Abstract

We study capital-skill complementarity in a multi-sector theory of aggregate technology featuring firms with arbitrary and heterogeneous multi-factor production functions. Importantly, the theory allows for distortions to factor choices and heterogeneous skill premia across firms. We characterize the elasticity of the aggregate skill premium to the price of equipment capital in terms of firm-level elasticities of substitution across factors, the elasticities of substitution across firms and sectors, and the distribution of factor intensities across firms. Using micro data from France, we estimate the two sets of elasticities assuming a firm-level production function featuring heterogeneous cross-factor elasticities. Applying our aggregation, we find that a 1% fall in equipment capital prices raises the observed skill premium by 0.06%. This response is mostly driven by the rise in aggregate skill demand due to market reallocations toward equipment- and skill-intensive firms.
1 Introduction

Technical change has long been recognized as an important driver of labor-market transformation. When embodied in capital, technical change typically reduces its price, altering the relative demand for different skill groups depending on their degree of complementarity with capital. In particular, if capital is relatively more complementary with skilled than unskilled labor at the firm level or if more capital-intensive firms are also more skill-intensive, then the observed rapid decline in the price of capital equipment may have increased the skill premium (the relative wage of more to less skilled workers). In either case, we can say that the aggregate production technology features capital-skill complementarity. Does the aggregate production technology indeed feature capital-skill complementarity?

Unfortunately, such simple empirical questions about the nature of aggregate technology are notoriously hard to answer in practice. There is indeed some evidence based on aggregate data in favor of capital-skill complementarity going back to the seminal work of Griliches (1969) and Krusell et al. (2000). Nevertheless, such evidence is fraught with concerns about potential mismeasurement, limited number of observations, and omitted variable bias (see, e.g., Acemoglu, 2002). One alternative is to rely instead on micro-level data, estimate the extent of capital-skill complementarity within firms, embed this within a macroeconomic model, and aggregate up to study the macro implications of those micro elasticities (in the spirit of Oberfield and Raval, 2021 and Baqaee and Farhi, 2019). However, even this approach is not without its own challenges: such aggregation exercises applied to other questions typically assume perfect factor markets and make strong assumptions on production technologies.

Taking the latter approach, in this paper we provide a strategy to generalize its underlying assumptions. We construct a theory of multi-factor and multi-sector aggregate technology that allows for arbitrary constant returns to scale firm-level production technologies, and arbitrary but exogenous distortions to firm-level factor prices. We characterize the response of the observed skill premium to changes in the price of equipment capital and show how it depends on generalized definitions of aggregate elasticities of substitution between factors. We then specify a flexible parametric form for the firm-level production function and estimate the key micro elasticities using rich micro-level data from France. Using our estimates and data on the distribution of factor intensities and skill premia across firms, we find a substantial degree of capital-skill complementarity in the aggregate French economy. We show that this aggregate complementarity is mostly

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1For more recent evidence, see Duffy et al. (2004) and Henderson (2009).
driven by labor reallocation toward equipment- and high-skill-intensive firms rather than 
by capital-skill complementarity at the firm level or by labor-market distortions.

We consider distortions to factor markets characterized by firm-specific deviations in 
factor prices from an unobserved, common, latent shadow factor price that clears the 
factor market. We treat these wedges between firm-specific and shadow factor prices as 
exogenous, but show that our approach also accommodates potential time variations in 
these wedges.\footnote{We provide two micro-foundations for these wedges in Appendix B.} In the presence of factor-market distortions, the observed skill premium 
depends on both the shadow skill premium, i.e., the relative shadow wage of high skilled 
to low skilled workers, and the distribution of skill premia across firms.

We show that the response of the observed skill premium to equipment price shocks 
depends on two distinct aggregate elasticities of substitution. The aggregate elasticity of 
substitution in factor demand accounts for the effect of changes in the price of a given 
factor on the demand for another factor. The aggregate elasticity of substitution in factor 
payments accounts for the effect of changes in the price of a given factor on the share 
of payments going to another factor. While these two aggregate elasticities coincide un- 
der perfect factor markets, they deviate in the presence of distortions. Thus, capital-skill 
complementarity may be a feature of factor demand, when the employment of low skilled 
labor is more substitutable with equipment compared to high skilled labor, or a feature 
of factor payments, when the payments to low skilled labor is more substitutable with 
equipment compared to high skilled labor. While the former characterizes the response 
of the shadow skill premium to changes in equipment prices, the response of the observed 
skill premium depends on both notions of capital-skill complementarity.

To perform the aggregation, we assume monopolistically competitive firms that oper- 
ate across multiple sectors, and nested constant elasticity of substitution product demand 
across sectors and firms within sectors. With these assumptions, we can fully charac- 
terize the aggregate elasticities of substitution for factor demand and factor payments in 
terms of micro-elasticities of substitution and the distribution of factor employment and 
payments across firms. As usual, the aggregate elasticities of substitution account both 
for the substitutability of factors in firm-level technology and also for the distribution of 
factor intensities across firms. In particular, even if capital and skill are not complements 
at the firm-level, aggregate complementarity emerges when capital and skill intensities 
positively covary across firms. Due to heterogeneity in firm-specific wages (wedges), the 
extent of this covariance may vary for employment and for payments. Thus, when falling 
capital prices shift labor toward capital-intensive firms, the effect on aggregate factor de- 
mand may be distinct from the effect on aggregate factor payments.
Our final theoretical result generalizes the analysis to allow for joint shocks to the price of equipment capital and to firm-specific wedges. We show that we can provide yet another generalization of the notion of the aggregate elasticity of substitution that captures the effect of shocks to firm-specific wedges to a given factor on the demand and payments to another factor. We again characterize these aggregate elasticities and show how they jointly determine the response of the shadow and observed skill premia to a joint shock to equipment prices and to labor market distortions.

To use our framework to assess the impact of falling equipment prices on the skill premium, we use matched employer-employee micro data from France spanning 1997-2007. In the data, we can measure variables that are key to our theoretical aggregation results, including equipment, low-skill labor, and high-skill labor intensities across firms, using worker occupation as a proxy for skill. We provide a set of stylized facts that play a central role in shaping aggregate capital-skill complementarity. Low-skill labor intensities fall sharply and monotonically across the firm size distribution, with the smallest and largest firms having a low-skill labor intensity of approximately 75% and 20%, respectively. Both high-skill labor and equipment intensities rise with firm size, with a particularly sharp increase in equipment intensities. This first set of stylized facts imply a strong negative correlation between equipment intensity and low-skill labor intensity, which will tend to generate capital-skill complementarity at the aggregate level. We also document that the skill premium is increasing in firm size among small firms, with employment below fifty workers, and then increasing in firm size among larger firms. This second stylized fact implies that a decline in the price of equipment, which increases the size of the low skill premia firms (since they are large and large firms are equipment intensive), will reduce the observed skill premium for given shadow wages.

We further use the data to estimate the key micro elasticities needed for our aggregation. For this exercise, we impose a functional form assumption on firm-level technology and consider the CRESH family of production functions (Hanoch, 1971). This family generalizes the CES production function by allowing varying degrees of substitutability among factor inputs without requiring ex ante assumptions about the nesting of factors (unlike the nested CES specifications). We estimate three factor-specific elasticity parameters of the CRESH production function using five-year changes in firm-specific factor employments, factor prices, and revenue, using two instruments related to firm-specific international trade connections and one instrument that moves the skilled and unskilled wages faced by firms. Across all specifications, we identify statistically significant capital-skill complementarity at the firm level. Finally, we also estimate the within-sector elasticity of substitution across firm products using five-year changes in sales and prices,
instrumenting for firm-specific price changes using real exchange rate movements and the initial allocation of firm imports across origin countries.

The estimated elasticities, when combined with the observed distribution of factor intensities, allow us to perform the main aggregation exercise. We measure the impact of the evolution of equipment prices on the skill premium in France. As a first step, we measure the aggregate elasticity of the observed skill premium using our micro estimates and the average firm-level shares across all years in our sample (1997-2007). In our baseline specification, we find that this elasticity is $-0.055$, so that a 1% decline in the price of equipment generates a 0.055% increase in the observed skill premium in France. In our main quantitative exercise, we feed in measured changes in equipment prices in each year, using year-specific measures of firm-level factor employment and factor payments and evaluate the cumulative impact on the skill premium. Since our theory takes the shadow wage of low-skill labor as the numeraire, the relevant shock is the price of equipment relative to the low-skilled shadow wage. We measure each in a model-consistent way. The cumulative change between 1997 and 2007 in this relative price is a fall of approximately 40 percent, driven almost equally by a decline in the price of equipment and a rise in the wage of low-skilled labor. We find that this generates an approximately 2.2% rise in the French skill premium.

**Literature.** Our paper contributes to a large literature in labor, trade, and macro economics that studies the implications of technological change on inequality across skill groups. Following Katz and Murphy (1992), many earlier studies considered a broad notion of skill-biased technical change, identified empirically as a residual to match the observed evolution of the skill premium given the observed evolution of the aggregate relative supply of skill (see Acemoglu, 2002; Card and DiNardo, 2002, for reviews).³

Our approach builds more directly on the studies of capital-embodied technical change, which focus on changes in the quality-adjusted price of capital equipment as a well-defined empirical proxy for the technological shock affecting the labor market (Greenwood and Yorukoglu, 1997; Hornstein et al., 2005). This strand of the literature in turn builds on earlier work on aggregate technology by Griliches (1969, 1970) who first hypothesized capital-skill complementarity. In a seminal paper, Krusell et al. (2000) use this idea to provide an account of the rise of the skill premium in the U.S. by calibrating a four-factor aggregate production function representation of the U.S. economy.

³A more recent approach in the literature emphasizes the importance of studying the endogenous assignment of production tasks to different skill groups for understanding the effect of technology on inequality (Costinot and Vogel, 2010; Acemoglu and Autor, 2011).
By providing a framework for explicit aggregation of micro-level firm responses, we also build on recent work by Oberfield and Raval (2021) and Baqee and Farhi (2019) who characterize the aggregate elasticity of substitution under the assumption of constant returns to scale production and perfect factor markets in a two-factor model with heterogeneous firms and multi-factor model without, respectively.\footnote{In another recent contribution, Lashkari et al. (2022) characterize the aggregate elasticity of substitution under arbitrary firm-level production functions with variable degrees of returns to scale, while still maintaining the assumption of perfect labor markets.} The relationship between micro-level and macro-level substitution is also widely studied in the context of the effects of trade liberalization on the skill premium (e.g., Parro, 2013; Burstein et al., 2013; Burstein and Vogel, 2017).

Alternative approaches to studying the effect of technical change on skill demand and the skill premium focus on the roles of information technology (e.g., Caroli and Van Reenen, 2001; Bresnahan et al., 2002; Akerman et al., 2015) or automation (e.g., Acemoglu and Restrepo, 2018, 2020); our contribution relative to this literature is aggregating from micro estimates to quantify macro implications. Our paper is also closely related to a number of recent empirical contributions. Caunedo et al. (2021) study the inequality consequences of capital-embodied technical change by studying the occupation-level elasticity of substitution between labor and capital. Lindner et al. (2021) use micro data from Hungary and Norway to study the effect of innovation and R&D on firm-level demand for skill assuming imperfect labor markets.\footnote{Our results documenting the empirical association between the firm-level skill premia and capital intensity are also related to the results of Aghion et al. (2017) who document a negative association between firm-level skill premia and R&D intensity.}

The paper is organized as follows. Section 2 presents our framework and theoretical results. Section 3 discusses our data and a number of stylized facts. Section 4 presents our strategy for estimating the firm-level production function along with the estimation results. Section 5 discusses the aggregation results and Section 6 concludes the paper.

\section{Theory}

\subsection{Environment}

\textbf{Firms and Production.} There is a large set of firms indexed by $i \in \mathcal{I}$ with arbitrary constant returns to scale production functions. We denote by $I_s \subseteq \mathcal{I}$ the set of firms in sector $s$. The goods market is monopolistically competitive within each sector, and we
assume nested CES preferences across sectors so that demand for firm \( i \in \mathcal{I}_s \) is given by

\[
Y_i = \Phi_i \left( \frac{P_i}{P_s} \right)^{-\epsilon} \left( \frac{P_s}{P} \right)^{-\eta} Y
\]

(1)

where \( \epsilon \) is the elasticity of substitution across firms within each sector, \( \eta \) is the elasticity of substitution across sectors, \( \Phi_i \) is a firm-specific demand shifter, \( P_s \) is the price index within sector \( s \), and \( P \) is the aggregate price index. With slight abuse of notation, we denote by \( P_i \) the price that firm \( i \) charges.

There are three factors \( X_f \) where \( f \in \{ \ell, h, e \} \): low-skill labor (\( X_\ell \)), high-skill labor (\( X_h \)), and capital equipment (\( X_e \)). Firms are also heterogeneous in terms of their production functions, which we characterize in terms of a collection of unit factor requirements \( (A_{\ell i}, A_{hi}, A_{ei}) \) for the three factors, such that the demand of firm \( i \) for factor \( f \) is given by \( X_{fi} = A_{fi} Y_i \). For some of the results below, we will consider a specific parametric assumption on the structure of these unit factor requirements, but more generally we allow for arbitrary functions of factor prices.

**Factor Markets.** In the aggregate, the two types of labor are in exogenous and fixed supply denoted by \( L \) and \( H \) whereas the aggregate supply of equipment \( E \) is infinitely elastic at an exogenous price. Firms face wedges in factor markets and pay firm-and-factor specific prices (which we sometimes refer to as wages). Given the fixed and exogenous wedge \( T_{fi} \), firm \( i \)'s factor-\( f \) price is \( W_{fi} = W_f T_{fi} \). We refer to \( W_f \) as the shadow price (or wage) of the factor, since it is unobservable in the data. The shadow factor price is common across firms and equates supply and demand,

\[
E = X_e \equiv \sum_i X_{ei}, \quad H = X_h \equiv \sum_i X_{hi}, \quad L = X_\ell \equiv \sum_i X_{\ell i}.
\]

We normalize \( W_\ell = 1 \). In Appendix B, we provide two alternative micro-foundations for these wedges: firm-and-labor-type specific compensating differentials in competitive factor markets and union bargaining with fringe competitive labor markets.

