Abstract

In an information-based equilibrium of an economy with uncertainty, where individuals are fully rational and sophisticated classical statisticians, autarky is an equilibrium, which minimizes aggregate welfare; individuals cannot recover the true uncertainty, and hold different assessments. Full exchange, which maximizes aggregate welfare but bears idiosyncratic risks, is also an equilibrium, robust to the individuals’ assessments, fully recoverable (assessments coincide with the truth), and for each individual yields higher expected gains than autarky. In autarky, under equilibrium assessments, expected gains are subjectively higher than in full exchange – autarky is subjectively Pareto superior. The social planner’s welfare criterion is different, ranking outcomes according to all possible distributions over uncertainty that could have generated her observations. The social planner can induce the switch to full exchange regime. Richer information cannot simultaneously account for more trade and more disparate individuals’ assessments.

∗jcopic@econ.ucla.edu. I am grateful to Andy Atkeson, Tiago Caruso, Aleh Tsivinsky, Romans Pancs, and to my students for participating in a bet that a favorable state will transpire. I am also grateful to Andrea Williamson for editorial assistance.
1 Introduction

Recently, there has been an emergence of various information-based explanations for economic inactivity during a downturn. Some of these quote heterogeneous beliefs, or expectations on the part of market participants; others focus on dissemination of information. Most of these studies focus on dynamic aspects of economic activity. Invariably, a common denominator of these approaches is that the beliefs of economic individuals are assumed to be in some ways distorted. The notions of economic equilibrium in these studies, therefore, involve some sense of agents’ myopia or some other manner in which these agents are irrational or unable to correctly process the information at hand, or there are some inherent frictions in the flow of information.¹

In this paper, we give an informational equilibrium explanation for economic inactivity, which is resolutely devoid of any notion of irrationality, myopia, or inability to correctly process the information at hand. The basis for this explanation is a notion of equilibrium where, instead of Bayesian decision makers who have prior beliefs, the individuals are imagined as classical statisticians who form asymptotically consistent assessments over uncertainty. The backdrop of our argument is that the individuals have at their disposal ideal datasets of partial observations of economic outcomes. In some outcomes, they may be unable to recover the true distribution over the possible states of the economy.² Consequently, it is possible for the individuals to hold different assessments over the uncertain parameters. In equilibrium, such an assessment is not only consistent with the individual’s observations, but also allows her to justify the data she observes as having resulted from optimal behavior by such consistent individuals. All individuals are sophisticated to the utmost in

¹For explanations based on erroneous beliefs see, for instance, Fostel and Geanakoplos 2008, Geanakoplos 2010, and Simsek 2013. For explanations based on dissemination of information see Amador and Weill 2010; and for a combination of the two see Angeletos and La’o 2013. See Kurz 2011, for a survey of dynamic macroeconomic models with agents’ heterogeneous beliefs.

²Uncertainty is here only regarding the probability distribution over the states of the economy – there is no model uncertainty. For that reason we use the term “recover” rather than “identify” as to not cause any confusion with the standard use of the latter term in Econometrics.
the sense that they all believe that each one of them might justify the data in this way, and believes that others believe so, and so on, *ad infinitum*. In equilibrium, these individuals are therefore rational economic optimizers, who are apt as rigorous classical statisticians and cunning in their sophisticated justification of the others’ behavior.\(^3\)

In our model of the economy, we imagine two individuals, a representative individual of a productive sector and a representative individual of a financial, or speculative sector, both of whom can either engage at a lower autarchic level of economic activity, or at a higher level of exchange. The productive individual can either choose an autarchic action, “autarky,” which may bear less risks but also results in a lower level of production. Alternatively, she may choose a risky action, “exchange,” such as taking a loan collateralized by the output of her production. That enables her to produce at a higher level, and in a normal state of the economy, her bet pays off so that she can fully repay the loan and obtain a higher profit than had she remained in autarky. In an extreme state of the economy (low or high), she cannot fully repay her loan, so that her productive output is seized, which results in lower gains relative to autarky.\(^4\) The speculative individual can also choose autarky, which can be imagined as a low-risk investment in government bonds, or she can choose exchange, which can be imagined as a risky investment in the productive sector. However, the speculative individual makes gains precisely by acquiring the distressed assets of the productive sector in the extreme states of the economy, and makes losses relative to autarky, when the state of the economy is intermediate. To complete the story,

\(^3\)This definition of equilibrium is called player equilibrium. The observation of each individual is given by the distribution of her signals, the actions she takes, and her payoffs. Here we do not explicitly construct the individuals’ ideal datasets – such a construction and a deeper discussion of this equilibrium notion can be found in Čopič 2014a.

\(^4\)Non-luxury cars are a good example fitting this description. In a low state of the economy, the demand for cars is low. In an extremely high state of the economy, the demand for cars may be high, however, consumers may instead buy luxury cars, so that the demand for non-luxury cars may nonetheless be low. More generally, any good for which there exists a substitute giffen good may correspond well to the productive sector described here.
the two individuals are ideal partners in the exchange so that the total welfare in
the economy is maximized in full exchange, when both individuals choose exchange.
When one chooses exchange while the other chooses autarky, the former is imagined as engaging with some other less ideal partner. This has an externality on the autarchic individual: in a favorable state, the autarchic individual is penalized (e.g., she might lose some of her market share to competitors), and in an unfavorable state, the externality on her is positive (e.g., her competitors’ relative market shares have been diminished). When both individuals choose autarky, neither of them faces any externalities or idiosyncratic risks.

Before making their choices, the individuals observe signals regarding the state of the economy. If the information of both individuals were aggregated, the state of the economy would be known. Full exchange is the unique equilibrium of such a benchmark economy. If the individuals’ assessments over the uncertain state of the economy coincide, then full exchange is still an equilibrium, while autarky is not; full exchange is also robust to the individuals’ assessments, even while the individuals can in fact recover the true distribution over uncertainty, so that their assessments must coincide with the truth – we call such an outcome fully recoverable. Under the truth, each individual perceives full exchange as superior to autarky in terms of her expected gains. However, when both individuals choose autarky, none of them can in equilibrium recover the true distribution of uncertainty. In turn, this allows the individuals to hold assessments which support autarky as an equilibrium in the sense described above. Under any such supporting assessments, each individual evaluates autarky as generating a higher expected gain to her than full exchange. The reason for the individuals’ inability to recover the truth in autarky is that, due to the low

\[5\]In the literature on Rational Expectations Equilibria, see e.g., Radner 1979, 1982, an outcome is said to be fully revealing if the individuals can deduce the state of the world from their private information and prices, i.e., if all the information is aggregated. In that sense, full exchange here is also fully revealing. In general, if an outcome is fully revealing, then it is fully recoverable, but an outcome may be fully recoverable and not be fully revealing.
level of economic activity, there is less variation in their observations of the outcome in the economy.

The intuition that the sort of data the individuals observe is endogenous to the level of economic activity, is related to the intuition in Ordoñez 2013. There, in a dynamic learning context, the economic agents learn about a change from a good state to a bad state more quickly than \textit{vice versa}. As a result busts are swift, while recoveries are slow. The reason is that in bad times, fewer signals are available from which the change in the state of the economy can be inferred. The workings of the present model are similar. However, instead of a dynamic learning story, we give a story based on an equilibrium argument, which is complementary to the dynamic story in Ordoñez 2013. While no statements can be made here regarding the swiftness of change of economic activity when the underlying state of the economy changes, the present model is independent of the assumptions on learning processes imputed on the individuals. The present model is also suitable for normative analysis.

