Econ 201B: Problem Set IV

Due on February 16, 2012

1. **Correlated Equilibrium**

   Characterize the set of distributions of action profiles that can be achieved by correlated equilibrium for the following Hawk-Dove game.

	$Hawk$	$Dove$
$Hawk$	$\frac{V-C}{2}, \frac{V-C}{2}$	$V, 0$
$Dove$	$0, V$	$\frac{V}{2}, \frac{V}{2}$

$$0 < V < C$$

2. **An Innovation Game and Supermodularity**

   Consider n firms which are independently making a decision regarding their R&D investments to develop a certain new technology. Firm i chooses the amount of resource $x_i \in R_+$ to spend for this R&D project. Suppose that the cost of x_i is $\frac{1}{2}kx_i^2$ where $0 < k < \frac{n-1}{2}$. Firm i succeeds in developing a new technology with probability $f(x_i) \in [0,1]$. Firm i's profit is 0 if it does not succeed and $g\left(\sum_{j\neq i} x_j\right) \geqq 0$ if it succeeds, where g is a continuous increasing function. That is, firm i's profit when succeeding depends on the amount of the other firms' investments. Thus firm i's expected profit given $x = (x_1, ..., x_n)$ is

$$\pi(x) = f(x_i)\, g\left(\sum_{j\neq i} x_j\right) - \frac{1}{2}kx_i^2.$$

   We assume that each firm maximizes its expected profit. Answer the following questions.

   (a) Explain why there may be multiple symmetric Nash equilibria for this game

   (b) Assume that $f(x_i) = \log(1 + x_i)$ and $g\left(\sum_{j\neq i} x_j\right) = \sum_{j\neq i} x_j$. Find all symmetric Nash equilibria.

   (c) Verify that the largest equilibrium in (b) is decreasing in k.

(d) Prove that there exists the largest equilibrium and the largest equilibrium is (weakly) decreasing in k when f is a continuous increasing function, without assuming any explicit functional form for f and g.

(e) Explain why some symmetric Nash equilibrium may be (locally) increasing in k.

3. **Increasing Difference**

"f on lattice $X \times Y \in \mathbb{R}^k$ satisfies increasing differences in (x, y) if and only if f satisfies increasing differences in any pair of $(x_i, y_j) \in \mathbb{R}^2$ given any x_{-i}, y_{-j}". Prove this statement.