Throughout, we are interested in the impact of an exogenous change in the price of equipment \( W_e \) on the observed skill premium \( \Psi \). The observed skill premium is defined as the average wage paid to high-skill workers relative to low-skill workers

\[
\Psi \equiv \frac{W_h}{W_\ell} = \frac{\sum_i W_{hi} X_{hi}}{\sum_i' W_{\ell i'} X_{\ell i'}} / \frac{X_h}{X_\ell}.
\]

(2)

where \( X_\ell \equiv \sum_i X_{\ell i} \) and \( X_h \equiv \sum_i X_{hi} \) are the aggregate demands for high- and low-skilled labor, respectively.
2.2 Micro and Macro Elasticities of Substitution

Each firm takes as given input prices and chooses its factor inputs to minimize costs. We define \( \theta_{fi} \equiv \frac{W_{fi}X_{fi}}{\left(\sum_{f'} W_{f'i}X_{f'i}\right)} \) as firm \( i \)'s factor-\( f \) intensity (i.e., \( i \)'s payments to factor \( f \) relative to \( i \)'s total factor payments). Let lower-case variables indicate the logarithm of upper-case variables, \( v \equiv \ln V \) for any variable \( V \), so that, e.g., \( a_{fi} \equiv \ln A_{fi} \). We define the elasticity of (factor) substitution between factors \( f \) and \( f' \) at the firm-\( i \)-level as

\[
\sigma_{ff',i} \equiv \frac{1}{\theta_{f'i}} \frac{\partial a_{fi}}{\partial w_{f'}}.
\]

(3)

and at the aggregate level as\(^6\)

\[
\sigma_{ff'} \equiv \frac{\partial \ln \left(\frac{X_f}{X_{f'}}\right)}{\partial w_{f'}}.
\]

(4)

where \( X_f \) stands for aggregate factor-\( f \) demand. Note that the variations in the price of factor \( f' \) in both definitions (3) and (4) correspond to shifts in the shadow factor price. Changes in shadow wages proportionally shift factor prices across all firms.

In the presence of factors wedges, we may define an alternative measure of substitutability across factors in terms of factor payments. Let \( \sigma_{ff'}^* \) denote the aggregate elasticity of factor payment substitution as

\[
\sigma_{ff'}^* \equiv 1 + \frac{\partial \ln \left(\frac{\theta_f}{\theta_{f'}}\right)}{\partial w_{f'}}.
\]

(5)

where \( \theta_f \equiv \frac{\left(\sum_{i} W_{fi}X_{fi}\right)}{\left(\sum_{f'} W_{f'i}X_{f'i}\right)} \) denotes the aggregate factor-\( f \) intensity of all factor payments in the economy. When there are no wedges (\( T_{fi} \equiv T_f \) for all \( i \)), the two aggregate elasticities of substitution defined in Equations (4) and (5) coincide \( \sigma_{ff'} \equiv \sigma_{ff'}^* \). More generally, however, the two elasticities may diverge due to the fact that reallocations across firms have distinct effects on aggregate factor demand and factor payments.

Let \( \Lambda_{fi} \equiv X_{fi} / \left(\sum_{i'} X_{f'i'}\right) \) and \( \Lambda_{fi}^* \equiv \frac{W_{fi}X_{fi}}{\left(\sum_{i'} W_{f'i}X_{f'i}\right)} \) denote the share of aggregate demand for and payments to factor \( f \) made by firm \( i \) and, with slight abuse of

\(^6\)The definition in Equation (4) is commonly referred to as the Morishima elasticity of substitution (MES), as contrasted from the Allen-Uzawa elasticity of substitution defined in Equation (3) (Blackorby and Russell, 1989). In our setting, it will prove more convenient to use the former to characterize the aggregate and the latter for the firm-level elasticity of substitution. With more than two factors, the two definitions are not identical—e.g., unlike the Allen-Uzawa elasticity, the Morishima elasticity is not symmetric between the two factors.
notation, let $\Lambda_{fs} \equiv \sum_{i \in I_s} \Lambda_{fi}$ and $\Lambda^*_{fs} \equiv \sum_{i \in I_s} \Lambda^*_{fi}$ denote the corresponding shares made by all firms in sector $s$. The following lemma characterizes the two aggregate elasticities of substitution using these definitions.

**Lemma 1.** The aggregate elasticities of substitution $\sigma_{ff'}$ and $\sigma^*_{ff'}$ between factors $f$ and $f'$ defined by Equations (4) and (5) are given by

$$
\sigma_{ff'} = \sum_i \left( \Lambda_{fi} \sigma_{ff',i} - \Lambda_{f'i} \sigma_{f'i,i} \right) \theta_{f'i} + \varepsilon \sum_i \left( \Lambda_{f'i} - \Lambda_{fi} \right) \theta_{f'i} - (\varepsilon - \eta) \sum_s \left( \Lambda_{fs}^{f'} - \Lambda_{fs} \right) \theta_{f's},
$$

(6)

$$
\sigma^*_{ff'} = \sum_i \left( \Lambda^*_{fi} \sigma_{ff',i} - \Lambda^*_{f'i} \sigma_{f'i,i} \right) \theta_{f'i} + \varepsilon \sum_i \left( \Lambda^*_{f'i} - \Lambda^*_{fi} \right) \theta_{f'i} - (\varepsilon - \eta) \sum_s \left( \Lambda^*_{fs}^{f'} - \Lambda^*_{fs} \right) \theta_{f's},
$$

(7)

where $\theta_{fs} \equiv \left( \sum_{i \in I_s} W_{fi} X_{fi} \right) / \left( \sum_{f',i' \in I_s} W_{f'i'} X_{f'i'} \right)$ denotes the factor-$f$ intensity of sector $s$.

**Proof.** See Appendix A. \qed

In the presence of exogenous distortions, Equations (6) and (7) provide decompositions of the two aggregate elasticities of substitution into three components: the first term accounts for the average of the within-firm elasticities of substitution across firms, the second term accounts for the effect of within-sector reallocations across firms, and the last term accounts for the effect of cross-sectoral reallocations. In the case of the aggregate elasticity of substitution for demand (payments), the averages across firms are weighted by their shares in aggregate demand for (payment to) factors.

To further unpack the contribution of within-firm substitution, consider two alternative examples for the specification of the firm-level production functions.

**Example 1 (CES Production Function).** Consider a CES production function with elasticity of substitution $\sigma$. The cross-factor elasticities of substitution in firm-level technology are given by $\sigma_{ff',i} \equiv \sigma$ for all $f \neq f'$ and $\sigma_{ff,i} = \sigma \left( \theta_{fi} - 1 \right) / \theta_{fi}$.

Assuming a single sector model $\eta = \varepsilon$, the CES production function implies that the aggregate elasticity of substitution for factor demand and factor payments are given by

$$
\sigma_{ff'} = \sigma + (\varepsilon - \sigma) \sum_i \left( \Lambda_{f'i} - \Lambda_{fi} \right) \theta_{f'i}, \quad \sigma^*_{ff'} = \sigma + (\varepsilon - \sigma) \sum_i \left( \Lambda^*_{f'i} - \Lambda^*_{fi} \right) \theta_{f'i}.
$$

If we assume an elasticity of demand that exceeds the elasticity of substitution across factors ($\varepsilon > \sigma$), the contribution of cross-firm reallocations to the aggregate elasticity of
substitution between two factors $f$ and $f'$ is positive if firms that are more intensive in factor $f'$ on average account for a larger share of aggregate payments to factor $f'$ compared to that to factor $f$ (Oberfield and Raval, 2021).

**Example 2** (CRESH Production Function (Hanoch, 1971)). Consider a production function implicitly defined through the following constraint on unit factor requirements:

$$\sum_{f \in \{\ell, h, e\}} (Z_{fi}A_{fj})^{\sigma_{f}-1} = 1, \tag{8}$$

where $Z_{fi}$ is firm-specific factor-augmenting productivity and where $\sigma_f > 0$ are factor-specific parameters. The production function defined in Equation (8) is a homothetic generalization of the standard CES production function that allows for different degrees of substitutability among different factor inputs. The CES production function is nested in this specification for the case of constant parameters $\sigma_f \equiv \sigma$ across all factors $f$.

The elasticity of substitution between factors $f$ and $f'$ for firm $i$ under the production function in Equation (8) satisfies

$$\sigma_{ff',i} = \frac{\sigma_f \sigma_{f'} \theta_{f,i}}{\bar{\sigma}_i} - \frac{\sigma_f}{\theta_{f,i}} I_{ff'}, \tag{9}$$

where we have defined $\bar{\sigma}_i \equiv \sum_f \theta_{f,i} \sigma_f$ as producer-specific weighted mean of $\sigma_f$, weighing by factor cost shares, and where $I_{ff'}$ is an indicator function that takes the value 1 when $f = f'$ and 0 otherwise. Let us now compare the firm-level elasticity of substitution between equipment and high-skill labor to that between equipment and low-skill labor. From Equation (9) it follows that the ratio of these two elasticities satisfies

$$\frac{\sigma_{he,i}}{\sigma_{\ell e,i}} = \frac{\sigma_h}{\sigma_\ell}. \tag{10}$$

Thus, parameters $\sigma_h$ and $\sigma_\ell$ characterize the relative substitutability of high- and low-skilled workers with equipment. If $\sigma_\ell > \sigma_h$, then low-skilled workers are more substitutable with equipment.

Assuming again a single-sector setting ($\eta = \varepsilon$) and no wedges, substituting Equation

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7Hanoch (1971) introduced this class of production functions and referred to them as CRESH (constant ratios of elasticities of substitution with homotheticity). The advantage of the specification in Equation (8), compared to a nested CES specifications like that of Krusell et al. (2000), is that it does not a require an a priori nesting structure between different factors in the firm-level production function.

8See Appendix A for the solution of the cost minimization problem and the derivation of Equation (9).
(9) into Equation (6) yields

\[
\sigma_{ff'} = \sigma_{f} \left[ 1 - \sum_i \left( \frac{\Lambda_{fi} \sigma_{f}'}{\sigma_i} - \Lambda_{fi} \frac{\sigma_{f}}{\sigma_i} \right) \theta_{fi} \right] + \varepsilon \sum_i \left( \Lambda_{fi} - \Lambda_{fi} \right) \theta_{fi}.
\]

We can also find the expression for the aggregate elasticity of substitution for factor payments \(\sigma_{ff'}^*\) in a similar fashion.

**Aggregate Elasticity of Substitution Between Labor and Equipment** We can use the framework developed so far to examine the aggregate elasticity of substitution between labor and equipment. Let \(\theta_n \equiv \theta_\ell + \theta_h\) denote the aggregate labor share in the economy. We can then define the aggregate elasticity of substitution between labor and equipment payments as

\[
\sigma_{ne}^* \equiv 1 + \frac{\partial \ln \left( \frac{\theta_n}{\theta_e} \right)}{\partial w_e} = \frac{\theta_\ell}{\theta_\ell + \theta_h} \sigma_{\ell e}^* + \frac{\theta_h}{\theta_\ell + \theta_h} \sigma_{he}^*,
\]

where \(\sigma_{\ell e}^*\) and \(\sigma_{he}^*\) are given by Equation (7). As expected, the aggregate elasticity of substitution between labor and equipment is a convex combination of the aggregate elasticities of the factor payments to the two skill groups, with each group’s weight given by the share of that skill group in labor payments.

### 2.3 The Skill Premium and the Equipment Price

**Perfect Labor Markets** Suppose first that all firms pay common factor prices, \(W_{fi} = W_f\). In this case the share of firm \(i\) in both aggregate factor demand and factor payments are equal \(\Lambda_{fi}^* = \Lambda_{fi}\), and the following lemma characterizes the general equilibrium response of the skill premium to a change in the price of equipment capital.

**Lemma 2.** Suppose that wages are equalized across firms \((T_{fi} \equiv T_f)\). The elasticity of the skill premium with respect to the price of capital equipment is given by

\[
\frac{d\psi}{dw_e} = \frac{dw_h}{dw_e} = -\frac{\sigma_{\ell e} - \sigma_{he}}{\sigma_{\ell h}},
\]

where the aggregate substitution elasticity of factor demand \(\sigma_{ff'}\) between factor \(f\) and \(f'\) is given by Equation (6).

**Proof.** See Appendix A.

The first equality in Equation (13) simply follows from the fact that we have abstracted away from firm-specific wedges and have taken the low-skill wage to be the numeraire.
In this case, the aggregate skill premium is identical to the relative marginal products of the two skill groups. Using Equation (6), we can write the second equality as

\[
\frac{dw_h}{dw_e} = -\frac{\sum_i (\Lambda_{li}\sigma_{\ell e,i} - \Lambda_{hi}\sigma_{he,i}) \theta_{ei} - \varepsilon \sum_i (\Lambda_{li} - \Lambda_{hi}) \theta_{ei} + (\varepsilon - \eta) \sum_s (\Lambda_{ls} - \Lambda_{hs}) \theta_{es}}{\sum_i (\Lambda_{li}\sigma_{\ell h,i} - \Lambda_{hi}\sigma_{hh,i}) \theta_{hi} + \varepsilon \sum_i (\Lambda_{hi} - \Lambda_{li}) \theta_{hi} - (\varepsilon - \eta) \sum_s (\Lambda_{hs} - \Lambda_{ls}) \theta_{hs}}.
\]

(13)

To unpack this result, let us first consider the case without heterogeneity, where all firms have identical production functions. In this case, under the CES production function (Example 1), we simply have \(\frac{d\psi}{dw_e} = 0\), and under the more general CRESH production function (Example 2), Equation (13) simplifies to

\[
\frac{d\psi}{dw_e} = -\frac{\sigma_{\ell e,i} - \sigma_{he,i}}{\sigma_{\ell h,i}} = -\frac{\sigma_{\ell} - \sigma_{h}}{\sigma_{\ell}\sigma_{h}}. \tag{14}
\]

Equation (14) shows that falling equipment prices raise the skill premium if the firm-level elasticity of substitution between equipment and low-skill labor exceeds that between equipment and high-skill labor. As we saw in Equation (10), the restriction \(\sigma_{\ell} > \sigma_{h}\) on the parameters of the production function is a necessary and sufficient condition for this result.

