Moral Hazard and Sorting in a Market for Partnerships

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Abstract

This paper incorporates two-sided moral hazard in an otherwise frictionless matching market for partnerships and examines how unobservability of the effort choices of the matched partners impacts the equilibrium sorting patterns. We find that the direction of this impact depends on whether unobservable effort and observable type are complements or substitutes: when they are *complements* (i.e.marginal products of effort are increasing in types), moral hazard favors *negative* sorting, and, conversely, when effort and type are *substitutes*, moral hazard favors *positive* sorting.

Keywords: moral hazard, partnerships, assortative matching

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1 Introduction

Economic situations in which different agents form bilateral partnerships in order to engage in a productive activity are widespread (examples include business partnerships, co-authorship, mergers, etc). When the potential partners are heterogenous in their abilities, the question arises as to what kind of matches are formed in which economic environments. Understanding the impact of various features of the economic environment on how agents are sorted into partnerships is important for empirical as well as policy analysis. It is well known that, in the absence of any frictions, sorting outcomes are determined by the properties of the production technology: when it is strictly supermodular (strictly submodular) in the partners' types, positive (negative) assortative matching obtains in all equilibria.[1] In this paper, we study how this link between the production technology and sorting patterns is impacted in the

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[1] This result is due to Becker (1973).

presence of two-sided moral hazard in the form of unobservable effort, and show that moral hazard may reinforce or weaken (and sometimes even reverse) the effect of existing technological complementarities on sorting patterns, depending on whether the partners' unobservable efforts and observable types are substitutes or complements.

Why would moral hazard have an effect on equilibrium sorting patterns? Intuitively, the presence of moral hazard may lead to an efficiency loss, the expected size of which may exhibit sub- or super-modularity if it is not additively separable in the types of the partners. In this paper we illustrate how this type of non-additivity arises. Consider a partnership which produces a stochastic output—a success or a failure, and the probability of success depends not only on the partners' types but also on their decision to work or shirk. When individual efforts are not observable, incentive provision for effort by both partners requires that they agree to implement punishments in the form of some inefficient action when the low output is realized.[2] Thus, the expected size of efficiency loss due to moral hazard is the *product* of the frequency with which low output is realized, and the size of the punishment implemented in that case. Both of these components – the frequency and the size of necessary punishment – depend on the partners' types. Consequently, the partners' types interact non-additively through the size of this loss.

How does the interaction of types through moral hazard cost affect sorting patterns? To answer this question, we focus on an environment in which there are no complementarities in the underlying technology, which implies that any sorting predictions arise solely due to the presence of moral hazard. Equilibrium matchings maximize the total surplus across matches and hence minimize the total expected moral hazard cost. This is achieved by matching the types that lead to high frequency of punishments with those that require smaller punishments. Therefore, whether the moral hazard favors positive or negative assortative matching is determined by how the types of the partners affect the frequency and the size of the inefficient punishments. Without loss of generality, suppose that higher types are more productive, and thus the frequency of inefficient punishments is declining in type. The size of the punishments, on the other hand, depends on how much the partners' individual deviations from working to shirking impact the total expected output, i.e. on the marginal products of the partners' efforts. The higher the marginal product of effort, the less attractive the deviations, and hence the smaller is the required punishment. Thus, if the marginal product of effort is increasing in type (i.e. effort and type are complements), then the size of the punishment is decreasing in type, and, to ensure that large punishments occur infrequently, lower types get matched with higher types; that is, moral hazard favors negative sorting. Conversely, if the marginal product of effort is decreasing in type (i.e. effort and type are substitutes), the size of the punishment is increasing in type, and therefore moral hazard favors positive sorting. One

[2] In a static model, such inefficient actions take the form of discarding recourses or "money burning"; while in the dynamic setting it could represent the possibility of the partnership's break-up or inefficient effort choices in the future.

natural situation in which effort and type are substitutes is where 'type' represents human capital (e.g. education or experience), and effort is the (unobservable) amount of task-specific information acquisition. In contrast, a classic example of effort and type being complements is when 'type' represents physical capital and effort stands for labor hours used to exploit this capital. Under this interpretation, our results suggest that moral hazard favors positive assortative matching in human-capital-intensive industries, but negative assortative matching in physical-capital-intensive industries.

In addition, we illustrate that the effect of moral hazard on sorting predictions not only varies across different production technologies, but also depends on the type of organizational structure. Following recent work by Franco et al. (2008), we consider the impact of moral hazard on the formation of teams within firms, in which workers have limited liability and the firm owner organizes workers in teams to maximize profit.[3] We find that, in many cases, the two environments—decentralized market vs. intra-firm formation of teams—lead to opposing sorting predictions. The driving force behind this result is that the optimal compensation scheme chosen by the firm owner requires making additional payouts to the workers after the realization of a *success* (which is in sharp contrast to the partnership problem, where the additional punishments are incurred in case of *failure*).

Our observation that moral hazard favors different sorting patterns in different technological environments and organizational structures may have interesting implications for appearance of partnerships as optimal organizational structure in certain industries. Suppose that the underlying technology exhibits some complementarity, which is perhaps a natural feature in team production models. Under this assumption, if effort and type are complements, there exists a conflict between the sorting patterns arising due to underlying complementarity in technology (which maximize the total output) and the sorting patterns which minimize the cost of moral hazard. In contrast, if effort and type are substitutes, such a conflict is not present, because both forces favor positive assortative matching. This suggests that partnerships may be more likely to arise in the second class of environments, e.g. in the human-capital intensive industries (such as law, accounting, consulting, etc.) as opposed to physical-capital-intensive industries (such as manufacturing), which is broadly consistent with empirical evidence[4].

The impact of moral hazard on sorting patterns was previously analyzed in economic environments different from the one we study here: Thiele and Wambach (1999) and Newman (2007) focus on one-sided moral hazard where entrepreneurs who are heterogeneous in wealth and whose effort is unobservable, are being sorted across projects with different levels of risk. Citanna and Chakraborty (2005) develop a model with two-sided moral hazard, but asymmetric roles of the partners: one of the tasks in the partnership is more effort intensive than the other. They assume that agents are heterogeneous in wealth, and have limited liability. The latter implies that, to facilitate incentive provision, richer agents have to be allocated to more

[3] We consider a modification of the model in Franco et al. (2008), for the purpose of drawing this contrast.
[4] See, for example, Levin and Tadelis (2005).

effort intensive tasks, which results in the matching of richer agents with the poorer ones. In contrast, in our paper, the partners' roles are symmetric, and the main mechanism through which moral hazard affects sorting predictions is very different from the one presented in Citanna and Chakraborty (2005).[5] Recent literature has also demonstrated that frictions other than moral hazard may impact the link between the technology and the sorting patterns: examples include search frictions (e.g. Shimer and Smith (2000)), incomplete information about the agents' types (e.g. Andersen and Smith (forthcoming), Kaya (2008)), borrowing constraints (e.g. Fernandez and Gali (1999)) or limited transferability of utility between the partners in the match (e.g. Legros and Newman (2007)).

The rest of this paper is organized as follows. Section 2 describes our modeling environment. Section 3 characterizes the optimal contract for a partnership of any two types. Section 4 derives our main results about matching predictions due to moral hazard. Section 5 illustrates that our results could be extended to a less stylized environment, where, instead of committing to a contract involving money burning, the partners play a repeated game without commitment. Section 6 illustrates that sorting implications of moral hazard may differ across different organizational structures, and Section 7 concludes.

2 Model

We consider a one-to-one matching market for partnerships with N participants. The participants are heterogeneous with types from $\mathcal{M} \subset \mathbb{R}$. We refer to typical elements of $\mathcal{M}$ as m, n. Throughout we assume that types are publicly known.

The interactions between the participants take place in two stages: in the first stage the participants of the market form partnerships (the matching stage), and in the second stage participants who are matched together in a partnership engage in a productive activity (production stage). Clearly, the interaction in the matching stage anticipates the outcome of the production stage. Below, we first describe the details of the production stage and then that of the matching stage. Our analysis in the subsequent sections also follows this order.

2.1 Production stage: Technology

If two participants i, j with types $m, n \in \mathcal{M}$, respectively, are matched together, they engage in a partnership game[6]: they simultaneously choose effort levels $e_i, e_j \in \{E, S\}$ where E stands for exerting effort and S stands for shirking. Choice of E entails an "effort cost" of $c > 0$ while shirking is costless. For notational convenience, we summarize this via the function $C : \{E, S\} \to \mathbb{R}$, so that $C(E) = c$ and $C(S) = 0$. The output resulting from the effort choices

[5] We further discuss the connection of our work with Citanna and Chakraborty (2005) in Section 4.2.

[6] This setup is a special case of a more general environment analyzed in Legros and Matsushima (1991). It is also related to the problem analyzed by Holmstrom (1982), though, as becomes clear below, it is different from Holmstrom (1982) in that effort choices are discrete and contracts are not restricted to be budget balanced.

is $y \in Y = \{\underline{y}, \bar{y}\}$ with $\bar{y} > \underline{y}$. The probability distribution over Y depends both on the types of the participants and the effort levels that they choose. This distribution is parameterized as follows:

$$Prob(\bar{y}|e_1, e_2, m, n) = \begin{cases} p(n,m) & \text{if } e_1 = e_2 = E \\ q_1(m,n) & \text{if } e_1 = E, e_2 = S \\ q_2(m,n) & \text{if } e_1 = S, e_2 = E \\ 0 & \text{if } e_1 = e_2 = S \end{cases} \quad (1)$$

We make the following assumptions:

A 1 For any $m, n \in \mathcal{M}$, $(p(m,n) - q_i(m,n))(\bar{y} - \underline{y}) > c$ and $q_i(m,n)(\bar{y} - \underline{y}) > c$, $i = 1, 2$.

A 2 $p(n,m)$ is strictly increasing in m and n, $q_i(m,n)$ are increasing in m and n, $i = 1, 2$.

A 3 For all $m, n \in \mathcal{M}$, $p(n,m) = p(m,n)$ and $q_1(m,n) = q_2(n,m)$.

A 4 For all $m, n \in \mathcal{M}$, $p(m,n), q_i(m,n) \in (0,1)$.

Assumption **A1** implies that exerting effort is socially optimal for each agent in any match. Assumption **A2** says that an increase in the type of a partner raises the probability of high output for any combination of efforts: that is, higher types are more productive. Assumption **A3** requires that the roles of partner 1 and partner 2 are symmetric. In terms of productivity, only the type of a partner matters and not whether he is called partner 1 or partner 2. Assumption **A4** guarantees that deviations from effort to shirk are not perfectly detectable since both output levels have positive probability under profiles where at least one of the partners exerts effort. Finally, we assume that $p(\cdot,\cdot)$, and $q_i(\cdot,\cdot)$ are twice continuously differentiable.

2.2 Production stage: Contracts

Once an output is realized, it is shared according to a contract which the partners commit to. Effort choices are not publicly observable, and therefore are not contractible. Hence, contracts can be written contingent only on the realized output. Correspondingly, a contract consists of effort level recommendations (e_1, e_2) for the two partners and a transfer scheme $(t_1, t_2) : Y \to \mathbb{R}^2$ specifying payments to each partner conditional on realized output. In what follows, we assume that enforceable transfers are those that satisfy a resource constraint: they can sum to no more than the total output produced in each contingency. That is, we consider transfers such that for all $y \in Y$, $t_1(y) + t_2(y) \leq y$. Notice that we allow for the total transfers to be strictly less than the realized output. In other words, we do not insist on balanced-budget contracts. If a contract involves $t_1(y) + t_2(y) < y$, we say that this contract involves "money burning".

Assuming that partners 1 and 2 have types m and n, respectively, a contract (e_1, e_2, t) is incentive compatible if

(2) $$e_1 \in \mathbf{argmax}_{e \in \{E,S\}} \sum_{y \in Y} Prob(y|e, e_2, m, n) t_1(y) - C(e)$$

and

(3) $$e_2 \in \mathbf{argmax}_{e \in \{E,S\}} \sum_{y \in Y} Prob(y|e_1, e, m, n) t_2(y) - C(e)$$

This is the usual definition of incentive compatibility: under such a contract, given the opponent's effort level and the types of both, each partner maximizes the expected transfers that he receives net of his effort cost, by choosing his part of the effort profile that is recommended. If a contract (e_1, e_2, t) is incentive compatible, we say that the transfer scheme t implements the effort profile (e_1, e_2).

Finally, we say that a payoff vector (v_1, v_2) is feasible for a pair of agents with types m, n, respectively, if there exists an incentive compatible contract (e_1, e_2, t) such that:

$$v_1 = \sum_{y \in Y} Prob(y|e_1, e_2, m, n) t_1(y) - C(e_1)$$

and

$$v_2 = \sum_{y \in Y} Prob(y|e_1, e_2, m, n) t_2(y) - C(e_2)$$

Thus, for each combination of partner types (m, n), the set of incentive compatible contracts generates the set of feasible payoff pairs. The properties of this set (namely, the dependence of its Pareto frontier on the types of the partners) determine the outcome of the matching stage, which we describe next.

