Econ 201A: Problem Set III

Due on October 26, 2012

1. **Revealed Preference and Utility.** Suppose that the optimal consumption vector for Alice is $x \in \mathbb{R}^L_+$ when the price is $p \in \mathbb{R}^L_{++}$ and x' when the price is p' Alice is maximizing her utility $u : \mathbb{R}^L_+ \to \mathbb{R}$. Answer the following questions.

   (a) Show $u(x) \geq u(x')$ if x is revealed preferred $(p \cdot x' \leq p \cdot x)$.

   (b) Show $u(x) > u(x')$ if x is strictly revealed preferred $(p \cdot x' < p \cdot x)$ and u is locally nonsatiated.

   (c) Show by an example that local nonsatiation is necessary for (b).

   (d) Show $u(x) > u(x')$ if x is revealed preferred to x' $(p \cdot x' \leq p \cdot x)$, $x' \neq x$, and u is strictly quasi-concave.

2. **More Abstract Approach to Choice and Preference**

   In many problems, an individual's choice set is just more than a budget set. So it is useful to think about (revealed) preference and choice in a more abstract setting. Let X be a set of a set alternatives. Let $\mathcal{B}^X$ be a collection of some subsets of X. *Choice rule* C is a mapping that assigns a nonempty subset $C(B) \subset B$ for every $B \in \mathcal{B}^X$. Here B corresponds to a "budget set" and $C(B)$ represents the individual's choices. Call $(\mathcal{B}^X, C)$ *choice structure*. A choice structure $(\mathcal{B}^X, C)$ satisfies the *weak axiom* if the following holds:

   If $x, y \in B \in \mathcal{B}^X,\ x \in C(B),$ and $y \in C(B')$ for $B' \in \mathcal{B}^X$ such that $x, y \in B',$ then $x \in C(B')$.

   (a) Let $\succeq$ be a preference on X and $C^{\succeq} : \mathcal{B}^X \to 2^X$ be a choice rule defined by

   $$C^{\succeq}(B) := \{x \in B | x \succeq y \text{ for all } y \in B\},$$

   where $C^{\succeq}(B) \neq \varnothing$ is assumed. Show that $(\mathcal{B}^X, C^{\succeq})$ satisfies the weak axiom.

   (b) Find an example of choice structure $(\mathcal{B}^X, C)$ for which the weak axiom is satisfied, but there is no rational preference $\succeq$ that rationalizes it (i.e. $C^{\succeq} = C$) (Hint: MWG, Proposition 1.D.2).

(c) Show that the weak axiom is equivalent to the following two properties when $\mathcal{B}^X$ contains all nonempty subsets of X.

  i. If $x \in B \subset B'$ and $x \in C(B')$ for $B, B' \in \mathcal{B}^X$, then $x \in C(B)$.[1]
  ii. If $x, y \in B \subset B'$, $x \in C(B)$ and $y \in C(B')$ for $B, B' \in \mathcal{B}^X$, then $x \in C(B')$.

[1] This property is often called IIA (*Independence of Irrelevant Alternative*).