In the presence of heterogeneity, Equation (12) shows that the intuition behind Equation (14) is still reflected in the response of the skill premium. The sign of the response of the skill premium to changes in equipment prices again depends on the difference between the degree of aggregate substitutability between equipment and the two labor types. Under the CES production function, and assuming a single sector model (\(\eta = \varepsilon\)), we find

\[
\frac{dw_h}{dw_e} = \frac{(\varepsilon - \sigma) \sum_i (\Lambda_{hi} - \Lambda_{li}) \theta_{ei}}{\varepsilon + (\varepsilon - \sigma) \sum_i (\Lambda_{hi} - \Lambda_{li}) \theta_{hi}}. \tag{15}
\]

Unlike the model with homogenous CES production functions, in the presence of heterogeneity the elasticity of the skill premium with respect to equipment may differ from zero. In particular, assuming \(\varepsilon > \sigma\), if firms with high equipment-intensities on average contribute a larger share of the wage bill of high-skilled workers, then a fall in equipment prices leads to a rise in the skill premium. In this case, there is complementarity between equipment and skill across firms. Under the CRESH production function, and assuming
again a single sector model ($\eta = \varepsilon$), we find

$$\frac{dw_h}{dw_e} = -\frac{\sigma_e \sum_i \left( \Lambda_{\ell i} \frac{\sigma_{\ell i} - \sigma_i}{\sigma_{\ell i}} + \Lambda_{hi} \frac{\sigma_{hi} - \sigma_h}{\sigma_{hi}} \right) \theta_{ei} + (\varepsilon - \sigma_e) \sum_i (\Lambda_{hi} - \Lambda_{\ell i}) \theta_{ei}}{\sigma_h \left[ 1 + \sum_i \left( \Lambda_{\ell i} \frac{\sigma_{\ell i}}{\sigma_{hi}} - \Lambda_{hi} \frac{\sigma_{hi}}{\sigma_{hi}} \right) \theta_{hi} \right] + \varepsilon \sum_i (\Lambda_{hi} - \Lambda_{\ell i}) \theta_{hi}}. \quad (16)$$

In this case, when the firm-level production function features sufficient complementarity between equipment and skill within firms, in the sense that $\sigma_h \leq \sigma_i \leq \sigma_{\ell}$, then the first term in the numerator of Equation (16) is positive. In such a setting, we have complementarity between equipment and skill both across and within firms.

**Imperfect Labor Markets** Let us now allow for the presence of firm-specific wedges $T_{fi} \neq T_f$ for the two skill types $f \in \{\ell, h\}$. In this case, the distribution of wage bills for each skill type across firms $\Lambda^*_f$ deviates from the distribution of employment $\Lambda_f$ for skill type $f \in \{\ell, h\}$ across firms. The following proposition characterizes the general equilibrium response of the skill premium under this scenario.

**Proposition 1.** The elasticity of the shadow skill premium and the observed skill premium with respect to the price of capital equipment are given by

$$\frac{dw_h}{dw_e} = -\frac{\sigma_{\ell e} - \sigma_{he}}{\sigma_{\ell h}}, \quad (17)$$

$$\frac{d\psi}{dw_e} = \frac{dw_h}{dw_e} - \sigma^*_{\ell h} \left( \frac{dw_h}{dw_e} + \frac{\sigma^*_{\ell e} - \sigma^*_{he}}{\sigma^*_{\ell h}} \right), \quad (18)$$

where the aggregate elasticities of substitution for factor demand and factor payments $\sigma_{ff'}$ and $\sigma^*_{ff'}$ are given by Equations (6) and (7), respectively.

**Proof.** See Appendix A.

Proposition 1 shows that in the presence of firm-specific labor wedges, the response of the shadow skill premium is given by an expression similar to the one in Equation (2). When employment $\Lambda_f$ and wage bill distributions differ $\Lambda^*_f$, the employment distribution determines the elasticity of the shadow skill premium since the shadow skill premium equates labor supply with labor demand.

Now the response of the observed skill premium deviates from that of the shadow skill premium, as seen in the second term on the right hand side of Equation (18). This term accounts for the contribution of labor reallocation across firms to the change in the observed skill premium at a fixed value of the shadow skill premium. To understand the
intuition for this second term, we can express equation (18) as

\[
\frac{d\psi}{dw_e} = \frac{dw_h}{dw_e} + \sum_i \left( \left( \Lambda^*_hi - \Lambda_{hi} \right) \frac{dx_{hi}}{dw_e} - \left( \Lambda^*_\ell i - \Lambda_{\ell i} \right) \frac{dx_{\ell i}}{dw_e} \right)
\]

where \( x_{fi} = \log X_{fi} \) is the logarithm of firm \( i \)'s employment of factor \( f \). If firms pay common wages, then \( \Lambda_{fi} = \Lambda^*_fi \) and labor reallocation has no direct effect on the observed skill premium (i.e., holding the general equilibrium shadow wages fixed); this explains why the second term is zero under the case of perfect labor markets covered in Lemma 2. If firms do not pay common wages, so that there exist firms for which \( \Lambda_{fi} \neq \Lambda^*_fi \), then labor reallocation has a direct effect on the observed skill premium. If skilled labor reallocates towards firms paying higher skilled wages (firms for which \( \Lambda_{hi} > \Lambda^*_hi \)) or unskilled labor reallocates towards firms paying lower unskilled wages (firms for which \( \Lambda_{\ell i} < \Lambda^*_\ell i \)), then the observed skill premium rises for a given shadow skill premium.

As a corollary of Proposition 1, we derive an expression for the general equilibrium elasticity of substitution between labor and equipment, which accounts both for the direct effect of equipment prices on substitution between equipment and labor as well as the endogenous response of the shadow skill premium.

**Corollary 1.** In response to an exogenous shock to the shadow price of equipment, the response of the share of labor in factor income \( \theta_n \equiv \theta_\ell + \theta_h \) satisfies

\[
\frac{d \ln (\theta_n / \theta_e)}{dw_e} = \sigma^*_ne - 1 - \left( \frac{\theta_\ell}{\theta_\ell + \theta_h} (\sigma^*_\ell h - 1) - (\sigma^*_eh - 1) \right) \sigma_{\ell e} - \sigma_{he} \sigma_{\ell h},
\]

where \( \sigma^*_ne \) is given by Equation (11) and the aggregate elasticities of substitution for factor demand and factor payments \( \sigma_{ff}' \) and \( \sigma^*_{ff}' \) are given by Equations (6) and (7), respectively.

**Proof.** See Appendix A.

Comparing Equations (19) and (11), we find the endogenous response of the labor share to falling equipment prices depends on that of the shadow skill premium. Assuming that the latter is positive (\( \sigma_{\ell e} > \sigma_{he} \)), then whether or not the labor share rises or falls may deviate from a prediction based solely on the elasticity of substitution between labor and equipment \( \sigma^*_ne \). In particular, the extent of this deviation depends on the aggregate elasticities of substitution between low- and high-skilled labor payments \( \sigma^*_{\ell h} \) and that between high-skilled labor and equipment payments \( \sigma^*_{eh} \). The intuition is simple: falling equipment prices (relative to the wages of low-skilled workers) raise the skill premium and the average wage. The more substitutable are the payments to high-skilled labor and equipment (the higher is \( \sigma^*_{eh} \)), the stronger the shift away from labor payments toward
equipment. In contrast, the more substitutable the two skill types are (the higher is $\sigma_{\ell h}^*$), the weaker the shift way from labor payments becomes since low-skilled labor receives some benefits from rising skill premia.

### 2.4 Joint Shocks to the Equipment Price and Labor Wedges

In this section, we now consider an additional exogenous shock that shifts heterogeneous labor wedges across firms. In particular, we assume that labor wedges $T_{fi}$ for $f \in \{L, H\}$ shift infinitesimally by an amount $d\tau_{fi} \equiv d\ln T_{fi}$ characterized by

$$d\tau_{fi} = \omega_{fi} d\tau,$$

where $\omega_{fi}$ is a firm-specific (potentially negative) weight satisfying $\sum_i \Lambda^*_f \omega_{fi} = 1$. In parallel to our definition of the aggregate elasticities of substitution (4) and (5), we can define the aggregate wedge elasticities capturing the effect of the shock to factor-$f'$ wedges on the relative demand for and payments to factor $f$ as

$$\sigma^\omega_{ff'} = \frac{\partial \ln \left( \frac{X_f}{X_{f'}} \right)}{\partial \tau_{f'}} \omega_{f'i},$$

$$\sigma^*\omega_{ff'} = 1 + \frac{\partial \ln \left( \frac{\theta_f}{\theta_{f'}} \right)}{\partial \tau_{f'}},$$

where the superscript $\omega$ indicates the fact that the elasticity is defined with respect to a given set of firm-specific weights ($\omega_{fi}$) following Equation (20). Note that the aggregate elasticities of substitution defined in Equations (4) and (5) are special cases of the aggregate wedge elasticity defined in Equation (21) corresponding to the case of $\omega_{fi} \equiv 1$. The next proposition characterizes the aggregate wedge elasticities in their general form.

**Lemma 3.** The aggregate wedge elasticities of substitution $\sigma^\omega_{ff'}$ between factors $f$ and $f'$ defined by Equations (21) are given by

$$\sigma^\omega_{ff'} = \sum_i \left( \Lambda^*_f \sigma_{ff' i} - \Lambda^*_{f'} \sigma_{f f' i} \right) \theta_{f'i} \omega_{f'i},$$

$$\sigma^*\omega_{ff'} = \sum_i \left( \Lambda^*_f \sigma^*_{ff' i} - \Lambda^*_{f'} \sigma^*_{f f' i} \right) \theta_{f'i} \omega_{f'i}$$

$$+ \varepsilon \sum_i \left( \Lambda^*_f - \Lambda^*_{f'} \right) \theta_{f'i} \omega_{f'i} - (\varepsilon - \eta) \sum_s \left( \Lambda^*_{f's} - \Lambda^*_{f's} \right) \theta_{f's} \omega_{f's},$$

$$+ \varepsilon \sum_i \left( \Lambda^*_{f'i} - \Lambda^*_f \right) \theta_{f'i} \omega_{f'i} - (\varepsilon - \eta) \sum_s \left( \Lambda^*_{f's} - \Lambda^*_{f's} \right) \theta_{f's} \omega_{f's},$$

where $\varepsilon$ and $\eta$ are firm-specific parameters.

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where we have defined the mean sector-specific wedge shock \( \omega_{fs} \) as

\[
\omega_{fs} \equiv \sum_{i \in I_s} \Lambda_{f i | s}^* \left( \frac{\theta_{fi}}{\theta_{fs}} \right) \omega_{fi},
\]

with \( \Lambda_{f i | s}^* \) accounting for the share of sector-s payments to factor \( f \) paid by firm \( i \).

**Proof.** See Appendix A.

Comparing the results of Lemmas 1 and 3, we find that the expressions for the aggregate wedge elasticities simply parallel those for the aggregate elasticities of substitution with one modification: they replace the firm-specific factor-\( f' \) intensity \( \theta_{fi}' \) with \( \theta_{fi}' \omega_{fi} \). This result is intuitive. The structure of the wedge shocks assumed in Equation (20) implies different pass-throughs from the overall factor-level wedge shock \( d\tau \) to firm-specific factor prices. In so far as we are interested in the aggregate effects of the shock, the heterogeneous pass-throughs translate into effective factor-intensities that are smaller or larger than the underlying firm-level factor intensities depending on the size of the pass-through.

The next proposition characterizes the responses of the shadow and observed skill premia to joint shocks to the shadow equipment prices and to labor wedges.

**Proposition 2.** Consider exogenous infinitesimal shocks to the shadow equipment price \( dw_e \) and firm-specific labor wedges \( d\tau_{fi} \equiv \omega_{ji} d\tau_{f} \) for \( f \in \{\ell, h\} \). The responses of the skill premia are given by

\[
dw_h = -\frac{\sigma_{\ell e} - \sigma_{\ell e}^*}{\sigma_{\ell h}} dw_e - \frac{\sigma_{\ell h}^*}{\sigma_{\ell h}} d\tau_h + \frac{\sigma_{h\ell}^*}{\sigma_{\ell h}} d\tau_{\ell},
\]

\[
d\psi = dw_h - \sigma_{\ell h}^* \left( dw_h + \frac{\sigma_{\ell e} - \sigma_{\ell e}^*}{\sigma_{\ell h}} dw_e + \frac{\sigma_{\ell h} - \sigma_{\ell h}^*}{\sigma_{\ell h}^*} d\tau_h - \frac{\sigma_{h\ell}^*}{\sigma_{\ell h}} d\tau_{\ell} \right),
\]

where the aggregate elasticities are given by Equations (6), (7), (22), and (23).

**Proof.** See Appendix A.

Proposition 2 completes our theoretical results by providing predictions about the responses of shadow and observed skill premia to the joint shocks to equipment prices and labor wedges across firms to the first order of approximation in the sizes of the shocks. To use this result in practice, we can discipline the changes in the labor wedges by the observed evolution of the skill premia across firms.
3 Data and Stylized Facts

3.1 Data sources

We exploit a number of French administrative data sources on firms, workers, and firm-level export and import transactions.

Balance sheet and administrative information for the near universe of French enterprises are retrieved from FICUS, a dataset jointly administered by the French Institute National de la Statistique et des Études Économiques (INSEE) and the Direction Générale des Finances Publiques (DGFiP). The detailed breakdown of firm capital by asset type is retrieved from the BRN dataset, which covers all firms that are subject to the BRN (Bénéfice Réel Normal) tax regime over the period 1993-2009. It includes over 60% of French firms, which account for 79% of employment and value added compared to the full set of firms contained in FICUS; moreover they account for over 90% of the value of trade flows in the Customs data set.

Firm-level international transaction data are provided by the French Directorate-General of Customs and Indirect Taxes (DGDDI), which provides information on the annul value of imports and exports by country of origin/destination and 8-digit CN product for all firms involved in international transactions (with simplified declarations below certain thresholds). We make use of the data over the period 1994-2007 and focus on manufacturing firms. Our final dataset describes the international trade flows of French firm concerning over 5,000 products from/to 161 countries.

Firm-level employment and wage bill data are computed aggregating worker-level data on hours worked and wages. These data, together with worker characteristics, are retrieved from the DADS (Déclarations Annuelles de Données Sociales) Poste, a matched employer-employee dataset provided by INSEE that covers the whole population of French workers.

Sectoral data on employment, investment, and capital by asset type are retrieved from INSEE National Accounts. Industry-level data on depreciation rates by asset type come from the EU KLEMS Database. Summary statistics are displayed in Table 1.

3.2 Variable definitions

In this section we provide a brief overview of how we compute the main variables of interest used in the analysis. For more details see Appendix C.

