Econ 201A: Problem Set II

Due on October 19, 2012

1. **Walrasian Demand with CES Utility Function.** Let $u(x, y) = (\alpha x^{\rho} + \beta y^{\rho})^{\frac{1}{\rho}}$, $\alpha, \beta > 0$, $\rho\,(\neq 0) < 1$ be a utility function of a consumer.

   (a) Derive the Walrasian demand given $(p^x, p^y, w) \gg 0$.

   (b) Derive the **elasticity of substitution** $\frac{d(y/x)}{d(p^x/p^y)} \frac{p^x/p^y}{y/x}$.

2. **Indirect Utility Function with Homothetic Preference (MWG. 3.D.3)** Suppose that $u : \mathbb{R}^L_+ \to \mathbb{R}$ is a differentiable, strictly quasi-concave utility function. Answer the following questions.

   (a) Show that $x(p, w)$ and $v(p, w)$ are homogeneous of degree 1 in w if $u(x)$ is homogeneous of degree 1.

   (b) What is the wealth elasticity of the (Walrasian) demand $\left(\frac{dx/x}{dw/w}\right)$ in this case?

   (c) Suppose that $x(p, w)$ is differentiable in $\mathbb{R}^{L+1}_{++}$ and $v(p, w)$ is homogeneous of degree 1 in w. Prove that $u(x)$ is homogeneous of degree 1 in $\mathbb{R}^L_{++}$ if the underlying preference is monotone. You may proceed as follows:

      i. Prove that $x(p, w)$ is homogeneous of degree 1 in w (use Roy's identity).

      ii. Prove $u(\alpha x(p, w)) = \alpha u(x(p, w))$ for any $(p, w) \gg 0$.

      iii. Show that for any $x \in \mathbb{R}^{L+1}_{++}$ there exists $(p, w) \gg 0$ such that $x = x(p, w)$. (The supporting hyperplane theorem (MWG, p.949) may be useful. If you cannot prove it, try to draw a graph to explain why this must be the case).