Indeed, the main contribution here is normative. When the individuals can hold a variety of statistically correct yet subjective assessments, then it is no longer evident how the efficiency of an outcome should be evaluated. After all, the individuals can’t tell what the true objective uncertainty is. An outside observer would presumably face similar recovery problems unless she were magically endowed with much richer observations of the data than the individuals themselves. One way to tackle this issue is by taking the perspective of each individual separately and considering her expected benefit from each outcome under any possible assessment that she might hold. That is, suppose an individual finds herself in a given equilibrium regime and compares her expected gains with that of another outcome. She will surely find one or the other preferrable if it is preferrable under all possible equilibrium supporting assessments that she could hold under the current equilibrium regime.

This idea that, instead of considering incentive-efficient allocations as in Holmstrom and Myerson 1983, one should take as a starting point the subjective welfare
from each individual’s perspective, to our knowledge dates at least as far as back as Wilson 1978. More recently, in a setting with heterogeneous priors, which are not restricted by equilibrium considerations, Brunnermeier, Simsek, and Xiong 2013 explore this idea to develop a belief-neutral welfare measure. Their welfare measure can be thought of as a Pareto criterion based on each individual’s subjective welfare criterion given all possible heterogeneous priors. They show that equivalently, their welfare measure can be thought of as a belief-neutral measure of social welfare from the perspective of a social planner.

In the present setting, where assessments are determined as a part of the equilibrium outcome, welfare must be measured differently. For example, each individual finds autarky preferrable to full exchange under any possible supporting assessment that she might hold under autarky. Therefore, in autarky, the outcome subjectively Pareto dominates full exchange. Ironically, the full exchange equilibrium is fully recoverable, robust to the individuals’ assessments over uncertainty, efficient, and, under the true distribution over uncertainty, it also maximizes each individual’s expected gains. Therefore, under full exchange, both individuals’ assessments agree with the true objective uncertainty, and both find it preferrable to autarky – the outcome thus subjectively (and objectively) Pareto dominates autarky. Such subjective Pareto evaluations are thus outcome dependent. Nevertheless, in the present economy, full exchange generates the highest total sum of individuals’ utilities in every state of the economy – it is efficient, while autarky is not. On the one hand, this suggests a different measure of welfare from the perspective of a social planner. We assume that the social planner can observe the total sum of the individuals’ expected gains. She will then consider some behavior, which is supportable in equilibrium, as welfare superior, if the total expected gains are under that behavior are higher than under any possible objective uncertainty that could in equilibrium have generated the observed total expected gains. Such welfare criterion is therefore based purely on equilibrium analysis and on what the social planner can in fact observe. On the
other hand, in the present example, a social planner can provide inducements to the individuals to jointly switch from autarky to full exchange, while balancing the budget.

A relationship with two classical studies is non-coincidental. According to Aumann 1976, when individuals have the same prior and their posteriors are common knowledge, their posteriors must coincide. According to Milgrom and Stokey 1982, when risk-averse traders begin at a Pareto-efficient allocation and have concordant posteriors, they can never agree on a non-null trade in a fully recoverable equilibrium. The full exchange equilibrium of the present economy tightly corresponds with these results. The individuals’ equilibrium assessments (analogous to priors) are common, after receiving their signals about the state of the economy, their conditional assessments (analogous to posteriors) are common knowledge and coincide. Trade is possible because the initial autarchic allocation is not Pareto-efficient while full exchange is. In contrast, in autarky, players’ conditional assessments may be common knowledge but, because their assessments are different, these conditional assessments do not coincide. Switching from autarky to exchange is not possible even though autarky is not Pareto-efficient, while full exchange is. Our example of autarky thus provides a counterpoint to these classical results.

The prediction of trade and the extent of disagreement in the individuals’ assessments can also be compared to the predictions from models with distorted priors. One of the pervasive predictions in that literature is that new information can make agents’ beliefs more disparate and induce more trade. In the present economy, richer information makes the individuals’ assessments coincide and autarky is then no longer an equilibrium outcome. More generally, in the context of the present notion of equilibrium of fully rational, consistent and sophisticated individuals, it is not possible to have richer information result in both, more disparate assessments

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and more trade.

As a final note on the exposition, we have chosen to distill our setting as to construct the simplest possible example to make our points. In our model, the set of possible states of the economy is discrete, there are only two representative individuals, each has only two possible actions, and there are no dynamic considerations. Moreover, we have purposefully constructed our economy so that in the full exchange equilibrium there is idiosyncratic risk but no aggregate risk (in autarky there is no idiosyncratic risk). The payoffs to the individuals are constructed in such a way that their equilibrium decision problems are essentially identical. While we have decided to go for the most abstract and stylized version of our story, our definitions are general and the transparency of our exposition makes it clear that our points hold under more general and realistic scenarios.\footnote{Of these modeling choices, perhaps the most radical abstraction is our exposition in the context of an entirely static economy. The individuals are imagined as rigorous classical statisticians and decision makers who have collected infinite datasets of observations of equilibrium play. Their consistent assessments, equilibrium behavior, and justifications of others' behavior are stable points of economies where such a process is envisioned explicitly. We feel that providing an explicit narration of such a dynamic process would only unnecessarily complicate our formal exposition, and make our points less clear. See Čopič 2014a for such a dynamic justification of equilibrium points given here. In the language of Čopič 2014b, autarky is positive, pooling, and informationally adverse, and by Theorem 1 and the discussion in Čopič 2014b, each individual must have at least 2 possible signals in such an outcome, so that the model given here is indeed the simplest possible model exhibiting equilibrium outcomes with present characteristics.}

In Section 2 we describe the basic economic environment. In Section 3 we provide the benchmark economy with correct assessments and the full exchange equilibrium. In Section 4 we describe the autarky equilibrium. In Section 5 we comment on the disagreement features of autarky and show how a central planner might be able to induce a switch from the inefficient autarky regime to the full exchange equilibrium in a way that balances the budget. In Section 6 we provide a more general discussion of the predictions of our model vis-à-vis richer information, and a discussion of our welfare measures, relative to the models with heterogeneous priors. Proofs are collected in the appendix.
2 The economic environment

There are three possible states of the economy: an intermediate state and two extreme states – low and high. We imagine the two extreme states as the low and the high points of the business cycle, and the intermediate state as the medium point of the business cycle. This is, however, for purely interpretative purposes: in a moment we shall assume that there is no aggregate risk.

There are two individuals, both of whom can either remain in autarky or engage in exchange. If both individuals choose autarky, then they both receive the same payoffs in all states and there is no idiosyncratic risk. In exchange, the two individuals differ. We imagine Individual 1 as the representative agent of the productive sector of the economy who prefers the intermediate, or normal, state of the economy. Individual 2 is the representative agent of the speculative, or financial, sector of the economy and prefers the extreme states of the economy. When both individuals choose to exchange, both can engage in a higher level of economic activity, but that brings some idiosyncratic risks. Non-autarchic engagement by either individual raises the aggregate welfare in the economy so that it is socially desirable, and full exchange by both individuals maximizes social welfare.

If an individual chooses exchange, we imagine that as a bet that one of her favorable states will perspire. Such a bet is on average still better for the individual than remaining in autarky, even if the other individual chooses autarky. If an individual chooses autarky while the other chooses to exchange, then the autarchic individual is penalized in a state that is unfavorable to her. As a simplifying assumption, in any outcome each individual obtains the same payoffs in the high and in the low states of the economy: to Individual 1 the extreme states are equally unfavorable;  

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8For example, the productive Individual 1 may either engage in a low level of production, or take a collateralized loan which she may only be able to repay fully in a favorable state. She is therefore facing some idiosyncratic risks that she might, to a certain extent, be able to trade away by buying insurance against the unfavorable states.
to Individual 2 the extreme states are equally favorable. Additionally, when both individuals choose to exchange, the aggregate welfare in the economy is normalized to 1 – in the socially optimal outcome of full exchange there is no aggregate risk.