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9Firms with revenues above a certain threshold must be affiliated with the BRN regime. In 2007, the thresholds were 763,000 euros if a firm operates in trade or real estate sectors and 230,000 euros otherwise.
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
<th>Reduced Sample</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Source</td>
<td>Obs. (Nb)</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>FICUS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Value Added</strong></td>
<td>FICUS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Low-Skill Working Hours</strong></td>
<td>DADS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Low-Skill Hourly Wage</strong></td>
<td>DADS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Low-Skill Wage Bill (W_L)</strong></td>
<td>DADS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>High-Skill Working Hours</strong></td>
<td>DADS</td>
<td>1,555,231</td>
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<tr>
<td><strong>High-Skill Hourly Wage</strong></td>
<td>DADS</td>
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</tr>
<tr>
<td><strong>High-Skill Wage Bill (W_H)</strong></td>
<td>DADS</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Equipment Stock (W_E)</strong></td>
<td>BRN</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Exports - Total</strong></td>
<td>Customs</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Imports - Total</strong></td>
<td>Customs</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Imports - Intermediate products</strong></td>
<td>Customs</td>
<td>1,555,231</td>
</tr>
<tr>
<td><strong>Imports - Equipment products</strong></td>
<td>Customs</td>
<td>1,555,231</td>
</tr>
</tbody>
</table>

Notes: Observations is the number of firm-year pairs in the manufacturing sector over the 1998-2007 period. The units for all other variables are euros except for those involving working hours. The full sample include all firms used in the aggregation exercise of Section 5. The reduced sample is restricted to the 2002-2007 period and the set of firms used in the estimation of the firm-level production elasticities (Table 3), which conditions on firms having a positive equipment capital stock, employing both low- and high-skill workers, and importing both intermediate and equipment products.

Firm revenue. Our baseline model doesn’t include intermediate inputs. Revenue and value added are therefore identical in the model, and we use value added in the data as the counterpart to revenue in the model. Annual value added for each firm i is obtained from FICUS as gross output less intermediate expenditures, where gross output is defined as the sum of sales and changes in inventories while intermediates include all the purchases of the firm.

Skill groups. Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on their occupation, using occupational classifications in French social security data (CS - catégories socioprofessionnelles). High-skill workers are those employed as Managers, Middle managers and professionals, and Qualified workers. Low-skill workers are employed as Clerks and Blue-collar.

Change in firm-and-skill-specific wages and changes in employment. In the model, all workers in a skill group are identical but in the data workers within the same group vary in observables. To measure five-year firm-and-skill specific wage changes in the data, \( \Delta w_{hit} \) and \( \Delta w_{\ell it} \), we correct changes in observed wages for both changes in returns to worker observable characteristics and changes in worker composition. To do so, we estimate a national Mincerian regression on log wage changes between \( t - 1 \) and \( t \) controlling for worker observables (age, sex, 2-digit CS category) including a firm-skill-time effect on a sample of workers in a given skill group who are employed by the same firm between \( t - 1 \) and \( t \). We additionally exclude atypical workers and those who did not meet a minimum hours-worked threshold. The year-on-year change in the firm-and-skill specific wage is the firm-skill-year fixed effect (the average log change in wages controlling
for observables on the estimation sample). We chain these together to obtain five-year changes. We measure five-year changes in employment, $\Delta x_{hit}$ and $\Delta x_{ilit}$, by deflating changes in firm-and-skill-specific wage bills by the corresponding wage change, $\Delta w_{hit}$ and $\Delta w_{hit}$.

**The level and change in equipment stock.** We measure the equipment stock on the basis of data on domestic investments in equipment products, retrieved from BRN, and transaction-level data on the imports of equipment, obtained from the customs dataset. We compute changes in firm-level equipment prices as the average of equipment product-, import-market-, and firm-specific year-on-year log unit price changes weighed by Sato-Vartia weights, where we use the series of equipment prices from the French national accounts for all firms’ domestic equipment purchases. We then chain these equipment price changes to construct a firm-specific equipment price level in each year, $P_{it}^e$. Equipment investment quantities are obtained by dividing firm-level equipment investment values by the firm-level equipment price. Equipment quantities are then used to obtain the equipment stock on the basis of the perpetual inventory method. Five-year log changes in equipment quantities are obtained directly from this series of firm-and-year-specific equipment stocks.

**User cost of Equipment.** We define the firm-level user cost of equipment $W_{eit}$ (the effective rental price of equipment capital) as follows:

$$W_{eit} = P_{it}^e \left( R_i^e + \delta_{st} - \frac{p_{s,t+1}^e - p_{s,t-2}^e}{3} \right)$$

where $P_{it}^e$ is the firm-specific price of equipment products purchased by firm $i$ in time $t$ defined above, $R_i^e$ is the required rate of return on equipment investments, and $p_{s,t}^e$ and $\delta_{st}$ are the log price and depreciation rate of equipment products at the sector level from the national accounts, respectively.

### 3.3 Stylized Facts

We now document two sets of stylized facts on how factor intensities and skill premia vary across firms.

We measure factor intensities, $\theta_{fi}$ for skilled and unskilled labor using wage bills for skilled and unskilled workers. For equipment, we use the payments to equipment $W_{eit}X_{eit}$ constructed as described in Section 3.2 above.

Figure 1 shows the average across firms of the factor intensity of each factor $f$ across the firm-size distribution together with their corresponding standard errors. Low-skill
labor intensity declines monotonically and substantially in firm size, from approximately 75% for the smallest firms to below 20% for the largest. Larger firms are, therefore, more intensive in the combination of high-skill labor and equipment. While high-skill intensity increases from approximately 20% to 35% from the smallest to largest firms, it is flat across most of the firm-size distribution except for increasing steeply between the first two and the last two size bins. Equipment intensity, on the other hand, rises with firm size throughout the size distribution from below 10% to above 40%.

We find in Figure 1 a strong negative correlation between equipment intensity and low-skill labor intensity and a weak positive correlation between equipment intensity and high-skill labor intensity across the firm size distribution. These correlations have important implications for the impact of a decline in equipment prices on the skill premium, even in the absence of capital-skill complementarity at the firm level and in the absence of imperfect labor markets. In this case, a decline in equipment prices induces factors to reallocate towards equipment-intensive firms and raises the relative demand for skilled labor, since equipment-intensive firms are relatively skill intensive. As we saw in our theoretical analysis in Section 2, in response to falling equipment prices this reallocation mechanism leads to an increase in the shadow and, therefore, observed skill premium.

We conclude this section by documenting the pattern of observed skill premia across the firm-size distribution. We construct the observed skill premium for each firm size bin averaging across all sample years. Figure 2 displays our results, including the corresponding standard deviation. Among small firms, the skill premium is increasing in size up to firms with fifty employees. Thereafter, the skill premium is monotonically decreasing in firm size. In this case, a decline in equipment prices induces labor to reallocate towards firms with lower skill premia (because they are larger and larger firms have higher equipment intensities according to Figure 1). As we saw in our theoretical analysis
in Section 2, in response to falling equipment prices this reallocation mechanism leads to a decrease in the observed skill premium holding shadow wages fixed.

Figure 2: Firm Size and the Skill Premium

Notes: The figure plots the average skill premium across firms by size bin and the corresponding standard deviation (in grey) over the 1998-2007 period. The firm-level skill premium is measured as the ratio of the observed average hourly wage of high skill vs low skill workers employed by firm, computed as the wage bill divided by the number of hours worked in each skill group.

## 4 Estimation

In Section 2, we provided a theory that characterizes the elasticity of the observed skill premium and the labor share with respect to the equipment price for arbitrary constant returns to scale firm-level production functions. In this section, we assume that the firm-level production function belongs to the CRESH family presented in Example 2, and specified in Equation (8) and estimate this production function using our micro data. We additionally use the data to estimate the key remaining elasticities that we need to perform the aggregation exercise presented in Section 2.

### 4.1 Estimating Equations

To perform the aggregation, we require the following micro elasticities: the demand elasticity across sectors $\eta$, the demand elasticity across firms within each sector $\varepsilon$, and the factor-specific elasticities $\sigma_{\ell}$, $\sigma_{h}$, and $\sigma_{e}$ that together characterize the CRESH production function. We impose $\eta = 1$ and estimate the remaining parameters.
Demand Elasticity $\varepsilon$. Given CES demand across firms within each sector, we have

$$\Delta q_{it} = \alpha_{st} - \varepsilon \Delta p_{it} + \Delta \phi_{it}$$  \hspace{1cm} (27)

where $\alpha_{st}$ is a sector- and time-$t$ fixed effect, $\Delta p_{it}$ is the change in firm $i$’s log price, and $\Delta \phi_{it}$ is the log change in firm $i$’s demand shifter.

In our baseline, we estimate Equation (27) using five-year changes in export quantities and prices at the firm level, where $\Delta x_t \equiv x_t - x_{t-5}$ for any variable $x$. We use export data since we do not observe unit prices for domestic sales. For each exporting firm $i$, we measure the firm-specific year-on-year log export price change as the average of product- and export-market-specific log unit price changes weighed by Sato-Vartia weights. We then chain together these year-on-year price changes to obtain five-year price changes, $\Delta p_{it}$. We then measure firm $i$’s five-year export quantity change $\Delta q_{it}$ by deflating firm $i$’s five-year change in log total export revenue by the constructed export price index $\Delta p_{it}$.

Production Function Elasticities $\sigma_k$, $\sigma_h$, and $\sigma_e$. The production function specified in Equation (8) implies that firm $i$’s factor demand satisfies

$$\Delta x_{eit} - \Delta x_{hit} = \beta_{eit} - \sigma_e \left( \Delta w_{eit} - \Delta w_{hit} \right) + \left( \frac{\sigma_e}{\sigma_e} - 1 \right) \left( \frac{\varepsilon}{\varepsilon} \Delta r_{it} - \Delta x_{eit} \right) + \nu_{eit}$$  \hspace{1cm} (28)

$$\Delta x_{eit} - \Delta x_{hit} = \beta_{hit} - \sigma_h \left( \Delta w_{eit} - \Delta w_{hit} \right) + \left( \frac{\sigma_h}{\sigma_e} - 1 \right) \left( \frac{\varepsilon}{\varepsilon} \Delta r_{it} - \Delta x_{eit} \right) + \nu_{hit},$$  \hspace{1cm} (29)

where $x_{f it}$ is the logarithm of employment of factor $f$ in firm $i$ at time $t$ and where we have replaced the change in firm $i$’s output with the change in revenue $r_{it}$ using the structure imposed by the CES demand for firm products. The structural residuals $\nu_{eit}$ and $\nu_{hit}$, therefore, depend not only on changes in firm-specific factor-augmenting productivities $\Delta z_{f i}$, but also the change in firm $i$’s demand shifter and the change in the relevant sectoral price index. See the proof of Equations (28) and (29) in Appendix A.

In our baseline, we estimate equations (28) and (29) using five-year changes in factor employment, prices, and revenues. We measure firm $i$’s employment and payment of each factor as described in Section 3.2 and we obtain the respective five-year changes by chaining the firm-specific year-on-year log changes. Relative to measuring the change in the average wage across workers employed in each year, our approach is not affected by changes in the composition of the firm’s workforce.\footnote{There is an alternative approach to estimating $\sigma_f$ that does not require the inclusion of output (or revenue) as an independent variable. There are two benefits of using equations (28) and (29). First, the inclusion of additional factors of production leave these estimating equations unchanged. Hence, our estimates of these parameters is robust to additional factors. Second, these equations can be estimated using}
4.2 Estimation Approach and Instruments

We first describe our overall estimation strategy and then describe our instruments.

**Estimation Strategy.** We estimate $\epsilon$ first using equation (27) via 2SLS. We then use the implied value $\epsilon$ to estimate $\sigma_\ell$, $\sigma_h$, and $\sigma_e$ using equations (28) and (29) via GMM, since the three structural parameters we are interested in identifying depend on parameter estimates from both equations.\(^{11}\) We bootstrap the full estimation so that our confidence intervals for $\sigma_\ell$, $\sigma_h$, and $\sigma_e$ incorporate the dispersion of our estimated $\epsilon$.

**Instrument for Demand Elasticity Estimation.** In general, firm-level price changes $\Delta p_{it}$ depend on firm-specific changes in demand shifters $\Delta \phi_{it}$, which is then correlated with the residuals on the right hand sides of Equations (28) and (29). To address this endogeneity problem, we construct an instrument that shifts firm $i$’s marginal cost independently of changes in its demand shifter.

We leverage the differences in firm-specific import exposure across source countries and changes in real exchange rates between France and its trading partners. If firm $i$ obtains a large share of its imports from a given source country $c$ and the real exchange rate of country $c$ depreciates relative to France, then firm $i$’s cost of imports falls, thereby reducing firm $i$’s marginal cost. We refer to this as the real-exchange-rate instrument and define it formally as

$$\Delta RER_{it} = \sum_c \frac{M_{c,i,t-5}}{M_{i,t-5}} \Delta \ln \left( \frac{NER_{ct}}{CPI_{ct}} \cdot \frac{CPI_{c,t}}{CPI_{FR,t}} \right)$$

(30)

Here, $NER_{ct}$ is the nominal exchange rate (defined as euros per country $c$ currency), $CPI_{ct}$ is the consumer price index (CPI) in the origin country $c$ and $CPI_{FR,t}$ is the CPI of France in year $t$. Exposure shares are constructed as firm $i$’s total import value in year $t-5$ from country $c$, $M_{c,i,t-5}$, relative to total imports by firm $i$ in $t-5$, $M_{c,t-5}$. A real depreciation of country $c$’s currency relative to the euro is reflected in an increase in $\Delta RER_{it}$.

**Instruments for Firm-Level Production Elasticities Estimation.** The structural residuals $\nu_{\ell i t}$ and $\nu_{hi t}$ in Equations (28) and (29) depend on the changes in firm-specific factor-augmenting productivities $\Delta z_{fi}$, but also the change in firm $i$’s demand shifter and the change in the relevant sectoral price index.\(^{12}\) To address these endogeneity problems, we construct instruments that shift firm $i$’s (i) the cost of equipment relative to low-skilled labor, and (ii) the cost of equipment relative to high-skilled labor, and (iii) revenue rela-

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\(^{11}\)In an analysis of the robustness of our results, we estimate equations (28) and (29) separately using 2SLS.

\(^{12}\)See the derivation of Equations (28) and (29) in Appendix A.
tive to its equipment stock. These instruments should be independent of changes in the
firm’s factor-augmenting productivities and its demand shifter.

