

# Econ 201B: Problem Set II

Due February 2, 2012

## 1. Bertrand Competition with Homogeneous Good

There are two firms  $i = 1, 2$  that provides the identical good to the market. Suppose that firm 1's marginal cost  $c_1$  is lower firm 2's marginal cost  $c_2$ . There is no fixed cost. The firms choose prices simultaneously. Firm  $i$ 's profit given  $p = (p_1, p_2) \in \mathbb{R}_+^2$  is  $\pi_i(p) = (p_i - c_i)D_i(p)$ , where the demand function is given as follows:

$$D_i(p) := \begin{cases} K (> 0) & \text{if } p_i < p_{-i} \\ \frac{K}{2} & \text{if } p_i = p_{-i} \\ 0 & \text{if } p_i > p_{-i} \end{cases}$$

- Show that there is no pure strategy Nash equilibrium in this game.
- Find a mixed strategy Nash equilibrium.

## 2. Rationalizability in First Price Auction

Consider a first price auction with 2 bidders, where each bidder can bid an integer value from \$0 to \$100. Suppose that the values of bidders are given by  $v_1 = 90 > v_2 = 80$ . Answer the following questions.

- Suppose that the winner is selected randomly when bids are tied. Delete strictly dominated actions iteratively (delete all strictly dominated actions in each round). Show that any bid  $b \geq v_1$  is deleted for bidder 1 and any bid  $b \geq v_1 - 1$  is deleted for bidder 2 in this process. How many rounds it would take for this process of iterated deletions to stop?
- Suppose that bidder 2 wins when bids are tied. Delete strictly dominated actions iteratively as in (a). Show that any bid  $b \geq v_1 + 1$  is deleted for bidder 1 in this process and any bid  $b \geq v_1$  is deleted for bidder 2. How many rounds it would take for this process of iterated deletions to stop?

### 3. Iterated Deletion of Dominated Actions

Delete strictly dominated actions iteratively in the Traveler's dilemma game. Describe each step carefully. Which action set survives this process of iterated deletions?