ and $u_1(A \cup B) = u_1(A) = u_1(x_1)$. Thus

$$U(A \cup B) = u_1(x_1) - u_2(z_2) + \kappa u_0(z_0) < u_1(x_1) - u_2(y_2) + \kappa u_0(y_0) = U(A)$$
$$U^*(A \cup B) = u_1(x_1) - u_2(z_2) + \kappa^* u_0(z_0) > u_1(x_1) - u_2(y_2) + \kappa^* u_0(y_0) = U^*(A)$$

and hence, $A \cup B \succ^* A$ and $A \succ A \cup B$.

(iii) $\kappa \geq \kappa^*$.

The rankings $A \cup B \succ^* A$ and $A \succ A \cup B$ are inconsistent with the assumption that \succeq desires more flexibility than \succeq^* , or that \succeq^* desires more commitment than \succeq . Thus $\kappa \ge \kappa^*$ must hold.

Proof of Theorem 2.4

Let \succeq be a regular preference represented by a triple (u, v, κ) . Let $u_0 = u$, $u_1 = u + v$, and $u_2 = v$. By Lemma A.2, u_0, u_1, u_2 are not redundant. Take $x_0, x_1, x_2 \in X$ that satisfy condition (14) in Lemma A.1.

Take any menu A and any $x_* \in \arg \max_{y \in A} u_1(y)$. Then $u_1(x_*) \ge u_1(y)$ for all $y \in A$. Fix any $\alpha \in (0, 1)$. Let $z = \alpha x_1 + (1 - \alpha) x_*$ and

$$B = \{z\} \cup (\alpha\{x_0, x_2\} + (1 - \alpha)A).$$

Then $u_1(z) > u_1(y)$ for all $y \in B$ such that $y \neq z$. Yet $u_0(z) < u_0(\alpha x_0 + (1 - \alpha)x_*)$ and $u_2(z) < u_2(\alpha x_2 + (1 - \alpha)x_*)$. Then $B \notin \mathcal{M}_z$ and by (13), $B \succ B \setminus \{z\}$. By Consistency, $C(B) = \{z\}$. As α can be arbitrarily small, then by Closed Graph, $x_* \in C(A)$.

Take any $y \in A$ such that $u_1(y) < u_1(x_*)$. Let $Y = \{x \in X : u_1(y) < u_1(x) < u_1(x_*)\}$. Then Y is a mixture space. When restricted to Y, the linear functions u_0, u_1, u_2 are not redundant. Therefore, x_0, x_1, x_2 can be found in Y. Let $B = \{x_0, x_1, x_2, y\}$. By (13), $B \succ B \setminus \{x_1\}$. Yet x_1 is not the perfect element in B. By Consistency, $C(B) = \{x_1\}$.

Let $A' = A \cup \{x_1\}$. Note that $x_* \in \arg \max_{z \in A'} u_1(z)$. Therefore $x_* \in C(A')$. Suppose that $y \in C(A)$. By WARP, $y \in C(A')$ because $x_* \in C(A') \cap A$, and $y \in C(B)$ because $x_1 \in C(B) \cap A'$. Yet $C(B) = \{x_1\}$. This contradiction implies that $y \notin C(A)$. Thus $x \in C(A)$ if and only if $u_1(x) \ge u_1(y)$ for all $y \in A$.

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