Before an individual decides whether to remain in autarky or not, she observes a private signal regarding the state of the economy. We assume that the low and the high states are equally likely, and that the intermediate state is as likely as the two extreme states combined. The signals are as follows: in the high state both individuals receive a high signal, in the low state both individuals receive a low signal, and in the intermediate state one individual receives a high signal while the other receives a low signal, i.e., their signals are perfectly negatively correlated. Therefore, if the information of both individuals were aggregated, the state of the economy would be known with certainty. The timing and the information structure are as follows:

- The state of the economy \( \omega \) is drawn, \( \omega \in \{\omega_L, \omega_M, \omega_H\} \), where \( \bar{Pr}(\omega_L) = \bar{Pr}(\omega_H) = \frac{1}{3} \), and \( \bar{Pr}(\omega_M) = \frac{1}{2} \).

- Each individual \( i \) receives a private signal \( \theta_i \) regarding the state of the economy, \( \theta_i \in \{\theta_L, \theta_H\} \), where \( \bar{Pr}(\theta_i = \theta_H \mid \omega_H) = \bar{Pr}(\theta_i = \theta_L \mid \omega_L) = 1 \), \( \bar{Pr}(\theta_i = \theta_H \mid \omega_M) = \bar{Pr}(\theta_i = \theta_L \mid \omega_M) = 0 \), and for \( \theta_i = \theta_L \) and \( \theta_j = \theta_H \), \( \bar{Pr}(\theta_i = \theta_L, \theta_j = \theta_H \mid \omega_M) = \bar{Pr}(\theta_i = \theta_L, \theta_j = \theta_L \mid \omega_M) = \bar{Pr}(\theta_i = \theta_L, \theta_j = \theta_H \mid \omega_M) = \bar{Pr}(\theta_i = \theta_H, \theta_j = \theta_L \mid \omega_M) = 0 \).

After the individual has received her signal, she decides whether to remain in autarky or not. As described above, unless both choose autarky, the individual obtains higher payoffs in a favorable state than in an unfavorable state, and these payoffs depend on the actions of the other individual. State \( \omega_M \) is favorable to Individual 1, while states \( \omega_L \) and \( \omega_H \) are favorable to Individual 2. The payoffs to the individuals are specified as follows:

- If both individuals remain in autarky, then the individual obtains a payoff \( y^* < \frac{1}{2} \) regardless of the state of the economy.
• If both individuals choose to exchange, then the individual obtains a payoff $x^*$ in a favorable state and $1 - x^*$ in an unfavorable state, where $1 > x^* > \frac{1}{2} > 1 - x^* > 0$. While there is no aggregate, risk there is idiosyncratic risk.

• If the individual is in autarky while the other is not, the autarchic one obtains a payoff $\bar{y}$ in an unfavorable state and $y$ in favorable state, where $\bar{y} < y < 1 - x^*$.

• If the individual is non-autarchic, while the other remains in autarchy, then the non-autarchic individual obtains a payoff $\bar{x}$ in an unfavorable state and $\bar{x}$ in a favorable state, where $\bar{x} \leq y^* \leq \bar{x}$.

• We assume that it is socially optimal for both individuals to exchange, i.e., $\bar{y} + x < 1$ and $\bar{y} + \bar{x} < 1$, and as mentioned before, $y^* < \frac{1}{2}$.

• We assume that it is on average better for the individual to exchange if the other individual is autarchic, which we write as:

$$\bar{x} - y^* = \alpha (y^* - \bar{x}),$$

where $\alpha > 1$.

To summarize, we can represent an individual $i$’s payoffs by the following payoff matrices ($i$’s actions are in rows, and $j$’s actions are in columns):

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Before an individual chooses autarky or exchange, she observes her private signal regarding the state of the economy. The combination of signals to both individuals completely determines the state, and the state that is favorable to one individual is
unfavorable to the other. We can therefore represent the problem that the two individuals face as the following normal-form game with uncertainty, where Individual 1 is the row player 1 and Individual 2 is the column player 2.

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<th>$(\theta_H, \theta_L)$</th>
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<td>$exc_1$</td>
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In what follows, we call the outcome where both individuals choose exchange regardless of their signal full exchange. We call the outcome where both individuals choose autarky regardless of their signal autarky.

Under our assumptions, the probability distribution $\bar{Pr}$ over the states of the economy $\Omega$ and the signals $\Theta$ is such that the probability of each of the four possible combinations of signals to the two individuals is uniform. The signal to an individual is entirely uninformative as to whether the economy is in a favorable or unfavorable state. That is, regardless of what her signal is, the state of the economy is equally likely to be favorable or unfavorable to her. Inadvertently, this is the setting with the highest possible dispersion of information.

### 3 Benchmark: economy with correct information

In the first benchmark case, suppose that the state of the economy were known to both individuals at the moment when they make their decisions. Then, it is immediate that in each state, the unique Nash equilibrium outcome is for both individuals to choose exchange. For example, in $\omega_L$, $exc_2$ is strictly dominant for
Individual 2, to which $exc_1$ is the only best reply by Individual 1. Therefore, if the information of the two individuals were aggregated prior to their decisions, full exchange would be the unique equilibrium outcome of this economy.

In a slightly more realistic second benchmark case, both individuals hold correct assessments over uncertainty. While an individual only observes her signal, she knows the likelihoods of the three states, and she knows the conditional probabilities with which individuals obtains their signals in each state. Given a strategy profile, the individual also knows the strategy of the other individual. As will be demonstrated in the next section, the assumption that the individuals know all that might in some cases not necessarily be unrealistic. In general, the individuals hold assessments over the likelihoods of states of the economy, signals, and the other individual’s strategy.

We show that the socially desirable outcome of full exchange is an equilibrium. This equilibrium is robust to the precise assumptions regarding the individuals’ assessments over uncertainty. Under our assumptions on the parameters of the economy, when both individuals’ assessments over uncertainty are correct, then at the stage when the individuals choose their actions they agree that full exchange is efficient from each individual’s perspective. That is, when an individual knows her own signal but does not yet know the full realization of the state of the economy, her expected payoff is highest under full exchange.

Since each individual’s choice is contingent on her private signal, an individual’s strategy is a contingent plan of action, $s_i : \Theta_i \rightarrow A_i$, where $A_i = \{exc_i, aut_i\}$ is the set of actions of individual $i$. She chooses an action for each of her signals so that $s_i(\theta_i)$ is the chosen action of individual $i$ when her signal is $\theta_i$. We denote by $S_i$ the set of individual $i$’s pure strategies; by $\Delta(S_i)$ we denote the set of $i$’s mixed strategies.\textsuperscript{9}

When the individuals’ assessments are correct, the appropriate notion of equilibrium is essentially Bayes-Nash equilibrium. When an equilibrium is independent of the

\textsuperscript{9}In this paper, we focus on pure strategies, as those suffice to make our point; mixed strategies are important here only for the definition of equilibrium.
individuals’ assessments, such an equilibrium is robust.\textsuperscript{10} In the following definition, \( u_i(a; \omega) \) denotes the utility of individual \( i \) when the action profile is \( a \) and the state of the economy is \( \omega \). In the following definitions, an outcome is the probability distribution over the states of the economy and the individuals’ signals, here given by \( \bar{Pr} \), along with the strategy profile \( s^* \).