2.3 Matching stage: Partnership formation

The outcome of the market for partnerships is a matching defined as a one-to-one map $M : N \to N$ such that for any $i, j \in N$, $i = M(j)$ if and only if $j = M(i)$. The expression $M(i) = j$ is understood to mean that participants i and j form a partnership under matching M.

We are interested in the "stable" matchings of the partnership market in the standard sense:[7] a matching is stable if no subsets of the participants can re-match among themselves so that each member of this subset improves their payoff.

Formally, an equilibrium consists of a matching $M : N \to N$ and a payoff assignment $v^* : N \to \mathbb{R}$ such that

(1) for all i, j with $i = M(j)$, $(v^*(i), v^*(j))$ is feasible given their types;

[7] See for example Roth and Sotomayor (1990)

(2) the matching is stable: for any $i, j \in N$, there exists no payoff vector w which is feasible for i and j such that $w(i) > v^*(i)$ and $w(j) > v^*(j)$.

Our goal is to describe the properties of an equilibrium matching M. More specifically, we are interested in whether M exhibits positive assortative matching (when higher types are matched with higher types) or negative assortative matching (when higher types are matched with lower types).[8] Clearly, which (if either) of these sorting patterns emerges as an equilibrium outcome depends on what sets of feasible payoffs are obtained under partnerships of various pairs of types. Note that part (2) of the definition of the equilibrium implies that any pair matched together in an equilibrium should receive a payoff pair that is on the Pareto frontier of payoffs that are feasible for them. Thus in the following section we derive the Pareto frontier for a given partnership.

3 Characterization of the set of feasible payoffs

In this Section we characterize the set of feasible payoff pairs for a partnership, in which partner 1 and partner 2 are of types $m \in \mathcal{M}$ and $n \in \mathcal{M}$ respectively. Without loss of generality, assume that $m \geq n$. The Pareto frontier of the set of feasible payoff pairs is found by solving the following problem:

$$
\begin{aligned}
W_{mn}(v) = & \max_{e_1, e_2, t_1(\cdot), t_2(\cdot)} \sum Prob(y|e_1, e_2, m, n) t_2(y) - C(e_2) \\
& \text{s.t.} \; \sum Prob(y|e_1, e_2, m, n) t_1(y) - C(e_1) = v \\
& \quad e_1 \in \mathbf{argmax}_e \sum Prob(y|e, e_2, m, n) t_1(y) - C(e) \\
& \quad e_2 \in \mathbf{argmax}_e \sum Prob(y|e_1, e, m, n) t_2(y) - C(e) \\
& \quad t_1(y) + t_2(y) \leq y, \quad y = \underline{y}, \overline{y}
\end{aligned} \tag{4}
$$

The first constraint is the individual rationality constraint for partner 1, the second and the third are the incentive compatibility constraints and the last is the resource constraint. We solve (4) in two steps: first, for any effort profile (e_1, e_2) we find the optimal transfers implementing (e_1, e_2); then we determine which effort profile delivers the highest value to partner 2.

Before we describe the solution in detail, a few important observations can be made. Notice that, due to the possibility of making transfers, the set of payoff pairs obtained by contracts implementing a particular effort profile (e_1, e_2) has a linear frontier with the slope -1.[9] Denote the level of this frontier, which is the highest total surplus that can be achieved while

[8] Formally, M exhibits positive (respectively, negative) assortative matching if $m(i) > m(j)$ implies $m(M(i)) \geq m(M(j))$ (respectively, $m(M(i)) \leq m(M(j))$), where $m(i)$ refers to the type of participant i and therefore $m(M(i))$ is the type of participant i's partner under matching M.

[9] To see this, suppose that a pair of values (v, w) can be attained via an effort profile (e_1, e_2) and a transfer scheme $t_i(\overline{y}), t_i(\underline{y})$ $(i = 1, 2)$ that implements (e_1, e_2). Then, for any Δ, the combination of payoffs $(v+\Delta, w-\Delta)$

implementing (e_1, e_2), by $S^{e_1e_2}(m,n)$. It is clear, then, that the solution to (4) has the form

$$W_{mn}(v) = -v + S(m,n), \tag{5}$$

where $S(m,n) = \max\{S^{SS}(m,n), S^{EE}(m,n), S^{ES}(m,n), S^{SE}(m,n)\}$, and that all the payoff pairs on the Pareto frontier in (4) are achieved by contracts specifying the same effort profile. It follows from (5) that, in the terminology of the matching literature, our model has *fully transferable utility*, which substantially simplifies the characterization of the matching equilibrium in the next Section. In order to provide full characterization of the Pareto frontier $W_{mn}(v)$ and to determine which effort combination is implemented along it, we find $S^{e_1e_2}(m,n)$ for all $(e_1, e_2) \in \{E, S\}^2$.

If effort profile $(e_1, e_2) = EE$ is implemented, the optimal transfer scheme solves the following problem

$$\begin{aligned} \max_{t_1(\cdot),t_2(\cdot)} \; & p(m,n)t_2(\bar{y}) + (1-p(m,n))t_2(\underline{y}) - c \\ \text{s.t. } \; & p(m,n)t_1(\bar{y}) + (1-p(m,n))t_1(\underline{y}) - c \geq v \\ & (p(m,n) - q_2(m,n))(t_1(\bar{y}) - t_1(\underline{y})) \geq c \\ & (p(m,n) - q_1(m,n))(t_2(\bar{y}) - t_2(\underline{y})) \geq c \\ & t_1(y) + t_2(y) \leq y, \quad y = \underline{y}, \bar{y} \end{aligned} \tag{6}$$

Observe that if the partner 1's individual rationality constraint is slack, both transfers to the first agent may be decreased and the transfers to the second agent increased by equal amounts to make the individual rationality constraint binding. This adjustment will satisfy the incentive and resource constraints, and, at the same time, increase the payoff to the second agent. Therefore, the individual rationality constraint necessarily binds, and the optimal contract also maximizes the total surplus

$$S^{EE}(m,n) = p(m,n)[t_1(\overline{y}) + t_2(\overline{y})] + (1-p(m,n))[t_1(\underline{y}) + t_2(\underline{y})] - 2c,$$

subject to the incentive compatibility and resource constraints.

Depending on the parameters of the model, the solution to (6) would either be budget balanced ($t_1(y)+t_2(y) = y$ for all y) or may involve money burning ($t_1(y)+t_2(y) < y$ for some y). If an incentive compatible balanced budget contract exists, it delivers the surplus

$$S^*(m,n) = p(m,n)\overline{y} + (1-p(m,n))\underline{y} - 2c. \tag{7}$$

Since $t_1(y)+t_2(y) \leq y$ for all y, this is the highest surplus that can potentially be achieved in the partnership. Note also that the same surplus would be obtained if effort levels were observable.

can be obtained via a contract consisting of the original effort profile (e_1, e_2) and transfers $(t_1(\bar{y})+\Delta, t_1(\underline{y})+\Delta)$ for agent 1 and $(t_2(\bar{y})-\Delta, t_2(\underline{y})-\Delta)$ for agent 2. Such a contract is incentive compatible since the effort levels and the spreads between individual transfers are the same as in the original contract. Moreover, this contract delivers the values $(v+\Delta, w-\Delta)$.

In other words, if budget balancing can be incentive compatible, there is no efficiency loss due to moral hazard. Since our goal is to analyze how inefficiencies resulting from moral hazard may affect sorting predictions, in the rest of the paper we restrict attention to the parameter values for which the solution to (6) requires money burning. Define

$$\eta(m,n) = \frac{c}{p(m,n) - q_1(m,n)} + \frac{c}{p(m,n) - q_2(m,n)} - \left(\bar{y} - \underline{y}\right). \tag{8}$$

The following assumption guarantees that the optimal contract involves money burning:

A 5 For any $m, n \in \mathcal{M}$, $\eta(m,n) > 0$.

Under assumption **A5**, any balanced budget transfer scheme would violate at least one of the incentive constraints,[10] and thus the optimal contract must involve money burning.[11] Correspondingly, when **A5** holds both incentive constraints bind. Also, to maximize the spread between the transfers in case of low and high output, money is burned only when low output is realized. This implies that $t_1(\bar{y}) + t_2(\bar{y}) = \bar{y}$, and therefore:

$$t_1(\underline{y}) + t_2(\underline{y}) = \underbrace{t_1(\bar{y}) + t_2(\bar{y})}_{\bar{y}} - \frac{c}{p(m,n) - q_2(m,n)} - \frac{c}{p(m,n) - q_1(m,n)} < \underline{y}.$$

The total surplus $S^{EE}(m,n)$ is then expressed as

$$\begin{aligned} S^{EE}(m,n) &= p(m,n)[t_1(\bar{y}) + t_2(\bar{y})] + (1 - p(m,n))[t_1(\underline{y}) + t_2(\underline{y})] - 2c \\ &= \underbrace{p(m,n)\bar{y} + (1 - p(m,n))\underline{y} - 2c}_{S^*(m,n)} - (1 - p(m,n))\eta(m,n) \end{aligned} \tag{9}$$

Notice that $S^{EE}(m,n)$ can be decomposed into two parts, the total surplus $S^*(m,n)$ that would be obtained if there were no moral hazard minus an additional term which appears due to moral hazard and is strictly positive (by assumption **A5**). This term represents the "expected moral hazard cost": inefficient punishments of size $\eta(m,n) > 0$ occur when low output is realized, i.e. with frequency $(1 - p(m,n))$.

The effort profile $(e_1, e_2) = ES$ can be implemented without money burning, and the partners obtain the total surplus of

$$\begin{aligned} S^{ES}(m,n) &= q_1(m,n)[\underbrace{t_1(\bar{y}) + t_2(\bar{y})}_{\bar{y}}] + (1 - q_1(m,n))[\underbrace{t_1(\underline{y}) + t_2(\underline{y})}_{\underline{y}}] - c \\ &= q_1(m,n)\bar{y} + (1 - q_1(m,n))\underline{y} - c. \end{aligned} \tag{10}$$

Money burning is not required in this case because the incentive constraints of both partners can be satisfied by increasing the spread between the transfers to agent 1 and reducing the

[10] This can easily be seen by adding up the incentive constraints, applying $t_1(y) + t_2(y) = y$ for all y and observing that the resulting inequality contradicts to **A5**.

[11] The necessary and sufficient condition for inefficiency derived in Legros and Matsushima (1991) boils down to our assumption **A5** when applied to our setting. We formally demonstrate this in Section 8.1 of the Appendix.

spread between the transfers to agent 2, while maintaining $t_1(y) + t_2(y) = y$ for all y.[12] Also note that $S^{ES}(m, n)$ can be rearranged as

$$S^{ES}(m,n) = S^*(m,n) - \underbrace{\left[(p(m,n) - q_1(m,n))(\bar{y} - \underline{y}) - c\right]}_{>0}, \tag{11}$$

which, similarly to (9), is the difference between the surplus in the frictionless environment $S^*(m, n)$ and the cost of incentive provision, which, by assumption **A1**, is strictly positive. Notice, however, that the source of inefficiency in (11) is different from the one in (9): while under effort profile EE the moral hazard cost appears due to money burning, under ES its driving force is the inefficient effort choice.

By symmetry, under the effort profile $(e_1, e_2) = SE$, the partners can achieve the total surplus of

$$S^{SE}(m,n) = S^*(m,n) - \left[(p(m,n) - q_2(m,n))(\bar{y} - \underline{y}) - c\right].$$

Note also that, by assumption **A2**, $S^{ES}(m, n) \geq S^{SE}(m, n)$ if $m \geq n$.

Finally, if none of the partners exerts effort the total surplus is $S^{SS}(m, n) = \underline{y}$. Assumption **A1** ensures that $S^{ES}(m, n) > S^{SS}(m, n)$. Thus the total surplus obtained in the optimal contract is given by

(12)

$$\begin{aligned} S(m,n) &= max\{S^{EE}(m,n), S^{ES}(m,n)\} \\ &= \underbrace{p(m,n)\bar{y} + (1 - p(m,n))\underline{y} - 2c}_{S^*(m,n)} - \underbrace{\min\left\{(1 - p(m,n))\eta(m,n), (p(m,n) - q_1(m,n))\bar{y} - c\right\}}_{\text{moral hazard cost}}. \end{aligned}$$

The following proposition summarizes the analysis of this section:

Proposition 1 *(*Characterization of the set of feasible payoff pairs)
*Suppose that assumptions **A1**-**A5** hold. Then for any $m, n \in \mathcal{M}$ such that $m \geq n$, the set of feasible payoffs for a partnership of types m and n is given by $\{(v, w)|v + w \leq S(m, n)\}$, where $S(m, n)$ is defined in (12). In addition, if*

$$(1 - p(m,n))\eta(m,n) \quad \leq \quad (>) \quad (p(m,n) - q_1(m,n))(\bar{y} - \underline{y}) - c, \tag{13}$$

where $\eta(m, n)$ is defined in (8), all the payoff pairs from the Pareto frontier of the set of feasible payoffs are obtained by the contracts implementing effort profile EE (ES).

