Econ 201B: Problem Set V

Due on March 1, 2012

1. **Public Good Problem with Private Cost**

   Consider the following public good problem with unknown private cost.

   - There are n players.
   - Players decide simultaneously whether to make a contribution or not given their cost of contribution. Player i's cost of contribution c_i is private information. It is a common knowledge that $c_1, ..., c_n$ are independently and uniformly distributed on $[0, 1]$.
   - A public good is provided if and only if at least 1 player contributes. Each player gains utility 1 from the provided public good. So player i's payoff is $1 - c_i$ if the public good is provided and i makes a contribution

   (a) Show that, if player i with cost c_i contributes in equilibrium, then player i with cost $c_i' < c_i$ must contribute as well in equilibrium.

   (b) Find all symmetric Bayesian Nash equilibrium.

   (c) Is there any asymmetric Bayesian Nash equilibrium?

2. **Campaign Spending**

   Suppose that n candidates are running for office. Only one candidate will be elected. Candidate i's value of winning is v_i, which is independently and uniformly distributed on $[0, 1]$. Given their value, the candidates choose simultaneously the amount of their campaign spending: $x_1, ..., x_n \geqq 0$. The candidate who spends most wins the election. Candidate i's payoff is $v_i - x_i$ when winning the election and $-x_i$ otherwise.

   Find a symmetric Bayesian Nash equilibrium.

3. **Knowledge Operator**

Prove the following properties of the knowledge operator or provide a counterexample.

(a) $K(E_1 \cap E_2) = K(E_1) \cap K(E_2)$

(b) $K(E_1 \cup E_2) = K(E_1) \cup K(E_2)$

(c) $K(E) \subset K(K(E))$

(d) $\lim_{n\to\infty} K(E_n) = K(\lim_{n\to\infty} E_n)$ for any decreasing sequence of events $E_1 \supset E_2 \supset E_3 \supset ...$

(e) $\lim_{n\to\infty} K(E_n) = K(\lim_{n\to\infty} E_n)$ for any increasing sequence of events $E_1 \subset E_2 \subset E_3 \subset ...$