\textbf{Definition 1.} An outcome \((\bar{P}, s^*)\) is a Bayes-Nash equilibrium outcome, if,

\[
\sum_{\theta_j \in \Theta_j, \omega \in \Omega} \bar{P}(\omega, \theta_j | \theta_i) u_i(s_i^*(\theta_i), s_j^*(\theta_j); \omega) \geq \sum_{\theta_j \in \Theta_j, \omega \in \Omega} \bar{P}(\omega, \theta_j | \theta_i) u_i(s'_i(\theta_i), s_j^*(\theta_j); \omega),
\]

\( \forall \theta_i \in \Theta_i, \forall s'_i \in \Delta(S_i) \).

An outcome \((\bar{P}, s^*)\) is a robust equilibrium outcome, or an \textit{ex-post} equilibrium outcome if:

\[
u_i(s_i^*(\theta_i), s_j^*(\theta_j); \omega) \geq u_i(s'_i(\theta_i), s_j^*(\theta_j); \omega),
\]

\( \forall \omega \in \Omega \) and \( \forall \theta_i \in \Theta_i, \theta_j \in \Theta_j \), such that \( \bar{P}(\theta_i, \theta_j | \omega) > 0 \), and \( \forall s'_i \in \Delta(S_i) \).

Our second definition refers to the efficiency of outcomes from an individual’s perspective.\textsuperscript{11}

\textbf{Definition 2.} Under correct assessments, an outcome \((\bar{P}, s^*)\) is \textit{i-efficient} relative to the outcome \((\bar{P}, s')\) if:

\[
\sum_{\theta \in \Theta, \omega \in \Omega} \bar{P}(\omega | \theta) u_i(s_i^*(\theta_i), s_j^*(\theta_j); \omega) \geq \sum_{\theta \in \Theta, \omega \in \Omega} \bar{P}(\omega | \theta) u_i(s'_i(\theta_i), s'_j(\theta_j); \omega).
\]

The outcome \((\bar{P}, s^*)\) is \textit{i-efficient} if \((\bar{P}, s^*)\) is \textit{i-efficient} relative to \((\bar{P}, s')\), \( \forall s'_i \in \Delta(S_i), s'_j \in \Delta(S_j) \).

\textsuperscript{10}This definition of robust equilibrium corresponds to the definition of \textit{ex-post} equilibrium in Bergemann and Morris 2005.

\textsuperscript{11}Efficiency described here is \textit{ex-ante} efficiency from an individual’s perspective, similar to Wilson 1978. When an outcome is efficient for both individuals then it maximizes the \textit{ex-ante} welfare. In a general game, there may be \textit{ex-ante} efficient outcomes in the sense of Holmstrom and Myerson 1983, i.e., outcomes that are Pareto-efficient in the \textit{ex-ante} sense, which are not efficient for either individual.
We now give our benchmark results. First, when the individuals hold correct assessments, autarky is not an equilibrium, while full exchange is a robust equilibrium of this economy. Second, under correct assessments, after observing her signal, each individual strictly prefers full exchange over any other possible outcome.

**Proposition 1.** In the above economy, under correct assessments $\bar{Pr}$, we have the following:

1. Full exchange is a robust equilibrium outcome.
2. Autarky is not a Bayes-Nash equilibrium outcome.
3. Full exchange is $i$-efficient for each individual $i \in N$.

## 4 Information, autarky, and subjective welfare

In this section, we show that when individuals hold certain assessments over uncertainty which are different from the truth, autarky is an equilibrium of this economy. Moreover, under such equilibrium assessments, autarky is by each individual subjectively perceived as efficient relative to full exchange.

When the individuals’ assessments are different, there are several issues that we must tackle. First, under what assumptions and circumstances is it possible for the individuals to hold different assessments over uncertainty? Second, what is meant by equilibrium when the individuals’ assessments are different? Third, how to assess the efficiency of different outcomes from an individual’s perspective? These issues are intimately connected.

If each individual were allowed to hold any assessment over uncertainty whatsoever (as argued by, e.g., Morris 1995), then, on the one hand, such assessments might be inconsistent with the actual probability distribution over the parameters that the individual observes. On the other hand, if an individual is assumed to observe (in
a probabilistic sense) all the parameters at the ex-post stage (i.e., the realized state of the world, the signals, as well as the actions chosen), then her assessment is completely determined and must equal the truth. That is, her assessment must be correct, in which case Proposition 1 above applies. That individuals should be able to observe all these parameters is unrealistic. Nevertheless, we require that their assessments are anchored by some aspects of the objective truth that the individuals can observe.

To begin with, we assume that the individuals know the payoff structure. That is, they know what payoffs each would obtain in a given state for a given profile of actions. They also know the information structure. That is, they know the set of possible states of the economy and the set of possible signals in each state. Thus, they know that there is no aggregate uncertainty, i.e., the combination of individuals’ signals completely determines the state of the economy. But they do not know the probability distribution over the states nor the conditional distribution over the signals in each state.

We assume that each individual knows her own signals and actions chosen, and then observes the variation in her own payoffs. But an individual doesn’t directly observe either the other individual’s signals or the likelihoods of the draws of the state of the world. She also doesn’t directly observe the other individual’s actions so that she in general doesn’t observe the other individual’s strategy. From the resulting probability distribution over her own payoffs, the individual can possibly deduce the distribution over the uncertain parameters. Or, if there is no variation in

\footnote{For instance, the individuals know that it is not possible that in a low state, one would receive a high signal while the other received a low signal.}

\footnote{We could additionally assume that each individual also observes the actions chosen by the other, and in the present example that would make no difference. Either in autarky or in full exchange, an individual can deduce the other’s strategy – in these outcomes the individuals’ contingent choices of actions are signal-independent and there are no other action profiles that would yield the same payoffs to the individual. In general, even if we assumed that the individuals observed one-another’s actions, that would still not imply that they directly observe one-another’s strategies: e.g., if an individual observed the other individual choose exc and aut each with probability \( \frac{1}{2} \), that could still in principle be a result of a variety of different strategies.}
her own payoffs, she faces an information recovery problem, in which case there is a variety of assessments over uncertainty that she can make.

We require that each individual’s assessment over uncertainty and the other individual’s strategy is consistent with the joint distribution over her own signals, chosen actions, and payoffs. Each individual then optimizes relative to her assessment. An individual $i$’s assessment is denoted by $(Pr^i, s^i)$, where $Pr^i$ is a joint distribution over the states of the economy and the signals to the individuals, and $s^i \in \Delta(S_1) \times \Delta(S_2)$ is the strategy profile; $V_i = \{y, x, y^*, \bar{x}, \bar{y}, 1-x^*, x^*\}$ denotes the set of possible payoffs to individual $i$. Given an outcome $(Pr, s)$, denote by $\hat{Pr}_{\Theta_i, A_i, V_i}[Pr, s]$ the induced joint probability over the individual $i$’s signals, actions, and payoffs.\footnote{That is, for $\theta_i \in \Theta_i, a_i \in A_i, v_i \in V_i$,} We first define consistency of an individual’s assessment.

**Definition 3.** An individual $i$’s assessment $(Pr^i, s^i)$ is $i$-consistent with the outcome $(Pr, s)$ if $\hat{Pr}_{\Theta_i, A_i, V_i}[Pr^i, s^i] \equiv \hat{Pr}_{\Theta_i, A_i, V_i}[Pr, s]$.