Whether the optimal contracts induce effort by both partners or by only one of them, depends on the properties of the production technology. In subsequent sections, we alternately make one of the following two assumptions:

[12]In this case, the incentive constraints are $q_1(m,n)(t_1(\bar{y}) - t_1(\underline{y})) \geq c$ and $(p(m,n) - q_1(m,n))(t_2(\bar{y}) - t_2(\underline{y})) \leq c$ for agent 1 and agent 2, respectively.

A 6 For all $m, n \in \mathcal{M}$ such that $m \geq n$:

$$(1 - p(m,n))\eta(m,n) \leq (p(m,n) - q_1(m,n))(\overline{y} - \underline{y}) - c$$

A 7 For all $m, n \in \mathcal{M}$ such that $m \geq n$:

$$(1 - p(m,n))\eta(m,n) > (p(m,n) - q_1(m,n))(\overline{y} - \underline{y}) - c$$

By Proposition 1, under assumption **A6**, effort profile EE is implemented in any partnership. Conversely, under assumption **A7**, effort profile ES is implemented.[13] Note that assumption **A1** ensures that if $q_1(m,n)$ and $q_2(m,n)$ are sufficiently small for all $m, n \in \mathcal{M}$, then assumption **A6** holds.[14] Intuitively, for such parameter values, incentives to exert effort are provided cheaply, because individual deviations lead to a large loss in total surplus and are not attractive, and thus effort profile EE is implemented in all partnerships. On the other hand, if $q_1(m,n)$ and $q_2(m,n)$ are sufficiently close to $p(m,n)$ for all $m, n \in \mathcal{M}$, assumption **A7** is satisfied. In this case, it becomes too expensive to induce effort by both partners and, at the same time, shutting one of them down leads to a relatively small loss in expected output, implying that all optimal contracts implement effort profile ES.

Alternatively, the values of $q_1(m,n)$ and $q_2(m,n)$ may be associated with the precision of the monitoring technology. When $q_1(m,n)$ and $q_2(m,n)$ are sufficiently small relative to $p(m,n)$, the likelihood ratios $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$, $i = 1, 2$ are large, implying that the probability of punishment when it is warranted $(1 - q_i(\cdot,\cdot))$—which is 1 minus the probability of committing a type II error when testing the hypothesis that one of the partners has deviated—is large relative to the probability of punishment when it is not warranted $(1 - p(\cdot,\cdot))$—which is the probability of type I error when testing the hypothesis that one of the partners has deviated. In this case, the monitoring technology is of "*high precision*" and assumption **A6** is satisfied, while when the said likelihood ratio is small, the monitoring technology is of "*low precision*" and assumption **A7** is satisfied. In the next Section, we take up these two cases (high precision and low precision monitoring) separately and describe the matching predictions under both types of technology.

[13]Notice that we abstract from the cases when optimal effort profiles differ across partnerships. By focusing only on the two extreme situations, we are able to understand how particular distortions created by moral hazard—inefficient compensation or inefficient effort—may affect matching patterns. Potentially, interaction between the two sources of inefficiency may have additional effects on equilibrium matchings, but their analysis appears to be cumbersome and does not contribute to the main message of the paper, and are therefore omitted.

[14]As $q_i(m,n)$ become sufficiently close to 0, the inequality in assumption **A6** converges to $2c - p(m,n)c \leq p(m,n)(\overline{y} - \underline{y})$, which strictly holds by assumption **A1**.

4 Matching predictions

In this section we present our main results. In particular, we demonstrate that the presence of moral hazard has non-trivial impact on sorting predictions and we provide economic intuition for why this happens.

Proposition 1 establishes that, for any $m, n \in \mathcal{M}$, the sum of payoffs $S(m,n)$ along the Pareto frontier of the feasible payoff set is constant, i.e. this is a fully transferable utility environment. In this case, it is well-known (Becker (1973)) that for all equilibrium matchings to be positive (negative) assortative, it is sufficient that the function $S(\cdot,\cdot)$ is strictly supermodular (submodular). When the function $S(\cdot,\cdot)$ is twice continuously differentiable, an equivalent definition of supermodularity (due to Topkis (1978)) relates to the cross partial derivatives: $S(\cdot,\cdot)$ is strictly supermodular (submodular) if and only if $\frac{\partial^2 S(m,n)}{\partial m \partial n} > (<) 0$.[15] Thus, our analysis in this section focuses on determining the sign of $\frac{\partial^2 S(m,n)}{\partial m \partial n}$.

In order to isolate the effects of moral hazard on sorting patterns, in the rest of the paper we assume that:

A 8 For all $m, n \in \mathcal{M}$: $\frac{\partial^2 p(m,n)}{\partial m \partial n} = 0$, $\frac{\partial^2 q_1(m,n)}{\partial m \partial n} = 0$ and $\frac{\partial^2 q_2(m,n)}{\partial m \partial n} = 0$

This assumption guarantees that, in the absence of moral hazard, the total optimal surplus $S^*(m,n)$ defined in (7) would be neither submodular nor supermodular, and, correspondingly, any matching predictions that might arise in the model with imperfect monitoring are solely driven by unobservability of partners' efforts.

Next, we establish under which conditions $S(m,n)$ defined in Proposition 1 exhibits strict supermodularity or submodularity. We conduct the analysis separately for the two cases introduced in the previous section: *high precision monitoring* and *low precision monitoring.*

4.1 High precision monitoring

Throughout this subsection we maintain assumption **A6**. It follows from Proposition 1 that, in this case,

$$S(m,n) = \underbrace{p(m,n)\bar{y} + (1-p(m,n))\underline{y} - 2c}_{S^*(m,n)} \ldots$$
$$- \underbrace{(1-p(m,n))}_{\text{frequency}} \underbrace{\left[\frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)} - (\bar{y} - \underline{y}) \right]}_{\eta(m,n),\ \text{size of punishment}}$$

[15]Strictly speaking, this characterization requires that $S(\cdot,\cdot)$ be twice continuously differentiable on $(a,b)^2$ where (a,b) is an open interval in $\mathbb{R}$. In this case, the conclusion is that the function $S(\cdot,\cdot)$ is supermodular over $(a,b)^2$. Therefore, we are able to use this result, for instance, under the assumption that $\mathcal{M}$ is an open interval.

As remarked above, under assumption **A8**, $S^*(m,n)$ exhibits no complementarities. Therefore, the possibility of super/sub modularity in $S(m,n)$ lies in the potential interaction of types of the two partners via the moral hazard cost, which is represented as the product of the frequency and size of the inefficient punishment. In this subsection we take a closer look at this interaction.

First notice that by assumption **A2**, the frequency of inefficient punishment – which occurs only when the output is low – is decreasing in the types of both partners. The effect of a change in a partner's type on the size of the punishment, however, is less straightforward. To see how this effect transpires, it is sufficient to analyze the term $\frac{c}{p(m,n)-q_2(m,n)}$. Notice that the denominator is related to the marginal product of effort for partner 1 when partner 2 is exerting effort: $(p(m,n)-q_2(m,n))(\overline{y}-\underline{y})$ measures the increase in expected output when partner 1 switches from shirking to working. If this term is increasing in type we say that *type and effort are complements*. If it is decreasing we say that *type and effort are substitutes*. Next, we take up these two cases separately.

4.1.1 Effort and type are complements

In this part we make the following assumption:

A 9 For all $m,n \in \mathcal{M}$, $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_2(m,n)}{\partial n} \geq 0$ and $\frac{\partial p(m,n)}{\partial m} - \frac{\partial q_2(m,n)}{\partial m} > 0$.

Under assumption A9, effort is complementary to type: its marginal product is higher when the agent is of higher type or when matched with a higher type partner.[16] In general, any technology of the form

$$Prob\{\overline{y}|e_1,e_2,m,n\} = mf_1(e_1,e_2) + nf_2(e_1,e_2) + p_0(e_1,e_2), \tag{14}$$

where $f_1(\cdot,\cdot)$ and $f_2(\cdot,\cdot)$ are increasing in both arguments, satisfies assumption **A9**. One canonical example that fits in this framework is where

$$p(m,n) = m+n+a_0; \qquad q_1(m,n) = m+a_1 \qquad q_2(m,n) = n+a_2,$$

with $a_i \in \mathbb{R}$, suitably chosen. A more broad interpretation of the functional form in (14) is that 'type' plays the role of capital which the agent brings to the match, while effort plays the role of labor, and capital and labor are complementary, as in the Cobb-Douglas production function.

Proposition 2 *Suppose that assumptions* ***A1-A6****,* ***A8*** *and* ***A9*** *hold. Then all equilibrium matchings are negative assortative.*

[16] Notice that the marginal product of effort of type m partner is given by $p(m,n)-q_2(m,n)$. We allow this marginal product to be constant in n, the type of the other partner; that is, marginal product of effort is allowed to be independent of the partner's type. Also, notice that, by symmetry of the partners' roles (assumption **A3**), it is implied that $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_1(m,n)}{\partial n} > 0$ and $\frac{\partial p(m,n)}{\partial m} - \frac{\partial q_1(m,n)}{\partial m} \geq 0$.

Proof: We need to verify that $\frac{\partial^2 S(m,n)}{\partial m \partial n} < 0$. Proposition 1 and assumption **A8** imply that

$$\frac{\partial^2 S(m,n)}{\partial m \partial n} = -\frac{\partial^2}{\partial m \partial n}\left[(1-p(m,n))\left(\frac{c}{p(m,n)-q_2(m,n)} + \frac{c}{p(m,n)-q_1(m,n)}\right)\right].$$

Then, by symmetry, the sign of $\frac{\partial^2 S(m,n)}{\partial m \partial n}$ is the negative of the sign of $\frac{\partial^2}{\partial m \partial n}(1-p(m,n))\frac{c}{p(m,n)-q_2(m,n)}$. For brevity, we drop reference to m and n, and express

$$\frac{\partial^2}{\partial m \partial n}\left[(1-p) \times \frac{c}{p-q_2}\right] =$$

$$(15) \qquad \underbrace{\frac{\partial}{\partial n}(1-p)}_{<0} \times \underbrace{\frac{\partial}{\partial m}\frac{c}{p-q_2}}_{<0} + \underbrace{\frac{\partial}{\partial m}(1-p)}_{<0} \times \underbrace{\frac{\partial}{\partial n}\frac{c}{p-q_2}}_{\leq 0} + (1-p) \times \frac{\partial^2}{\partial m \partial n}\frac{c}{p-q_2}$$

Assumption **A9** implies that the first two additive terms in the sum above are positive. The last term is further expressed as

$$\frac{\partial^2}{\partial m \partial n}\frac{c}{p-q_2} = \frac{1}{(p-q_2)^3} \times \frac{\partial}{\partial m}(p-q_2) \times \frac{\partial}{\partial n}(p-q_2) > 0,$$

which is also positive. Thus $\frac{1-p(m,n)}{p(m,n)-q_2(m,n)}$ is strictly supermodular, $S(m,n)$ is strictly submodular, and, hence, all equilibrium matchings are negative assortative. □

The intuition behind Proposition 2 is as follows. An equilibrium matching maximizes the total surplus. Hence, it minimizes the sum of the costs of incentive provision $(1-p(n,m))\eta(m,n)$ across matched pairs. On the one hand, the frequency of inefficient punishments $1-p(n,m)$ is decreasing in the types of both partners. On the other hand, the size of the punishment $\eta(m,n)$ is inversely related to $p(m,n)-q_2(m,n)$, which is increasing in m and n by assumption **A9**. The monotonicity in m implies that higher types have less incentive to deviate (because their deviations have larger effects on the total expected output), and thus the optimal contract punishes them less in the event of low output. By the same token, the monotonicity of $p(m,n)-q_2(m,n)$ in n implies that the partners of the higher types are faced with lower punishments. To minimize the cost of incentive provision, lower types, who lead to more frequent punishments, should be matched with higher types, who induce smaller size punishment. Thus the interaction between the frequency and the size of punishment acts in favor of negative matching. In addition, the types of the partners interact through their joint effect on the size of punishments (the last term in (15)), which further pushes an equilibrium matching towards negative assortative.

Alternatively, the mechanism through which the interaction between the frequency and the size of inefficient punishments generates matching predictions can be described in terms of the willingness and ability of each type agent to transfer additional payoffs to his partner to be able to switch to a higher type one. Notice that in a stable matching, those who are willing and able to pay more for such a switch end up with higher type partners. Switching to a higher type partner is good for all types because, by assumption **A9**, it reduces the size of inefficient punishments. However, this reduction in the size of the punishment is more important for the lower types since they have to go to these punishments more often. Moreover, such a switch will decrease the frequency of punishment, which is, once again, more important for a lower type agent since the size of the total inefficient punishment is decreasing in type (assumption **A9**). This is why lower types would be willing to 'bid more' for a higher type partner, which leads to negative assortative matching in equilibrium.