Our first instrument is intended to shift firm $i$’s equipment price. Its logic is similar
to our real-exchange-rate instrument. A decrease in the transport cost of shipping type-
k equipment from origin country $c$ to France will reduce the equipment cost of a firm $i$
for which this country-equipment pair constitutes a large share of equipment imports.
Following Hummels et al. (2014), we create an instrument that leverages differences in
firm-specific equipment import exposure across source countries and predicted changes
in shipping costs between France and its trading partners induced by changes in oil and
jet fuel prices. Using French micro-data, we obtain mode-of-transport frequencies and
weight-value ratios by HS6 product. Exploiting different sources, we retrieve data on
oil and jet fuel prices, weighted distances between France and all other countries, and
transportation charge elasticities (more details in Appendix). For each origin country $c$
and equipment product $k \in K_e$, we compute predicted five-year changes in transport
costs and construct a firm-specific average weighting by initial equipment import shares.
We refer to this as the equipment transit-cost instrument and define it formally as

$$\Delta ET C_{it} = \sum_{c} \sum_{k \in K_e} \frac{M_{cki,t-5}}{M_{ti,t-5}} \Delta \ln T C_{ckt}$$

Here, $\ln T C_{ckt}$ is the predicted transport cost of equipment product $k$ from country $c$ to
France (given oil and fuel prices, the distance between France and country $c$, and the
predicted shipping mode). Exposure shares are constructed as firm $i$’s import value of
equipment products in year $t - 5$ from country $c$, denoted by $M_{cki,t-5}$, relative to the
value of total imports of equipment products in year $t - 5$ by firm $i$, denoted by $M_{e,c,t-5}$.

Our second instrument is intended to shift firm revenues. Its logic is also similar to our
real-exchange-rate instrument. An increase in productivity for origin country $c$ in produc-
ing intermediate input $k$ will reduce the marginal cost of a firm $i$ for which this country-
input pair constitutes a large share of intermediate imports. Again following Hummels et
al. (2014), we create an instrument that leverages differences in firm-specific intermediate
good import exposure across source countries and intermediates and changes in world
export supplies. For each exporting country $c$ and HS6 product $k$ in the set of interme-
diate products $K_I$, we measure the value of total exports towards all countries excluding
France in year $t$, denoted by $Exp_{ckt}$. We use this to construct a firm-$i$-specific average of
changes in export supplies weighting by initial intermediate good and origin country im-
port shares. We refer to this as the world-export-supply instrument and define it formally

$$\Delta ET C_{it} = \sum_{c} \sum_{k \in K_I} \frac{Exp_{ckt}}{M_{ki,t-5}} \Delta \ln T C_{ckt}$$
as
\[
\Delta WES_{it} = \sum_c \sum_{k \in \mathcal{K}_t} \frac{M_{cki,t-5}}{M_{i,t-5}} \Delta \ln \text{Exp}_{ckt}
\]  

(32)

Here, \( \Delta \ln \text{Exp}_{ckt} \) is the change in exports of intermediate product \( k \in \mathcal{K}_t \) from origin country \( c \) to the rest of the world (excluding France). Exposure shares are constructed as firm \( i \)'s import value of intermediate good \( k \) in year \( t - 5 \) from country \( c \), \( M_{cki,t-5}, \) relative to the value of total imports of intermediate goods in year \( t - 5 \) by firm \( i \), \( M_{i,t-5} \).

Our third instrument is intended to shift the wage of skilled and unskilled workers facing each firm. A growth in French output of a sector \( s \) that is particularly skilled (or unskilled) intensive will raise the skilled (unskilled) wage in a region with a large share of employment in that sector. This will raise the effective skilled (unskilled) wage facing firm \( i \) if it has a large share of its employment in such regions. Following this intuition, our third instrument leverages differences in the spatial compositions of firm production, the industrial mix of these French regions, and the skill intensities and growth of these industries. We refer to this as the skill-specific-wage instrument and define it formally as

\[
\Delta \text{SSW}_{it} = \sum_r \sum_s \left( \frac{X_{ri,t-5}}{X_{ni,t-5}} \right) \cdot \left( \frac{X_{rs,t-5}}{X_{ns,t-5}} \right) \cdot \left( \frac{X_{rh,t-5}}{X_{ls,t-5}} \right) \cdot \Delta \ln \text{GO}_{st}
\]  

(33)

Here, \( \Delta \ln \text{GO}_{st} \) is the change in gross output for sector \( s \) at the national level in France. Exposure shares are constructed as the product of three components: (i) firm \( i \)'s employment in region \( r \), \( X_{ni,t-5} \), relative to its total employment, \( X_{ni,t-5} \) across all domestic regions; (ii) employment in sector \( s \) in region \( r \), \( X_{ns,t-5} \), relative to total employment in region \( r \), \( X_{n,t-5} \); and (iii) employment of skilled labor in sector \( s \), \( X_{rs,t-5} \), relative to employment of unskilled labor in sector \( s \), \( X_{ls,t-5} \), both defined at the national level.

4.3 Estimation Results

**Demand Elasticity Estimates.** Since we estimate Equation (27) using export quantities and prices, in our baseline we restrict the sample of firms to those exporting on average at least 5% of their gross output over the period. Column 1 of Table 2 displays the OLS estimate of \( \varepsilon \) in equation (27) while columns 2 - 6 display the 2SLS estimates together with standard errors clustered by firm. Whereas the OLS estimate is close to one, the 2SLS estimates range from 2.2 to 3.6. Column 2 represents our baseline estimate, \( \varepsilon = 2.92 \), which is close to the median value of 2.7 reported by Broda and Weinstein (2006) for products at the five-digit level. In our baseline specification, the first-stage KP \( F \)-statistic is approximately 13 and it remains broadly stable across specifications.
### Table 2: Between-Firm Demand Elasticity Estimates

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>2SLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$-\Delta p$</td>
<td>0.978</td>
<td><strong>2.920</strong></td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.794)</td>
</tr>
<tr>
<td>Observations</td>
<td>72,202</td>
<td>45,435</td>
</tr>
<tr>
<td>Year FE</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Year-Sector FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Exp/GO $\geq$ 5%</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>IV</td>
<td>-</td>
<td>RER</td>
</tr>
<tr>
<td>KP F stat</td>
<td>-</td>
<td><strong>12.75</strong></td>
</tr>
</tbody>
</table>

Notes: This table reports estimation results of equation (27). The dependent variable is the 5-year change in firm $i$’s export quantity and the independent variable is $-1 \times$ the corresponding change in firm $i$’s export price. Columns 2-6 use the RER instrument defined in Equation (30) and column 6 additionally uses the SSW instrument defined in equation (33). Year fixed effects are included in column 4 and year-sector fixed effects are included in all other columns. Controls indicates the inclusion of the average import share in the pre-sample period (1996-7) interacted with a time-varying coefficient. Exp/GO $\geq$ 5% indicates that the sample is restricted to firms that export at least 5% of gross output. We report the Kleibergen-Paap rk Wald (KP F stat) of the first stage regression. Robust standard errors are clustered at the firm level and reported in parenthesis.

In column 3, we omit the sample restriction that firms export on average at least 5% of their gross output and the estimate increases to 3.6. In column 4 we replace year-sector fixed effects with year fixed effects, and our estimate falls slightly to 2.2. In column 5, we additionally control for the average import share interacted with time dummies, following Adao et al. (2022). Our estimated elasticity of substitution is largely unchanged including this set of controls. Finally, in column 6, we use both our real-exchange-rate instrument and our skill-specific-wage instrument and our estimate falls slightly to 2.3.\(^{13}\)

---

**Firm-Level Production Elasticity Estimates.** The top panel of Table 3 displays coefficient estimates in equations (28) in the odd columns and (29) in the even columns. The bottom panel displays the corresponding estimates of the structural parameters of interest, $\sigma_\ell$, $\sigma_h$, and $\sigma_e$, estimated using GMM for different values of $\epsilon$ and using different fixed effects. Columns 1 and 2 displays our baseline estimates of $\sigma_\ell = 0.97$, $\sigma_h = 0.84$, and $\sigma_e = 1.11$, using our baseline estimate of $\epsilon = 2.92$ and including year effects. We find that $\sigma_\ell > \sigma_h$, which implies that equipment is more substitutable with low- than high-skilled labor: at the firm level, production exhibits equipment-skill complementarity. This difference in

---

\(^{13}\)Table D.1 replicates Table 2 using an alternative instrument in which exposure shares are constructed as firm $i$’s import value of intermediate goods only. Results are robust.
Table 3: Firm-Level Production Elasticities

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_e - \Delta w_i$</td>
<td>-0.968</td>
<td>-0.969</td>
<td>-0.967</td>
<td>-1.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.179)</td>
<td>(0.165)</td>
<td>(0.179)</td>
<td>(0.456)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\Delta \gamma}{\Delta x_e}$</td>
<td>-0.129</td>
<td>-0.113</td>
<td>-0.140</td>
<td>-0.0123</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.055)</td>
<td>(0.070)</td>
<td>(0.163)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \gamma - \Delta x_e$</td>
<td>-0.841</td>
<td>-0.864</td>
<td>-0.824</td>
<td>-0.798</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.147)</td>
<td>(0.145)</td>
<td>(0.143)</td>
<td>(0.339)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 35,142  35,142  35,142  35,142  35,142  35,142  35,142  35,142
Year FE: Yes  Yes  Yes  No
Year-Sector FE: No  No  Yes  Yes
$\epsilon$: 0.9679  0.9689  0.9674  1.0012
       (0.179)  (0.165)  (0.179)  (0.456)
$\sigma_h$: 0.8412  0.864  0.8236  0.7979
               (0.147)  (0.145)  (0.143)  (0.339)
$\sigma_e$: 1.1107  1.0921  1.1255  1.0136
            (0.247)  (0.215)  (0.252)  (0.672)
Pr[$\sigma_{\ell} \leq \sigma_h$]: 0.005  0.005  0.005  0.015
                      2.92  2.321  3.627  2.92

Notes: This table reports regression coefficients for the system of equations (28) and (29) estimated using GMM including year effects (columns 1-6) or year-sector effects (columns 7-8) for a given value of $\epsilon$. Corresponding point estimates of $\sigma_{\ell}$, $\sigma_h$, and $\sigma_e$ are displayed. Bootstrap standard errors are reported in parenthesis, obtained by bootstrapping the estimation of $\epsilon$ and of this system 200 times and Pr[$\sigma_{\ell} \leq \sigma_h$] reports the share of bootstrapped estimates in which $\sigma_{\ell}$ is lower than $\sigma_h$. If $\sigma_{\ell} > \sigma_h$ in each of our 200 replications, we set this value to 1/200.

The parameters is statistically significant, as shown in the bottom panel of Table 3.

In columns 3 and 4, we use the lowest estimate of $\epsilon$ reported in Table 2 (conditional on sector-year fixed effects) and in columns 5 and 6 we use its highest estimate. In columns 7 and 8 we include year-sector effects instead of year effects. Across all columns, we find that $\sigma_{\ell} > \sigma_h$, and that the difference between $\sigma_{\ell}$ and $\sigma_h$ is statistically significant.14

Figure 3 displays the implications of these estimates for firm-level elasticities of substitution between low-skilled labor and equipment $\sigma_{\ell e,i}$ and between high-skilled labor and equipment $\sigma_{he,i}$ across the firm size distribution, as given by Equation (9). The levels of $\sigma_{\ell e,i}$ and $\sigma_{he,i}$ decline across the firm-size distribution because larger firms have higher equipment intensities $\theta_{ei}$, as shown in Panel (c) of Figure 1, and $\sigma_e$ is greater than both $\sigma_{\ell}$ or $\sigma_h$, as shown in Table 3. This implies that $\sigma_i$ increases in firm size, as shown in Figure 3.

5 Aggregation Results

14In Table D.2 in Appendix, we perform a joint GMM estimation of the demand elasticity and the production elasticities on the subsample of firms that are simultaneously importers and exporters. Our conclusions are vastly unchanged and we obtain very similar estimates for all parameters, featuring in particular the same ranking for production elasticities.
Figure 3: Firm Size and Elasticities of Substitution Between Factors

![Graph showing elasticities of substitution between low-skilled labor and equipment, high-skilled labor and equipment, and the aggregate elasticity of substitution between labor and equipment across firm size distribution.]

Notes: This figure displays firm-level elasticities of substitution between low-skilled labor and equipment $\sigma_{\ell,e,i}$ and between high-skilled labor and equipment $\sigma_{h,e,i}$ across the firm size distribution. The elasticities are constructed using equation (9), our baseline estimates of $\sigma_{\ell}$, $\sigma_{h}$, $\sigma_{e}$ and the values of factor intensities $\theta_{fi}$, measured in 2003, for each $f$ constructed within the relevant firm-size bin.

Finally, we quantify the effects of changes in equipment prices on the skill premium and the labor share over time in France. As a first step, Table 4 displays the elasticity of the observed skill premium with respect to equipment prices $d\psi/dw_e$ constructed using Equation (18), the elasticity of the labor share with respect to equipment prices $d\ln(\theta_n/\theta_e)/dw_e$ constructed using Equation (19), and the aggregate elasticity of substitution between labor and equipment $\sigma_{ne}^*$ constructed using Equation (11), under various parameterizations, all calculated using shares averaged across all years.

Column 1 is our baseline specification, corresponding to the micro elasticities presented in Columns 1 and 2 of Table 3. Using average shares over the sample period, a decrease in the equipment price of 1% increases the skill premium by approximately 0.06% and decreases the labor share by approximately 0.20%. The fact that the labor share rises implies that the elasticity of substitution between equipment and labor is greater than one. The third row of Table 4 confirms this; $\sigma_{ne}$ is approximately 1.4.

Columns 2 and 3 replace $\epsilon$ with our lower and upper bound of estimates from Table 2 and the corresponding estimates of $\sigma_{\ell}$, $\sigma_{h}$, and $\sigma_{e}$ from Columns 3-4 and 5-6 of Table 3, respectively. Finally, Column 4 uses values of $\sigma_{\ell}$, $\sigma_{h}$, and $\sigma_{e}$ estimated including year-sector fixed effects from Columns 7 and 8 of Table 3. Qualitative results—a decline in the price of equipment raises the skill premium and lowers the labor share—are robust across specifications. Quantitative elasticities are largely robust as well, with the elasticity of the skill premium ranging from $-0.046$ and $-0.084$ and the elasticity of the labor share ranging from $0.133$ and $0.270$. 

Page 28
Finally, we turn to quantifying the impact of factual equipment price changes on the evolution of the skill premium and the labor share in France in our sample period. Doing so requires both an elasticity of the skill premium with respect to the equipment price, presented in Table 4 using firm shares averaged across the sample, together with a change in the equipment price. Because we normalize the shadow wage of low-skilled labor in our theory in Section 2, we must measure this relative price change.