For example, take the full exchange outcome, and take individual 1. When she receives the signal $\theta_L$, she takes the action $exc_1$. She then observes her payoff $x^*$ with the likelihood $\frac{1}{2}$, and that could only perspire if the state had been $\omega_M$ and individual 2 took the action $exc_2$; She observes her payoff $1-x^*$ with the likelihood $\frac{1}{2}$ and that could only perspire when the state was $\omega_L$ and individual 2 took the action $exc_2$. Hence, she deduces that, conditional on her signal, the states $\omega_L$ and $\omega_H$ occur with equal likelihoods, and that individual 2 observes her signals $\theta_L$ and $\theta_H$ with equal likelihoods. Similarly, when individual 1 receives the signal $\theta_H$, she deduces that the states $\omega_L$ and $\omega_H$ occur with equal likelihoods, and that individual 2 observes her signals $\theta_L$ and $\theta_H$ with equal likelihoods. Since individual 1 observes each of her signals with a likelihood $\frac{1}{2}$, it follows that she must correctly assess the
other individual’s strategy, and the likelihoods of each state and of each signal. A
similar argument is true for individual 2. Therefore, in full exchange each individual
can only hold a correct assessment.

Next we define what is meant by an individual to play optimally given her assessment.

**DEFINITION 4.** An individual $i$’s assessment $(Pr^i, s^i)$ is $i$-incentive-compatible, $i$-IC, if:

$$\sum_{\theta_j \in \Theta_j, \omega \in \Omega} Pr^i(\omega \mid \theta_i, \theta_j) u_i(s^i_i(\theta_i), s^i_j(\theta_j); \omega) \geq \sum_{\theta_j \in \Theta_j, \omega \in \Omega} Pr^i(\omega \mid \theta_i, \theta_j) u_i(s'_i(\theta_i), s'_j(\theta_j); \omega),$$

$\forall \theta_i \in \Theta_i, \forall s'_i \in \Delta(S_i)$.

In equilibrium, each individual must also be able to justify the observables as
resulting from optimal behavior relative to such a consistent assessment by the other
individual. This is true by default in the benchmark case of the previous section:
if both individuals’ assessments are correct, then in particular, an individual can
justify the outcome by asserting that the other individual’s assessment is correct and
that the other individual is optimizing, and so on. The next definition of equilibrium
satisfies these requirements.\(^\text{15}\)

**DEFINITION 5.** An outcome $(Pr, s)$ is a player equilibrium outcome if there exist
assessments $(Pr^1, s^1)$ and $(Pr^2, s^2)$ such that,

1. $(Pr^i, s^i)$ is $i$-IC, $i = 1, 2$,

2. $(Pr^i, s^i)$ is $i$-minimally consistent with $(Pr, s)$, $i = 1, 2$, and

---

\(^{15}\)This definition of player equilibrium is from Čopić 2014a, where the epistemic foundations of
equilibrium are given. Player equilibrium is characterized by a common belief in minimal consistency
and incentive compatibility and a common belief in player’s assessments – see Theorem 4 in Čopić
2014a. Player equilibrium can be thought of as a stable point of a process whereby individuals
have accumulated an infinite amount of aforementioned data from equilibrium behavior and can
similarly justify the others’ behavior.
3. \((Pr^j, s^j)\) is \(j\)-minimally consistent with \((Pr^i, s^i)\), \(i, j = 1, 2\).

Any such assessments \((Pr^i, s^i)\) are called supporting assessments. Given a player equilibrium outcome \((Pr, s)\), we denote by \(O_{Pr,s}^{Pr,s}\) the set of all possible supporting assessments for individual \(i\), \(O_{Pr,s}^{Pr,s} = \{(Pr^i, s^i) / \exists (Pr^j, s^j) \text{ s.t. 1-3 hold}\}\).

When \(|O_{Pr,s}^{Pr,s}| = 1\) for both individuals we say that the outcome \((Pr, s)\) is fully recoverable. Of course, a fully recoverable outcome is a Bayes-Nash equilibrium outcome, since the individuals’ assessments must in that case be correct. By the above discussion, full exchange is fully recoverable.

A player equilibrium is therefore an outcome where individuals can hold assessments over uncertainty such that: (i) each individual is optimizing, (ii) each individual can justify her assessment as being consistent with what she can observe, and (iii) each individual maintains that the other individual’s assessment must be consistent with her own. The first equilibrium requirement is indisputable. The second requirement is indisputable if equilibrium is thought of as a stable point of a process whereby the individuals have accumulated the data from equilibrium behavior. The third requirement merits some explanation.

The interpretation is that, from an individual’s perspective, her assessment is the truth, and the assessment that she imputes on the other individual must be consistent with that truth. Under requirement (iii), even if both individuals communicated their assessments to one another, they would still have no reason to change them, and requirement (iii) guarantees that the individuals can justify one another’s behavior in an infinite regress – formally, there is a common belief in consistency and optimality. Requirement (iii) therefore guarantees that equilibrium is devoid of myopia, unsophistication, or any other sense of irrationality. Requirement (iii) guarantees

\[^{16}\text{The way that the individuals can justify one another’s behavior is very similar to the way they can do so in a Bayes-Nash equilibrium outcome. For example, if individual 1 considers the data she observes, then she is optimizing. Insofar as she can tell, her assessment is the truth. By requirement (iii), individual 2’s assessment is consistent with that truth, and individual 2 is optimizing.}\]
that even if the individuals communicate their assessments to one another they will still have no reason to change such assessments.

When the supporting assessments of both individuals coincide with the truth, the resulting player equilibrium outcome is a Bayes-Nash equilibrium outcome. For example, that is the case when the outcome is fully recoverable. As we observed above, full exchange is fully recoverable. Therefore, full exchange is a player equilibrium outcome where the only supporting assessments are such that both individuals hold correct assessments over uncertainty. We summarize these statements in the next proposition.

**Proposition 2.** Full exchange \((\bar{P}_r, s^*)\) is a fully recoverable player equilibrium outcome, and the only supporting assessments are given by \((Pr^i, s^i) = (\bar{P}_r, s^*), i = 1, 2\).

While autarky is not a Bayes-Nash equilibrium outcome, it is an equilibrium outcome in the sense of player equilibrium. It is not fully recoverable, so that there is for each individual a set of possible supporting assessments, and autarky is only supportable by assessments that are different. We formally state this in the following proposition. We denote the autarky outcome by \((\bar{P}_r, \bar{s})\).

**Proposition 3.** Autarky \((\bar{P}_r, \bar{s})\) is a player equilibrium outcome. The supporting assessments \((Pr^i, s^i)\) are such that \(s^i \equiv \bar{s}\), and \(Pr^i\) are given by \(Pr^i_{\Theta_j}(\theta_L) = \bar{P}_{r\Theta_i}(\theta_L) = \frac{1}{2}, Pr^i_{\Theta_j}(\theta_H) = \bar{P}_{r\Theta_i}(\theta_H) = \frac{1}{2}, i, j \in \{1, 2\}\), and,