To be more concrete, consider a simple technology where $p(m,n) = m + n$, $q_1(m,n) = m$ and $q_2(m,n) = n$. In this case, the total size of the inefficient punishment for a matched pair of types m, n is $\frac{c}{m} + \frac{c}{n} - (\overline{y} - \underline{y})$. Take two types $n > n'$. If any participant—independent of his type—switches from an n'-type partner to an n-type partner, the total inefficient punishment in the partnership will fall by $\frac{c}{n'} - \frac{c}{n}$ and the frequency of inefficient punishment will fall by $n - n'$. Therefore, to make such a switch, an agent of type m would be willing to leave a value of $(1 - m)\left(\frac{c}{n'} - \frac{c}{n}\right) + (n - n')\frac{c}{m} + u_{n'}$ to the n-type agent, where $u_{n'}$ is the payoff of a type n' agent. Notice that this is decreasing in m: exactly because the reduction in size as well as the reduction in frequency is more important for a lower type agent.

Remark 1 Observe that if the marginal product of the agent's effort depends on his own type but not on the type of his partner (i.e. $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_2(m,n)}{\partial n} = 0$), the last two terms in (15) disappear, but the first term remains strictly positive, implying that $S(m,n)$ is strictly submodular. In other words, the non-trivial matching predictions persist even if there is no interaction between an agent's effort and his partner's type.

4.1.2 Effort and type are substitutes

In this part, we analyze how matching predictions change if assumption **A9** is replaced with the following assumption:

A 10 For all $m, n \in \mathcal{M}$, $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_2(m,n)}{\partial n} < 0$ and $\frac{\partial p(m,n)}{\partial m} - \frac{\partial q_2(m,n)}{\partial m} \leq 0$.

If assumption **A10** holds, an agent's effort is a substitute for his and (weakly) his partner's type. The general form of technology that satisfies **A10** is

$$Prob\{\overline{y}|e_1, e_2, m, n\} = f_1(m + h_1(e_1, e_2)) + f_2(n + h_2(e_1, e_2)),$$

where $f_i(\cdot)$ and $h_i(\cdot,\cdot)$ are increasing, and $f_i(\cdot)$ is also concave. This could happen, for instance, if 'type' represents the amount of accumulated knowledge (i.e., education or experience), effort is exerted to acquire some project-specific knowledge (information) and the probability of success is a decreasing returns to scale function of total knowledge.

When assumption **A10** replaces assumption **A9**, higher types, as well as their partners, lead to bigger-size punishments. Thus, to minimize the cost of incentive provision, higher types should be matched with higher types so that bigger size punishments occur less frequently. This, in contrast to the effects described in the previous section, acts in favor of positive assortative matching. There is still, however, the interaction between the two types through the size of punishments, which pushes towards negative matching, as the last term in (15) remains positive. Therefore, under assumption **A10**, moral hazard can generally lead to either positive or negative matching in equilibrium. Proposition 3 below establishes sufficient conditions for each type of matching to occur.

Proposition 3 *Suppose that assumptions **A1-A6**, **A8** and **A10** hold. Then, if* $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$ *is increasing in* m *and* n *for* $i = 1, 2$*, all equilibrium matchings are positive assortative. On the other hand, if* $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$ *is decreasing in* m *and* n *for* $i = 1, 2$*, all equilibrium matchings are negative assortative.*

Proof: Observe that $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$ is inversely related to $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)} = \frac{1}{L_i(m,n)-1}$. Dropping for brevity the reference to m and n and using assumption **A8**, we can express the cross-partial derivative in (15) as[17]

$$\text{(16)} \qquad \frac{\partial^2}{\partial m \partial n}\frac{1-p}{p-q_2} = -\left[\frac{\partial}{\partial m}(p-q_2)\cdot\frac{\partial}{\partial n}\frac{1-p}{p-q_2} + \frac{\partial}{\partial n}(p-q_2)\cdot\frac{\partial}{\partial m}\frac{1-p}{p-q_2}\right]/(p-q_2).$$

By assumption **A10**, $\frac{\partial}{\partial m}(p-q_2)$ is negative and $\frac{\partial}{\partial n}(p-q_2)$ is non-positive. Thus, if $\frac{1-p}{p-q_2}$ is strictly decreasing in m and n (which corresponds to $L_2(m,n)$ increasing in m and n), then $\frac{\partial^2}{\partial m \partial n}\frac{1-p}{p-q_2} < 0$, which leads to positive assortative matching. The argument for the second part is analogous.[18] □

Intuitively, the results of Proposition 3 can be explained as follows: Moral hazard leads to non-trivial matching predictions because the size and the frequency of inefficient punishments vary across agents. As discussed earlier, the interaction between sizes of punishments induced by different types (which appears in the last term of (15)) pushes toward negative assortative matching, while, under assumption **A10**, the interaction between the frequency and size (the first two terms of (15)) pushes toward positive assortative matching. Thus, the bigger is the variation in the frequencies of inefficient punishments across types (for a given variation in sizes), the more likely it is that the types will be sorted positively in equilibrium. Observe that the likelihood ratio $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$ is inversely related to $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)}$, where both

[17] To verify this, observe that for any $f(m,n)$ and $g(m,n)$ such that $f_{mn}(m,n) = 0$ and $g_{mn}(m,n) = 0$ for all m, n, the following holds: $\frac{\partial}{\partial m}\frac{f}{g} = \frac{f_m - g_m\frac{f}{g}}{g}$, and thus $\frac{\partial^2}{\partial m \partial n}\frac{f}{g} = -\left[g_m g \frac{\partial}{\partial n}\frac{f}{g} + g_n\left(f_m - g_m\frac{f}{g}\right)\right]/g^2$, which simplifies to $\frac{\partial^2}{\partial m \partial n}\frac{f}{g} = -\left[g_m\frac{\partial}{\partial n}\frac{f}{g} + g_n\frac{\partial}{\partial m}\frac{f}{g}\right]/g$.

[18] Identity (16) holds as long as **A8** is satisfied. Thus it can also be used to verify the result of Proposition 2: assumption **A9** implies that $L_i(m,n)$ is decreasing, and thus $\frac{\partial^2}{\partial m \partial n}\frac{1-p}{p-q_i} > 0$, implying negative assortative matching. Even though such a proof may be more concise than the one presented in the previous section, it does not shed light on the economic mechanisms that lead to matching predictions. That is why we use a more involved— but at the same time more intuitive—argument while establishing the result of Proposition 2.

the numerator and the denominator are decreasing in m and n. Therefore, $L_i(m,n)$ may be increasing only if $1-p(m,n)$ declines with types sufficiently fast, that is, only if the probability of inefficient punishments varies a lot across different types. Thus increasing likelihood ratio is associated with positive assortative matching. In contrast, for $L_i(m,n)$ to be decreasing, it is necessary that $1-p(m,n)$ declines with types sufficiently slow, which leads to relatively little variation in the frequency of punishment across types. Thus decreasing likelihood ratio is associated with negative assortative matching.

Remark 2 Finally, it is important to point out that if, as in Remark 1, the marginal product of the agent's effort depends on his own type but not on the type of his partner (i.e. $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_1(m,n)}{\partial n} = 0$), there is no ambiguity in matching predictions driven by assumption **A10**. In this case, there is no interaction between the agents' types through the size of punishment, and positive matching necessarily obtains in any equilibrium.

To sum up, Propositions 2 and 3 illustrate that, under sufficiently precise monitoring—i.e. when assumption **A**6 holds—the effects of moral hazard on matching predictions depend on the properties of the underlying technology. When unobservable effort and observable type are complements, moral hazard leads to negative assortative matching, but when effort and type are substitutes, moral hazard may lead to positive assortative matching. Recall that these results are derived under the assumptions of no complementarity in underlying technology (assumption **A8**). Had such complementarity been present (i.e. if $\frac{\partial^2 p(m,n)}{\partial m \partial n} \neq 0$), moral hazard could either reinforce it or act against it, depending on the technological properties.[19] In fact, in the following example we illustrate a situation where introduction of moral hazard reverses Becker's prediction. In this example, even though the underlying technology exhibits supermodularity, the equilibrium matchings are negative assortative.

Example 1 Suppose that $p(m,n) = m + n + x(m,n)$, $q_1(m,n) = m + x(m,n)$ and $q_2(m,n) = n + x(m,n)$; where $x(m,n) = am^\alpha n^\alpha$ with $a > 0$, $\alpha \in (0,1)$ and $m,n \in (\underline{\theta}, \overline{\theta})$. Also, assume that $\bar{y} = 1$ and $\underline{y} = 0$. Here, it is possible to choose c to satisfy assumptions **A1** and **A5**, and to choose $\underline{\theta}$ and $\overline{\theta}$ to satisfy assumption **A4**.

The first best surplus for a pair of types m, n obtained in the absence of moral hazard is given by:

$$S^*(m,n) = p(m,n) - 2c = m + n + am^\alpha n^\alpha,$$

which is clearly supermodular. Therefore, in the absence of moral hazard all equilibrium matchings are positive assortative. In the presence of moral hazard the total surplus defined in (12) becomes:

[19] Namely, the expression for $\frac{\partial^2 S(m,n)}{\partial m \partial n}$ would have an additional non-zero term $\frac{\partial^2 p(m,n)}{\partial m \partial n} \times \left(\frac{c}{p(m,n)-q_1(m,n)} + \frac{c}{p(m,n)-q_2(m,n)} - (\bar{y} - \underline{y}) \right)$ in (15), which has the same sign as $\frac{\partial^2 p(m,n)}{\partial m \partial n}$.

$$S(m,n) = 1 - 2c - (1 - m - n - x(m,n))\left(\frac{c}{m} + \frac{c}{n}\right)$$

Consequently,

$$-\frac{\partial^2 S(m,n)}{\partial m \partial n} = \frac{c}{m^2} + \frac{c}{n^2} + \left(\frac{c}{n} + \frac{c}{m}\right) a(1-\alpha)\alpha m^{\alpha-1} n^{\alpha-1} > 0$$

Therefore, $S(m,n)$ is strictly submodular and all equilibrium matchings are negative assortative.

4.2 Low precision monitoring

Now we turn to the alternative assumption **A7**. As shown in Section 3, under this restriction

$$S(m,n) = q_1(m,n)\bar{y} + (1 - q_1(m,n))\underline{y} - c$$

Recall that this surplus is obtained via a contract that prescribes effort only by the higher type partner, and the only loss of efficiency is due to inefficient effort—no inefficient punishments are necessary. Moreover, in this case, in an equilibrium, exactly half of the population exerts effort in the production stage, while the other half remains idle. Given this, the total surplus is maximized under matchings which have the following property: if agents are ranked according to their types, each agent that is ranked above the median will be matched with an agent that is ranked below the median. This type of matching will guarantee that the most productive half of the population exerts effort and the least productive half remains idle. Therefore, any stable matching will exhibit this property, since stable matchings maximize the total surplus across matchings. That is, all equilibrium matchings will feature "across the median matches".[20]

This type of sorting predictions are obtained in a recent paper by Citanna and Chakraborty (2005) in a model where the roles of the partners are exogenously asymmetric. In their model, the agents are heterogenous with respect to their wealth levels and they write contracts that respect limited liability to provide incentives for effort. Since the limited liability constraint is less stringent for richer agents, it is more efficient to allocate them to the less effort-intensive roles where it is necessary to provide larger wedges between payments in case of success and in case of failure. In some sense, when assumption **A7** holds in our model, the partners' roles become endogenously asymmetric, and the matching predictions resemble those in Citanna and Chakraborty (2005).

[20] Notice that this type of matching has a flavor of "negative sorting". In particular, it would be equivalent to negative assortative matching if there were only two types of agents. When there are more types, it rules out positive assortative matching. For instance, if there are 4 agents with types $m_1 > m_2 > m_3 > m_4$, all possible matchings except the positive assortative matching (where m_1 matches m_2 and m_3 matches m_4) satisfy this condition. In particular, a matching where m_1 matches m_3 and m_2 matches m_4 (as well as the negative assortative matching where m_1 matches m_4) would be an "accross the median" matching.

5 Money burning versus dynamic incentives

The assumption that two partners that are matched together can commit to discard some of the output may appear unrealistic. Even though this is a necessary assumption to obtain our results when the matched partners are assumed to engage in a one-shot productive relationship, it is possible to dispense with it if the partners are assumed to play an infinitely "repeated partnership game" once they are matched.