We construct model-consistent year-on-year changes in the equipment price and low-skill shadow wage using data from our estimation sample. The aggregate price of equipment is obtained by averaging across our firm-specific log equipment price changes, weighted by Sato-Vartia ideal weights (based on total equipment investment). The change in the low-skilled shadow wage is obtained by averaging across our firm-specific log change in the observed hourly low-skill wage (among workers who stay employed by the same firm between the two years, thereby measuring the change in the shadow wage), using Sato-Vartia ideal weights (based on the low-skill wage bill of stayers). The cumulative log change in the relative equipment price is approximately 41 log points over the period 1997-2007, with changes in equipment prices and changes in low-skill shadow wages each accounting for nearly half of this decline.\(^{15}\)

---

**Table 4: Aggregate elasticities**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{d\psi}{dw_e} )</td>
<td>-0.055</td>
<td>-0.046</td>
<td>-0.060</td>
<td>-0.084</td>
</tr>
<tr>
<td></td>
<td>[-0.084, -0.017]</td>
<td>[-0.075, -0.013]</td>
<td>[-0.102, -0.013]</td>
<td>[-0.169, -0.014]</td>
</tr>
<tr>
<td>( \frac{d\log(\theta_n/\theta_e)}{dw_e} )</td>
<td>0.196</td>
<td>0.133</td>
<td>0.270</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>[0.063, 0.392]</td>
<td>[0.007, 0.298]</td>
<td>[0.094, 0.958]</td>
<td>[-0.093, 0.733]</td>
</tr>
<tr>
<td>( \sigma^*_{ne} )</td>
<td>1.368</td>
<td>1.250</td>
<td>1.508</td>
<td>1.334</td>
</tr>
<tr>
<td></td>
<td>[1.190, 1.736]</td>
<td>[1.013, 1.560]</td>
<td>[1.176, 2.800]</td>
<td>[0.825, 2.376]</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>2.92</td>
<td>2.321</td>
<td>3.627</td>
<td>2.92</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma_L )</td>
<td>0.841</td>
<td>0.864</td>
<td>0.824</td>
<td>0.798</td>
</tr>
<tr>
<td>( \sigma_h )</td>
<td>1.111</td>
<td>1.092</td>
<td>1.126</td>
<td>1.014</td>
</tr>
</tbody>
</table>

Notes: \( \frac{d\psi}{dw_e} \) and \( \frac{d\log(\theta_n/\theta_e)}{dw_e} \) are the elasticities of the observed skill premium and the labor share with respect to the equipment price and \( \sigma^*_{ne} \) is the elasticity of substitution between equipment and labor. These elasticities are constructed using firm factor shares averaged across all sample years. Column 1 is our baseline, columns 2 and 3 use our lowest and highest estimates of \( \varepsilon \), and column 4 uses our estimates of \( \sigma_L, \sigma_h, \) and \( \sigma_e \) including year-sector FE. The table displays 95% confidence intervals obtained by bootstrapping the estimation of \( \varepsilon \) and each \( \sigma_f \) 200 times.
Panel (a) of Figure 4 shows the model-predicted impact of the fall in the relative equipment price on the skill premium between 1997-2007 (along with the associated 95% confidence interval). To compute the predicted change in the skill premium between years $t - 1$ and $t$, we compute the aggregate elasticities using the distribution of factor intensities in year $t - 1$ and use the relative equipment price change between $t - 1$ and $t$ as the corresponding shock. We then compute the implied cumulative change in the skill premium starting from 1997. The confidence interval is constructed by re-estimating the micro-level elasticities of substitution across bootstrapped samples of the data (200 times). Between 1997 and 2007, the decline in the price of equipment relative to the shadow wage of low-skill labor generates a 2.2 log point increase in the skill premium, which is statistically different from zero. Panel (b) displays the robustness of our quantitative results across the estimation specifications displayed in Table 3. Consistent with the elasticities in Table 4, we obtain very similar results across specifications.

We obtain a similar result for the predicted change in the skill premium when we perform a simpler exercise multiplying the total cumulative change in the relative equipment price by the average aggregate elasticity over the entire period from column 1 of Table 4. The results are displayed in Table D.3 (Micro approach): We obtain again a total 2.2 log point increase of the skill premium (and a 8 log point decline in the labor share). We can contrast these results against the corresponding predicted changes when for the relative equipment price we directly use the official published series (in this case we proxy the aggregate change in the low-skilled shadow wage with the change in the nominal hourly minimum wage). This simple quantification delivers similar conclusions over the period (Macro approach). At the same time, it allows us to extend the time frame of the analysis and obtain the predicted changes over three decades. Our model generates an approximately 4.2% rise in the skill premium and a 15% fall in the labor share in France between 1990 and 2019.

6 Conclusion

Does the aggregate production technology feature capital-skill complementarity? If so, is this driven primarily by capital-skill complementarity at the firm level, by labor-market distortions, or by a positive correlation between firm-level capital intensity and skill intensity?

of the aggregate series “Other machinery and equipment and weapons systems (gross)” taken from French national accounts. This is due in part because we use substantially more detailed import price changes and in part because we make use of detailed industry- and equipment-type-specific domestic price deflators weighted by the observed investments of the firms in our (manufacturing) sample.
To make progress on these questions, we have characterized the response of the observed skill premium to changes in the price of equipment capital in a multi-factor, multi-sector framework featuring arbitrary constant returns to scale production technologies at the firm-level and arbitrary distortions to firm-level factor prices. We have shown that generalized definitions of the aggregate elasticities of substitution between factors—the aggregate elasticity of substitution in factor demand and the aggregate elasticity of substitution in factor payments—shape the answers to these questions.

To leverage our theoretical results, we used matched employer-employee data from France to document a set of stylized facts that play a central role in shaping aggregate capital-skill complementarity. We further relied on the data to measure firm-level factor intensities and wages for each skill group, and estimate the micro-level elasticities of substitution, under a particular functional form assumption on firm-level technology. We identified statistically significant capital-skill complementarity at the firm level.

The estimated elasticities, when combined with the observed distribution of factor intensities, imply that a 1% decline in the price of equipment generates a 0.055% increase in the observed skill premium in France. Together with an approximately 40 percent decline in the relative price of equipment, capital-skill complementarity generated an approximately 2.2% rise in the French skill premium between 1997 and 2007. We found that the majority of aggregate capital-skill complementarity is driven by a positive correlation between firm-level capital intensity and skill intensity.
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A Proofs and Derivations

Proof for Equation 9. Let us first find the output elasticity of factor $f$:

$$\frac{X_f \partial Y}{Y \partial X_f} = \left( \frac{\sigma_f - 1}{\sigma_f} \right) \frac{(Z_f A_{f_i})^{\frac{\sigma_f - 1}{\sigma_f}}}{\sum_p \left( \frac{\sigma_{p'} - 1}{\sigma_{p'}} \right) (Z_{p'} X_{f_{p_i}})^{\frac{\sigma_{p'} - 1}{\sigma_{p'}}}}.$$

We compute the factor cost shares through solving the problem of minimizing unit costs for the firm:

$$\min_{(A_f)} \sum_f W_{f_i} A_{f_i} + \Xi_i \left( 1 - \sum_{f \in \{\ell, h, e\}} (Z_{f_i} A_{f_i})^{\frac{\sigma_f - 1}{\sigma_f}} \right),$$

where $\Xi_i$ is the Lagrange multiplier on the constraint defining the production function in Equation (8), where we have substituted for the unit factor-$f$ requirement as $A_{f_i} \equiv \frac{X_{f_i}}{Y_i}$.

The first order condition with respect to factor $f$ gives us:

$$W_{f_i} = \Xi_i \frac{\sigma_f - 1}{\sigma_f} A_{f_i} \left( Z_{f_i} A_{f_i} \right)^{\frac{\sigma_f - 1}{\sigma_f}}.$$

Using the constraint in Equation (8) then yields:

$$\Xi_i = \sum_f \frac{\sigma_f}{\sigma_f - 1} W_{f_i} A_{f_i}.$$

Moreover, the cost share of factor $f$ is given by:

$$\theta_{f_i} \equiv \frac{W_{f_i} A_{f_i}}{\sum_{f'} W_{f_{p_i}} A_{f_{p_i}}} = \frac{\left( \frac{\sigma_f - 1}{\sigma_f} \right) (Z_f A_{f_i})^{\frac{\sigma_f - 1}{\sigma_f}}}{\sum_{f'} \left( \frac{\sigma_{p'} - 1}{\sigma_{p'}} \right) (Z_{p'} X_{f_{p_i}})^{\frac{\sigma_{p'} - 1}{\sigma_{p'}}}} = \frac{X_f}{Y} \frac{\partial Y}{\partial X_f}.$$

It follows that the ratio of payments to factors $f$ and $f'$ satisfy:

$$\frac{W_{f_i}}{W_{f_{p_i}}} = \frac{\left( \frac{\sigma_f - 1}{\sigma_f} \right) Z_f A_{f_i}^{\frac{1}{\sigma_f}}}{\sum_{f'} \left( \frac{\sigma_{p'} - 1}{\sigma_{p'}} \right) (Z_{p'} X_{f_{p_i}})^{\frac{1}{\sigma_{p'}}}}.$$

It follows that the ratio of payments to factors $f$ and $f'$ satisfy:

$$\frac{W_{f_i}}{W_{f_{p_i}}} = \left( \frac{\sigma_f - 1}{\sigma_f} \right) Z_f A_{f_i}^{\frac{1}{\sigma_f}}.$$

(34)
Thus, assuming constant factor-augmenting productivities, we find:

\[ dw_f - dw_{f'} = -\frac{1}{\sigma_f} da_{fi} - \frac{1}{\sigma_{f'}} da_{f'i}. \]  

(35)

Using the constraint (8) again, we find:

\[ 0 = \sum_f \left( \frac{\sigma_{f}^{-1}}{\sigma_f} \right) (Z_f A_{fi}) \frac{\sigma_{f}^{-1}}{\sigma_f} da_{fi} = \sum_f \left( \frac{\sigma_{f}^{-1}}{\sigma_f} \right) \theta_{f} da_{fi}. \]  

(36)

Combining Equations (35) and (36) yields:

\[ da_{fi} = \sigma_f \sigma_h \frac{\theta_{hi}}{\sigma_i} dw_h + \sigma_e \sigma_e \theta_{ei} \frac{\theta_{ei}}{\sigma_i} dw_e, \]  

(37)

\[ da_{hi} = \sigma_h \left( 1 - \sigma_h \frac{\theta_{hi}}{\sigma_i} \right) dw_h + \frac{\sigma_h \sigma_e \theta_{ei} \theta_{hi}}{\sigma_i} dw_e, \]  

(38)

\[ da_{ei} = \sigma_e \left( 1 - \sigma_e \frac{\theta_{ei}}{\sigma_i} \right) dw_e + \frac{\sigma_h \sigma_e \theta_{hi} \theta_{ei}}{\sigma_i} dw_h, \]  

(39)

which together lead to the desired result.\[ \square \]

Proof for Lemma 1. To compute the aggregate elasticities of substitution defined in Equations (4) and (5), first note that

\[ d \ln x_f = \sum_i \Lambda_{fi} (dy_i + d \ln a_{fi}), \]  

(40)

\[ d \ln \theta_f = \sum_i \Lambda^*_{fi} (ds_i + d \ln \theta_{f'i}), \]  

(41)

where we have \( \theta_f = \sum_i S_i \theta_{f'i} \) with \( S_i \) standing for the share of firm \( i \) in all factor payments, and where we have let \( s_i \equiv \ln S_i \) denote the log of the share of firm \( i \) in all factor payments. Given that we have CRS technology at the producer level, we can express \( \theta_{f'i} \) in terms of unit factor requirements \( A_i \), as \( \theta_{f'i} = W_f A_{fi}/C_i \), where \( C_i \equiv \sum_{f'} W_{f'} A_{fi} \) is the minimum unit cost of producer \( i \). This allows us to write \( d \ln \theta_{f'i} = dw_f + da_{f'i} - dc_i \).

Let us first compute the aggregate elasticity of substitution for factor demand \( \sigma_{f f'} \) in Equation (4). We have \( da_{f'i} = \sum_{f'} \sigma_{f f'} \theta_{f'i} dw_{f'} \), and with monopolistic competition and CES demand in Equation (1) and cost minimization, we obtain

\[ dy_i = dy - \varepsilon (dc_i - dc_s) - \eta (dc_s - dc), \]

where we have \( dc_s \equiv \frac{1}{S_s} \sum_{i \in I_s} S_i dc_i \) and \( dc \equiv \sum_i S_i dc_i \). Combining these expressions with

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Equation (40) gives us:
\[
d \ln x_f = \sum_i \Lambda_{fi} (da_{fi} - \varepsilon dc_i) + (\varepsilon - \eta) \sum_s \Lambda_{fs} dc_s + \eta dc,
\]
where we have used the result that \( dc_s = \sum_f \theta_{fs} dw_f \), and with slight abuse of notation, defined \( \Lambda_{fs} \equiv \sum_{i \in I_s} \Lambda_{fi} \). Equation (6) immediately follows from the above result.

Next, we compute the aggregate elasticity of substitution for factor payments \( \sigma_{ff}^* \) in Equation (5). We have \( d \ln \theta_{fi} = da_{fi} + dw_f - dc_i \) and, under the assumption of constant and equal firm-level markups, the share of input costs paid by producer \( i \), \( S_i \), can be expressed as \( S_i = C_i^{1-\varepsilon} / \sum_i C_i^{1-\varepsilon} \times \), in which case we have
\[
ds_i = (1 - \varepsilon) (dc_i - dc_s) + (1 - \eta) (dc_s - dc).
\]

Using again the definition of (3), Equation (41) can be expanded as:
\[
d \ln \theta_f = dw_f + \sum_i \Lambda_{fi}^* \left( \sigma_{ff}^* - \varepsilon \right) dw_f + (\varepsilon - \eta) \sum_s \Lambda_{fs}^* \theta_{fs} dw_{f'} + (\eta - 1) \eta dc,
\]
where we have again defined \( \Lambda_{fs}^* \equiv \sum_{i \in I_s} \Lambda_{fi}^* \). Equation (7) immediately follows from the above result.

