Econ 201A: Problem Set I

Due on October 12, 2012

Note: You can work in group. Each group needs to submit only one answer. Write down the names of all the members of your group at the top of its first page. The size of a group should not exceed 3.

1. **Strict Preference Representation of Rational Preference.** Let $\succeq^*$ be any rational preference on X and $\succ^*$ be the strict preference relation derived from $\succeq^*$ by $x \succ^* y \Leftrightarrow \{x \succeq y \text{ and } y \not\succeq x\}$.

   (a) Prove that $\succ^*$ is asymmetric ($x \succ^* y \rightarrow y \not\succ^* x$) and negatively transitive ($\{x \not\succ^* y \text{ and } y \not\succ^* z\} \rightarrow x \not\succ^* z$).

   (b) Let $\succ$ be any preference relation that satisfies asymmetry and negative transitivity. Define a preference relation $\succeq^{\succ}$ by $x \succeq^{\succ} y \Leftrightarrow y \not\succ x$. Show that $\succeq^{\succ}$ is a rational preference.

   (c) Derive $\succeq^{\succ^*}$ from $\succ^*$ as in (b). Show that $\succeq^*$ and $\succeq^{\succ^*}$ are identical preference relations.

2. **Preference over Sets**

   Let X be any finite set of objects and $\succeq$ be a preference relation over X. Let 2^X be a collection of all subsets of X. We can derive a preference $\dot{\succeq}$ over 2^X as follows.

   $$A \dot{\succeq} B \Leftrightarrow \text{For any } y \in B, \text{ there exists } x \in A \text{ such that } x \succeq y \ \ (*) .$$

   (a) Show that $\dot{\succeq}$ is complete and transitive if $\succeq$ is complete and transitive.

   (b) Show that $A \dot{\sim} A \cup B$ if $A \dot{\succeq} B$.

   (c) Suppose that $X = \{steak, \ pasta, \ ice \ cream\}$ and a preference $\succeq'$ on 2^X satisfies the following properties: (1) $\{steak\} \succ' \{pasta\} \succ' \{ice \ cream\}$ and (2) $\{pasta\} \succ' \{steak, ice \ cream\} \succ' \{pasta, ice \ cream\}$. The idea is as follows. This DM (decision maker) is very hungry, so (1) represents the preference of this decision maker if he can choose only one dish out of three. At the same time, he is health conscious and prefers to avoid to have an ice cream for dessert by any mean. But he also knows that he will be tempted to order an ice cream if it's available. So he prefers to exclude ice cream from options in advance. In fact, he would be happy to have pasta rather than steak if he can commit not to have an ice cream for a dessert ($\{pasta\} \succ' \{steak, ice\ cream\}$). Can this preference on 2^X be derived from some rational preference on X through $(*)$? Explain.

3. **Existence of Continuous Utility Function**

   Let $\succeq$ be a rational and continuous preference on $\mathbb{R}_+^L$. Suppose that $\succeq$ satisfies local nonsatiation. The purpose of this question is to provide an elementary proof for the existence of a continuous utility function that represents $\succeq$ in this case.

   (a) Show that, for closed ball $B_n = \left\{x \in \mathbb{R}_+^L | \, \|x\| \leq n\right\}$, there exists the least preferred bundle $\underline{x}_n$ in it (hint: use the definition of compactness based on open sets).

   (b) Define $u_n : \mathcal{U}(\underline{x}_n) \to \mathbb{R}$ by $u_n(x) = \min_{y \in \mathcal{U}(x)} \|y - \underline{x}_n\|$ as in class, where $\mathcal{U}(x)$ is the upper contour set at x. Prove that u_n is a continuous function (hint: use Maximum theorem).

   (c) Complete the proof by showing that there exists a continuous function $u : \mathbb{R}_+^L \to \mathbb{R}$ that represents $\succeq$.