In the Appendix (Section 8.2) we formally develop such a model of a repeated partnership game in which the technology of production is identical to the one in our main model described in Section 2. Once two agents are matched together, they play a repeated game in every stage of which they simultaneously choose effort levels. After the realization of output, they make simultaneous transfers to each other but they cannot discard resources. We also assume that the partners cannot commit to contracts: the transfers that they make in equilibrium need to be self-enforcing. The history of outputs and transfers are publicly observable, while the effort levels are not. Therefore, this is a repeated game with imperfect monitoring. Finally, we assume that the partners discount their future payoffs using a common discount factor δ. We take the set of feasible payoff vectors for a matched pair to be the set of "public perfect equilibrium" (PPE) payoff vectors of this game.[21]

In this context, we establish the following result, which is an analog of Propositions 2 and 3 in the main text:

Proposition 4 *(Matching predictions in a repeated partnership game under precise monitoring) Assume **A1**-**A5**, **A6** and **A8** hold.*

*(a) If, additionally, **A9** holds, then there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, all equilibrium matchings are negative assortative.*

*(b) If additionally, **A10** holds, then there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, all equilibrium matchings are positive (negative) assortative if $L_i(m,n) = \frac{1-q_i(m,n)}{1-p(m,n)}$ is increasing (decreasing) in m and n, $i = 1, 2$.*

The main difficulty with establishing the results of Proposition 4 is due to the fact that we do not have the exact characterization of the Pareto frontiers for δ strictly smaller than 1, even if it is arbitrarily close to 1. In particular, we know that these frontiers are not linear.[22]

[21] PPE are equilibria in which all strategies are conditioned solely on publicly observable occurrences – in this case, the history of observed outputs. See Appendix for the formal definition.

[22] It can be shown that for each (large enough) δ, there is a piece of the frontier which is linear with slope -1, but close to the axes the frontier becomes strictly concave. The reason, roughly, is as follows: when the value of one of the partners is very low (lower than $(1-\delta)\frac{\bar{y}}{2}$) it is not possible to give him incentives to make the necessary transfers to achieve this value using trigger strategies: he is better off not making the transfer in case of success, keeping his half of the output and getting 0 payoff from then on. Therefore, to deliver such a value, the partners need to use suboptimal effort choices. That is why, the level of the frontier $(v + W(v))$ is lower at

Therefore, for any $\delta < 1$, the environment is not one of transferable utility, and hence the technique of using the link between increasing/decreasing differences and matching patterns is not valid. Instead, we use a generalized version of this link, established by Legros and Newman (2007), to show our result. In Lemma 2 in the Appendix we show that if the Pareto frontier of the limit set (as $\delta \to 1$) of equilibrium payoff vectors has transferable utility and satisfies increasing/decreasing differences, then, for all sufficiently large δ, the Pareto frontiers of the actual sets of equilibrium payoffs satisfy *generalized* increasing/decreasing difference, which allows us to apply the results in Legros and Newman (2007) and conclude that the equilibrium matchings are positive/negative assortative.

Then, in order to characterize the properties of the limit Pareto frontier, we employ the methodology developed by Fudenberg et al. (1994). Proposition 6 in the Appendix establishes that the limit Pareto frontier coincides with the Pareto frontier of feasible payoffs in the static partnership game that we analyze in the main text (see Proposition 1). This result is intuitive: in this setup, money burning is replaced by moving to an equilibrium whose payoffs are off-the Pareto frontier and commitment to contracts is replaced by implementation via trigger strategies (since transfers are observable). Since we know that the Pareto frontier obtained in the static model has transferable utility and satisfies increasing (decreasing) differences when assumption **A9** (**A10**) holds, the argument in the above paragraph implies that the first (second) statement of Proposition 4 applies. The formal analysis supporting the above discussion is included in the Appendix (Sections 8.2.3-8.2.4).

6 Matching under an alternative organizational structure

This section draws a parallel between our paper and recent work by Franco et al. (2008), who analyze the effects of moral hazard on matching patterns for a different organizational structure: many workers hired by one large firm, as opposed to our environment with many agents participating in a market for partnerships. The main observation made in this section is that, under some restrictions on the parameter values, matching predictions that arise due to moral hazard vary across different organizational structures.

Suppose that, instead of participating in the market for partnerships, all the agents work in one firm, and the firm owner (the principal) organizes them in teams and offers a wage profile contingent on all observable variables. Assume that workers' outside opportunity is given by $\underline{u} \geq 0$ and that they have limited liability, in the sense that wages cannot be below zero in any state.

Then the profit $\Pi(m, n)$ extracted by the firm owner form a team of workers with types m

points close to the axes. As $\delta \to 1$, the "concave" part of the frontier disappears and the limit frontier becomes linear with slope -1.

and n is found by solving the following problem:

$$\begin{aligned}
\Pi(m,n) = & \max_{e_1,e_2,w_1(\cdot),w_2(\cdot)} \sum Prob(y|e_1,e_2,m,n)(y - w_1(y) - w_2(y)) \\
\text{s.t. } & e_1 \in \mathbf{argmax}_e \sum Prob(y|e,e_2,m,n)w_1(y) - C(e) \\
& e_2 \in \mathbf{argmax}_e \sum Prob(y|e_1,e,m,n)w_2(y) - C(e) \\
& \sum Prob(y|e_1,e_2,m,n)w_i(y) - C(e_i) \geq \underline{u},\ i = 1,2 \\
& w_i(y) \geq 0 \text{ for all } y,\ i = 1,2
\end{aligned} \tag{17}$$

where $w_1(y)$ and $w_2(y)$ are output-contingent wages of agent 1 and agent 2 respectively. The principal collects the output net of the total wage payment. The first two constraints are the incentive compatibility constraints for the workers, while the last two constraints are the individual rationality and limited liability constraints.[23]

As in the solution for the partnership's optimal contract, we find the optimal compensation scheme implementing any given effort profile, and then choose the effort profile that delivers the highest profit. To illustrate our point, we restrict attention to cases where the following assumption holds for all $m, n \in \mathcal{M}$:

$$p(m,n)\frac{c}{p(m,n) - q_1(m,n)} - c \geq \underline{u} \quad \text{and} \quad p(m,n)\frac{c}{p(m,n) - q_2(m,n)} - c \geq \underline{u} \tag{18}$$

As we show below, this assumption guarantees that for the contract implementing the effort profile (E, E), the individual rationality constraints do not bind (because the workers' outside value is sufficiently low).[24]

Let $\Pi^{e_1,e_2}(m,n)$ stand for the principal's maximum profit generated by a team of types (m,n), conditional on implementing effort profile (e_1, e_2). If none of the workers is asked to exert effort then $\Pi^{SS}(m,n) = \underline{y} - 2\underline{u}$. If only the first agent exerts effort, the principal can capture the full surplus net of the agents' outside options by setting $w_1(\underline{y}) = \underline{u}$, $w_1(\overline{y}) = \frac{c}{q_1(m,n)} + \underline{u}$ and $w_2(y) = \underline{u}$ for all y. Thus

$$\begin{aligned}
\Pi^{ES}(m,n) = & q_1(m,n)\left(\overline{y} - \frac{c}{q_1(m,n)}\right) + (1 - q_1(m,n))\underline{y} - 2\underline{u} \\
= & q_1(m,n)\overline{y} + (1 - q_1(m,n))\underline{y} - c - 2\underline{u} \\
> & \underline{y} - 2\underline{u} = \Pi^{SS}(m,n),
\end{aligned}$$

[23] This is a modification of the model studied in Franco et al. (2008). They analyze the model with continuous effort choice, where productivity differences arise endogenously due to differences in effort costs. In our case, discreteness of the effort allows for a clearer comparison of the two environments, while still capturing the main mechanism leading to contrasting matching predictions.

[24] If the individual rationality constraints were binding for all the workers in all matches, the profit of the firm owner would be independent of the sorting structure, implying that moral hazard would have no impact on matching predictions.

where the last inequality is a direct implication of assumption **A1**. Similarly, if only the second agent exerts effort, $\Pi^{SE}(m,n) = q_2(m,n)\overline{y} - c - 2\underline{u}$, which, by assumption **A2**, is smaller than $\Pi^{ES}(m,n)$ if $m \geq n$.

If both agents exert effort, in the optimal contract either the limited liability constraint or the participation constraint of the agents bind. It is easy to see that the optimal contract, ignoring the participation constraint, is where the principal sets $w_i(\underline{y}) = 0$ and $w_i(\overline{y}) = \frac{c}{p(m,n)-q_{-i}(m,n)}$, $i = 1, 2$, and hence both incentive compatibility constraints bind. By the assumption stated in (18), the participation constraint is also satisfied by this contract. Therefore, the maximal profit that the principal earns while implementing effort profile EE is given by:

$$(19)\quad \Pi^{EE}(m,n) = p(m,n)\left(\overline{y} - \frac{c}{p(m,n) - q_1(m,n)} - \frac{c}{p(m,n) - q_2(m,n)}\right) + (1 - p(m,n))\underline{y}$$

In the rest of this section, we focus on the set of parameters, for which $\Pi^{EE}(m,n) \geq \Pi^{ES}(m,n)$ for all $m \geq n$, because the most interesting comparisons between the two environments are drawn in this case.[25] It is easy to verify that this occurs if the following inequality holds for all $m, n \in \mathcal{M}$ such that $m \geq n$:

(20)

$$2\underline{u} \geq p(m,n)\left(\frac{c}{p(m,n) - q_1(m,n)} + \frac{c}{p(m,n) - q_2(m,n)}\right) - c - (p(m,n) - q_1(m,n))(\overline{y} - \underline{y}),$$

i.e. if the monitoring technology is sufficiently precise.[26]

The optimal matching structure maximizes the total profit of the firm owner $\sum_{\text{all matches}} \Pi(m,n)$, or minimizes the cost of total payouts (wages paid when high output is realized), which under condition (20) and assumption **A8** simplifies to

$$\sum_{\text{all matches}} p(m,n)\left(\frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)}\right).$$

Recall that equilibrium matchings in the partnership model minimize the total cost associated with inefficient punishments, which, in case of precise monitoring, is given by

$$\sum_{\text{all matches}} (1 - p(m,n))\left(\frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)} - (\overline{y} - \underline{y})\right).$$

Observe that, the payouts in the firm owner's problem, as well as the inefficient punishments in the partnership model, are determined by the wedge between payments to a given type worker, which are necessary to bind his incentive constraint. Therefore, for a given pair of

[25] If effort profile ES is chosen for all pairs in both organizational structures, the principal's profit $\Pi(m,n)$ coincides with the surplus $S(m,n)$ obtained in the partnership up to an additive constant $2\underline{u}$, and thus the matching structures in the two models have the same properties.

[26] It is easy to see that condition (20) is compatible with (18) since $(p(m,n) - q_1(m,n))(\overline{y} - \underline{y}) > c$, by assumption A1.

workers who are matched together, the necessary payouts in the firm owner's problem and the necessary punishment in the partnership model are the same up to an additive constant. Now, while the payouts in the firm owner's problem are made when *high* output is realized (i.e. with probability $p(m,n)$), the inefficient punishments in the partnership model happen when *low* output occurs (i.e. with probability $1-p(m,n)$). This distinction has important implication for matching predictions. Namely, in order to maximize profit, the principal would like to team the workers who generate larger size payouts with the workers who lead to smaller probability of these payouts: that is *lower* type workers. Similarly, in equilibria of the partnership model the workers who generate larger size punishments are matched with workers that lead to smaller probability of these punishments, who, in sharp contrast to the firm's problem, are the *higher* types. This implies that opposite matching patterns may arise in the two models. This, nevertheless, will not always be so because, in addition to the interaction between the frequency and the size of payouts, there is also the effect of both workers' types on the size of the payout (described by $\frac{\partial^2}{\partial m \partial n}\frac{c}{p(m,n)-q_i(m,n)}$, $i=1,2$), which, as in the model of partnerships, always favors negative sorting. However, if this effect is relatively weak (or disappears, as in the case when the marginal product of effort depends only on the agent's own type – see Remarks 1 and 2), the two organizational structures generate the opposite matching predictions. The following proposition formally summarizes how matching patterns chosen by the firm owner vary with the properties of technology.