**Proof of Lemma 2.** We keep the aggregate supply of high, \( H \), and low skilled labor, \( L \), constant. Hence, we have
\[
0 = d \ln \left( \frac{L}{H} \right) = \left[ \frac{\partial \ln (X_l / X_h)}{\partial w_h} \right]_{w_e} dw_h + \left[ \frac{\partial \ln (X_l / X_e)}{\partial w_e} \right]_{w_h} dw_e + \left[ \frac{\partial \ln (\theta_l / \theta_h)}{\partial w_h} \right]_{w_e} dw_h + \left[ \frac{\partial \ln (\theta_l / \theta_e)}{\partial w_e} \right]_{w_h} dw_e,
\]
which gives us Equation (12) using the definition (4).

**Proof of Proposition 1.** We first derive Equation (18). We have
\[
-d \psi = d \ln \left( \frac{\bar{W}_l L}{\bar{W}_h H} \right) = d \ln \left( \frac{\theta_h}{\theta_l} \right) = \left[ \frac{\partial \ln (\theta_l / \theta_h)}{\partial w_h} \right]_{w_e} dw_h + \left[ \frac{\partial \ln (\theta_l / \theta_e)}{\partial w_e} \right]_{w_h} dw_e,
\]
This gives us
\[
-d \psi = (\sigma_{lh}^* - 1) \frac{dw_h}{dw_e} + (\sigma_{le}^* - \sigma_{he}^*) dw_e,
\]
where all the aggregate elasticities are defined as before. Equation immediately (44) leads to the desired result in Equation (18).
Next, we compute the response of the shadow skill premium \( \frac{d\omega_{h}}{d\tau_{f}} \) in derive Equation (17). From the labor market clearing condition we have

\[
0 = d \ln \left( \frac{L}{H} \right) = \frac{\partial \ln \left( X_{f} / X_{h} \right)}{\partial \omega_{h}} \left. \right|_{w_{e}} d\omega_{h} + \left[ \frac{\partial \ln \left( X_{f} / X_{c} \right)}{\partial w_{e}} \left. \right|_{w_{h}} - \frac{\partial \ln \left( X_{h} / X_{c} \right)}{\partial w_{e}} \left. \right|_{w_{h}} \right] d\omega_{e},
\]

leading to the desired result.

**Proof of Corollary 1.** Since \( \theta_{c} = 1 - \theta_{h} \), we can write

\[
\frac{1}{\theta_{e}} \frac{d \ln \theta_{h}}{d \omega_{e}} = \frac{d \ln \left( \theta_{h} / \theta_{c} \right)}{d \omega_{e}} = \frac{\partial \ln \left( \theta_{h} / \theta_{c} \right)}{\partial w_{h}} \frac{\partial \ln \left( \theta_{h} / \theta_{c} \right)}{\partial \omega_{e}} + \frac{\partial \ln \left( \theta_{h} / \theta_{e} + \theta_{h} / \theta_{e} \right)}{\partial \omega_{h}} \frac{\partial \ln \left( \theta_{h} / \theta_{e} + \theta_{h} / \theta_{e} \right)}{\partial \omega_{e}}
\]

\[
= \sigma_{ne}^{*} - 1 + \left[ \frac{\theta_{e}}{\theta_{e} + \theta_{h}} \left( \frac{\partial \ln \left( \theta_{h} / \theta_{e} \right)}{\partial w_{h}} + \frac{\partial \ln \left( \theta_{h} / \theta_{e} \right)}{\partial \omega_{h}} \right) \frac{\partial \ln \left( \theta_{h} / \theta_{e} \right)}{\partial \omega_{h}} \right] + \frac{\theta_{h}}{\theta_{e} + \theta_{h}} \frac{\partial \ln \left( \theta_{h} / \theta_{e} \right)}{\partial \omega_{h}} \frac{\partial \ln \left( \theta_{h} / \theta_{e} \right)}{\partial \omega_{h}}
\]

leading to the desired result using Equation (19).

**Proof for Lemma 3.** Using Equations (40) and (41), and taking steps similar to those for the proof of Lemma 1, we find:

\[
d \ln x_{f} = \sum_{i} \Lambda_{fi} \left( da_{fi} - \varepsilon dc_{i} \right) + (\varepsilon - \eta) \sum_{s} \Lambda_{fs} dc_{s} + \eta dc.
\]

We now have that \( da_{fi} = \sum_{f'} \theta_{f'i} \sigma_{ff'i} \omega_{ff'i} d\tau_{f'} \) and \( dc_{i} = \sum_{f'} \theta_{f'i} d\tau_{f'} \). Substituting these expressions in Equation (45), we find

\[
d \ln x_{f} = \sum_{i} \Lambda_{fi} \left( \sigma_{ff'i} - \varepsilon \right) \omega_{f'i} d\tau_{f'} + (\varepsilon - \eta) \sum_{s} \Lambda_{fs} \theta_{f's} \omega_{f's} d\tau_{f'}.
\]

The above expression immediately leads to Equation (22).

From the definition of \( \theta_{f} \), Equations (41) follows. We can now write \( d \ln \theta_{fi} = d\tau_{f_i} + \)
\[ da_{fi} - dc_i, \] leading to:

\[ d \ln \theta_f = d \tau_f + \sum_i \Lambda_{fi}^* (da_{fi} - \epsilon dc_i) + (\epsilon - \eta) \sum_s \Lambda_{fs}^* dc_s + (\eta - 1) \ dc, \tag{47} \]

where we have used the normalization \( \sum_i \Lambda_{fi} \omega_{fi} = 1 \) in the first term on the right hand side. Substituting again for the expressions for \( da_{fi}, dc_i, \) and \( dc_s, \) we find

\[ d \ln \theta_f = d \tau_f + \sum_i \Lambda_{fi}^* \sum_f \theta_{fi} \omega_{fi} d \tau_f + (\epsilon - \eta) \sum_f \Lambda_{fs}^* \theta_{fs} \omega_{fs} d \tau_f + (\eta - 1) \ dc. \tag{48} \]

Equation (23) immediately follows.

**Proof of Proposition 1.** We first derive Equation (26). The change in the skill premium satisfies

\[-d \psi = d \ln \left( \frac{\theta_f}{\theta_h} \right) = \frac{\partial \ln (\theta_f / \theta_h)}{\partial \omega_h} dw_h + \left[ \frac{\partial \ln (\theta_f / \theta_e)}{\partial \omega_e} - \frac{\partial \ln (\theta_h / \theta_e)}{\partial \omega_e} \right] dw_e + \frac{\partial \ln (\theta_f / \theta_h)}{\partial \tau_h} d \tau_h - \frac{\partial \ln (\theta_h / \theta_f)}{\partial \tau_f} d \tau_f,\]

leading to the desired result.

\[ 0 = d \ln \left( \frac{L}{H} \right) = \frac{\partial \ln (X_l / X_h)}{\partial \omega_h} dw_h + \left[ \frac{\partial \ln (X_l / X_e)}{\partial \omega_e} - \frac{\partial \ln (X_h / X_e)}{\partial \omega_e} \right] dw_e + \frac{\partial \ln (X_l / X_h)}{\partial \tau_h} d \tau_h - \frac{\partial \ln (X_h / X_l)}{\partial \tau_l} d \tau_l, \]

leading to Equation (25).

**Proof of Equations (28) and (29).** Writing Equation (34) in differences, we find for \( f \in \{ \ell, h \} : \)

\[ \Delta w_{eit} - \Delta w_{fit} = \frac{1}{\sigma_f} (\Delta x_{eit} - \Delta y_{eit}) - \frac{1}{\sigma_e} (\Delta y_{eit} - \Delta y_{fit}) + \left( 1 - \frac{1}{\sigma_e} \right) \Delta z_{eit} - \left( 1 - \frac{1}{\sigma_f} \right) \Delta z_{fit}, \]

where we have define the residual \( \tilde{v}_{fit} \equiv \left( 1 - \frac{1}{\sigma_e} \right) \Delta z_{eit} - \left( 1 - \frac{1}{\sigma_f} \right) \Delta z_{fit} \) in the second
equality. We can write this result as

\[
\Delta x_{eit} - \Delta x_{fit} = -\sigma_f (\Delta w_{eit} - \Delta w_{fit}) + \left(\frac{\sigma_f}{\sigma_e} - 1\right) (\Delta y_{it} - \Delta x_{eit}) + \sigma_f \tilde{v}_{fit}. \tag{49}
\]

Since we do not observe the growth in firm-level quantity \(\Delta y_{it}\), we use the demand system in Equation (1) to write it in terms of revenue. The demand system implies:

\[
\Delta p_{it} = -\frac{1}{\varepsilon} (\Delta y_{it} - \Delta y - (\varepsilon - \eta) \Delta p_{st} - \eta \Delta p - \Delta \phi_{it}),
\]

which leads to the following relationship between growth in revenue and the growth in quantities:

\[
\Delta r_{it} = \Delta p_{it} + \Delta y_{it} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) (\Delta y_{it} + \Delta \tilde{\phi}_{it}),
\]

where we have define \((\varepsilon - 1) \Delta \tilde{\phi}_{it} \equiv \Delta \phi_{it} + (\varepsilon - \eta) \Delta p_{st} + \eta \Delta p + \Delta y\). Substituting for \(\Delta y_{it}\) in Equation (49), we find the desired results with the choice of

\[
v_{fit} \equiv \sigma_f \tilde{v}_{fit} + \left(\frac{\sigma_f}{\sigma_e} - 1\right) \Delta \tilde{\phi}_{it}.
\]

\[\square\]

### B Microfoundations for the Labor Wedges

#### B.1 Compensating Differentials

Here, we provide a microfoundation for firm- and factor-specific wages stemming from variations in compensating differentials across firms. In this microfoundation, the labor market is perfectly competitive.

Suppose that firms are characterized by variations in amenities that are specific to worker skill. More specifically, if a high-skill (or low-skill) worker’s utility is employed by firm \(i\), then her real income is divided by \(T_{hi}\) (or \(T_{\ell i}\)). The market equilibrium then leads to firm-specific compensating differentials that are equivalent to amenity differences.

For two firms \(i\) and \(i'\) to have positive employment of skilled (or unskilled) labor in a competitive labor market, their wages must satisfy \(W_{hi}/T_{hi} = W_{hi'}/T_{hi'}\) (or \(W_{\ell i}/T_{\ell i} = W_{\ell i'}/T_{\ell i'}\)). Letting \(W_h\) and \(W_\ell\) denote the shadow wages of skilled and unskilled labor, which we set equal to the wage of a (potentially counterfactual) firm \(i'\) with \(T_{hi'} = 1\) or
\( T_{\ell i' = 1} \), we then have \( W_{hi} = W_{h}T_{hi} \) and \( W_{\ell i} = W_{\ell}T_{\ell i} \) for all \( i \), exactly as in our model of Section 2.

### B.2 Unions and Efficient Bargaining

Consider an industry with a share \( 1 - \chi \) of unionized firms, and a share \( \chi \ll 1 \) of non-unionized firms that operate with a fringe of competitive labor markets for high and low skilled workers. In the fringe competitive labor market, high and low skilled workers have wage rates \( W_{h} \) and \( W_{\ell} \), respectively.

**Union Bargaining** Following Kramarz (2008), we consider a strongly efficiency bargaining model of unions, in which firm-level unions engage in Nash bargaining with the firm over both labor inputs and wages.\(^\text{16}\) Given firm \( i \)'s choice of

\[
(W_{\ell i}, X_{\ell i}, W_{hi}, X_{hi}) \equiv \arg\max \left( 1 - \zeta \right) \ln \left( \Pi_i - \Pi_i^0 \right) + \zeta \ln \left[ (W_{\ell i} - W_{\ell}) X_{\ell i} + (W_{hi} - W_{h}) X_{hi} \right],
\]

where \( \zeta \) stands for the bargaining parameter of the unions, \( W_{h} \) and \( W_{\ell} \) are the competitive wage rates that stand for the outside options of the workers, and where the profit of the firm is given by

\[
\Pi_i = R_{i} (X_{\ell i}, X_{hi}) - W_{\ell i}X_{\ell i} - W_{hi}X_{hi},
\]

and \( \Pi_i^0 \) stands for the outside profits of the firm. In the equation above, \( F \) is the production function of the firm and \( X_{ei} \) stands for the firm’s equipment decision that is made anticipating the outcome of the bargaining game.

Let \( R_{i} \) stand for firm-\( i \)'s revenues. The first order conditions with respect to the two types of labor inputs give:

\[
\frac{1 - \zeta}{\Pi_i - \Pi_i^0} \left( W_{\ell i} - \frac{\partial R_{i}}{\partial X_{\ell i}} \right) = \frac{\zeta}{(W_{\ell i} - W_{\ell}) X_{\ell i} + (W_{hi} - W_{h}) X_{hi}} (W_{\ell i} - W_{\ell}),
\]

\[
\frac{1 - \zeta}{\Pi_i - \Pi_i^0} \left( W_{hi} - \frac{\partial R_{i}}{\partial X_{hi}} \right) = \frac{\zeta}{(W_{\ell i} - W_{\ell}) X_{\ell i} + (W_{hi} - W_{h}) X_{hi}} (W_{hi} - W_{h}).
\]

\(^{16}\)Breda (2015) provides additional evidence that the relevant aspects of union bargaining in France does indeed happen at the firm level.
Both first order conditions with respect to wages yield:

\[
1 - \xi \frac{\Pi_i - \Pi^0_i}{\Delta \Pi_i - \Delta \Pi^0_i} = (W_{\ell i} - W_{\ell}) (X_{\ell i} + (W_{hi} - W_{h}) X_{hi}),
\]

\[
\Rightarrow \xi \left( \frac{\Pi_i - \Pi^0_i}{\Delta \Pi_i - \Delta \Pi^0_i} \right) = (1 - \xi) \frac{(W_{\ell i} - W_{\ell}) X_{\ell i} + (W_{hi} - W_{h}) X_{hi}}{\Delta \Pi^0_i - \Delta \Pi^0_i},
\]

where \(\Delta \Pi_i \equiv R_i (X_{\ell i}, X_{hi}) - W_{\ell} X_{\ell i} - W_{h} X_{hi} - \Pi^0_i \) stands for the surplus of the firm and \(\Delta \Pi^0_i \) for the surplus of labor. Combining the conditions yields the following equations that determine firm labor inputs:

\[
\frac{\partial R_i}{\partial X_{\ell i}} = W_{\ell}, \quad \frac{\partial R_i}{\partial X_{hi}} = W_{h}.
\]

Note that the setting above leaves the allocation of rents between the two labor groups unspecified. We add the assumption that the union distributes the rents equally across all firm employees to reach:

\[
W_{\ell i} = W_{\ell} + \xi \frac{\Delta \Pi_i}{X_{\ell i} + X_{hi}}, \quad W_{hi} = W_{h} + \xi \frac{\Delta \Pi_i}{X_{\ell i} + X_{hi}}.
\]