---

By requirement (iii), individual 1 may assert that individual 2 could impute on individual 1 the individual 1’s actual assessment, whereby individual 1 optimizes and is consistent if the individual 2’s assessment were the truth, and so on, *ad-infinitum*. If requirement (iii) here is dropped then the resulting notion would coincide with the Self-confirming equilibrium for games with uncertainty as defined in Dekel et al 2004. See Čopić 2014a, 2014b for a more detailed discussion of the epistemic foundations and other related notions of equilibrium. Note that a player equilibrium outcome only requires the existence of such supporting assessments but is silent whether or not the individuals in fact hold such assessments.
By Proposition 3, the individuals’ supporting assessments in autarky must be different. That is, in autarky it is common knowledge among the individuals that their assessments are different. Nevertheless, both individuals are still perfectly rational, consistent, and sophisticated. Whatever the truth might be, their assessments are consistent with the truth given what they can observe, and the individuals can justify one-another’s behavior. In autarky, there are a number of individuals’ assessments satisfying Definition 5. By Proposition 3, these are given by,

\[
O_{1}^{Pr,s} = \{ Pr^1(\theta_L,\theta_L), Pr^1(\theta_H,\theta_H) \leq \frac{1}{1+\alpha}, Pr_{\Theta_1}(\theta_L) = Pr_{\Theta_1}(\theta_H) = \frac{1}{2} \}, \quad (3)
\]

\[
O_{2}^{Pr,s} = \{ Pr^2(\theta_L,\theta_L), Pr^2(\theta_H,\theta_H) \geq \frac{\alpha}{1+\alpha}, Pr_{\Theta_2}(\theta_L) = Pr_{\Theta_2}(\theta_H) = \frac{1}{2} \}. \quad (4)
\]

From the objective perspective, i.e., given the true distribution over uncertainty \(\bar{Pr}\), the question of efficiency of an outcome to an individual remains the same. But the objective perspective might no longer be valid: given their data, the individuals or a central planner are in autarky unable to recover the objective uncertainty. Whether an outcome is efficient or not to an individual depends on the individual’s assessment over uncertainty. Still, in equilibrium an individual’s possible assessments must satisfy conditions (i)-(iii). Therefore, to evaluate efficiency of an equilibrium outcome from an individual’s subjective perspective, only her possible equilibrium supporting assessments should be considered.

**Definition 6.** A player equilibrium outcome \((Pr, s)\) is subjectively \(i\)-efficient relative to an outcome \((Pr, s')\) if \((Pr^i, s^i)\) is \(i\)-efficient relative to \((Pr^i, s')\), \(\forall (Pr^i, s^i) \in \)
The outcome \((Pr, s)\) is subjectively efficient relative to \((Pr, s')\) if it is subjectively \(i\)-efficient for both individuals.

Therefore, an equilibrium outcome is subjectively \(i\)-efficient relative to another outcome if it is \(i\)-efficient, regardless of what equilibrium supporting assessment \(i\) might hold – the individual only evaluates the efficiency by the assessments pertaining to the outcome \((Pr, s)\). The interpretation is that, when an individual finds herself in an outcome \((Pr, s)\), it is in principle not known what assessment the individual holds so that the efficiency of that outcome vis-à-vis another outcome must be considered for any possible assessment she might hold under \((Pr, s)\). Any possible assessment that she might hold in that outcome leads the individual to conclude that the outcome is, from her perspective, efficient. Of course, if an outcome is fully recoverable, then the subjective \(i\)-efficiency and the \(i\)-efficiency evaluations coincide.

We now consider the subjective efficiency of autarky relative to full exchange under the assessments specified in Proposition 3. We show that for a wide set of parameters, autarky is subjectively \(i\)-efficient for both individuals.

**Proposition 4.** Suppose that \(\alpha(1 - x^*) + x^* \leq (1 + \alpha)y^*\), where \(\alpha = \frac{y - y^*}{y^* - x^*}\). Then, autarky is subjectively efficient relative to full exchange.

We summarize our narrative up to this point as follows: should the individuals find themselves in autarky, such an outcome is supportable in equilibrium; furthermore, under any equilibrium assessments supporting autarky, each individual is convinced that, given her implied expected payoff, autarky is preferable to full exchange. In contrast, under the objective uncertainty \(Pr\), full exchange is preferred by both individuals.
5 Disagreement and policy intervention

As discussed above, an equilibrium can be imagined as an outcome where the individuals optimize and have consistent assessments, and where each individual’s assessment can take the role of the truth so that the other individual’s assessment must be consistent with that truth. Additionally, their supporting assessments have the property that, even if they had communicated their assessments to one another, neither one would have any reason to change her assessment. In what follows, denote by $U_{Pr,s}^i$ the (subjective) expected payoff that individual $i$ obtains in the outcome $(Pr,s)$.

When $\alpha(1 - x^*) + x^* < (1 + \alpha)y^*$, under any assessment that the other individual $j$ might hold in autarky, $i$ would have preferred full exchange over autarky – the argument for this follows the argument in the proof of Proposition 4. We thus have the following relationships:

$$U_{Pr,s}^i > U_{Pr,s}^{i,j}, \forall Pr \in O_{Pr,s}^i, \forall Pr' \in O_{Pr,s}^j, \forall i, j \in \{1, 2\}, j \neq i.$$ (5)

For example, consider Individual 1. Under any assessment that she may hold in autarky, her subjective evaluation is that autarky delivers superior expected gains to her than full exchange; under any assessment that Individual 2 may hold in autarky, Individual 1’s expected gains would be higher in full exchange than in autarky. In autarky, the assessments of Individual 1 project that if the two individuals were to switch regime to full exchange, Individual 2 would reap more than all the social gains from such a switch in regime. In the exact same way, the relationships given by (5) describe welfare considerations of Individual 2.

Autarky therefore supports disagreement in a very strong sense of subjective efficiency: in autarky, the individuals disagree in their subjective assessments of uncertainty and both are convinced that autarky is more efficient than full exchange.
Player equilibrium survives the criterion that the individuals communicate their assessments to one another. Hence, the individuals can maintain their disagreement even if their assessments are common knowledge.

Due to the relationships given by (5), in autarky there is disagreement among the individuals in an even stronger sense. Consider a narrative whereby Individual 2 were to suggest that the two individuals should jointly deviate to full exchange. Individual 1 might then make the following argument: “I must conclude that you have realized that I have made the correct estimate, i.e., that my assessment is the truth. In that case, you will be better off in the full exchange regime. However, I will be worse off, so that the only reason why you are suggesting the switch to full exchange is out of your self-interest, which is contrary to my interest.”

In autarky, there is disagreement about the redistributive properties of equilibrium. This makes it impossible for the individuals to jointly switch to the robust and efficient equilibrium outcome of full exchange. As described above, the reason is that in autarky, under any assessment that an individual might hold, she is convinced more than the total social gain from the switch in regime will go to the other individual. Ironically, under the objective truth, full exchange is preferred over autarky also from the perspective of each individual (Proposition 1). Since full exchange is fully recoverable, were the individuals to switch to full exchange, they would both be in complete agreement about that being preferrable.

From the policy perspective, how costly is it for the central planner to induce the individuals to switch from autarky to full exchange? An individual would under any circumstance be willing to switch to the efficient regime if the central planner were to provide a subsidy which compensated the individual under her assessment

\footnote{Here we assume that the central planner observes, for example the total sum of individuals' expected payoffs, i.e., the GDP of this stylized economy. If the central planner observed nothing at all about the interaction, then the central planner's problem makes no sense. We could also assume that the central planner observes the distribution over the individuals' actions, as in Copić 2014b), and that would make no difference in the ensuing argument.}
that was most favorable to autarky. The individual’s assessment is most favorable to autarky when she assigns all the probability mass to the unfavorable states – such an assessment is an equilibrium assessment by (3) and (4). We therefore have the following proposition:

**Proposition 5.** Suppose $\alpha(1 - x^*) + x^* < (1 + \alpha)y^*$. Then, for the individuals to surely switch from autarky to full exchange, the central planner must provide each with a subsidy in the amount of $y^* + x^* - 1$.

Proposition 5 gives conditions under which the individuals switch from autarky to full exchange even under the assessments supporting the autarky, which are most favorable to autarky. Of course, in practice, the individuals might be less pessimistic than that, and smaller inducements might suffice.