Proposition 5 *(Matching patterns within a firm)*
*Suppose that **A1** - **A6** and **A8** hold and that conditions (18) and (20) is satisfied for all $m \geq n$.*

*(a) If, additionally, assumption **A9** holds, i.e. effort and type are complements, then the optimal matching of workers' types within the firm exhibits positive (negative) assortative matching if a worker's marginal product of effort is independent of the type of his partner (i.e. $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_2(m,n)}{\partial n} = 0$) or if $\frac{p(m,n)}{p(m,n)-q_i(m,n)}$ is increasing (decreasing) for all $m,n \in \mathcal{M}$.*[27]

*(b) If, additionally, assumption **A10** holds, i.e. effort and type are substitutes, then the optimal matching of workers' types within the firm exhibits negative assortative matching.*

The sharp contrast between the matching predictions arising within the firms and in the market for partnerships has a potentially interesting application regarding the appearance

[27] In contrast to Proposition 3, we cannot formulate (a) of Proposition 5 in terms of the likelihood ratio, since monotonicity of $L_i(m,n)$ is not equivalent to the monotonicity of $\frac{p(m,n)}{p(m,n)-q_i(m,n)}$ (and it is the latter that stipulates the results). Nevertheless, the intuition for the results in (a) relies on the same mechanisms as outlined after Proposition 3: under assumption **A9**, increasing $\frac{p(m,n)}{p(m,n)-q_i(m,n)}$ implies that the frequency of payouts varies a lot across agents of different types, and thus the interaction between the frequency and the size of payouts becomes the driving force of sorting of types in teams; in contrast, when $\frac{p(m,n)}{p(m,n)-q_i(m,n)}$ is decreasing, there is not much variation in frequency, and thus the matching patterns are driven by the interaction between the sizes of payouts.

of different organizational structures in different technological environments. Consider, for example, the case when effort and type are complements (i.e. assumption **A9** holds). Suppose also that the underlying technology has some degree of complementarity (i.e. $\frac{\partial^2 p(m,n)}{\partial m \partial n} > 0$), which is perhaps a natural feature in the team production models. Under assumption **A9**, firms benefit from complementarity in $p(m, n)$ more than partnerships do. The reason is that, from the perspective of the firm owner, positive matching simultaneously maximizes the gains from complementarity in the underlying technology and minimizes the losses due to unobservable effort. The partnership market, however, faces a trade-off between these two channels: on the one hand, the gains from embodied technological complementarities are maximized via positive matchings, but, on the other hand, the interaction between the frequency and the size of inefficient punishment due to moral hazard is fully exploited via negative matchings. Thus, when assumption **A9** holds (e.g. when 'type' can be viewed as physical capital), complementarity in the underlying technology gives a comparative advantage to the firms and might make them more likely to arise. In contrast, if assumption **A10** holds (e.g. when 'type' represents human capital), the firms are at relative disadvantage, which might make individual partnerships more likely to appear. This observation could potentially explain why partnerships have been prominent in human-capital-intensive professional services (such as law, accounting, consulting, etc.), while the corporate form or production dominates across physical-capital-intensive industries, such as manufacturing.[28] Of course, the above intuition is only suggestive. To formally support it, one needs to develop a rigorous model of organizational structure, in which large firms and small partnerships could coexist. This analysis, however, is outside the scope of our paper and remains an open question for future research.

7 Discussion

In this section we discuss how our results might change under some alternative modeling choices:

Heterogeneity in costs: It is crucial for our results that the agents are heterogenous with respect to their productivities. If they were heterogenous only with respect to costs, then the frequency of inefficient punishments in a given match would not vary with types, and therefore, we would not get matching predictions. On the other hand, adding differing costs (declining in type) on top of differing productivities would reinforce our results when effort and type are complements and weaken them when effort and type are substitutes. This is because, the size of the inefficient punishment required to satisfy incentive compatibility is proportional to the

[28] Levin and Tadelis (2008) offer an alternative explanation for this observation, which relies on the differences in the partnerships' and firms' objectives (total profit versus profit per partner) and the choice of employees / partners quality in the environment, where prospective clients have imperfect information about the quality of the final product (and there are no informational frictions between the partners or employees).

effort cost. Therefore, in the former case, the size of the punishment will be decreasing even faster with types and in the latter it would be increasing slower (and maybe even decreasing) with types.

Balanced-budget contracts, renegotiation proofness: If we rule out the assumption of commitment to discarding of resources in our main model, it is not possible for a matched pair to write a contract that implements effort by both partners. This is due to our assumption **A5**. In this case, in every match one of the partners will work and one will remain idle. Therefore, the sorting patterns will mimic those in the case of "imprecise monitoring" analyzed in Section 4.

Similar results would be obtained in the repeated model described in Section 5, if attention is restricted to the payoffs generated by "renegotiation proof" equilibria of the repeated partnership game played among matched partners - that is, if in that repeated game, continuation values are restricted to come from the Pareto frontier. This is because in the repeated game, moving to inefficient equilibria after low output is analogous to discarding of resources after low output in the static model. Renegotiation-proofness rules out such punishments.

Non-transferability: It may be of interest to know how our results would change if the output of a partnership was not divisible and each partner would automatically receive half of the output. In this case, the static partnership game described in the main model turns into a typical prisoner's dilemma. Therefore, it has a unique equilibrium in which no partner exerts effort and hence matching patterns are immaterial for maximization of surplus. On the other hand, if the interaction between the partners is modeled as a repeated prisoner's dilemma game, there is a non-degenerate set of payoffs that can be obtained by a matched pair. In this case, even though the utility is not fully transferable, it is possible to transfer utility between agents via equilibria prescribing asymmetric effort choices. Therefore, this model would fall into the realm of models studied in Legros and Newman (2007), where utility can be transferred at a rate different than 1. With some additional simplifying restrictions on the modeling environment, it can be shown that such modification of the model favors positive assortative matching.[29] The intuition for this result is simple and is based on Legros and Newman (2007): the only way an agent transfers utility to his partner is via working harder and allowing the partner to shirk on some occasions. Therefore, it is harder for a lower type to transfer utility to his partner. Hence, lower types are unable to bid for higher type partners as efficiently as the higher type partners would. This breaks down the rationale for negative assortative matching even in the cases where total surplus would be maximized under such a matching.

[29]We make this statement based on a canonical example where the effort and type are complements and there are only two types. Therefore, in the case of divisible output, the equilibria under this technology would exhibit negative assortative matching, regardless of whether there is precise or imprecise monitoring. We are able to show that, when output is indivisible, in the case of imprecise monitoring, where only one of the partners in each match exerts effort, the equilibrium matchings are positive assortative. For the case where the monitoring technology is precise enough, we have numerical results suggesting that positive assortative matching would obtain in equilibrium.

8 Appendix

8.1 Relationship between assumption A5 and the necessary and sufficient condition for inefficiency in partnerships provided in Legros and Matsushima (1991).

Theorem 2 in Legros and Matsushima (1991) states that the solution to the partnership problem satisfy budget balancing if and only if an index of the likelihood of deviations (measured as the average gain from deviations divided by a measure of the closeness of distributions induced by deviations) is bounded from above for all possible deviations. In our model, since there are only two output outcomes, $p(m,n) \in (0,1)$ and $q_i(m,n) \in (0,1)$, $i = 1,2$ for all $m,n \in \mathcal{M}$, there exist (possibly, mixed) deviation strategies which induce identical distributions over output outcomes. Namely, there exist $\alpha_1 \in [0,1]$ and $\alpha_2 \in [0,1]$, such that

$$(1-\alpha_1)p(m,n) + \alpha_1 q_2(m,n) = (1-\alpha_2)p(m,n) + \alpha_2 q_1(m,n), \tag{21}$$

where α_1 and α_2 are the probabilities with which partner 1 and partner 2 deviate.[30] Thus, the index of the likelihood of deviations is bounded from above for all possible strategies if an only if for for all deviations (α_1, α_2) that satisfy (21), the total utility gain from these deviations is negative. Such utility gain is measured as[31]

$$\Delta = \alpha_1 \left(c - (p(m,n) - q_2(m,n)) \frac{\overline{y} - \underline{y}}{2} \right) + \alpha_2 \left(c - (p(m,n) - q_1(m,n)) \frac{\overline{y} - \underline{y}}{2} \right).$$

Using (21), it can be rearranged as

$$\Delta = \alpha_1 (p(m,n) - q_2(m,n)) \left(\frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)} - (\overline{y} - \underline{y}) \right),$$

which is negative if and only if assumption **A5** is violated.

8.2 Alternative model: the repeated partnership game

8.2.1 Description of the game

Assume that in the stage game every partner is entitled to half of the realized output $\frac{y}{2}$. After the realization of the output, the partners simultaneously make transfers $t_1(y)$ and $t_2(y)$ to each other. Here, $t_i(\cdot)$ represents the net transfer that player i receives, and hence the total payment to player i is $\frac{y}{2} + t_i(y)$. In contrast to the model in the main text, assume that the partners cannot commit to money burning. That is, the sum of the ex-post payments to each of the players is equal to the total output (no money burning), i.e. $t_1(y) + t_2(y) = y$. In addition, suppose that agents have limited liability, in the sense that transfers cannot exceed

[30] Equivalently, we can say that the *pairwise identifiability condition* (due to Fudenberg et al. (1995)) is violated in our model.

[31] Here we assume, without loss of generality, that each partner is entitled to one half of the realized output.

the total output a partner is entitled to in one period, i.e. $t_i(y) \in \left[-\frac{y}{2}, \frac{y}{2}\right]$.[32] Throughout, t refers to a profile of transfers and e refers to an effort profile (e_1, e_2). For convenience we use the notation $\bar{t}_i = t_i(\bar{y})$ and $\underline{t_i} = t_i(\underline{y})$, $i = 1, 2$.

In the repeated game, each player discounts the future with a common discount factor δ. Each period, the realized output y and the transfers t are publicly observable while the choice of effort by each player is not. The public outcome $h(\tau)$ in period τ consists of the realized output $y(\tau) \in Y$ and transfers $t(\tau) \in \mathbb{R}^4$. A public history of length τ is therefore $h^\tau = (h(1), ..., h(\tau))$. Let $\mathcal{H}^\tau$ represent the set of all public histories of length τ and $\mathcal{H} = \bigcup_{\tau=1}^{\infty} \mathcal{H}^\tau \bigcup \{h_0\}$ represent the set of all public histories. Here h_0 is the null history. A pure public strategy for player i is a map $\sigma_i : \mathcal{H} \to \{E, S\} \times \mathbb{R}^2$ that maps each public history to a stage game strategy of player i, consisting of effort choice and output-contingent transfers. We focus on the pure strategy public perfect equilibria (PSPPE) of this game.[33] A PSPPE is a public strategy profile $\sigma = (\sigma_1, \sigma_2)$ such that σ_1 is a best response to σ_2 and vice versa.

Let $W_{mn}(\delta)$ represent the set of PSPPE payoff vectors of the repeated partnership game for discount factor δ when the partners have types m and n, respectively. Also define the Pareto frontier of $W_{mn}(\delta)$ by

$$\mathcal{W}^\delta_{mn}(v) = \sup\{w | \exists v' \geq v \text{ such that } (v', w) \in W_{mn}(\delta)\}$$

Let $W_{mn} = \lim_{\delta \to 1} W_{mn}(\delta)$ be the limit set of the PSPPE payoff vectors, where the limit is with respect to the Hausdorff distance. Finally, define the Pareto frontier of $\mathcal{W}_{mn}$ by

$$\mathcal{W}_{mn}(v) = \sup\{w | \exists v' \geq v \text{ such that } (v', w) \in W_{mn}\}$$

That is, $\mathcal{W}_{mn}(v)$ is the maximum payoff that player 2 can get among equilibria where player 1's payoff is at least v as $\delta \to 1$.

8.2.2 The matching game: additional definitions

In the definition of an equilibrium given in the main text, only the definition of a feasible payoff vector is adjusted: the payoff pair (v, w) is feasible for a pair of agents i, j if $w \leq \mathcal{W}^\delta_{\kappa(i)\kappa(j)}(v)$. Given this definition of feasibility, the equilibrium definition is unchanged.

In environments where the sum of payoffs to a matched pair may vary along the Pareto frontier of achievable payoffs (non-transferable utility case), Legros and Newman (2007) introduce the following generalization of increasing (decreasing) differences property. They also establish that, in such an environment, this property is sufficient for all equilibrium matchings to be positive (negative) assortative.

[32] In fact, the assumption we need is that the enforceable transfers are bounded. We use this particular bound because it seems to be a natural one used in the literature. Boundedness guarantees the existence of a solution for some of the intermediate optimization problems we solve in what follows.

[33] Allowing for mixed strategies complicates the characterization of the equilibrium payoff set because if $q_i > p/2$ then a randomization between effort 0 and 1 eases the incentive constraints of partner $-i$, while reducing the expected output. How these effects balance out depends on the parameters.