**Firm Decision Making**  Anticipating the union bargaining, the firm chooses its equipment inputs solving the following problem:

\[
\max_{X_{ci}} \Pi^0_i + (1 - \xi) \Delta \Pi_i,
\]

\[
= \Pi^0_i + (1 - \xi) \left[ R_i (X_{\ell i}, X_{hi}) - W_{\ell} X_{\ell i} - W_{h} X_{hi} \right],
\]

\[
= \Pi^0_i + (1 - \xi) \left[ \Phi_i F (X_{\ell i}, X_{hi}, X_{ci})^{1 - 1/\xi} - w_{E,i} X_{ci} - W_{\ell} X_{\ell i} - W_{h} X_{hi} \right],
\]

which suggests that the choice of firm equipment is also efficient, satisfying:

\[
\Phi_i \frac{\partial}{\partial X_{ci}} F (X_{\ell i}, X_{hi}, X_{ci})^{1 - 1/\xi} = W_{e}.
\]

**Summary**  The key result is that we can get the exact same predictions about the response of the wage premium in the competitive fringe. However, the observed wage premium could potentially behave differently due to the rent sharing. To simplify the setting, let us assume as is customary \(\Pi^0_i \equiv 0\). We now have (note that \(X_h \equiv \sum_i X_{hi}\) and
\( X_{\ell} \equiv \sum_i X_{\ell i} \):

\[
\Delta \ln \left( \frac{W_h}{W_\ell} \right) = \Delta \ln \left( \frac{\sum_i W_{hi} X_{hi}}{\sum_i W_{\ell i} X_{\ell i}} \right) - \Delta \ln \left( \frac{X_h}{X_\ell} \right),
\]

\[
= \Delta \ln \left( \frac{\sum_i W_{hi} X_{hi} + \tilde{\xi} \sum_i X_{hi} \Pi_i}{\sum_i W_{\ell i} X_{\ell i} + \tilde{\xi} \sum_i X_{\ell i} \Pi_i} \right) - \Delta \ln \left( \frac{X_h}{X_\ell} \right),
\]

\[
= \Delta \ln \left( \frac{W_h H \left( 1 + \tilde{\xi} \sum_i X_{hi} \Pi_i / W_h \right)}{W_\ell L \left( 1 + \tilde{\xi} \sum_i X_{\ell i} \Pi_i / W_\ell \right)} \right) - \Delta \ln \left( \frac{X_h}{X_\ell} \right),
\]

\[
= \Delta \ln \left( \frac{W_h}{W_\ell} \right) + \Delta \ln \left( \frac{1 + \tilde{\xi} \sum_i X_{hi} X_{hi} R_i - W_{hi} X_{hi}}{1 + \tilde{\xi} \sum_i X_{\ell i} X_{\ell i} R_i - W_{\ell i} X_{\ell i}} \right).
\]

The first term above is the change in the shadow skill premium. The second term is the contribution of unions.

## C Empirical Appendix

### C.1 Measuring firm-and-skill-specific wages and employment

Our model-consistent measures of firm- and skill-specific wages are computed exploiting data retrieved from DADS Poste. This dataset provides, for each worker, the observed wage in \( t \) and \( t - 1 \) as well as the number of hours worked, the duration of the employment contract and a set of individual characteristics.\(^{17}\) In the model, all workers in a skill group are identical. As a result, we need to correct the observed changes in firm- and skill-specific wages for worker-level unobservables and changes in returns to worker observable characteristics. To do so, we estimate a national Mincerian regression on log wage changes between \( t - 1 \) and \( t \) controlling for worker observables (age, sex, 2-digit occupation category) and a firm-skill-time effect on a sample of workers in a given skill group who are employed by the same firm between \( t - 1 \) and \( t \). We exclude from our sample atypical workers, cross-border workers, trainees and workers who were employed for a period shorter than a month or for less than 160 hours over the year. Worker-level hourly wages are computed by dividing individual wage bills by the reported number of hours worked. Finally, we drop observations reporting an hourly wage below 80% of the minimum wage.

\(^{17}\) The dataset is at the job-spell level. Starting in 2002, a unique worker identifier is provided so that job spells can be correctly aggregated at the worker level. Before 2002, we cannot implement this aggregation and we consider each spell as a separate worker.
We estimate the following linear regression separately for low- and high-skilled workers
\[
dwage_{jit} = b_t X_{jit} + \gamma_{fit} + dw_{jit},
\]
where worker \( j \) is classified as either \( h \)- or \( \ell \)-skilled depending on its occupation (catégories socioprofessionnelles) at time \( t \), as defined in the main text. The year-on-year change in the firm-and-skill specific wage is the estimated firm-skill-year fixed effect: \( dw_{fit} = \hat{\gamma}_{fit} \) with \( f \in \{h, \ell\} \). We chain the year-on-year changes together to obtain five-year changes. We further measure five-year changes in employment, \( \Delta x_{hit} \) and \( \Delta x_{\ell it} \), by deflating changes in firm-and-skill-specific wage bills by the corresponding wage change, \( \Delta w_{hit} \) and \( \Delta w_{\ell it} \).

C.2 Measuring equipment stock

The data that we use to build our measure of the stock of equipment come from the BRN and the Customs data sets. Firm-level data available in BRN include a breakdown of tangible capital by asset type. Our measure of domestic equipment includes the following two asset categories: Machinery, equipment and tools (AR - Installations techniques, matriel et outillage industriels) and Other tangible fixed assets (AT - Autres immobilisations corporelles). Investments. Since the version of BRN available to external researchers does not include information on investments, we compute a proxy for investments in each asset category by exploiting data on the book value of the stock, \( K_{it} \), of disposed assets, \( U_{it} \), and on the value of the depreciation account, \( D_{it} \). Assuming an average useful life \( n \) of 3 years, we compute our proxy for equipment investment as:18

\[
I_{it}^e = \frac{1}{(1 - \frac{1}{n})} \times \left[ (K_{it} - K_{it-1}) + (D_{it-1} - D_{it}) + \sum_{v=1}^{t-1} \frac{I_{it}^e}{n} + \psi_{it} + U_{it} - \lambda_{it-1} \right],
\]

where

\[
\psi_{it} = \begin{cases} 
\min \left( \frac{K_{i0}}{n}, K_{i0} - D_{i0} - \frac{K_{i0}}{n}(t - 1) \right) & \text{if } K_{i0} - D_{i0} - \frac{K_{i0}}{n}(t - 1) > 0 \\
0 & \text{otherwise}
\end{cases}
\]

We tested our proxy against real investment data, only accessible to INSEE employees, obtaining a correlation above 0.9. We are grateful to Jocelyn Boussard for his help.
For the imported part of equipment investment, we rely on Customs data, which provide information on partner country-, CN8 product-, firm-, and year-specific transaction value and quantities (in kgs). We identify equipment products as the ones that belong to the following two BEC Rev. 4 broad categories: i) *Capital goods - except transport equipment* (41), and ii) *Capital goods (except transport equipment); parts and accessories* (42). Subsequently, we obtain a proxy for domestic equipment investment by subtracting imported equipment from the total value of equipment investment obtained above.

**Prices.** Prices for imported equipment products are defined as unit values at the firm-, year-, product-, and country of origin-level. Conversely, we do not have firm-level information on domestic prices. Therefore we exploit sector- asset-, and year-specific data provided by the French Statistical Institute (Insee). Our proxy for equipment price is computed aggregating data on 3 broad ESA 2010 asset categories, namely *Transport equipment* (N1131G), *Computer hardware* (N11321G) and *Other machinery and equipment and weapons systems* (N110G).

**Aggregation.** At this point we have yearly firm-level data on domestic equipment investment value as well as product-, country-, firm- level transaction data on imported equipment investments, together with their relative prices. For the purpose of this study, we need to compute a firm-level aggregate price for equipment, accounting for the relevant heterogeneity in product-country combinations and entry-exit dynamics that characterize the firms in the sample. The aggregation is implemented via a slightly modified version of the Sato-Vartia price aggregation. We treat domestic investments as a individual equipment product originating from France, we then define the share of equipment products $k$ coming from country of origin $c$ as $S_{ckit}^e = I_{ckit}^e / \hat{I}_{it}^e$, where $\hat{I}_{it}^e$ is the total firm-level equipment investment at time $t$ on common (product-country) varieties between $t - 1$ and $t$. Similarly we define the share of equipment products $k$ coming from country of origin $c$ at $t - 1$ as $S_{ckit-1}^e = I_{ckit-1}^e / \hat{I}_{it-1}^e$, where $\hat{I}_{it-1}^e$ is the total firm-level equipment investment at time $t - 1$ on common (product-country) varieties between $t - 1$ and $t$.

Armed with the shares just obtained, we define product-country ideal weights $\omega_{ckit}^e$
as:

\[
\omega_{ckit}^e = \begin{cases} 
    \frac{S_{ckit}^e - S_{ckit-1}^e}{S_{ckit} - S_{ckit-1}} & \text{if } S_{ckit}^e \neq S_{ckit-1}^e \\
    S_{ckit}^e & \text{if } S_{ckit}^e = S_{ckit-1}^e 
\end{cases}
\]

and we compute the firm-level Sato-Vartia log price indices as:

\[
p_{it}^e = \sum_{ck} d p_{ckit} \frac{\omega_{ckit}^e}{\sum_{ck} \omega_{ckit}^e}
\]

Subsequently, we compute equipment investment quantities \(Q_{it}^e\) using equipment investment values, \(I_{it}^e\), and the price indices computed in the previous stage: \(Q_{it}^e = I_{it}^e / P_{it}^e\). Finally, we compute the equipment stock \(X_{it}^e\), applying the perpetual inventory method:

\[
X_{it}^e = \begin{cases} 
    \frac{1}{\text{depr}^e_{st}} \frac{\sum Q_{it}^e}{T_i} & \text{if } t = 1 \\
    (1 - \text{depr}^e_{st}) Q_{it-1}^e + Q_{it}^e & \text{if } t > 1
\end{cases}
\]

where \(\text{depr}^e_{st}\) is a sector-specific depreciation rate obtained from the EU KLEMS dataset and, following Mueller (2008), we set the initial capital stock for each firm \(i\) as the average investment over all \(T_i\) years in which the firm is present in the sample divided by the depreciation rate.
## D Additional Tables and Figures

Table D.1: Between-Firm Demand Elasticity Estimates (Alternative Instrument)

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th></th>
<th>2SLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$- \Delta p$</td>
<td>0.978</td>
<td>2.325</td>
<td>2.696</td>
</tr>
<tr>
<td>(0.010)</td>
<td>(0.589)</td>
<td>(0.860)</td>
<td>(0.396)</td>
</tr>
<tr>
<td>Observations</td>
<td>72,202</td>
<td>44,020</td>
<td>53,171</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Year-Sector FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Exp/GO≥ 5%</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>IV</td>
<td>-</td>
<td>RER ($K_I$)</td>
<td>RER ($K_I$)</td>
</tr>
<tr>
<td>KP F stat</td>
<td>-</td>
<td>13.06</td>
<td>8.34</td>
</tr>
</tbody>
</table>

Notes: This table reports estimation results of equation (27). The dependent variable is the 5-year change in firm $i$’s export quantity and the independent variable is $-1 \times$ the corresponding change in firm $i$’s export price. Columns 2-6 use the RER instrument defined in Equation (30), with the difference that exposure shares are based on the import of intermediate products only. Column 6 additionally uses the SSW instrument defined in equation (33). Year fixed effects are included in column 4 and year-sector fixed effects are included in all other columns. Controls indicates the inclusion of the average import share in the pre-sample period (1996-7) interacted with a time-varying coefficient. Exp/GO≥5% indicates that the sample is restricted to firms that export at least 5% of gross output. We report the Kleibergen-Paap rk Wald (KP F stat) of the first stage regression. Robust standard errors are clustered at the firm level and reported in parenthesis.
Table D.2: Joint Estimation of Between-Firm Demand Elasticity and Firm-Level Production Elasticities

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta w_e - \Delta w_\ell)</td>
<td>-0.922</td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon - 1) (\Delta r - \Delta x_e)</td>
<td>-0.0705</td>
<td>0.0467</td>
<td></td>
</tr>
<tr>
<td>(\Delta w_e - \Delta w_h)</td>
<td>-0.817</td>
<td>0.128</td>
<td></td>
</tr>
<tr>
<td>(\Delta p)</td>
<td>-1.927</td>
<td>0.418</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>26,845</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_\ell)</td>
<td>0.922</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_h)</td>
<td>0.817</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_e)</td>
<td>0.992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>1.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Pr[\sigma_\ell \leq \sigma_h])</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports regression coefficients for the system of equations (27), (28), and (29) jointly estimated using GMM. Corresponding point estimates of \(\sigma_\ell\), \(\sigma_h\), \(\sigma_e\), and \(\varepsilon\) are displayed. Robust standard errors are clustered at the firm level and reported in parenthesis. \(\Pr[\sigma_\ell \leq \sigma_h]\) reports a one-sided Wald test.
Table D.3: Predicted Change in the Skill Premium and Labor Share

<table>
<thead>
<tr>
<th></th>
<th>Macro approach</th>
<th>Micro approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_e$</td>
<td>0.0209</td>
<td>0.0094</td>
</tr>
<tr>
<td>$\Delta w_\ell$</td>
<td>0.7876</td>
<td>0.3386</td>
</tr>
<tr>
<td>$\Delta w_e - \Delta w_\ell$</td>
<td>-0.7667</td>
<td>-0.3292</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>0.0422</td>
<td>0.0181</td>
</tr>
<tr>
<td>$\Delta \log(\theta_n / \theta_e)$</td>
<td>-0.1503</td>
<td>-0.0645</td>
</tr>
</tbody>
</table>

Notes: This table reports the predicted log change in the skill premium ($\psi$) and the labor share ($\theta_n / \theta_e$) in response to the observed fall in the equipment price relative to the low skill shadow wage. The predicted change is obtained by multiplying the total cumulated change in the relative equipment price by the average aggregate elasticities over the entire period from column 1 of Table 4. In the ‘Micro approach’ we use the change in prices computed on our micro data. In the ‘Macro approach’, we use the log change in the price deflator of the aggregate series “Other machinery and equipment and weapons systems (gross)” taken from French national accounts, and we proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage (‘Smic’) sourced from the French Ministry of Labour.