Econ 201B: Problem Set I

Due January 26, 2012

1. **Third Price Auction**

   Suppose that $n \geq 3$ bidders with values $v_1 > v_2 > ... > v_n > 0$ play a sealed-bid *third price auction*: the highest bidder wins the object and pays the third highest bid. Answer the following questions.

   (a) Is it a weakly dominant action for bidder i to bid v_i?

   (b) Is there any weakly dominant action for any bidder?

   (c) Find a Nash equilibrium.

   (d) Is there any Nash equilibrium where bidder 1 is not the winner?

2. **A Model of Party Competition in Election**

   Suppose that elections will be held simultaneously at n districts. There are two parties: party A and party B, and there is one candidate from each party at each district. Party A's resource for the election is $W_A > 0$ and party B's resource is $W_B > 0$. If A and B spend x_k and y_k at district k respectively, then the candidate from party A wins with probability $f_k(x_k, y_k) \in [0, 1]$ and the candidate from party B wins with probability $1 - f_k(x_k, y_k)$. Party A and B choose $(x_1, ..., x_n) \geq 0$ and $(y_1, ..., y_n) \geq 0$ simultaneously subject to the constraint: $\sum_{k=1}^{n} x_k \leq W_A$ and $\sum_{k=1}^{n} y_k \leq W_B$. Each party maximizes the expected number of districts that their candidates win. Assume that each $f_k : \mathbb{R}_+^2 \to [0, 1]$ is twice differentiable and satisfies $\frac{\partial^2 f_k}{(\partial x_k)^2} \leq 0$ and $\frac{\partial^2 f_k}{(\partial y_k)^2} \geq 0$ for (a) and (b).

   (a) Show that there exists a Nash equilibrium in this game.

   (b) Show that each party's expected payoff (=number of districts) is the same in any Nash equilibrium.

   (c) Suppose that f_k is given by the same function f in all districts. Assume that f is homogeneous of degree 0 and satisfies $\frac{\partial f}{\partial x} > 0$, $\frac{\partial f}{\partial y} < 0$, $\lim_{x \to 0} \frac{\partial f(x,y)}{\partial x} = \infty$, $\lim_{y \to 0} \frac{\partial f(x,y)}{\partial y} = -\infty$ and $\frac{\partial^2 f}{(\partial x)^2} \leq 0$ and $\frac{\partial^2 f}{(\partial y)^2} \geq 0$ in

$\mathbb{R}^2_{++}$. Also assume that $f(0, y) = 0$ for any $y > 0$, $f(x, 0) = 1$ for any $x > 0$ and $f(0, 0) = 0.5$. Find all Nash equilibria.

3. **Learning Nash Equilibrium in Simple Cournot Model**

   Consider the simple Cournot model we discussed in the class with firm 1 and 2, the inverse demand function $P(q_1, q_2) = \max\{12 - q_1 - q_2, 0\}$ and linear cost function $3q_i$ for $i = 1, 2$.

   (a) Consider a hypothetical dynamic adjustment process in which each firm chooses a best response to the other firm's production in the previous period. This generates a sequence of production profiles $q(t) = (q_1(t), q_2(t)), t = 0, 1, 2, ...$ given any starting production profile $(q_1(0), q_2(0))$, where $q_i(t)$ is a best response to $q_{-i}(t-1)$ for $i = 1, 2$ and $t = 1, 2, ..$. Prove that this process converges to the Cournot-Nash equilibrium given any starting point. (Hint: note that $q_i(t) \leq \frac{9}{2}$ for all $t \geq 1$, then $q_i(t) \geq \frac{9}{4}$ for all $t \geq 2$...)

   (b) Consider a different dynamics in which each firm best responds to the past average productions of the other firm. This generates a sequence of production profile $q(t) = (q_1(t), q_2(t)), t = 0, 1, 2, ..$given any starting production profile $(q_1(0), q_2(0))$, where $q_i(t)$ is a best response to $\frac{\sum_{t=0}^{t-1} q_{-i}(k)}{t}$ for $i = 1, 2$ and $t = 1, 2, ...$ Prove that this process converges to the Cournot-Nash equilibrium given any starting point.