Definition 1 *A type-dependent utility possibilities frontier* $\mathcal{W}^{\delta}_{mn} : \mathbb{R} \to \mathbb{R}$ *satisfies generalized increasing (decreasing) differences if for all* $m > m'$ *and* $n > n'$

$$\forall v, v' : \mathcal{W}^{\delta}_{m,n'}(v) = \mathcal{W}^{\delta}_{m',n'}(v') \Rightarrow \mathcal{W}^{\delta}_{m,n}(v) > \mathcal{W}^{\delta}_{m',n}(v')$$

8.2.3 Characterization of equilibrium payoffs of the partnership game

Once a matching is formed, the partners in a match play a repeated partnership game described in the previous section. As is well-known in this setting, the set of equilibrium payoff vectors is difficult to characterize. However, it is possible to bound the equilibrium payoff set using techniques introduced in Fudenberg et al. (1994). The following proposition is a direct application of Fudenberg et al. (1994)'s result to our setting and establishes that the Pareto frontier of the PSPPE payoffs of the repeated partnership game converges to the Pareto frontier of the static partnership game discussed in the main text:

Proposition 6 *Define*

$$\eta(m,n) = (\bar{y} - \underline{y}) - \frac{c}{p(m,n) - q_2(m,n)} - \frac{c}{p(m,n) - q_1(m,n)} \tag{22}$$

For any $m \geq n$*, let* $\mathcal{W}_{mn}(v) = -v + S(m,n)$ *where*

$$\begin{aligned} S(m,n) =& p(m,n)\bar{y} + (1 - p(m,n))\underline{y} - 2c - \\ & \min\left\{(1 - p(m,n))\eta(m,n), (p(m,n) - q_1(m,n))(\bar{y} - \underline{y}) + c\right\} \end{aligned} \tag{23}$$

For any $\varepsilon > 0$ *there exists* $\bar{\delta}_{mn}(\varepsilon) < 1$ *such that for any* $\delta > \bar{\delta}_{mn}(\varepsilon)$ *and for all* v*:*

$$\mathcal{W}^{\delta}_{mn}(v) \in (\mathcal{W}_{mn}(v) - \varepsilon, \mathcal{W}_{mn}(v)]$$

Proposition 6 introduces the Pareto frontier ($\mathcal{W}(\cdot)$) of a set that bounds the equilibrium payoff vectors, which turns out to coincide with the Pareto frontier of the feasible payoffs in the static model from the main text, and states that the true Pareto frontier of the equilibrium payoff vectors converges to this bound as the discount factor δ approaches 1.

Proof: We start by deriving the limit Pareto frontier (Steps 1-5), and then establish convergence, which involves minor modification of the Fudenberg et al. (1994) result (Step 6).

Step 1: Methodology of characterizing the limit Pareto frontier:

In the Fudenberg et al. (1994) characterization, the bounding set that is shown to be the limit of the set of equilibrium value vectors is the intersection of the largest half-spaces in each direction whose boundary values can be decomposed on these hyperplanes. To be more specific, a few definitions should be introduced.

• A *half space* $H(\lambda, k)$ *with direction* $(\lambda, 1-\lambda)$ *and level* k is the set

$$H(\lambda, k) = \{v \in \mathbb{R}^2 | \lambda v_1 + (1-\lambda)v_2 \leq k\}.$$

For brevity, define $u_i(e,t) = E\{\frac{y}{2} + t_i(y)|e\} - C(e_i)$ where $E\{\cdot|e\}$ represents the expectation taken with respect to y using the distribution induced by the effort profile e.

• A value vector $v = (v_1, v_2)$ is *decomposable on a set* $W \in \mathbb{R}^2$ if there exists an effort profile e, transfers t and continuation value vectors $\gamma(y) \in W$ for each $y \in \{\bar{y}, \underline{y}\}$ such that

(PK) $v_i = (1-\delta)u_i(e,t) + \delta E\{\gamma(y)|e\}$

(IC) $v_i \geq (1-\delta)u_i((e_i', e_{-i}), t) + \delta E\{\gamma_i(y)|e_i', e_{-i}\}$ for any $e_i' \in \{0, 1\}$

The first condition is the promise keeping condition: it guarantees that the current payoff and the expected continuation payoff average to v. The second condition is the standard incentive compatibility condition. Strictly speaking, the definition should also include the conditions stipulating that the transfers are also incentive compatible. That is,

$$\forall y : (1-\delta)t_i(y) + \delta\gamma_i(y) \geq 0 \tag{24}$$

The constraint takes this form because the transfers are observable and deviations can be punished by switching to the worst equilibrium with payoffs $(0,0)$. Notice that as $\delta \to 1$, (24) becomes $\gamma_i(y) \geq 0$. Since we are characterizing the limit case, in what follows, we ignore this constraint keeping in mind that $v, w \geq 0$.

Then, *the largest half-space in direction* λ *whose boundary values can be decomposed on itself by effort profile* e *and transfers* t is $H(\lambda, k^*(\lambda, e, t))$ where $k^*(\lambda, e, t)$ is characterized by the following linear programming problem:

$$\begin{aligned} k^*(\lambda, e, t) &= \max_x \quad \lambda[u_1(e,t) + E_y\{x_1(y)|e\}] + (1-\lambda)[u_1(e,t) + E_y\{x_2(y)|e\}] \\ &\text{s.t.: } \; u_1(e,t) + E_y\{x_1(y)|e\}] \geq u_i(e_i', e_{-i}, t) + E_y\{x_i(y)|e_i', e_{-i}\} \\ &\qquad \lambda x_1(y) + (1-\lambda)x_2(y) \leq 0 \end{aligned} \tag{25}$$

This can be seen by noting that if the continuation values $x_i(y)$ are obtained via the normalization $x_i(y) = (\gamma_i(y) - v)\frac{\delta}{1-\delta}$ from unnormalized continuation values $\gamma_i(y)$, the first constraint is equivalent to the condition (IC), the objective function is nothing but $\lambda v_1 + (1-\lambda)v_2$ for some $v = (v_1, v_2)$ for which (PK) is satisfied. Finally, the last constraint guarantees that the unnormalized continuation values γ come from the hyperplane $H(\lambda, k^*(\lambda, e, t))$. Define

$$k^*(\lambda) = \max_{e,t} k^*(\lambda, e, t) = \max_e \max_t k^*(\lambda, e, t).$$

Therefore, $H(\lambda, k^*(\lambda))$ is *the largest half space in direction* λ.

Fudenberg et al. (1994) show that the set of equilibrium payoffs is bounded by

$$W = \bigcap_{\lambda} H(\lambda, k^*(\lambda))$$

Obviously, W includes the set of PSPPE payoff vectors $W(\delta)$, and below we also show (using FLM methodology) that $W(\delta)$ converges to W as $\delta \to 1$.

Steps 2-5 below are devoted to characterizing the set W for our repeated partnership game. We characterize $k^*(\lambda, e)$ for each possible combination of efforts[34](Steps 2 and 3), then find $k^*(\lambda) = \max_e k^*(\lambda, e)$ (Step 4), and, finally, characterize W and its Pareto frontier $\mathcal{W}(v)$ (Step 5). Since all the analysis is carried out for a given partnership (m, n), we drop reference to the partners' types throughout.

Step 2: Characterizing $k^*(\lambda, EE,)$

First, we find $k^*(\lambda, EE, t)$ for all t and then maximize it with respect to t to find $k^*(\lambda, EE)$. The linear programming problem (25) described in the previous section becomes:

$$\begin{aligned} k^*(\lambda, EE, t) = &\max_{x_1, x_2} \lambda \left(\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_1 + (1-p)\underline{t}_1 + px_1(\bar{y}) + (1-p)x_1(\underline{y}) \right) + ... \\ &... + (1-\lambda) \left(\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_2 + (1-p)\underline{t}_2 + px_2(\bar{y}) + (1-p)x_2(\underline{y}) \right) \\ \text{subject to:} \quad &\frac{1}{p - q_2} \left(c - \frac{(p-q_2)(\bar{y} - \underline{y})}{2} \right) - (\bar{t}_1 - \underline{t}_1) \leq x_1(\bar{y}) - x_1(\underline{y}) \\ &\frac{1}{p - q_1} \left(c - \frac{(p-q_1)(\bar{y} - \underline{y})}{2} \right) - (\bar{t}_2 - \underline{t}_2) \leq x_2(\bar{y}) - x_2(\underline{y}) \\ &\lambda x_1(y) + (1-\lambda) x_2(y) \leq 0 \quad \text{for all} \quad y \in Y \end{aligned}$$

Denote the left hand side of the IC constraint for agent i by L_i. That is,

$$L_1 = \frac{1}{p - q_2} \left(c_1 - \frac{(p-q_2)(\bar{y} - \underline{y})}{2} \right) - (\bar{t}_1 - \underline{t}_1).$$

$$L_2 = \frac{1}{p - q_1} \left(c_n - \frac{(p-q_2)(\bar{y} - \underline{y})}{2} \right) - (\bar{t}_2 - \underline{t}_2).$$

To characterize the solution to this problem, it is convenient to distinguish between two separate cases,

$$\lambda L_1 + (1-\lambda) L_2 \leq 0 \tag{26}$$

and

$$\lambda L_1 + (1-\lambda) L_2 > 0. \tag{27}$$

[34]In fact, we omit the characterization of $k^*(\lambda, SS)$ noting that it is trivially equal to 0.

If λ and t satisfy the first inequality then it is possible to choose $x_1(\cdot), x_2(\cdot)$ in such a way that $\lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y}) = \lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) = 0$ and incentive constraints are satisfied.[35] Therefore, in this case

(28)
$$k^*(\lambda, EE, t) = \lambda\left(\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_1 + (1-p)\underline{t}_1\right) + (1-\lambda)\left(\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_2 + (1-p)\underline{t}_2\right)$$

Thus (26) implies that orthogonal implementation is possible.

On the other hand, if λ and t are such that (27) holds, then

$$\lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) < \lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y})$$

and it would be optimal to choose a combination of $x_1(\bar{y})$ and $x_2(\bar{y})$ that satisfies $\lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y}) = 0$. Obviously, $\lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) = 0$ is not feasible any more, and in the optimal solution both incentive constraints must bind. Therefore

(29)
$$\begin{aligned} k^*(\lambda, EE, t) =& \lambda\left[\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_1 + (1-p)\underline{t}_1 - (1-p)L_1\right] + \ldots \\ &\ldots(1-\lambda)\left[\frac{p\bar{y} + (1-p)\underline{y}}{2} - c + p\bar{t}_2 + (1-p)\underline{t}_2 - (1-p)L_2\right]. \end{aligned}$$

Note that if (26) holds with equality then both incentive constraints must bind and therefore equations (29) and (28) deliver the same value.

The next step is to choose the transfer profile $t^*(\lambda)$ that maximizes the level of the hyperplane in the direction λ when EE is the effort profile. This is done in the following Lemma.

Lemma 1 *Define for all y:*

(30)
$$t_1^*(y) = -t_2^*(y) = \begin{cases} \frac{y}{2} & \text{if } \lambda \geq \frac{1}{2} \\ -\frac{y}{2} & \text{if } \lambda < \frac{1}{2} \end{cases}$$

Then $t^* \in \mathbf{argmax}_t k^*(\lambda, EE, t)$.

Proof: The result follows from observing that t^* defined in (1) maximizes (28) and minimizes the left hand side of (26). □

This Lemma implies that for large δ and when v, w is on the frontier, one of the agents receives all of the current output if both agents work. Therefore, $k^*(\lambda, EE)$ can be expressed as follows:

(31)
$$k^*(\lambda, EE) = \begin{cases} \lambda(p\bar{y} + (1-p)\underline{y} - c) + (1-\lambda)(-c) - (1-p)\max\{0, \eta_1(\lambda)\} & \text{if } \lambda > \frac{1}{2} \\ \lambda(-c) + (1-\lambda)(p\bar{y} + (1-p)\underline{y} - c) - (1-p)\max\{0, \eta_2(\lambda)\} & \text{if } \lambda \leq \frac{1}{2} \end{cases}$$

[35] If (26) holds with strict inequality, at least one of the incentive constraints will be slack.

where

$$\eta_1(\lambda) = \lambda \frac{c - (p-q_2)(\bar{y} - \underline{y})}{p - q_2} + (1-\lambda)\frac{c}{p-q_1}$$

and

$$\eta_2(\lambda) = \lambda \frac{c}{p - q_2} + (1-\lambda)\frac{c - (p-q_1)(\bar{y} - \underline{y})}{p-q_1}$$

Note that $\eta_1(\lambda)$ and $\eta_2(\lambda)$ are the values of $\lambda L_1 + (1-\lambda)L_2$ evaluated at the corresponding optimal t's. The term $\max\{0, \eta_1(\lambda)\}$ in equation (31) becomes positive when the condition (27) holds: ICs are binding and orthogonal implementation is not possible.