In the present example there is no aggregate risk under full exchange. The central planner could then, instead, subsidize the individuals contingent on their payoffs under full exchange and thus balance the budget. For example, the central planner could promise each individual to compensate her in case of a loss, and in turn take some fraction $\beta$ of the individual’s extra earnings in case of a gain. Since full exchange is efficient while autarky is not, it follows that for the central planner to balance the budget, $\beta < 1$, so that both individuals should presumably be eager to take such a bet. The central planner could thus induce the change from the expectations-driven inefficient autarky to the efficient full exchange equilibrium.

We have here further simplified our model by assuming that the payoffs to the individuals are symmetric in the sense that in a state favorable to Individual 1, she obtains payoffs that are exactly like the payoffs obtained by Individual 2 in a state favorable to her. Of course, in reality this need not be the case. But it is quite evident that if the sort of conditions described in propositions 3 and 4 hold for both individuals, all our results remain essentially the same – this simplification has here been made for purely expositional reasons.
There is one sense in which this simplification is relevant: if the amount of the necessary subsidy by the central planner varies across the individuals, then she will still be able to recoup the subsidy on average, but not necessarily in every state of the economy. That occurs if one individual requires a larger subsidy while the other has a smaller gain in the full exchange equilibrium. But this is indeed no more problematic than the symmetric case: the only reason for the central planner to be unable to provide a subsidy in either case is if the central planner is unable to borrow in the short run.

Of course, such policy interventions presuppose that the central planner were able to enforce the change in the individuals’ behavior. If that were not the case, both individuals might simply collect their subsidies, remain in autarky, and leave the central planner with a broken budget. Suppose that for whatever reason, the central planner is unable to recoup the subsidy, or that the amount of subsidy is relevant in the short run, before it can be recovered. Then the magnitude of idiosyncratic risk in the efficient outcome matters. That is, the lower the difference $x^* - (x^* - 1)$, the lower the magnitude of idiosyncratic risk, and of course, the lower the amount of subsidy $x^* + y^* - 1$. Note that there can be some idiosyncratic risk for the necessary subsidy to equal zero – by Proposition 4, when $x^* < \frac{\alpha(1+\alpha)}{\alpha-1}$, autarky is no longer subjectively efficient.

Everything else equal, what determines the magnitude of idiosyncratic risk is how much of it can be traded away. We do not here explicitly model the manner in which the idiosyncratic risk is traded away. But we can revisit our interpretation of Individual 1 as the productive sector and Individual 2 as the financial sector of the economy. Suppose, then, that the way in which Individual 1 engages in exchange is by taking a collateralized loan, where her production serves as a collateral. We can then imagine $\omega_L$ as the state in which her realized production is low, and $\omega_H$ as the state in which the demand for her product is low.\textsuperscript{18} Under normal circumstances, in

\textsuperscript{18}Under the interpretation that $\omega_H$ is an extremely good state of the economy, that can be the
the state \( \omega_M \), Individual 1 makes large profits. A specific way in which the Individual 1 may be able to trade away the idiosyncratic risk is by insuring against the states \( \omega_L \) and \( \omega_H \). For example, she might go short some number of call options on her product at a high price (to insure against \( \omega_H \)), and go long some number of put options at a low price (to insure against \( \omega_L \)). For the financial sector, making the collateralized loan to the productive sector is on average more efficient than a safer investment under autarky, e.g., into government bonds.

In what amount Individual 1 can insure against the extreme states of the economy then depends on the availability and prices of such possible insurance schemes. That is, it depends on the counterpart of that trade taken up by Individual 2, i.e., the financial sector. The sort of examples that fit the present model are those where the idiosyncratic risk cannot be traded away entirely. One explanation for that is that the prices of insurance schemes are such that at least one of the individuals would only be willing to trade away some of the idiosyncratic risk. In that sense, our example here is stylized, as we do not model the prices in the market for idiosyncratic risk explicitly. We take it as a premise that, in the economy studied here, some residual idiosyncratic risk is a property of the efficient full exchange equilibrium outcome.

Nevertheless, in some cases, where the reasons for residual idiosyncratic risks are of institutional nature, there may exist remedies for the problems described in this paper. If the residual idiosyncratic risks is a result of poor access to financial markets by the productive sector, then improving the ability of the productive sector to trade away its risks would remedy the problem described here. Alternatively, if such residual idiosyncratic risk stemmed from issues with pricing, e.g., a collusive scheme or monopolistic power in one of the sectors of the economy, then ensuring more competition in that sector would likely resolve the problem. In the most favorable situation, all the idiosyncratic risk in the efficient and robust equilibrium of full case for example if for the product of Individual 1, there is a substitute commodity, which is a giffen good.
exchange can be traded away. In that case \( x^* = \frac{1}{2} \) and autarky is evidently no longer individually efficient under \textit{any} assessments; it is also no longer a player equilibrium outcome. Once there are no limits to trading away idiosyncratic risks in the full exchange equilibrium, then the subject of the present study has inadvertently run its course.

6 Discussion: predictions and welfare

Many recent prominent in finance involve agents with heterogeneous beliefs. In these models, the agents’ beliefs are heterogeneous due to some pre-specified distortion – a bias or some other sort of irrationality, such as optimism or pessimism. One prediction that these models share is that when agents have heterogenous beliefs, and there is new information, that induces more trade and more disparate beliefs. This is the case in Miller 1977, Harrison and Kreps 1978, and more recently, Scheinkman and Xiong 2003, Fostel and Geanakoplos 2008, Geanakoplos 2010, Angeletos and La’O 2013, and Simsek 2013. In the context of the present model such a prediction is impossible. More generally, in the context of an equilibrium model, which is a stable point with individuals who accumulate an infinite amount of stationary data and do not suffer from any behavioral problems, such a prediction is impossible. Here is why.

Assume that the individuals’ assessments of uncertainty are a result of an asymptotically consistent estimation procedure. Then the only possible circumstance under which their assessments can be heterogeneous is when these individuals’ datasets are comprised of partial observations of the outcome. In that case, the individuals face recovery problems, and there are many assessments that satisfy asymptotic consistency with the true probability distribution over the uncertain parameters. As shown, there may exist equilibrium outcomes, which are only supportable under such differing assessments. In the present economy, autarky is such an equilibrium
outcome.

In the context of the present model we must imagine public information as meaning that the individuals observations of the outcome are richer, e.g., an individual may also observe the payoffs to the other individual. But richer information necessarily implies that the recovery problem faced by the individuals is diminished. Hence, for each outcome the set of consistent assessments of an individual is smaller so that the individual’s incentive constraints become more difficult to satisfy and justify. Consequently, fewer outcomes are supportable in equilibrium and the individuals’ assessments are closer to the truth, and must also be closer to one another. Therefore, public information cannot possibly make the individuals’ assessments more disparate but must necessarily make them more similar. Hence, in any model where trade is a result of the individuals’ assessments being disparate, the release of public information will result in less trade, rather than more.

Of course, in the present case, due to externalities, exchange occurs when the individuals’ assessments coincide, and does not occur when the individuals’ assessments are disparate. Public information will then increase the amount of trade in the sense that with sufficiently rich information, autarky will no longer be supportable in equilibrium. However, exchange is then the result of a precisely opposite effect of information: that the individuals’ assessments have become more similar, in fact, the same. We could also construct examples where more information results in a social-welfare inferior allocation, but the fact that more information reduces disparity in beliefs is unavoidable in the context of the present notion of equilibrium.