Correspondingly, the hyperplane associated with the optimal transfer schedule for $\lambda \geq \frac{1}{2}$ [the hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, EE)$] passes through either A_1 or B_1 defined below—namely the one which delivers a lower level in this direction λ.[36]

(32)
$$\begin{aligned} A_1 : &\quad \big(p\bar{y} + (1-p)\underline{y} - c, -c\big) \\ B_1 : &\quad \left(p\bar{y} + (1-p)\underline{y} - c - \frac{1-p}{p-q_2}(c - (p-q_2)(\bar{y} - \underline{y})), -c - \frac{1-p}{p-q_1}c\right) \end{aligned}$$

For $\lambda \leq \frac{1}{2}$ the corresponding hyperplane passes through the lower one of the following two points:

(33)
$$\begin{aligned} A_2 : &\quad \big(-c, p\bar{y} + (1-p)\underline{y} - c\big) \\ B_2 : &\quad \left(-c - \frac{1-p}{p-q_2}c, p\bar{y} + (1-p)\underline{y} - c - \frac{1-p}{p-q_1}(c - (p-q_1)(\bar{y} - \underline{y}))\right) \end{aligned}$$

Step 3: Characterizing $k^*(\lambda, ES)$ and $k^*(\lambda, SE)$

For the effort profile ES and transfers t, the linear programming problem (25) can be written as

(34)

$$\begin{aligned} k^*(\lambda, ES, t) = \mathbf{argmax}_x \quad & \lambda[q_1(\bar{y}/2 + \bar{t}_1) + (1-q_1)(\underline{y}/2 + \underline{t}_1) - c + q_1 x_1(\bar{y}) + (1-q_1)x_1(\underline{y})] + ... \\ & ...(1-\lambda)[q_1(\bar{y} + \bar{t}_2) + (1-q_1)(\underline{y}/2 + \underline{t}_2) + q_2 x_2(\bar{y}) + (1-q_2)x_2(\underline{y})] \\ \text{subject to:} \quad & x_1(\bar{y}) - x_1(\underline{y}) \geq \frac{1}{q_1}\left(c - \frac{q_1(\bar{y} - \underline{y})}{2}\right) - (\bar{t}_1 - \underline{t}_1) \\ & x_2(\bar{y}) - x_2(\underline{y}) \leq \frac{1}{p-q_1}\left(c - \frac{(p-q_1)(\bar{y} - \underline{y})}{2}\right) - (\bar{t}_2 - \underline{t}_2) \\ & \lambda x_1(y) + (1-\lambda)x_2(y) \leq 0; \quad y \in \{\underline{y}, \bar{y}\} \end{aligned}$$

If there were no incentive constraints, it would always be possible to enforce (ES, t) orthogonally, which would deliver

(35) $k^*(\lambda, ES, t) = \lambda[-c + q_1(\bar{y} + \bar{t}_1) + (1-q_1)(\underline{y} + \underline{t}_1)] + (1-\lambda)[q_1(\bar{y} + \bar{t}_1) + (1-q_1)(\underline{y} + \underline{t}_2)]$.

[36] By this we mean, the inner product $(\lambda, 1-\lambda) \times (v, w)$ is minimized.

Since the incentive constraints in (34) bound $x_1(\bar{y}) - x_1(\underline{y})$ from below and $x_2(\bar{y}) - x_2(\underline{y})$ from above, it is always possible to choose such x that $\lambda x_1(y) + (1-\lambda)x_2(y) = 0$ for any y and both incentive constraints are satisfied.[37] Thus (35) is also a solution to the linear program (34).

To maximize $k^*(\lambda, ES, t)$ with respect to t we need to set $\bar{t}_1 = -\bar{t}_2 = -\frac{\bar{y}}{2}$ and $\underline{t}_1 = -\underline{t}_2 = -\frac{\underline{y}}{2}$ if $\lambda < \frac{1}{2}$ and similarly $\bar{t}_1 = -\bar{t}_2 = \frac{\bar{y}}{2}$ and $\underline{t}_1 = -\underline{t}_2 = \frac{\underline{y}}{2}$ otherwise. Therefore,

$$k^*(\lambda, ES) = \begin{cases} \lambda[q_1\bar{y} + (1-q_1)\underline{y} - c] & \text{if } \lambda \geq \frac{1}{2} \\ \lambda(-c) + (1-\lambda)[q_1\bar{y} + (1-q_1)\underline{y}] & \text{otherwise} \end{cases}.$$

The level of the largest half-space in direction λ decomposed on itself by SE can be straightforwardly determined in a similar way:

$$k^*(\lambda, SE) = \begin{cases} \lambda[q_2\bar{y} + (1-q_2)\underline{y}] + (1-\lambda)(-c) & \text{if } \lambda \geq \frac{1}{2} \\ (1-\lambda)[q_2\bar{y} + (1-q_2)\underline{y} - c] & \text{otherwise} \end{cases}.$$

Step 4: Characterizing $k^*(\lambda)$

For each λ the level of largest half space in the direction λ is found as

$$k^*(\lambda) = \max\{k^*(\lambda, EE), k^*(\lambda, ES), k^*(\lambda, SE)\} \tag{36}$$

It is convenient to first characterize $\max\{k^*(\lambda, ES), k^*(\lambda, SE)\}$ and then compare it with $k^*(\lambda, EE)$.

For $\lambda \geq \frac{1}{2}$ the hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, ES)$ passes through D_1 and hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, SE)$ passes through G_1 defined as follows:

$$\begin{aligned} D_1: &\quad \big(q_1\bar{y} + (1-q_1)\underline{y} - c, 0)\big) \\ G_1: &\quad \big(q_2\bar{y} + (1-q_2)\underline{y}, -c)\big) \end{aligned} \tag{37}$$

For $\lambda \leq \frac{1}{2}$ the corresponding hyperplanes pass through D_2 and G_2:

$$\begin{aligned} D_2: &\quad \big(-c, q_1\bar{y} + (1-q_1)\underline{y}\big) \\ G_2: &\quad \big(0, q_2\bar{y} + (1-q_2)\underline{y} - c)\big) \end{aligned} \tag{38}$$

It is easy to see that for $m > n$ the points G_1, G_2 lie below the line connecting the points D_1 and D_2.[38] Therefore, for $\lambda \geq \frac{1}{2}$, the hyperplane passing through D_1 should be chosen and for $\lambda \leq \frac{1}{2}$ the hyperplane passing through D_2 should be chosen. This is intuitive because it implies that whenever only one of the agents works it is the more efficient one.

In the light of this discussion, we get that whenever $m > n$:

$$k^*(\lambda) = \max\{k^*(\lambda, EE), k^*(\lambda, ES)\}.$$

[37] We only need to make sure that $x_1(\bar{y}) - x_1(\underline{y})$ and $x_2(\bar{y}) - x_2(\underline{y})$ are sufficiently far away from each other.

[38] To see this note that the line connecting D_1 and D_2 has slope -1 and level $f(1,0) - c$ while the lines through G_1 and G_2 with slope -1 have levels $f(0,1) - c < f(1,0) - c$.

Step 5: Characterization of $\mathcal{W}(\cdot)$

Recall that

$$\mathcal{W} = \bigcap_{\lambda}\{(v,w)|\lambda v + (1-\lambda)w \leq k^*(\lambda)\}$$

Proposition 7 *Define*

$$\eta = \frac{c-(p-q_2)(\bar{y}-\underline{y})/2}{p-q_2} + \frac{c-(p-q_1)(\bar{y}-\underline{y})/2}{p-q_1}$$

Then,

$$\mathcal{W}(v) = -v + p\bar{y} - 2c - \min\{2(1-p)\eta, (p-q_1)(\bar{y}-\underline{y})+c\}$$

Proof: First, notice that $\overline{A_1A_2}$, $\overline{B_1B_2}$ and $\overline{D_1D_2}$ all have slopes -1. Also, all points $A_1, A_2, B_1, B_2, D_1, D_2$ lie outside of the positive orthant. Next, observe that $\overline{A_1A_2}$ lies above $\overline{B_1B_2}$ by assumption **A5**, and by assumption **A1**, $\overline{A_1A_2}$ lies above $\overline{D_1D_2}$. Finally, observe that $p\bar{y}+(1-p)\underline{y}-2c-2\eta$ and $q_1\bar{y}+(1-q_1)\underline{y}-c$ are the levels of $\overline{B_1B_2}$ and $\overline{D_1D_2}$, respectively. □

Step 6: Convergence

The following proposition reproduces the result of Fudenberg et al. (1994) that $\mathcal{W}$ is the limit of the equilibrium payoff set as δ converges to 1 and the proof of Proposition 6 follows directly from it.

Proposition 8 *(Fudenberg et al. (1994)) Let $V \subset intW$ be smooth*[39] *and convex. Then there exists $\bar{\delta}$ such that for any $\delta > \bar{\delta}$, $V \subset W(\delta)$.*

Proof: We note that the Pareto frontier of the repeated partnership game is obtained using $(\bar{t}_i, \underline{t}_i) \in \{-\bar{y}/2, \bar{y}/2\} \times \{\underline{y}/2, -\underline{y}/2\}$. That is, restricting attention to equilibria that use only these transfers does not shrink the equilibrium set. Therefore, the proof directly follows from Fudenberg et al. (1994).□

8.2.4 Matching patterns: proof of Proposition 4

Proposition 4 follows from Proposition 6 above, Propositions 2 and 3 in the main text and the following result which proves that if the limit frontier $\mathcal{W}_{mn}(v)$ satisfies increasing (decreasing) differences, then for large δ, $\mathcal{W}^{\delta}_{mn}(v)$ also does.

Lemma 2 *Assume $S(m,n)$ exhibits (decreasing) differences. Then there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, $\mathcal{W}^{\delta}_{mn}(\cdot)$ exhibits generalized increasing (decreasing) differences.*

[39] A smooth set is closed, with non-empty interior and its boundary is twice continuously differentiable.

Proof: Let $T = \{\kappa(i)|i \in N\}$. That is T is the set of types of the N agents in the economy. First assume increasing differences, i.e.

$$(S(m,n) - S(m,n')) - (S(m',n) - S(m',n')) > 0$$

Let $\varepsilon = \inf\{S(m,n) + S(m',n') - S(m,n') - S(m',n')|m > m', n > n' \in T\}$ and $\varepsilon^* = \frac{1}{4}\varepsilon$. For each m,n there exists $\delta_{mn}(\varepsilon^*)$, such that for all $\delta > \delta_{mn}(\varepsilon^*)$, $\sup_v |\mathcal{W}^\delta(v) - \mathcal{W}(v)| < \varepsilon^*$. Now, let $\bar{\delta} = \max\{\bar{\delta}_{mn}(\varepsilon^*)|m,n \in T\}$. Since T is finite, $\varepsilon^* > 0$ and $\bar{\delta} < 1$.

Take $\delta > \bar{\delta}$, $m > m', n > n'$ and v, v' such that

$$\mathcal{W}^\delta_{mn'}(v) = \mathcal{W}^\delta_{m'n'}(v')$$

Then,

$$\mathcal{W}_{mn'}(v) - \mathcal{W}_{m'n'}(v') > -2\varepsilon^*$$

Moreover,

$$\mathcal{W}^\delta_{mn}(v) - \mathcal{W}^\delta_{m'n}(v') > \mathcal{W}_{mn}(v) - \mathcal{W}_{m'n}(v') - 2\varepsilon^*$$

Also,

$$(\mathcal{W}_{mn}(v) - \mathcal{W}_{m'n}(v')) - (\mathcal{W}_{mn'}(v) - \mathcal{W}_{m'n'}(v')) = (S(m,n) - S(m'n)) - (S(m,n') - S(m',n')) \geq 4\varepsilon^*$$

The last three inequalities together imply

$$\mathcal{W}^\delta_{mn}(v) - \mathcal{W}^\delta_{m'n}(v') > 0$$

which establishes generalized increasing differences. The proof for generalized decreasing differences is analogous.□

References

ANDERSEN, A. AND L. SMITH, "Assortative Matching and Reputation," *Review of Economic Studies* (forthcoming).

BECKER, G., "A Theory of Marriage: Part I," *Journal of Political Economy* 81 (1973), 813–46.

CITANNA, A. AND A. CHAKRABORTY, "Occupational Choice, Incentives and Wealth Distribution," *Journal of Economic Theory* 122 (2005).

FERNANDEZ, R. AND J. GALI, "To each according to...? Markets, tournaments and the matching problem with borrowing constraints," *Review of Economic Studies* 66 (1999), 799–824.

FRANCO, A., M. MITCHELL AND G. VERSHCHAGINA, "Incentives and the Structure of Teams," mimeo (2008).

Fudenberg, D., D. K. Levine and E. Maskin, "The Folk Theorem with Imperfect Public Information," *Econometrica* 62 (1994).

———, "Balanced-Budget Mechanisms with Incomplete Information," mimeo (1995).

Holmstrom, B., "Moral Hazard in Teams," *The Bell Journal of Economics* 13 (1982).

Kaya, A., "Two-sided Matching with Private Information," mimeo (2008).

Legros, P. and H. Matsushima, "Efficiency in Partnerships," *Journal of Economic Theory* 55 (1991), 296–322.

Legros, P. and A. F. Newman, "Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities," *Econometrica* 75 (2007).

Levin, J. and S. Tadelis, "Profit Sharing and the Role of Professional Partnerships," *Quarterly Journal of Economics* 123 (2008), 1111–1159.

Newman, A. F., "Risk-bearing and Enrepreneurship," *Journal of Economic Theory* 137 (2007), 1–25.

Roth, A. E. and M. O. Sotomayor, *Two-Sided Matching: A study in game-theoretic modeling and analsis*, Econometric Society Monographs (Cambridge University Press, 1990).

Shimer, R. and L. Smith, "Assortative Matching and Search," *Econometrica* 68 (2000), 343–69.

Thiele, H. and A. Wambach, "Wealth Effects in the Principal Agent Model," *Journal of Economic Theory* 89 (1999), 247–60.

Topkis, D. M., "Minimizing a Submodular Function on a Lattice," *Operations Research* 26 (1978).