The second distinction between the present model and the models of heterogeneous beliefs is in the evaluation of welfare. In a recent study, Brunnermeier, Simsek and Xiong 2013 propose a measure of welfare for models with distorted beliefs. They propose a belief neutral measure, which takes into account the expected welfare under any convex combination of agents’ beliefs.\footnote{The motivation for their welfare measure comes partly from the literature in the decision theory} Since beliefs are determined ex-
ogenously, such a measure makes sense from a perspective of a neutral social planner. Such a measure can also be thought of as a Pareto criterion based on any beliefs that agents might hold.

In the present model, the individuals’ assessments are determined in equilibrium. It no longer makes sense to compare outcomes *per se* – where an outcome is defined as \((Pr, s)\) – since some of these outcomes are not plausible. We have also restricted the welfare comparison to equilibrium strategy profiles, since those are the only ones that are incentive-feasible. The Pareto criterion is then different – it is based on possibly subjective assessments: an equilibrium strategy profile \(s\) is (subjectively) Pareto superior to \(s'\) if the outcome \((Pr', s)\) is \(i\)-efficient relative to \((Pr', s')\), for all \(Pr' \in O_{i}^{Pr,s}\), for all \(i \in N\). The idea is that when an individual assesses her own expected welfare, she will do so according to some equilibrium-supporting assessment. We could define a more general Pareto criterion for non-equilibrium outcomes, where for a non-equilibrium outcome \((Pr, s)\), \(O_{i}^{Pr,s} \equiv \{(Pr', s') | (Pr', s')i\text{-consistent with } (Pr, s)\}\). For the social planner one possible welfare criterion is then simply the Pareto criterion itself. In the economy considered in this paper autarky is subjectively Pareto superior to full exchange.

There is another possibility, based on the objective welfare, which we embraced here. Suppose that the social planner observes some statistic of the equilibrium outcome. First, there may in general be several different outcomes that are consistent with the social planner’s observation. Second, the social planner might not necessarily care about the individuals’ subjective welfare evaluations, but rather what outcomes might objectively be socially superior. The social planner must therefore consider the set of all possible objective outcomes that are supportable in equilibrium and are consistent with her observation. When the objective outcome is \((Pr, s)\),

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pointing to the fact that under agents’ conflicting beliefs a Pareto criterion might be problematic – see Mongin 1997, Gilboa, Samet, and Schmeidler 2004, and Gilboa, Samuelson, and Schmeidler 2012.
denote this set by $O_{SP}^{Pr,s}$. For instance, in the economy presented here, we assume that the social planner observes the total sum of the individuals’ expected payoffs, so that in autarky, $O_{SP}^{Pr,s} = \{(P, s) \mid (P, s) \text{ is supportable in equilibrium}\}$. Among other things, this yields a set of possible objective distributions over uncertainty. The social planner then compares these possible outcomes to another outcome that is supportable in equilibrium by considering any possible objective distribution over uncertainty. Formally, given an equilibrium outcome $(P_r, s)$, of which the social planner observes only a statistic, assuming equal welfare weights, the social planner concludes that a strategy profile $s^*$ may be socially superior if:

$$\sum_{i \in N} U_i^{Pr',s^*} \geq \sum_{i \in N} U_i^{Pr',s'}, \forall (Pr', s') \in O_{SP}^{Pr,s}.$$  

As we have shown above, in the economy considered here autarky is subjectively Pareto superior to full exchange, but full exchange is socially superior to autarky. Of course, the most desirable situation is when for the outcome that the socially efficient outcome is superior in terms of both welfare evaluations. When that is not the case, as in the example here, the appropriate welfare measure depends on the social planner’s ability to provide inducements to the individuals to switch to the socially efficient outcome.

### 7 Appendix

**Proof of Proposition 1.** 1 follows is immediate to verify. To see 2, take Individual 1, and her type $\theta_H$. If she deviates from $aut_1$, then when $\theta_2 = \theta_L$ her payoff increases by $\bar{x} - y^*$, and when $\theta_2 = \theta_H$ her payoff decreases by $y^* - x$. Since $\bar{x} - y^* = \alpha(y^* - x)$, where

\[\text{when the set of these possible distributions over uncertainty is convex, then this welfare measure can be thought of as the belief-neutral welfare measure where the social planner evaluates welfare according to all possible distributions over uncertainty that could have the generated equilibrium behavior.} \]
α > 1, it follows that under \( \bar{Pr} \) she would have incentives to deviate, so that autarky is not a Bayes-Nash equilibrium outcome. Point 3 is also immediate to verify.

**Proof of Proposition 3.** First note that in autarky, in order for \( i \)-consistency to hold, individual \( i \) must correctly assess the other individual’s strategy, i.e., \( s^i \equiv s \); she must also correctly assess the likelihoods of her own signals, i.e., \( Pr^i_\Theta (\theta_i) = \frac{1}{2}, \theta_i \in \{ \theta_L, \theta_H \} \); but her payoffs do not vary with signals to the other individual, so that for consistency with \( \bar{Pr} \), conditional on her own signal, she can make any assessment regarding the likelihoods of the signals to the other individual. In order for the equilibrium requirement (iii) to hold, that is, for \( Pr^j \) to be \( j \)-consistent with \( Pr^i \), it must be that \( Pr^i_\Theta (\theta_j) = \frac{1}{2}, \theta_j \in \{ \theta_L, \theta_H \} \).

In autarky, the incentive constraints of Individual 1 are given by,

\[
Pr^1(\theta_L, \theta_L)(\bar{x} - y^*) + Pr^1(\theta_L, \theta_H)(\bar{x} - y^*) \leq 0,
\]

\[
Pr^1(\theta_H, \theta_L)(\bar{x} - y^*) + Pr^1(\theta_H, \theta_H)(\bar{x} - y^*) \leq 0,
\]

and the incentive constraints of Individual 2 are given by,

\[
Pr^2(\theta_L, \theta_L)(\bar{x} - y^*) + Pr^2(\theta_H, \theta_L)(\bar{x} - y^*) \leq 0,
\]

\[
Pr^2(\theta_L, \theta_H)(\bar{x} - y^*) + Pr^2(\theta_H, \theta_H)(\bar{x} - y^*) \leq 0.
\]

Since \((\bar{x} - y^*) = \alpha(x - y^*)\), the claim follows.

**Proof of Proposition 4.** Take Individual 1 and suppose her signal is \( \theta_H \). In autarky, \((\bar{Pr}, \bar{s})\), any of her supporting assessments must put a sufficiently large probability mass on the draw of signals \((\theta_H, \theta_L)\). From Proposition 3,

\[
Pr^1(\theta_H, \theta_L) \geq \frac{1}{2} + \frac{\alpha}{1 + \alpha}.
\]
The expected payoff to the individual in autarky is $y^*$. Since $x^* > 1 - x^*$, in full exchange her subjective expected payoff (conditional on $\theta_H$) is decreasing in the probability mass that she assigns to the event $(\theta_H, \theta_L)$. Therefore, her subjective expected payoff in full exchange is highest when $P^i(\theta_H, \theta_L) = \frac{1}{2} \times \frac{\alpha}{1 + \alpha}$. By a similar argument when she observes the signal $\theta_L$, we conclude that in autarky, the highest subjective expected payoff that she can assign to full exchange equals,

$$\frac{\alpha}{1 + \alpha} (1 - x^*) + \frac{1}{1 + \alpha} x^*.$$

Thus, her subjective expected payoff will be higher in autarky than in full exchange under any supporting assessment if,

$$\frac{\alpha}{1 + \alpha} (1 - x^*) + \frac{1}{1 + \alpha} x^* \leq y^*.$$

A similar argument can be applied to Individual 2. □

**References**


